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Bianca Barbaro, Patrizio Tirelli

Forbearance vs foreclosure in a general equilibrium model

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Abstract

We build a business cycle model characterized by endogenous firms dynamics, where banks may prefer debt renegotiation, i.e. non-performing exposures, to outright borrowers default. We find that debt renegotiations only do not have adverse effects in the event of financial crisis episodes, but a large share of non-performing firms is associated with a sharp deterioration of economic activity in two cases. First, if there are congestion effects in banks ability to monitor non-performing loans. Second, if such loans adversely affect the commercial banks' moral hazard problem due to their opacity. Aggressive interest rate reductions and quantitative easing limit defaults and the output contraction caused by a financial crisis, without adverse effects on the entry of new, more productive firms. The model shows that the observed long-run trend in the share of non-performing loans might be caused by the persistent reduction in technological advancements which drive firm entry rates and firms turnover.

Keywords: Non-Performing Loans, DSGE Model, Financial Frictions, Quantitative Easing, Firms Entry.

JEL Codes: E32, E44, E50, E58.

Non Technical Summary

The last financial crisis was characterized by a large increase in the share of Non-Performing Loans (NPLs henceforth) in the banks' balance sheets. The role of NPLs has been widely discussed in the empirical literature, and, the effects of tolerating forborne loans on aggregate macroeconomic variables is still debated. If, on one hand, a high share of NPLs threatens the stability of the financial system and hampers the efficient allocation of loans, it is also true that forbearance can save from failure firms that are temporarily unprofitable and reduce the output losses. A better understanding of the consequences of an increase in NPLs is therefore crucial for both regulators and central bankers.

To investigate the issue, we develop a dynamic stochastic general equilibrium model with endogenous firms dynamics, where banks have an incentive to renegotiate loans to otherwise insolvent firms. To motivate debt renegotiations, we assume that incumbent firms inherit their installed capital before shock realizations are observed.

In the benchmark version of the model, our results suggest that the increase in the share of NPLs looks more like a symptom rather than a cause of the severity of the crisis. We cannot detect an adverse cyclical response to the financial crisis due to loan-contract renegotiations. By way of contrast, our model predicts that economies characterized by a larger equilibrium share of non-performing loans find it more difficult to reap the benefits of technology shocks.

Our conclusions change dramatically if there are congestion effects in banks ability to monitor non-performing loans and if NPLs add opacity to the balance sheet of commercial banks, worsening their moral hazard problem. If the share of NPLs reduces the investors' trust in the financial system, a higher level of forbearance substantially worsens the recession caused by a financial crisis.

Turning to monetary policy analysis we run simulations with different policy rules, finding that accommodative monetary policies should not be blamed for encouraging excessive bank leniency on less efficient firms. Quantitative easing policies are shown to have a strong and favourable impact on macroeconomic performance, on entry rates, on the profitability of commercial banks in the aftermath of a financial crisis. These results are particularly strong when we assume that NPLs adversely affect the commercial banks' moral hazard problem.

1 Introduction

We build a DSGE model characterized by endogenous firms dynamics, where the firms' exit rate is mitigated because banks prefer to avoid the default of some borrowers and choose to hold "non-performing" loans in their portfolios. This innovation has implications for the model-predicted effects of shocks and monetary policy actions.

The productivity slowdown that occurred over the past decade attracted growing interest in the consequences of inefficient banks' lending. The phenomenon was first spotted in Japan in the 1990s and the early 2000s (Caballero et al., 2008; Peek and Rosengren 2005). Adalet et al. (2018) show that, abstracting from cyclical effects, the rising capital share allocated to non-performing firms creates a congestion effect that limits productivity-enhancing capital reallocation and creates barriers to entry of new firms. According to Banerjee and Hofmann (2018), the phenomenon, exacerbated in economic downturns, is at least partly explained by the downward trend in interest rates, that gradually reduced the creditors' opportunity cost of "evergreening" loans to insolvent firms.

Within the EMU, the financial crisis caused an unprecedented increase in the share of non-performing and forborne loans[1], intensifying regulators' concerns for the stability of the financial system (Bofondi and Ropele, 2011) and the potential barriers to productivity growth and innovation created by the share of loans locked in by potentially insolvent borrowers. Storz et al. (2017) and Andrews and Petroulakis (2017) document that inefficient credit allocation and excessive leveraging of weak firms hampered recovery and productivity growth within the European Monetary Union. Acharya et al. (2019) point at the risk that strongly accommodative monetary policies in the Eurozone might exacerbate the productivity slowdown by distorting loans allocation towards less efficient firms.

So far, empirical research had the lion's share in the field. Our contribution instead is theoretical, and we incorporate in a dynamic stochastic general equilibrium model the financial frictions that may induce banks to live with non-performing loans. In our view, the role of non-performing loans should be carefully assessed. While there is little doubt that in the long run a high share of non-performing loans increases dispersion of firms efficiency and adversely affects the efficiency of the economy, it is less obvious that bank forbearance cannot play a valuable role when

[1] From the 2018 *Final Report - Guidelines on management of non-performing and forborne exposures* of the European Banking Authority.

bankruptcies would otherwise occur in consequence of a combination of adverse demand shocks and nominal rigidities. This potential trade-off is captured in the empirical work of Gropp et al. (2020) who find that higher regulatory forbearance to close banks during the crisis is associated both with lower output losses during the crisis and with slower post-crisis output and productivity growth.

To address the potentially controversial effects of banks forbearance we need a specific set of modelling assumptions. First, we need to model firms behaviour, accounting for both endogenous entry and endogenous exit. Second, we must characterize banks' incentives to re-negotiate debt contracts. Following Hopenhayn (1992) and Asturias et al. (2017), in our model the technology is characterized by idiosyncratic firm efficiency, decreasing returns to scale, and fixed production costs. In each period, a cohort of new entrants (NEs) drives stochastic productivity growth. Before shock realizations are observed, incumbent firms inherit installed productive capacity and the bank loan needed to finance it. The combination of fixed production cost and the predetermined loan is crucial to identify their profitability threshold. But this cutoff is not sufficient to identify bankruptcy decisions, which depend on banks' incentives to choose between forbearance and foreclosure. Following the standard costly state verification approach (Townsend 1979), we assume that loans repossession is possible conditional to a monitoring cost. As a result, banks find it profitable to renegotiate debt service payments for a fraction of firms that cannot meet the profitability threshold.

The rest of the model is essentially based on Gertler and Karadi (2011, GK henceforth), where a moral hazard problem imposes an endogenous balance sheet constraint on commercial banks, and nominal rigidities create room for monetary policy actions.

Our model generates a large amplification mechanism. The introduction of endogenous firm dynamics is sufficient to obtain a relatively large fall in investment, amplifying the output fall relative to the standard GK model. The investment fall essentially happens because bankruptcies increase, the flow of new entrants shrinks, decreasing returns to scale strengthen the intermediate goods producers' incentive to scale down their size in response to the shock. Inefficient allocation of factor inputs, due to predetermined capital, plays an important role in deepening the output contraction.

Our results so far suggest that the increase in the share of non-performing loans looks more like a symptom rather than a cause of the severity of the crisis.

We cannot detect an adverse cyclical response due to loan-contract renegotiations unless two hitherto neglected issues are borne to the forefront. The first one is the existence of some non-negligible congestion effects that raise banks monitoring costs during the crisis (Fell et al. 2018). The second one is the potential endogeneity of the commercial banks' moral hazard problem. There is a consensus that non-performing loans are opaque and difficult to value, with a minimal if not negative yield. Hence they might hinder a bank's ability in accessing liquidity. Ceteris paribus, a high share of non-performing loans could adversely affect commercial banks supply of loans (Balgova and Plekhanov 2016; Huljak et al. 2020). Our simulations suggest that this latter channel has potentially devastating effects in determining the consequences of a crisis, leading to an increase in the loan-deposit interest rate spread which is very close to what was observed in the Eurozone at the height of the 2011 crisis. We also find that debt renegotiations have adverse effects when permanent technology shocks, modelled as a permanent increase in the productivity distribution of new entrants, hit the economy. Relative to a model where debt renegotiations are artificially forbidden, we observe a much slower pace of convergence to the new, more efficient steady state. This happens because debt renegotiation generate a steady state where non-performing firms are relatively less productive and more exposed to the fall in prices caused by the shock. As a result, we observe a stronger initial increase in bankruptcies as well as a larger entry flow. This is a new, hitherto unexplored implication of the existence of non-performing firms.

Our analysis of monetary policies reaches two key results. The first one is that discretionary monetary expansion stimulates firms turnover and is beneficial for entry flows. Moreover, by raising labor costs it also limits the survival of non-performing firms. The second result is that a strong monetary response to the financial crisis, including quantitative easing actions is beneficial to counteract the adverse effects on firm exit and entry flows. This result carries over to situations where debt renegotiations cause congestion effects of banks ability to manage non-performing loans and even when non-performing loans are assumed to harm banks access to credit. In this regard, the effectiveness of monetary policy is directly linked to its ability to dampen the interest rate spread in the aftermath of the financial crisis.

One important part of the paper is devoted to a discussion of the fundamental drivers of the share of non-performing loans in the steady state of our model economy. Banerjee and Hofmann (2018) document that the number of non-performing firms followed a secular trend and cannot be ascribed to a single financial crisis episode. To rationalize this finding, they emphasize the role of factors that reduced financial pressure, including lower real interest rates. We find that financial pressures, per se, do not matter in so far as they act symmetrically across all incumbent firms and do not affect the relative demand for loans from non-performing incumbents. We do find, however, that a reduction in the long run growth rate, which permanently lowers interest rates, also has a powerful positive effect on the share of non-performing firms, but this happens because the slower productivity growth weakens firms turnover, limiting exit rates in steady state. We also find that a reduction in fixed production costs, i.e. market deregulation raises the share of non-performing firms. This simply happens because, ceteris paribus, banks entirely appropriate the fixed cost reduction when they choose to avoid the bankruptcy of the insolvent firm.

We contribute to the growing field of studies on endogenous entry. The seminal work of Bilbiie, Ghironi and Melitz (2012) studies the role of endogenous entry in propagating business cycle fluctuations focusing on extensive margins, other studies include different levels of competition (Jaimovich and Floetotto, 2008; Etro and Colgiaco 2010). Siemer (2014) and Bergin et al. (2017) study the interaction between financial shocks and endogenous entry. The distinctive feature of our contribution is that exit decisions are also endogenous and explained by financial frictions. Rossi (2019) also studies the effect of financial frictions on endogenous exit decisions, but her modelling strategy does not allow for non-performing banks exposures. Our characterization of endogenous firm dynamics is akin to Piersanti and Tirelli (2020), but in their model entry/exit flows are restricted to the capital goods producing sector of the economy and financial frictions do not impact on profitability thresholds.

Theoretical work on the role of non-performing loans in business cycle models is still in its infancy. Hamano and Zanetti (2017) find that a reduction in operational firms costs during post-recession recoveries allow the survival of production units that would fail to survive in an efficient equilibrium. This is obviously related to empirical evidence concerning the survival of "zombie" firms. In our model, this result is entirely driven by financial frictions. The rest of the paper is organized as follows: Section 2 describes the model economy, section 3 presents the results and section 4 concludes.

2 The Model Economy[2]

The backbone of the model essentially follows GK. In this economy, households consume, supply labor services and hold their wealth in the form of bank deposits. Bank loans to intermediate good producers are used to purchase capital goods. Commercial banks are subject to a moral hazard problem, and this introduces a wedge between the real return on bank deposits and the expected return from loans. Monopolistically competitive retailers allow introducing nominal rigidities. Capital goods producers and consumers demand the same bundle of retail goods.

The sequence of events is as follows. At the end of time $t-1$, the η_{t-1} intermediate goods producers borrow from financial intermediaries in order to buy capital. Their decisions are based on expectations of idiosyncratic efficiency and systemic shocks. Immediately thereafter, systemic shocks are observed and intermediate producers, including potential new entrants, learn their idiosyncratic productivity. Some new firms decide to enter the market and the mass of operating firms is

$$\eta_t = NE_t + INC_t$$

where INC_t defines incumbents surviving out of the η_{t-1} producers. The remaining $\eta_{t-1} - INC_t$ firms go bankrupt. This is the crucial time when debt renegotiations occur, and some firms manage to survive even if they cannot fully honour the loan contract.

Upon entry, *NEs* obtain the bank loans needed to purchase capital goods at the price Q_{t-1}. Then all the η_t firms hire labor at the consumption wage rate w_t, produce and sell their goods to retailers at the relative price p_t^m, households and capital goods producers purchase the retail goods, the intermediate firms sell their depreciated capital to capital goods producers at the relative price Q_t and repay their loans.

The modelization of firm dynamics is based on Asturias et al. (2017) but we allow firms to use both labor and capital inputs. Following Piersanti and Tirelli (2019), in every period a stochastic trend drives average efficiency of potential new entrants, whereas the average efficiency of incumbent firms is subject to a gradual depreciation.[3] In addition, we introduce substantial innovations concerning firms interactions with the financial sector, and we give insights on the specific role

[2]See the Appendix for a full derivation of the model.

[3]This assumption facilitates the calibration of steady state entry/exit flows.

played by commercial banks in driving exit decisions. The set of *INCs* includes some low-productivity firms that cannot repay their capitalized loans but still yield a return that is higher than the one the bank would obtain from foreclosure. In this case, forbearance applies and the loan is *de facto* renegotiated.

Right from the outset, we wish to clarify the rule adopted for the identification of non-performing firms. Models who abstract from financial frictions typically allow firms to operate with negative current profits insofar as the present value of discounted profits is non-negative. Assuming that debt renegotiations may occur when the present value of the firm is negative would require keeping track of firm-specific debt dynamics. To preserve tractability, we posit that all bank loans are short term, i.e. last one period, and that firms cannot carry over unserviced debt. By way of contrast, banks may offer within-period interest rate renegotiations to firms whose current profits would be negative at the contractual loan rate.

To preserve simplicity, we assume that renegotiations are based on a standard scheme, where the firm optimally chooses the scale of production and the bank receives all revenues available after payment of the fixed cost and of the wage bill. The bank incentive to renegotiate lies in the higher expected payoff relative to the alternative of repossessing the loan and paying the bankruptcy monitoring cost. The firm incentive to produce lies in the expectation of non-negative profits in future periods.

Finally, we assume that *NEs* optimally choose their capital stock at the beginning of each period. Relative to *INCs*, who inherit a bank loan whose amount was chosen before observing shock realizations, *NEs* have a second-mover advantage in the choice of their capital stock. We made this choice to neglect the accumulation of bad loans to potential new entrants and to sharpen the focus on renegotiations of loans to incumbent firms, in line with the empirical literature on NPLs.[4]

2.1 Households

Household members can be workers or bankers. Workers supply labor, l_t, at the competitive real wage rate, w_t. Bankers and workers randomly switch roles. Centralized decisions implement full consumption risk sharing within the household. Individual preferences are based on $t$he standard consumption bundle c_t and on

[4]Our results carry over to a version of the model where *NEs* capital is predetermined to shocks realizations.

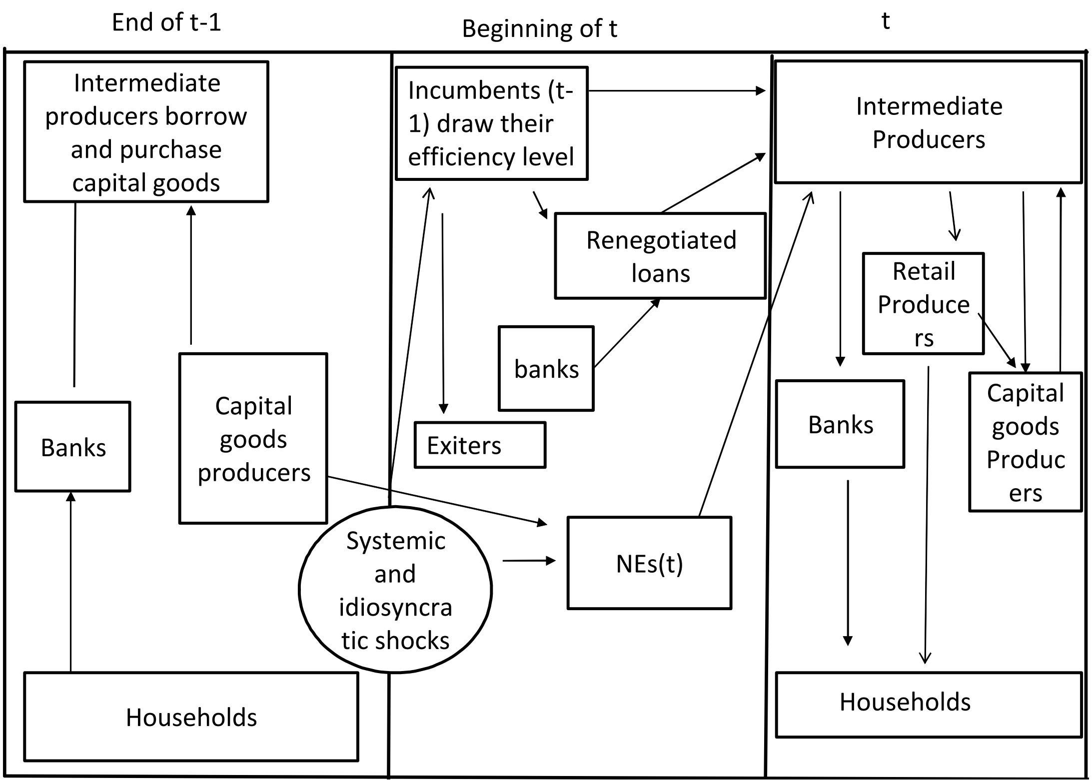

Figure 1: Sequence of events

labor effort

$$E_t \sum_{j=0}^{\infty} \beta^t \left(ln(c_{t+j} - hc_{t+j-1}) - \frac{l_{t+j}^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

The flow budget constraint is:

$$c_t + D_t = w_t l_t + r_{t-1}^d D_{t-1} + \Pi_t^{B,F} \tag{2}$$

Where r_t^d is the risk-free real remuneration on bank deposits D_t, and $\Pi_t^{B,F}$ is the flow of profits from bank ownership. Standard first order conditions apply:

$$\lambda_t = \frac{1}{c_t - hc_{t-1}} - \frac{\beta h}{c_{t+1} - hc_t} \tag{3}$$

$$l_t = \left(\frac{\lambda_t w_t}{} \right)^{\frac{1}{\varphi}} \tag{4}$$

$$\lambda_t = \beta r_t^d E_t\{\lambda_{t+1}\} \tag{5}$$

2.2 The Intermediate Sector

In a perfectly competitive market, the intermediate good producer j^f ,$(f = NE, INC)$, is characterized by the following production function:

$$\begin{aligned} y_t^{j^f} &= A_t^{j^f} \left(z_t^{j^f} \right)^{\gamma} : \\ z_t^{j^f} &= [(\Xi_t k_t^{j^f})^{\alpha} (l_t^{j^f})^{1-\alpha}] \end{aligned} \tag{6}$$

where $\gamma < 1$ defines decreasing return to scale, $A_t^{j^f}$ is the idiosyncratic efficiency level, $z_t^{j^f}$ is a standard bundle of capital $\left(k_t^{j^f}\right)$ and labor $\left(l_t^{j^f}\right)$ inputs, Ξ_t is a stochastic measure of capital quality.

$$\ln(\Xi_t) = \rho^{\Xi} \ln(\Xi_{t-1}) + \sigma^{\Xi} \varepsilon_t^{\Xi}, \quad \varepsilon_t^{\Xi} \sim \epsilon(0,1). \tag{7}$$

Firm profits in terms of retail goods are:

$$\Pi_t^{j^f} = p_t^m y_t^{j^f} - r_t^k b_t^{j^f} + Q_t \Xi_t k_t^{j^f} (1-\delta) - w_t l_t^{j^f} - \phi_t^{j^f} \tag{8}$$

Where p_t^m is the intermediate-good relative price in terms of retail goods, r_t^k is the real interest rate on bank loans b_t^j used to purchase the capital stock. All firms choose to supply goods up to the point where the marginal cost equals p_t^m. Finally, ϕ^{j^f} is a fixed cost that grows at the deterministic rate g^{ϕ}. No firm operates at $\Pi_t^{j^f} < 0$.

2.2.1 New Entrants

Potential *NEs* draw their idiosyncratic efficiency levels from a Pareto distribution:

$$f_t(A_t^{j^{NE}}) = \int_{\underline{e}_t}^{+\infty} \frac{\xi \underline{e}_t^{\xi}}{(A_t^{j^{NE}})^{\xi+1}} d(A_t^{j^{NE}}) = 1; \tag{9}$$

Note that

$$\underline{e}_t = \underline{e}_{t-1} g_t^{\underline{e}}$$

and

$$ln(g_t^{\underline{e}}) = (1-\rho_z) ln(g^{\underline{e}}) + \rho_z ln(g_{t-1}^{\underline{e}}) + \sigma^{g^{\underline{e}}} \varepsilon_t^{\underline{e}}, \quad \varepsilon_t^{\underline{e}} \sim \mu(0,1) \tag{10}$$

define the support of the technology frontier and its stochastic trend. The capital and labor inputs used by *NE* firms are

$$k_t^{j,NE} = \frac{\alpha\,\gamma p_t^m \left(y_t^{j,NE}\right)}{\left[Q_{t-1} r_t^k - \Xi_t (1-\delta) Q_t\right]} \tag{11}$$

$$l_t^{j,NE} = \frac{(1-\alpha)\,\gamma p_t^m \left(y_t^{j,NE}\right)}{w_t} \tag{12}$$

Using conditions (6), (8), (11), (12), we obtain the cost per unit of the bundle z_t^{jf}, p_t^z:

$$p_t^z = \left[\frac{(Q_{t-1} r_t^k - \Xi_t (1-\delta) Q_t)}{\Xi_t \alpha}\right]^{\alpha} \left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)},$$

defined in retail goods, and the firm supply function

$$y_t^{jNE} = \left(A_t^{jNE}\right)^{\frac{1}{1-\gamma}} \left\{\frac{\gamma\,(p_t^m)}{p_t^z}\right\}^{\frac{\gamma}{1-\gamma}} \tag{13}$$

Using conditions (6), (11), (12), and the firm supply when profits are nil:

$$y_t^{jNE}\,\{\Pi_t = 0\} = \frac{\phi_t^{NE}}{p_t^m\,(1-\gamma)}, \tag{14}$$

we obtain the idiosyncratic efficiency cutoff associated to the zero-profit condition:

$$\hat{A}_t^{NE} = \left[\frac{\phi_t^{NE}}{p_t^m (1-\gamma)}\right]^{1-\gamma} \left(\frac{p_t^z}{p_t^m \gamma}\right)^{\gamma}. \tag{15}$$

The interpretation of (15) is now straightforward; the idiosyncratic efficiency of the marginal firm increases in the production values of fixed and variable costs, respectively $\frac{\phi_t^{NE}}{p_t^m}$ and $\frac{p_t^z}{p_t^m}$. The subset of the $f_t(A_t^{jNE})$ firms characterized by $A_t^{j,NE} \geq \hat{A}_t^{NE}$ defines the *NEs* mass:

$$NE_t = \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\xi \underline{e}_t^{\xi}}{(A_t^{j,NE})^{\xi+1}} d(A_t^{jNE}) = \left(\frac{\underline{e}_t}{\hat{A}_t^{NE}}\right)^{\xi};\, \hat{A}_t^{NE} \geq \underline{e}_t \tag{16}$$

Condition (16) shows that the mass of New entrants increases in the support of the technology frontier $\underline{e}_t$ and decreases in the productivity cutoff $\hat{A}_t^{NE}$, where the latter is essentially driven by variations in p_t^m and p_t^z.

We obtain total *NEs* production using conditions (13), (16): [5]

$$Y_t^{NE} = \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{jNE}\right)^{\frac{1}{1-\gamma}} \left[\frac{\gamma\left(p_t^m\right)}{p_t^z}\right]^{\frac{\gamma}{1-\gamma}} \frac{\xi \underline{e}_t^{\xi}}{(A_t^{j,NE})^{\xi+1}} d(A_t^{jNE}) = \frac{NE_t \xi \phi_t^{NE}}{\xi(1-\gamma)-1} \quad (17)$$

Thus, for a given scaling factor $\frac{\xi\phi_t^{NE}}{\xi(1-\gamma)-1}$, variations in NE_t directly map into Y_t^{NE}.

NEs demands for factor inputs amount to

$$K_t^{NE} = \frac{\alpha\,\gamma p_t^m\,\left(Y_t^{NE}\right)}{\left[Q_{t-1}r_t^k - \Xi_t(1-\delta)Q_t\right]}$$

$$L_t^{NE} = \frac{(1-\alpha)\,\gamma p_t^m\,\left(Y_t^{NE}\right)}{w_t}$$

2.2.2 Incumbents

At the end of period $t-1$, the

$$\eta_{t-1} = NE_{t-1} + INC_{t-1} \quad (18)$$

firms borrow from commercial banks the amount

$$b_{t-1}^{j,\eta} = Q_{t-1}k_t^{j,\eta},$$

that is used to purchase capital goods from the capital goods producers.

The choice of $b_{t-1}^{j,\eta}$ depends on expected shocks realizations: at the beginning of period t systemic shocks are observed and the η_{t-1} firms draw their idiosyncratic $A_t^{j,\eta}$ from the following Pareto distribution

$$f_t(A_t^I) = \int_{\hat{A}_{t-1}^I(1-\delta^{inc})}^{+\infty} \frac{\xi(\hat{A}_{t-1}^I(1-\delta^{inc}))^{\xi}}{(A_t^I)^{\xi+1}} d(A_t^I), \quad (19)$$

where $\hat{A}_{t-1}^I$ denotes the lower bound of the distribution that characterized the INC_{t-1} firms, and the term $(1-\delta^{inc}) < 1$ implies that, on average, the knowledge capital of incumbent firms' is subject to obsolescence, as in Piersanti and Tirelli (2019).

[5] Our standard calibrations ensure that $\xi(1-\gamma)-1 > 0$.

This sequence of events allows to characterize the optimal $\overline{b}^{\eta}_{t-1} = Q_{t-1}k^{j,\eta}_t$ as the solution to the representative η_{t-1} firm problem:

$$E_{t-1}\{\Pi^{\eta}_t\} = E_{t-1}\{p^m_t y^{\eta}_t - r^k_t \overline{b}^{\eta}_{t-1} + Q_t \Xi_t k^{\eta}_t (1-\delta) - w_t l^{\eta}_t\} - \phi^I_t \tag{20}$$

subject to the expected value of (6).

From (19) it is clear that the expected efficiency of the η_{t-1} firms is $\frac{\xi}{\xi-1}\hat{A}^I_{t-1}(1-\delta^{inc})$ but, since firms do not bear bankruptcy costs, their choice of capital is based on the expected efficiency of the average incumbent firm that will earn non-negative profits in period t:

$$E_{t-1}\left\{A^{I^P}_t\right\} = E_{t-1}\left\{\int_{\hat{A}^{I^P}_t}^{+\infty} A^I_t \frac{\xi}{(A^I_t)^{\xi+1}} d(A^I_t)\right\} = \frac{\xi}{\xi-1} E_{t-1}\left\{\hat{A}^{I^P}_t\right\},$$

where $\hat{A}^{I,P}_t$ is the efficiency cut-off, such that $E_{t-1}\{\Pi^{\eta}_t\} = 0$.

The firm characterized by $\overline{A^{I^P}_t} = E_{t-1}\left\{A^{I^P}_t\right\}$ is expected to hold the optimal capital endowment after idiosyncratic efficiency shocks are observed, and its expected production amounts to

$$\overline{y^{I^P}_t} = \overline{A^{I^P}_t}^{\frac{1}{1-\gamma}} E_{t-1}\left\{\frac{\gamma p^m_t}{p^z_t}\right\}^{\frac{\gamma}{1-\gamma}},$$

The borrowing choice of the representative η_{t-1} firm is defined by

$$\frac{\overline{b}^{\eta}_{t-1}}{Q_{t-1}} = \overline{y^{I^P}_t} E_{t-1}\left\{\frac{\alpha p^m_t \gamma}{r^k_t + \frac{Q_t}{Q_{t-1}}\Xi_t(1-\delta)}\right\} \tag{21}$$

After shocks realizations, continuing firms will optimally hire labor conditionally to their idiosyncratic efficiency, to the capital stock choice, and to the real wage in production units.

$$l^{j,I}_t = \frac{(1-\alpha)\gamma p^m_t y^{j,I}_t}{w_t} \tag{22}$$

Their supply function is

$$y^{j,I}_t = \left[A^j_t \left(\frac{\overline{b}^{\eta}_{t-1}}{Q_{t-1}}\right)^{\alpha\gamma} \left(\frac{(1-\alpha)\gamma p^m_t}{w_t}\right)^{(1-\alpha)\gamma}\right]^{\frac{1}{1-(1-\alpha)\gamma}}; \tag{23}$$

Right from the outset, it is interesting to compare the firm supply decision when capital is predetermined to the case where it is chosen conditional to idiosyncratic and systemic shocks, given by:

$$y_t^{j^*} = \left(A_t^{j^*}\right)^{\frac{1}{1-\gamma}} \left[\frac{\gamma p_t^m}{p_t^z}\right]^{\frac{\gamma}{1-\gamma}} \tag{24}$$

The assumption of predetermined capital implies that the supply function is less sensitive to individual efficiency, because $\frac{1}{1-(1-\alpha)\gamma} < \frac{1}{1-\gamma}$, and is inversely related to the real wage in production units instead of the current price of the total input bundle in production units, $\frac{p_t^m}{p_t^z}$.

Profitable Incumbents: The profitability threshold, $\hat{A}_t^{I^P}$, is obtained by plugging (22) and (23) into the zero profit condition

$$[1-(1-\alpha)\gamma]\left[\hat{A}_t^{I^P}\left(\frac{\overline{b}_{t-1}^{\eta}}{Q_{t-1}}\right)^{\alpha\gamma}\left(\frac{(1-\alpha)\gamma p_t^m}{w_t}\right)^{(1-\alpha)\gamma}\right]^{\frac{1}{1-(1-\alpha)\gamma}} =$$

$$= \frac{\phi_t^I}{p_t^m} + \left[r_t^k - \frac{\Xi_t Q_t(1-\delta)}{Q_{t-1}}\right]\frac{\overline{b}_{t-1}^{\eta}}{p_t^m} \tag{25}$$

Note that when the capital choice is not predetermined the productivity cutoff amounts to

$$\hat{A}_t^{I^*} = \left[\frac{\phi_t^I}{p_t^m(1-\gamma)}\right]^{1-\gamma}\left(\frac{p_t^z}{p_t^m\gamma}\right)^{\gamma}. \tag{26}$$

In comparison with (26), condition (25) pinpoints the twofold role of predetermined debt. First, the loan repayment net of the residual value of capital is akin to the fixed cost, unambiguously raising the productivity cutoff. Thus, the larger the size of the loan, the higher the productivity cutoff. Second, the larger the stock of borrowed capital the greater the firm size and its ability to generate revenues. This latter effect brings down $\hat{A}_t^{I^P}$.

After aggregation, the mass, production and labor demand of profitable incumbents respectively are:

$$
\begin{aligned}
INC_t^P &= \int_{\hat{A}_t^{I,P}}^{+\infty} \frac{\xi(\hat{A}_{t-1}^I(1-\delta^{inc}))^{\xi}}{(A_t^I)^{\xi+1}} d(A_t^I) \\
&= \eta_{t-1}\left(\frac{\hat{A}_{t-1}^I}{\hat{A}_t^{I,P}}(1-\delta^{inc})\right)^{\xi} \\
Y_t^{I,P} &= \int_{\hat{A}_t^{I,P}}^{+\infty} A_{j,t}^I\left[\left(\Xi_t k_{j,t-1}^I\right)^{\alpha}\left(l_{j,t}^I\right)^{1-\alpha}\right]^{\gamma} dF(A_{j,t}^I) \qquad (27) \\
L_t^{I,P} &= \frac{(1-\alpha)\gamma p_t^m}{w_t} Y_t^{I,P}
\end{aligned}
$$

Debt forbearance, "non-performing" and defaulting firms: When a firm does not meet the profitability condition $\hat{A}_t^{I,P}$, the financial intermediary retains the option of repossessing the capitalized loan, $r_t^k \overline{b}_{t-1}^{\eta}$, conditional to payment of a stochastic monitoring cost μ_t.

$$
\ln \mu_t = (1-\rho^{\mu})\ln \mu_t^* + \rho^{\mu}\ln \mu_{t-1} + \sigma^{\mu}\varepsilon_t^{\mu}, \quad \varepsilon_t^{\mu} \sim \epsilon(0,1), \quad corr\left(\varepsilon_t^{\mu}, \varepsilon_t^{\Xi}\right) > 0
$$

where μ_t^* denotes the deterministic component of the monitoring cost, the correlation between μ_t and Ξ_t captures possible congestion effects on banks monitoring capacity due to the severity of financial crisis episodes.[6]

The alternative option to outright default is a renegotiation of the "debt contract". One might characterize debt re-negotiation as the outcome of a potentially complex bargaining process. Here we focus on a simpler alternative, where it is assumed that, net of operational costs, all t-period revenues are transferred to the bank.[7]

The bank accepts debt renegotiation when firm efficiency is such that:

$$
[1-(1-\alpha)\gamma]\, p_t^m y_t^{j,NP} + \frac{\Xi_t Q_t(1-\delta)\overline{b}_{t-1}^{\eta}}{Q_{t-1}} - \phi_t^I \geq r_t^k \overline{b}_{t-1}^{\eta} - \mu_t.
$$

Loan renegotiations occur if $\hat{A}_t^{I,P} > A_t^j > \hat{A}_t^I$, where $\hat{A}_t^I$ denotes the productivity

[6]The deterministic component μ_t^* is driven by the deterministic trend identified for output.

[7]This implies that the firm participation constraint is met by the present value of profits in future periods.

cutoff for the set of firms that are granted debt renegotiation:

$$[1-(1-\alpha)\gamma]\left[\hat{A}_t^{I^P}\left(\frac{\overline{b}_{t-1}^{\eta}}{Q_{t-1}}\right)^{\alpha\gamma}\left(\frac{(1-\alpha)\gamma p_t^m}{w_t}\right)^{(1-\alpha)\gamma}\right]^{\frac{1}{1-(1-\alpha)\gamma}}-\frac{\phi_t^I}{p_t^m}=$$

$$=\left[r_t^k-\frac{\Xi_t Q_t(1-\delta)}{Q_{t-1}}\right]\frac{\overline{b}_{t-1}^{\eta}}{p_t^m}-\frac{\mu_t}{p_t^m} \tag{28}$$

We can now identify the mass of both INC_t and "Non Performing" INC_t^{NP} firms:

$$INC_t=\eta_{t-1}\int_{\hat{A}_t^I}^{\infty}\frac{\xi(\hat{A}_{t-1}^I(1-\delta^{inc})g_z)^{\xi}}{(A_t^I)^{\xi+1}}d(A_t^I)$$

$$INC_t^{NP}=\eta_{t-1}\int_{\hat{A}_t^I}^{\hat{A}_t^{I,P}}\frac{\xi(\hat{A}_{t-1}^I(1-\delta^{inc})g_z)^{\xi}}{(A_t^I)^{\xi+1}}d(A_t^I) \tag{29}$$

Note that the fraction of non-performing incumbents amounts to

$$\frac{INC_t^{NP}}{INC_t^{NP}+INC_t^{P}}=1-\left(\frac{\hat{A}_t^I}{\hat{A}_t^{I,P}}\right)^{\xi}$$

where

$$\frac{\hat{A}_t^I}{\hat{A}_t^{I,P}}=\left\{1-\frac{\mu_t}{\left[\phi_t^I+\left[r_t^kQ_{t-1}-\Xi_tQ_t(1-\delta)\right]\frac{\overline{b}_{t-1}^{\eta}}{Q_{t-1}}\right]}\right\}^{1-(1-\alpha)\gamma}$$

Hence, the the fraction of non-performing incumbents essentially depends on how large the monitoring cost μ_t is relative to the predetermined costs, $\left[\phi_t^I+\left[r_t^kQ_{t-1}-\Xi_tQ_t(1-\delta)\right]\frac{\overline{b}_{t-1}^{\eta}}{Q_{t-1}}\right]$.

Exiting firms, EX_t, are :

$$EX_t=\eta_{t-1}\left[1-\left(\frac{\hat{A}_{t-1}^I(1-\delta^{inc})}{\hat{A}_t^I}\right)^{\xi}\right]$$

Total production and labor demand originating from the non-performing firms are

$$Y_t^{I,NP}=\int_{\hat{A}_t^I}^{\hat{A}_t^{I,P}}A_t^I\left[\left(\frac{\Xi_t\overline{b}_{t-1}^{\eta}}{Q_{t-1}}\right)^{\alpha}\left(l_{j,t}^I\right)^{1-\alpha}\right]^{\gamma}dF(A_t^I) \tag{30}$$

$$L_t^{I,NP} = \frac{(1-\alpha)\gamma P_t^m}{w_t} Y_t^{I,NP}$$

2.3 Financial intermediaries

The representative banker's balance sheet is:

$$L_t^b = NW_t + D_t \tag{31}$$

where NW_t defines the banker's net wealth and

$$L_t^b = \overline{b}_{t-1}^{\eta} \eta_{t-1} + Q_{t-1} K_t^{NE}$$

is the amount of extended loans.

Proceedings from re-negotiated loans are

$$\Pi_t^{NP} = \left[1 - (1-\alpha)\gamma\right] P_t^m Y_t^{I,NP} + INC_t^{INP} \left[\frac{(1-\delta)\,\Xi_t Q_t \overline{b}_{t-1}^{\eta}}{Q_{t-1}} - \phi_t^I \right]$$

Bank returns from defaulting firms amount to $\left(r_t^k \frac{Q_t b_{t-1}^{\eta}}{Q_{t-1}} - \mu_t \right) EX_t$. The average return on loans, r_t^b, is:

$$r_t^b L_t^b = r_t^k \left[INC_t^P \overline{b}_{t-1}^{\eta} + Q_{t-1} K_t^{NE} \right] + \Pi_t^{NP} + \left(r_t^k \overline{b}_{t-1}^{\eta} - \mu_t \right) EX_t \tag{32}$$

In each period the banker can divert a fraction $\lambda_{b,t}$ of funds and exit the market so, for lenders to be willing to supply funds to the banker, the incentive compatibility constraint must be:

$$V_t^b \geq \lambda_{b,t} L_t^b \tag{33}$$

where V_t^b implicitly defines the banker's continuation value. The lower V_t^b, the smaller the amount of bank deposits and the supply of loans. This, in turn, raises the return on loans and ensures that the incentive-compatibility constraint of the banker is satisfied.

The standard characterization of the banker's moral hazard problem simply parameterizes $\lambda_{b,t}$ at a time-invariant value. In the following we consider an extension which links it to the expected evolution of non-performing firms:

$$\lambda_{b,t} = \lambda_b \left[1 + \alpha_{\lambda_b}\left(\frac{INC^{NP}_{t+1}}{INC^{NP}} - 1\right)\right] \tag{34}$$

The underlying intuition here is that non-performing loans add opacity to the bank balance sheet, and that the number of nonperforming firms, as opposed to the average size of the loan is better suited to capture this effect, because it is relatively more difficult for the market to monitor the specific features of an increasing number of non-performing loans (Suarez and Sánchez Serrano, 2018).

Condition (33) implies that bankers are subject to a leverage constraint which generates the following law of motion for banks equity capital, NW_t:[8]

$$NW_t = \theta_b \left[(r^b_t - r^d_{t-1})\Phi^b_{t-1} + r^d_{t-1}\right] NW_{t-1} + \omega \Xi_t K_{t-1} Q_t \tag{35}$$

where ω is the households' transfer to new bankers, and $\Phi^b_{t-1} = \frac{Q_{t-1}K_{t-1}}{NW_{t-1}}$ is bank leverage. The amount of bank deposits constrains the banks leverage ratio, and generates a loan rate spread $r^b_t - r^d_{t-1}$ such that the representative banker's continuation value matches the incentive to divert funds.

2.4 Retailers

There is a continuum of monopolistic retailers who buy intermediate output Y^m_t from intermediate firms and resell it as a differentiated non-durable final good.

Retailers are characterised by Calvo nominal rigidities. In each period the firm is able to adjust its price with probability $1 - \Gamma_h$.The pricing problem of retailers is:

$$max_{P_t(f)} \;\; E_t \sum_{i=0}^{\infty} \Gamma_h^{C,I} \beta^{C,I} \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{P^h_t(f)}{P^h_{t+i}} - \tilde{P}^h_t\right] Y^h_{t+1}(f)$$

$$s.t. \quad Y_{t+1}(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_{t+1}$$

The solution of retailers' pricing problem is:

$$E_t \sum_{i=0}^{\infty} (\beta\Gamma)^i \frac{\lambda_{t+1}}{\lambda_t} Y_{t+i} \left[(1-\varepsilon)\left(\frac{P_{t+i}}{P^*}\right)^{\varepsilon} + \varepsilon \left(\frac{P_{t+i}}{P^*}\right)^{(1+\varepsilon)} \tilde{P}_t\right] \tag{36}$$

[8]See the Appendix for a proof.

where P_t^* is the optimal price level and P_t is the price index.

2.5 Capital Producing Firms

At the end of period t, capital producing firms buy residual capital from intermediate good producers, choose the amount of gross investment I_t, and sell accumulated capital back to intermediate goods producers. In doing this, they are subject to standard investment adjustment costs

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

Where $f(1) = f'(1) = 0,\ f''(1) > 0$

Profit maximization yields the standard first order condition:

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + f'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right) - E_t\beta\frac{\lambda_{t+1}}{\lambda_t}f'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{37}$$

The law of motion for aggregate capital is

$$K_t = I_t + (1-\delta)\Xi_t K_{t-1} \tag{38}$$

2.6 Market Clearing and Resource Constraints

The aggregate resource constraint is

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + INC_t\,\phi^I + NE_t\,\phi^{NE} + EX_t\mu_t \tag{39}$$

where

$$Y_t = Y_t^{NE} + Y_t^{I,P} + Y_t^{I,NP}$$

The input resource constraints are

$$L_t = L_t^{NE} + L_t^{I,P} + L_t^{I,NP}$$

$$K_{t-1} = \eta_{t-1}\frac{\overline{b}_{t-1}^{\eta}}{Q_{t-1}} + K_t^{NE}$$

2.7 Monetary Policy

Monetary policy evolves accordingly to the following Taylor rule,

$$\frac{r_t^n}{r^n} = \left(\frac{r_{t-1}^n}{r^n}\right)^{\rho_i} \left[(\pi_t)^{\kappa^\pi} \left(\frac{Y_t}{Y}\right)^{\kappa^y}\right]^{1-\rho_i} exp(\sigma^r \varepsilon_t^r); \quad \varepsilon_t^r \sim \mu(0,1), \tag{40}$$

where π_t is the inflation rate of the consumption price index and ε_t^r is an interest rate shock. The following condition relates the nominal interest rate to the rate on bank deposits

$$\frac{r_t^n}{\pi_t} = r_t^d$$

Following GK, we also consider a policy scenario where the central bank engages in Quantitative Easing (QE) policies. Essentially we assume that the Central Bank can sell government-backed securities that households treat as perfect substitutes for bank deposits. The proceedings are then used to supply loans, L_t^{CB}, to intermediate goods producers. In this framework the total value of loans amounts to

$$\bar{b}_{t-1}^\eta \eta_{t-1} + Q_{t-1} K_t^{NE} = L_t^{CB} + L_t^b$$

The Central bank sets her loans as a fraction of total intermediate assets:

$$L_t^{CB} = \Psi_t \left[\bar{b}_{t-1}^\eta \eta_{t-1} + Q_{t-1} K_t^{NE}\right].$$

The quantitative easing policy rule is[9]

$$\Psi_t = \nu \left[\ln(r_t^b - r_{t-1}^d) - \ln(r_{ss}^b - r_{ss}^d)\right]. \tag{41}$$

The intuition here is that a crisis lowers the bankers' continuation value, causing a reduction in deposits and an increase in the spread on loans returns. The QE policy raises the supply of loans, strengthening the relative price of capital goods and lowering the cost of capital for intermediate goods producers.

2.8 Calibration

All parameters and shock processes are reported in Table (1). Firms returns to scale, $\gamma = 0.8$, are set at the lower bound of Basu and Fernald (1997) estimates,

[9] See Foerster (2015) for a detailed derivation of QE policies.

and the tail index of the Pareto distribution, $\xi = 6.1$ is set as in Asturias et al. (2017). The deterministic quarterly growth rate, g, is set at 1.0025, implying a yearly productivity growth rate at 1%. We calibrate the discount factor $\beta = 0.9925$ in order to obtain a steady state value of the risk-less rate of deposits $r^d = 1.0101$. The investment adjustment cost is set to $\gamma_i = 3.14$, following Justiniano, Primiceri and Tambalotti (2010).

We set the detrended support of the NEs distribution, z, the depreciation rate of firms efficiency, δ^{inc}, the detrended fixed production costs, ϕ^I and ϕ^{NE}, and the detrended banks monitoring cost to calibrate the values of some variables that characterize firm dynamics in steady state. The firm exit rate, $\frac{EX}{\eta} = \frac{NE}{\eta}$, is set at 10% on annual basis (Bilbiie et al., 2012). The steady state number of firms, η, is normalized at 1. The fixed costs of production amount to 5% of total GDP (Bilbiie et al., 2012; Etro and Colciago 2010).

The stochastic process for μ_t is only suggestive and meant to gauge the potential consequences of bank monitoring congestion. concerning the We set $\sigma^\mu = 0.5$, and fix at 0.9 both the autoregressive parameter ρ^μ and the $corr\left(\varepsilon_t^\mu, \varepsilon_t^\Xi\right)$ parameter.

Following Banerjee and Hofmann (2018), the share of non-performing firms, $\frac{INC^{NP}}{\eta}$, is set at 8%.[10] Clementi and Palazzo (2016) document that the relative size of new entrant firms is about 60%. We obtain a similar result by calibrating the NEs fixed cost. We calibrate the moral hazard parameter λ_b as in GK, and set α_{λ_b} to a value such that our financial crisis experiment replicates the spread observed by Gilchrist and Mojon (2018) for the Euro Area during the financial crisis. All the remaining parameters are borrowed from GK.

3 The Steady State

In this section, we characterize the model steady state and we investigate the specific role played by the pre-determined capital assumption, which is crucial to generate situations where commercial banks are induced to renegotiate debt contracts. Based on the set of parameters presented in Table (1), we essentially compare three scenarios. The first one is our baseline model, characterized by

[10]The empirical literature on non-performing firms focuses on firms that are not "young" and that remain insolvent for several quarters. The total amount of firms unable to service interest payments, our definition of non-performing firms, is certainly larger. Our results would not change even if we tripled $\frac{INC^{NP}}{\eta}$.

Table 1: Calibrated Parameters.

	Households	
φ	0.276	Inverse Frisch elasticity of labor supply
β	0.9925	Discount rate
δ	0.025	Depreciation rate
	2.6	Relative utility weight of labor
h	0.815	Habit parameter
g	1.0025	Gross BGP rate
	Intermediate Good Firms	
ϕ^{NE}	0.05	Entry cost
ϕ^{I}	0.11	Fixed production cost for incumbents
γ	0.8	Decreasing return index
α	0.33	Capital share
ξ	6.1	Pareto distribution shape parameter
$\underline{e}$	0.8614	Technology frontier initial value
H	0.025	Share of NEs over total firms in steady state
H_{NP}	0.08	Share of INC^{NP}s over total firms in steady state
	Financial Intermediaries	
λ_b	0.338	Fraction of capital that can be diverted
θ_b	0.9725	Survival probability of banks
ω	0.002	Proportional transfer to the entering bankers
μ	0.0208	Repossession cost for defaulting firms' debt
α_λ	1.4	Moral hazard parameter
	Capital Producing Firms	
γ_i	3.14	Inverse elasticity of net investment to the price of capital
	Retail Firms	
Γ	0.779	Probability of keeping prices fixed
ε	5	Elasticity of substitution
	Central Bank	
κ_y	0	Output gap coefficient of the Taylor rule
κ_y	3.1	Inflation coefficient of the Taylor rule
ρ_i	0.8	Smoothing parameter of the Taylor rule
ν	100	Quantitative Easing Parameters
	Exogenous Processes	
ρ_Ξ	0.66	Persistence of capital quality shock
ρ_μ	0.9	Persistence of μ shock
$\rho_{\Xi,\mu}$	0.9	Correlation μ and Ξ shocks

pre-determined capital and debt renegotiations. The second one, labelled efficient allocation (EA) model, is characterized by the assumption that all intermediate producers choose their capital stock *after* observing systemic shocks and their idiosyncratic efficiency. The third one, labelled efficient re-allocation (ERA) model, maintains the assumption of pre-determined capital, but it allows the opening of a secondary capital market after shocks have been observed, thus allowing the efficient reallocation of capital services towards more efficient firms.[11]

In Table 2 we report selected variables as ratios to the corresponding values obtained for the baseline model. The EA model is characterized by a steady state allocation where firms optimally select their capital stock, whereas in the baseline model predetermined capital is inefficiently concentrated in the hands of the less efficient firms. This has several far-reaching implications. First, the productivity cutoff for surviving incumbents is substantially lower in the EA model. This essentially happens because in the baseline model the predetermined capital stock generates a "fixed cost" effect that can be borne only by firms characterized by a relatively high idiosyncratic productivity. Second, capital misallocation limits the scale of production that can be attained by the more efficient firms. In fact, average firm size is much larger in the EA model. Third, the larger amount of output observed in the EA model is associated with higher salaries. This, in turn, worsens the relative position of NE firms, that efficiently choose their size even in our baseline model. As a result, the EA model is characterized by a higher productivity cutoff for NE firms, whose number falls.
The ERA model is characterized by outcomes that are quite close to the ones obtained in the EA model, showing that the predetermined capital friction would be almost irrelevant if a capital reallocation scheme could be properly designed.

Note that in all these models the steady state entry rate amounts to

$$\frac{NE}{\eta} = 1 - \left(1 - \delta^{inc}\right)^{\xi}$$

therefore the fall in NEs is matched by a corresponding decrease in the number of incumbents.

[11] The full derivation of these models is in Appendix D.

Var	Y	C	I	NE	$\frac{Y}{\eta}$	$\hat{A}^{NE}$	$\hat{A}^{I}$	$\frac{NE}{\eta}$
SS^{EA}	2.57%	2.63%	1.99%	−8.21%	11.75%	1.41%	−7.54%	1
SS^{ERA}	2.62%	2.85%	2.13%	−8.16%	11.75%	1.41%	−7.54%	1

Table 2: Steady state percentage change with perfect capital allocation hypothesis SS^{EA} and ex-post capital reallocation hypotesis SS^{ERA}

3.1 Long Run Effects of Structural Changes

We implement here a simple comparative statics exercise to assess the effects of a change in some key parameters that determine the share of non-performing incumbents, $\frac{INC^{NP}}{\eta}$ in our model. The steady state solution for $\frac{INC^{NP}}{\eta}$ is:[12]

$$\frac{INC^{NP}}{\eta} = \left(\frac{1-\delta^{inc}}{g_z}\right)^{\xi} \left[1 - \left(\frac{\widetilde{\hat{A}^{I}}}{\widetilde{\hat{A}^{I,P}}}\right)^{\xi}\right] \tag{42}$$

$$\frac{\widetilde{\hat{A}^{I}}}{\widetilde{\hat{A}^{I,P}}} = \left\{1 - \frac{\mu}{\phi^{I}\left[1 + \frac{\alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}}{\left\{1-[(1-\alpha)\gamma]-\alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\right\}}\right]}\right\}^{1-(1-\alpha)\gamma} \tag{43}$$

Straightforward manipulations allow to obtain the elasticity of $\frac{INC^{NP}}{\eta}$ to the technology growth rate, g_z:

$$\frac{\partial\left(\frac{INC^{NP}}{\eta}\right)}{\partial g_z}\frac{g_z}{\left(\frac{INC^{NP}}{\eta}\right)} = -\xi$$

To grasp intuition, note that a reduction g_z unambiguously reduces the flow of NEs. Correspondingly, the share of incumbents

$$\frac{INC}{\eta} = \left(\frac{1-\delta^{inc}}{g_z}\right)^{\xi} \tag{44}$$

[12]Note that $\widetilde{x}$ identifies the de-trended steady state value of variable x_t.

must rise. The share of profitable firms,

$$\frac{INC^P}{\eta} = \left(\frac{1 - \delta^{inc}}{g_z} \frac{\widetilde{\hat{A}^I}}{\widetilde{\hat{A}^{I,P}}} \right)^{\xi}, \tag{45}$$

also increases, but to a lesser extent because $\frac{\widetilde{\hat{A}^I}}{\widetilde{\hat{A}^{I,P}}} < 1$. The fraction of non-performing incumbents must therefore unambiguously increase.

Additionally, the elasticity of the technology growth rate to the deterministic growth rate of the economy g is

$$\frac{\partial g_z}{\partial g} \frac{g}{g_z} = 1 - \alpha\gamma$$

Given our calibration for the tail index of firms productivity distribution ξ, for the capital share α, and for the return of scale γ, a fall in the long run growth rate raises the share on non-performing incumbents by a factor of $\xi(1 - \alpha\gamma) = 4.49$, suggesting that the growth slow down of the last twenty years might have had an important role in raising the share of non-performing incumbents. Thus our model provides a rationale for the inverse correlation between interest rate and the share of non-performing incumbents, but only to the extent that the interest rate change is driven by a change in the long run growth rate. By contrast, an interest rate fall caused by an increase in the subjective discount rate, β, has no effect on firms turnover (Table 2) and on the share of non-performing incumbents, just like changes in the loan-deposits spread. These changes do not matter because they act symmetrically across all incumbent firms and do not affect the relative demand for loans from non-performing incumbents.

Andrews and Petroulakis (2017) emphasize the importance of poorly designed insolvency regimes as a con-cause that raised the share of non-performing loans. In our model, this effect is captured by the bank monitoring cost μ. From equation (43) it is easy to see that a fall in μ increases the share of incumbents. According to our numerical calculation a 90% fall in μ lowers $\frac{INC^{NP}}{\eta}$ by approximately 0.89 percentage points. The reduction in μ implies an initial saving in bankruptcy costs which is less than 0.2%, but the effect on firms efficiency is very strong, in the new steady state the total number of firms falls by 2.8%, and output per firm increases by 2.71%. Furthermore, investment falls by 3.76%, implying that firms efficiency on average increases. Total output falls, but consumption increases. Let us now

Var	β (+0.5%)	μ (−90%)	$\phi^{NE} = \phi^I$ (−1%)
Y	4.86%	−0.16%	0.43%
C	2.51%	0.72%	0.44%
I	1.73%	−3.76%	0.43%
η	4.86%	−2, 8%	10, 72%
$\frac{Y}{\eta}$	$\sim 0\%$	2, 71%	−10, 31%
NE	4.86%	−2, 8%	10, 72%
INC^{NP}	4.86%	−90%	20.45%
$\frac{NE}{\eta}$	$\sim 0\%$	$\sim 0\%$	$\sim 0\%$
$\frac{INC^{NP}}{\eta}$	$\sim 0\%$	−89%	9.72%

Table 3: Long Run Effects

consider the effect of a fall in the fixed cost ϕ^I. Generalized reductions in fixed production costs are often interpreted as the consequence of market deregulations (Ègert and Gal, 2018). Here we show that 1% fall in ϕ^I raises output, consumption and investment by about 43%. The firm turnover rate is not affected, but the number of firms increases by 10%. This, in turn, entails a fall in output per firm of approximately the same size. This should be hardly surprising, in our model intermediate goods producers are fully competitive, and the reduction in fixed costs implies that less efficient firms will survive in the market. What is perhaps less obvious is that the fall in ϕ^I is associated with a large increase in the share of non-performing incumbents, +9.72%.

From conditions (44) and (45) we know that the reduction in ϕ^I has no effect on the incumbents share but, by lowering the cutoff $\frac{\widetilde{\hat{A}^I}}{\widetilde{\hat{A}^{I,P}}}$, it does increase the share of non-performing incumbents. This latter effect requires some discussion. Note that, since in our model incumbent firms' capital is predetermined, the term $\phi^I + \frac{\left[r^{k,\eta} - (1-\delta)\right]\widetilde{b}^\eta}{g}$ defines the true "fixed cost" faced by incumbent firms, where the net contractual repayment on the bank loan

$$\left[\left(r^k\right) - (1-\delta)\right]\widetilde{b}^\eta = \frac{\alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\phi^I g}{\left\{1 - [(1-\alpha)\gamma] - \alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\right\}}$$

also depends on ϕ^I. The variation in ϕ^I therefore has a twofold positive effect on both $\widetilde{\hat{A}^{I,P}}$ and $\widetilde{\hat{A}^{I}}$. First, there is a direct effect on profitability. Second, the lower ϕ^I reduces net borrowing costs and this contributes to pushing down the productivity cut-offs $\widetilde{\hat{A}^{I}}$ and $\widetilde{\hat{A}^{I,P}}$, but the response of $\widetilde{\hat{A}^{I}}$ is unambiguously stronger.

4 Simulation Results

In this section we discuss a set of numerical simulations. First, we analyse IRFs to the different non-policy and policy shocks discussed above, also considering the effectiveness of alternative monetary policy regimes. Then we consider the potential effects of making the banks moral hazard problem endogenous to the number of non-performing incumbents, Finally, we investigate the implications of permanent changes in structural parameters of the model which determine the share of non-performing incumbents.

4.1 Financial Crisis Experiment

Our model differs from GK in several dimensions, i.e. firms heterogeneity and entry/exit flows, inefficient allocation of capital across incumbents, and debt renegotiation between commercial banks and a subset of firms. To shed light on the specific role that debt forbearance plays in determining our results, we compare our results (NPL model) with GK and with those obtained in two alternative models where: i) incumbent firms can optimally choose their capital stock after shock realizations are observed, so that the allocation of capital is efficient and there is no incentive to debt re-negotiation (EA); ii) bank loans are predetermined to shock realizations, but debt re-negotiation is artificially forbidden, so all non-profitable firms are forced to exit the market (No_NPL model).[13] We consider a negative capital quality shock (equation (7)) and maintain that the Taylor rule is based on pure inflation targeting ($\kappa^y = 0$). From Figure 2, it is easy to see that the NPL and GK models exhibit a similar pattern in the output and investment dynamics, but in the NPL model the amplification effect is unambiguously stronger and more persistent. In addition, we observe a stronger fall in inflation. The different dynamic patterns observed for the price of intermediate goods suggest that this

[13] In these two models, we retain the same calibration chosen for NE and η in the NPL model.

latter result is essentially due to the dampening effect that decreasing returns to scale have on marginal costs.

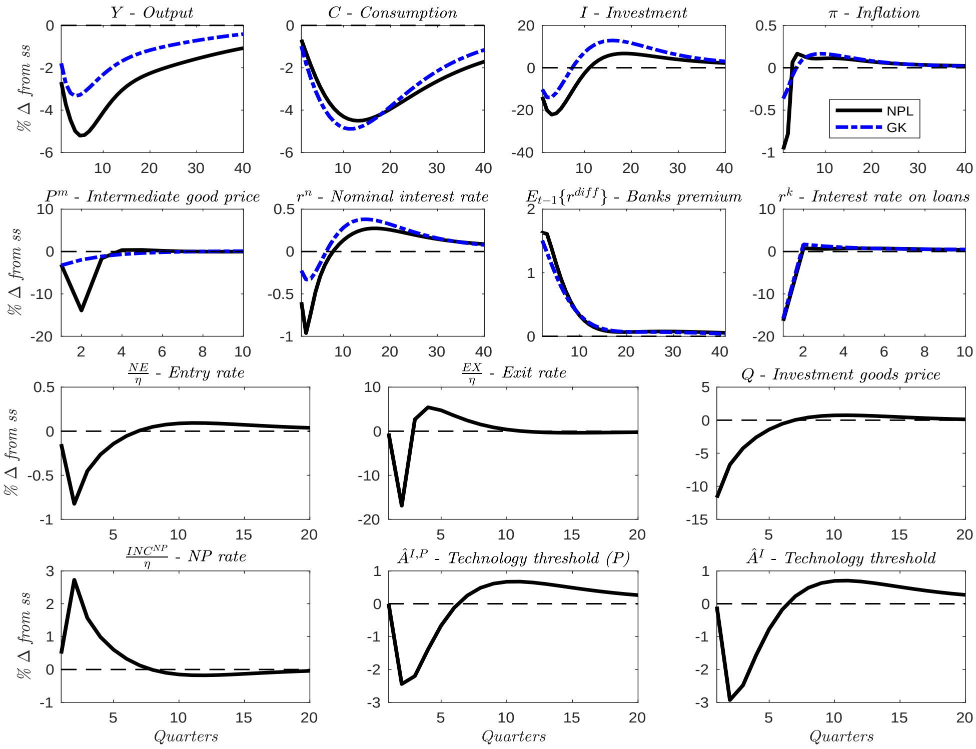

Figure 2: Responses to a negative capital quality shock.

From equation (8), it is clear that the fall in the price of intermediate goods reduces profitability for all intermediate goods producers. The flow of new entrants immediately shrinks (Figure 2), but we do not observe a symmetrical adjustment in exit rates. In fact, exit rates immediately contract. In our model defaults are driven by condition (28). The fall in p_t^m and in the market valuation of capital, Q_t, unambiguously push up the cut-off $\hat{A}_t^I$, potentially raising exit rates, but these effects are dominated by the fall in factor prices that works in the opposite direction. Note that the same effects determine the productivity cutoff for profitable incumbents, $\hat{A}_t^{I,P}$, and the co-movements between the cut-offs $\hat{A}_t^I$ and $\hat{A}_t^{I,P}$ in equation (29) are crucial to identify dynamics in the number of non-performing

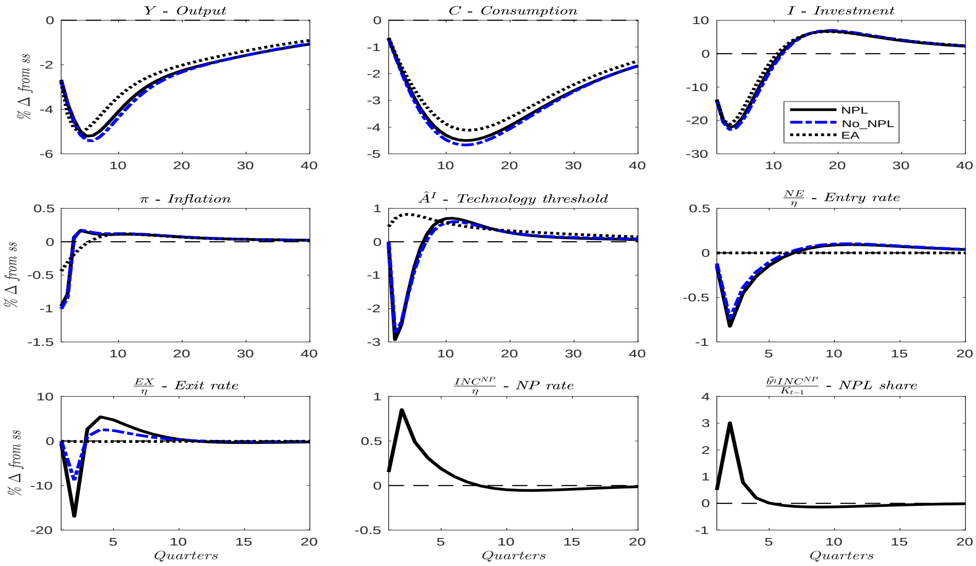

Figure 3: Responses to a negative capital quality shock.

firms, INC_t^{NP}. In fact the fraction of non-performing firms exhibits a sharp and prolonged increase.

Figure 3 allows us to draw a comparison between the NPL, No_NPL and EA. We do not plot here the IRFs for the ERA model, where a secondary capital market allows to efficiently reallocate capital, as they overlap with those obtained for the EA model. The output contraction is deeper in the NPL and the No_NPL models, mainly due to the stronger fall in consumption (Figure 3). This latter effect is driven by the differences in the real rates on deposits which, due to the interest rate rule, closely follow inflation differentials. The NPL-No_NPL models are initially characterized by a severely deflationary outcome, then inflation quickly rebounds and remains for several quarters above the level predicted by the EA model. Note that the output contraction is marginally stronger in the No_NPL model, suggesting that debt renegotiations provide a minimal recession-dampening effect.

To rationalize the specific pattern of the EA model, one should focus on the Incumbent firms cutoffs $\hat{A}^I$. The η_{t-1} firms are not saddled with predetermined capital when the shock hits the economy, and the cut-off $\hat{A}_t^I$ immediately increases because profitability conditions deteriorate with the fall in p_t^m. In subsequent

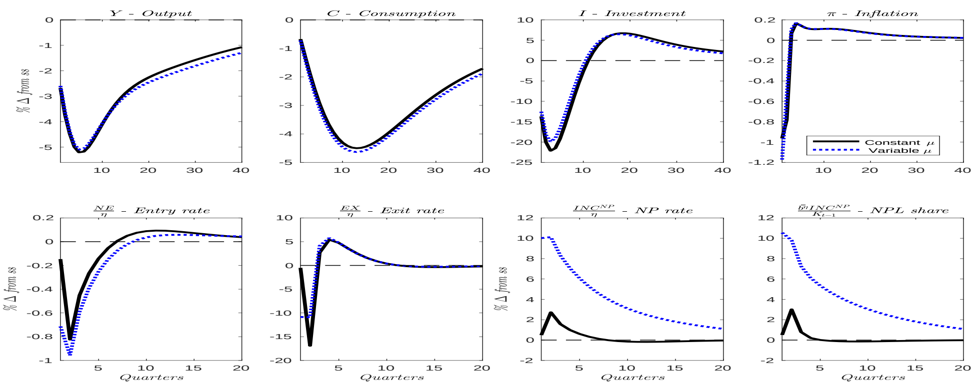

Figure 4: Responses to a negative capital quality shock.

periods, $\hat{A}^I$ falls as both p^m and the prices of factor inputs return to steady state equilibrium.

The increase in incumbent firms efficiency, and the efficient capital allocation, unambiguously reduce inflationary pressures relative to what we observe for the NPL-No_NPL models, where the η_{t-1} firms cannot adjust their capital stock. In this case, capital reallocation to more productive incumbent firms does not occur and the immediate outcome is that the rental price r_t^k collapses. This, in turn, facilitates the survival of less productive firms. In the first few periods, total production is similar across the different models, but in the EA model production is allocated to fewer, more productive firms. Over time, the inefficient allocation of factor inputs raises inflationary pressures in the NPL-No_NPL models, triggering a more contractionary monetary stance that causes the slack of demand (and production) relative to the EA model.

The EA model is characterized by complete stability of entry and exit flows in percentage of total firms. This happens because, in the absence of pre-determined capital, the choice of factor inputs is symmetrical across incumbents and new entrants.[14] The NPL and the No_NPL models are characterized by an initial contraction in exit rates, which is stronger for the NPL model. After a few quarters, exit rates rebound and become positive, an effect which is larger for the NPL model. The different exit rate patterns detected for the NPL and No_NPL models are obviously due to the initial surge in the number of non-performing incumbents.

[14]See the Appendix for a proof.

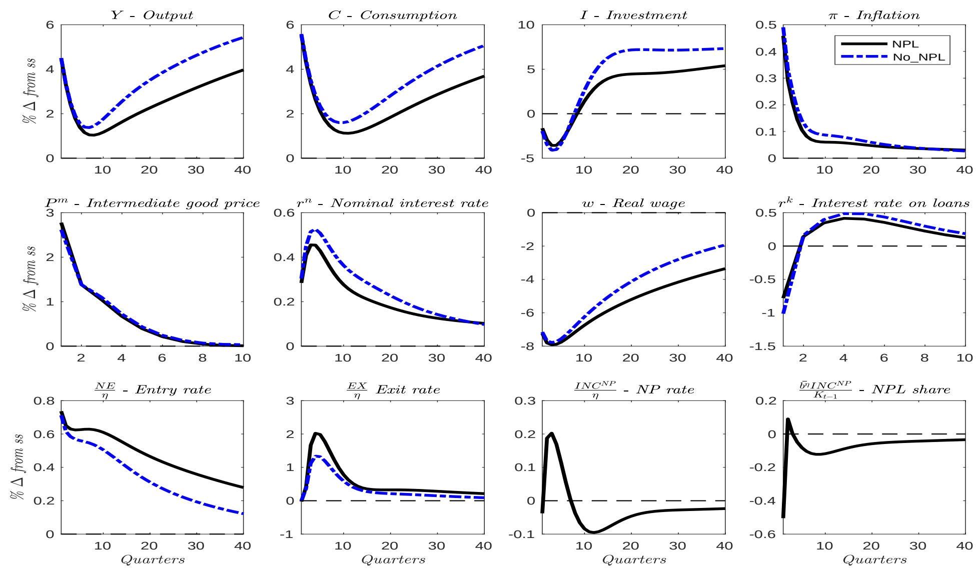

Figure 5: Responses to techonology innovation.

In the NPL model, debt renegotiations do not seem to cause any significant congestion effect on firm entry relative to the No_NPL model.

By contrast, allowing for congestion effects in bank monitoring costs does have important implications (Figure 4). We observe a large and persistent increase in the share on non-performing incumbents (and loans). This is associated with milder contractions in output and investment, followed by less favourable paths during the recovery. The stronger initial fall in inflation suggests that the large increase in the number of INC_t^{NP} firms does cause a supply congestion that deters entry, especially in the initial phase of the crisis.

4.2 Technology Shocks

The technology shock is modelled as a rightward shift of the potential *NE*s' pdf due to the impact of ε_t^z on $\underline{e}_t$(see conditions (9) and (10)).

For any given entry threshold, $\hat{A}_t^{NE}$, this causes an inflow of a larger mass of more productive *NE*s in the market (see condition (16)). The shock has been normalized to generate a 7.14% permanent increase in the de-trended values of Y, C, I. In Figure 5 we show that the economy is characterized by an initial consumption boom and by a contraction in investment. Consumption, output and investment follow an inverse hump-shaped pattern, which is driven by the interest rate response to the surge in inflation. In spite of the increase in demand, the persistent surge in the flow of *NE*s triggers a process of creative destruction. In fact, incumbent firms are confronted with a price of intermediate goods that falls relative to the cost of the production bundle Z_t. As a result, they are forced to restore competitiveness by scaling down production. The number of profitable incumbents inevitably falls, and both defaults and non-performing incumbents increase.

The No_NPL model is characterized by a faster convergence to the new steady state. Furthermore, we observe a milder increase in the number of exits. This happens for a simple reason. The two models in steady state are characterized by identical calibrations for the rental price of capital and for the exit rate but in the No_NPL model the incumbents' productivity cutoff is unambiguously higher because all incumbents are able to service their debt. This in turn implies that, relative to the benchmark model, the incumbent firms cutoff is less sensitive to the adverse cost dynamics described above.

These latter effects dominate, and for this reason, we observe the increase in $\hat{A}_t^I$ (Figure 4). Entry and exit rates increase, and we observe a sharp contraction in the share of both INC^{NP} and NPLs. These latter effects obtain because the cutoff associated with the total number of incumbents outperforms the increase in the cutoff of profitable firms.

4.3 Monetary Policies

In this section, we address one fundamental question, concerning the effects of monetary policies on endogenous firm dynamics. Are expansionary/accommodative policies a hindrance to growth because they limit creative destruction stemming from *NE*s flows?

To begin with, consider the implications of a negative interest rate shock (Figure 6). The output expansion is associated with a sharp increase in the incumbent firms cutoff, $\hat{A}_t^I$. Note that the output expansion is associated with an increase both in the price of intermediate goods and in the shadow price of capital. These two variations would bring down the $\hat{A}_t^I$, allowing survival of relatively less productive firms. By contrast, both the real wage and the rental price of capital increase.

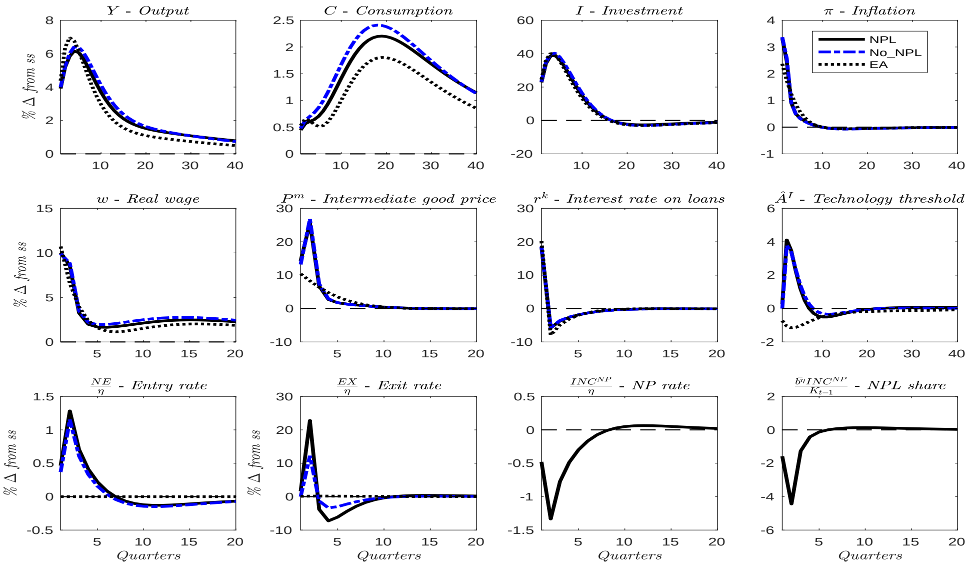

Figure 6: Responses to a monetary policy shock.

These latter effects dominate, and for this reason, we observe the increase in $\hat{A}_t^I$ (Figure 4). Entry and exit rates increase, and we observe a sharp contraction in the share of both INC^{NP} and NPLs. These latter effects obtain because the cutoff associated with the total number of incumbents outperforms the increase in the cutoff of profitable firms.

The shock has stronger expansionary effects in the No_NPL version of the model.[15] The two models exhibit identical entry flows, but the exit rate is unambiguously stronger in the NPL model.

The next step is the comparison of three policy regimes under a common capital quality shock: i) pure inflation targeting; ii) a standard Taylor rule as in (40); iii) A Taylor rule supplemented by a quantitative easing policy rule (41). Results are depicted in Figure 7. The more accommodative the monetary stance, the milder the contraction in output, consumption and investment. Similar results obtain for entry and exit flows. The increases in the number of INC_t^{NP} and in the share of non-performing loans is also inversely related to the strength of the monetary accommodation.

[15] The magnitude of IRFs responses is almost identical to those obtained under a standard GK model. Results available upon request.

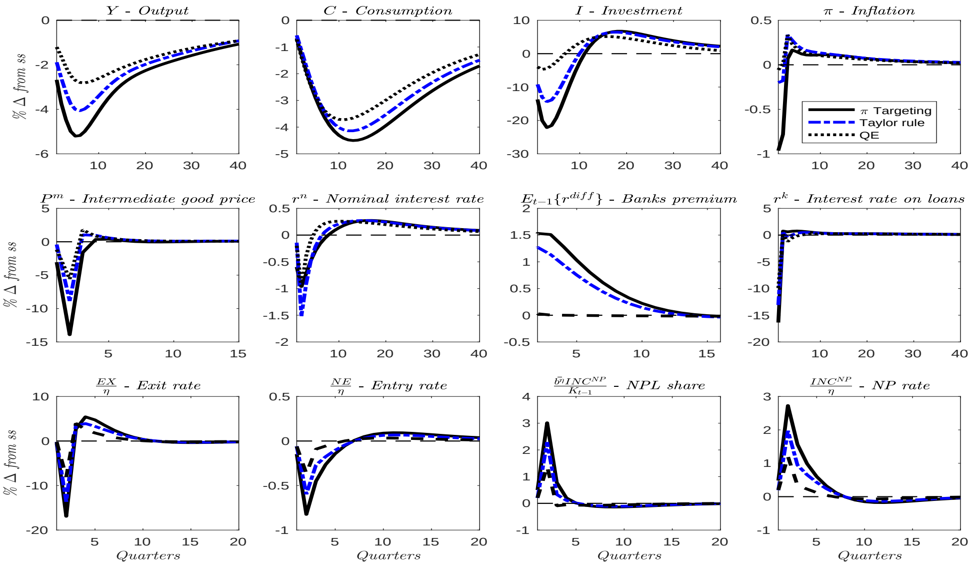

Figure 7: Responses to a negative capital quality shock.

4.4 Non-Performing Loans and the Bankers' Moral Hazard Problem

In this section, we run a financial crisis experiment under the assumption that the increase in the number of non-performing firms worsens the moral hazard problem of banks, as discussed in section 2.3.

4.4.1 Financial Crisis

We run a financial crisis experiment using a negative capital quality shock and compare this new endogenous moral hazard model with the NPL model (Figure 8). Endogenous moral hazard clearly amplifies the effect of a negative shock to capital quality. We observe a more severe GDP contraction and a decrease in investment which is 50% larger. The magnitude of the recession is driven by the spread between loans and deposit rates increases, whose increase doubles the one obtained in the benchmark NPL model.

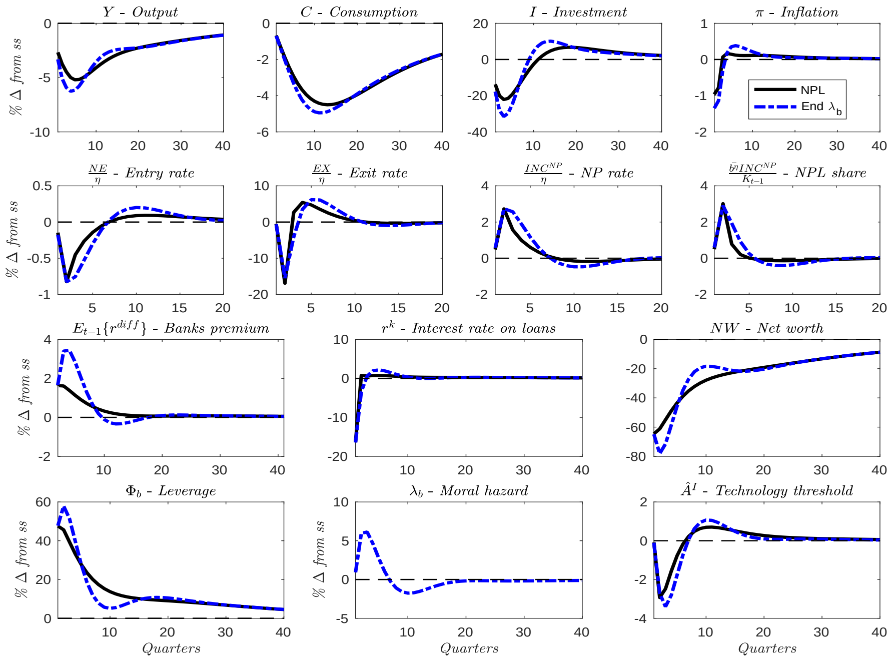

Figure 8: Responses to a negative capital quality shock..

4.4.2 Monetary Policy and Moral Hazard

Given the relevance of the endogenous moral hazard effect, we want to understand the effectiveness of less restrictive monetary policy (i.e. Quantitative easing) in this scenario. As shown in Figure 7, accommodating monetary policies can help to reduce the losses of a financial crisis if compared with inflation targeting monetary policies. The simulation in Figure 9 shows that the unconventional monetary policy can drastically reduce output losses. In fact, the Central Bank is very effective in limiting the loan rate spread and the response of the model is almost comparable to the one obtained with the one obtained in the benchmark NPL model.

4.4.3 Technology Shock and Moral Hazard

The response of the main variables to a shift in the technology frontier is not particularly sensitive to the endogenous nature of moral hazard. However, we can

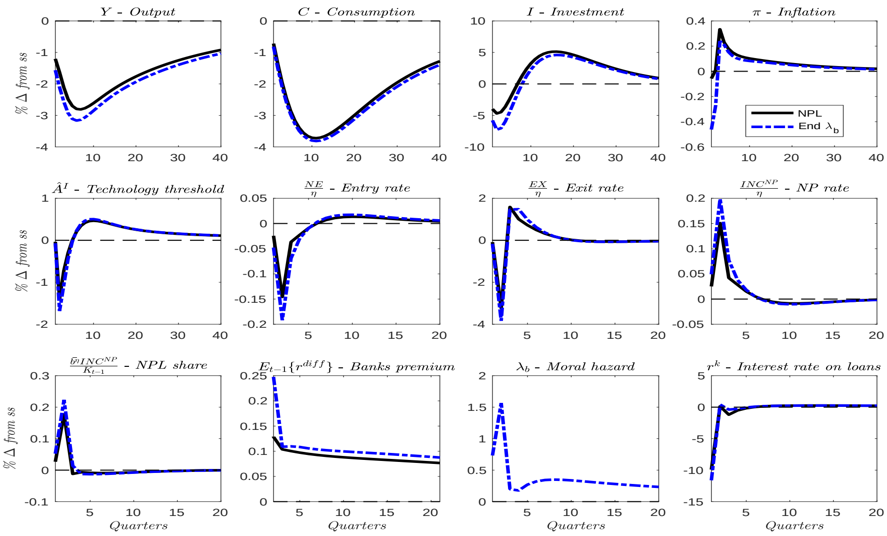

Figure 9: Responses to a negative capital quality shock with Quantitative Easing.

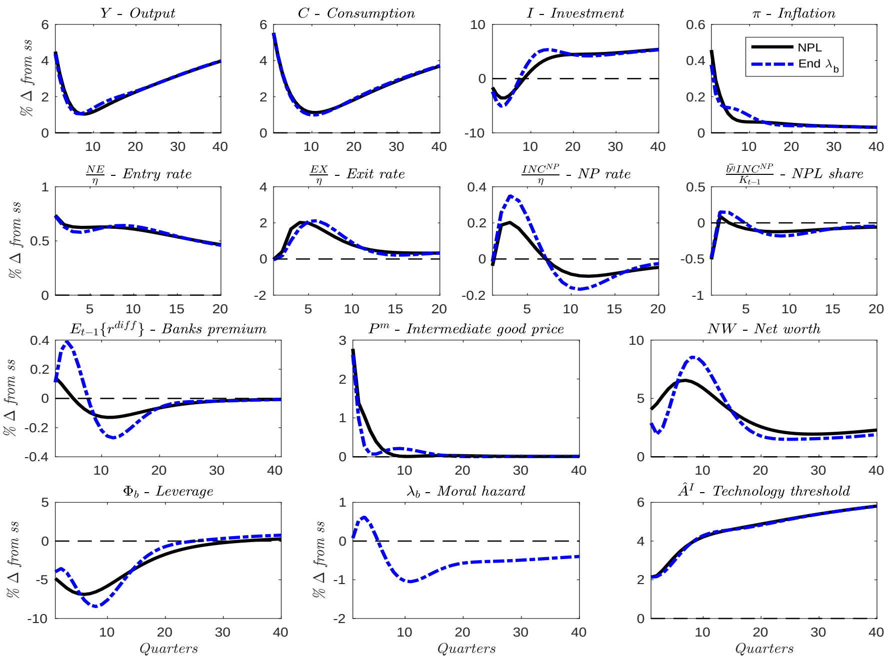

Figure 10: Responses to a technology innovation.

observe some differences in firms dynamic. The increase in technology requirements, allow bankers to earn a higher return on loans and save a bigger share of Non-performing firms. This brings to an increase in the moral hazard parameter that creates a bigger reduction in investment. This phenomenon has no consequences on the evolution of output and consumption, since, the reduction in firms exit reduce the increase in inflation, increasing the willingness to consume.

5 Conclusions

Our models provide new insights on the causes and consequences of non-performing loans in a DSGE model. We essentially downplay the specific role of debt renegotiations in specific financial crisis episodes, but our models predict that economies characterized by a larger equilibrium share of non-performing loans find it more difficult to reap the benefits of technology shocks.

Debt renegotiations and non-performing loans can lead to a dramatically worse macroeconomic performance if they add opacity to the balance sheet of commercial banks, worsening their moral hazard problem. Accommodative monetary policies should not be blamed for encouraging excessive bank leniency on less efficient firms. Quantitative easing policies are shown to have a strong and favourable impact on macroeconomic performance, on entry rates, on the profitability of commercial banks in the aftermath of a financial crisis.

We cannot confirm the view that relates looser financial pressures to the conspicuous increase in the share of non-performing loans observed over the last three decades. According to our results, the phenomenon could be explained by the pace of technological innovation, which causes firms exit rates, by market deregulations that lower fixed production costs, and bank monitoring costs. Future empirical research could put our findings at test.

Our results concerning the relatively favourable outcomes obtained in the ERA model suggest that future research should also investigate the design of efficient risk-sharing schemes amongst intermediate goods producers, which could alleviate the adverse effects caused by pre-determined capital allocation.

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Appendix A. Set of detrended equations[16].

The modelled economy follows a Balanced Growth Path (BGP), the stationary variables in steady state are the labor L_{ss}, the set of interest rate r^d_{ss}, r^k_{ss}, r^b_{ss}, i, the set of prices and inflation Q, p^m, π^c, Π^*_t, a set of financial sector variables, Φ^b, μ^b, ν^b, m, n. The number of firms η, NE, INC, INC^{NP}, INC^P is not subject to the BGP rate. The other variables grow at the endogenous rate g. Further, fixed costs of production grows at the rate g and the technology frontier $\underline{e}_t$ and the technology requirement $\hat{A}^{NE}$, $\hat{A}^I$, $\hat{A}^{I,P}$ grow at the exogenous rates $g_z = g^{(1-\alpha\gamma)}$. In order to compute the deterministic steady state, we have to identify the relation that binds the different growth rates. To clarify notation, a generic variable x_t has a corresponding de-trended value in $\widetilde{x}_t$. Stationary variables keep the original notation. Stationary values of fixed cost (ϕ^{NE} , ϕ^I and μ) and technology frontier ($\underline{e}$) will lose the time index. The complete set of de-trended equations is the following:

Households.

Consumption FOC:

$$\widetilde{\lambda}_t = \frac{1}{\widetilde{C}_t - h\widetilde{C}_{t-1}} - \frac{\beta h}{\widetilde{C}_{t+1} - h\widetilde{C}_t} \tag{46}$$

labor FOC:

$$\widetilde{w}_t = \frac{(L_t)^{\phi}}{\widetilde{\lambda}_t} \tag{47}$$

Intertemporal discount factor:

$$\widetilde{\Lambda}_t = \frac{\widetilde{\lambda}_{t-1}}{\beta\widetilde{\lambda}_t g_t} \tag{48}$$

Interest rate on deposits:

$$r^d_t = \frac{1}{E_t\left\{\widetilde{\Lambda}_{t+1}\right\}} \tag{49}$$

Risk-free interest rate:

$$r^n_t = E_t\left\{\frac{\pi_{t+1}}{\widetilde{\Lambda}_{t+1}}\right\} \tag{50}$$

[16]Term $\widetilde{x}$ defines de-trended value for variable x.

Intermediate good Producers.

New Entrants technology thresholds:

$$\widetilde{\hat{A}_t^{NE}} = \left[\frac{\phi^{NE}}{(1-\gamma)}\right]^{1-\gamma} (p_t^m \gamma^\gamma)^{-1} \left[\left[\frac{(Q_{t-1} r_t^k - (1+\delta)\Xi_t Q_t)}{\Xi_t \alpha}\right]^\alpha \left[\frac{\widetilde{w}_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^\gamma \tag{51}$$

Profitable Incumbents technology thresholds:

$$\widetilde{\hat{A}_t^{I,P}} = \left\{\frac{\phi^I + \frac{\left[Q_{t-1} r_t^k - \Xi_t Q_t (1-\delta)\right]\widetilde{b}_{t-1}^\eta}{g_t Q_{t-1}}}{\left[1-(1-\alpha)\gamma\right]}\right\}^{1-(1-\alpha)\gamma} (p_t^m)^{-1} \left[\left(\frac{g_t Q_{t-1}}{\Xi_t \widetilde{b}_{t-1}^\eta}\right)^\alpha \left(\frac{\widetilde{w}_t}{(1-\alpha)}\right)^{1-\alpha}\right]^\gamma \tag{52}$$

Non-Performing Incumbents technology thresholds:

$$\widetilde{\hat{A}_t^{I}} = \left\{\frac{\phi^I - \mu + \frac{\left[Q_{t-1} r_t^k - \Xi_t Q_t (1-\delta)\right]\widetilde{b}_{t-1}^\eta}{g_t Q_{t-1}}}{1-\left[(1-\alpha)\gamma\right]}\right\}^{1-(1-\alpha)\gamma} (p_t^m)^{-1} \left[\left(\frac{g_t Q_{t-1}}{\Xi_t \widetilde{b}_{t-1}^\eta}\right)^\alpha \left(\frac{\widetilde{w}_t}{(1-\alpha)\gamma}\right)^{1-\alpha}\right]^\gamma \tag{53}$$

New Entrants:

$$NE_t = \left(\frac{e}{\widetilde{\hat{A}_t^{NE}}}\right)^\xi \tag{54}$$

Incumbents:

$$INC_t = \eta_{t-1} \left(\frac{\widetilde{\hat{A}_{t-1}^I}}{\widetilde{\hat{A}_t^I} g_{z,t}} (1-\delta^{inc})\right)^\xi \tag{55}$$

Profitable Incumbents;

$$INC_t^P = \eta_{t-1} \left(\frac{\widetilde{\hat{A}_{t-1}^I}}{\widetilde{\hat{A}_t^{I,P}} g_{z,t}} (1-\delta^{inc})\right)^\xi \tag{56}$$

Non-Performing Incumbents:

$$INC_t^{NP} = \eta_{t-1} \left[\left(\frac{\widetilde{\hat{A}_{t-1}^I}(1-\delta^{inc})}{\widetilde{\hat{A}_t^I} g_{z,t}}\right)^\xi - \left(\frac{\widetilde{\hat{A}_{t-1}^I}(1-\delta^{inc})}{\widetilde{\hat{A}_t^{I,P}} g_{z,t}}\right)^\xi\right] \tag{57}$$

Number of firms:

$$\eta_t = NE_t + INC_t \tag{58}$$

Exit:

$$EX_t = \eta_{t-1}\left[1 - \left(\frac{\widetilde{\widehat{A}^I_{t-1}}(1-\delta^{inc})}{\widetilde{\widehat{A}^I_t} g_{z,t}}\right)^{\xi}\right] \tag{59}$$

New Entrants' output:

$$\widetilde{Y_t^{NE}} = \frac{\xi}{\xi(1-\gamma)-1} NE_t\phi^{NE} \tag{60}$$

Profitable Incumbents' output:

$$\widetilde{Y_t^{I,P}} = INC_t^P \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}(\widetilde{\widehat{A}_t^{I,P}})^{\frac{1}{1-\gamma}}\left\{\left(\frac{\Xi_t\tilde{b}^\eta_{t-1}}{g_tQ_{t-1}}\right)^{\alpha\gamma}\left[\frac{(1-\alpha)\gamma p_t^m}{\widetilde{w}_t}\right]^{(1-\alpha)\gamma}\right\}^{\frac{1}{1-(1-\alpha)\gamma}} \tag{61}$$

Non-Performing Incumbents' output:

$$\widetilde{Y_t^{I,NP}} = \frac{\xi\left[1-(1-\alpha)\gamma\right]}{\xi\left[1-(1-\alpha)\gamma\right]-1}\left[INC_t\left(\widetilde{\widehat{A}_t^I}\right)^{\frac{1}{1-(1-\alpha)\gamma}} - INC_t^P\left(\widetilde{\widehat{A}_t^{I,P}}\right)^{\frac{1}{1-(1-\alpha)\gamma}}\right]\cdot$$

$$\cdot\left\{\left(\frac{\Xi_t\tilde{b}^\eta_{t-1}}{g_tQ_{t-1}}\right)^{\alpha\gamma}\left[\frac{(1-\alpha)\gamma p_t^m}{\widetilde{w}_t}\right]^{(1-\alpha)\gamma}\right\}^{\frac{1}{1-(1-\alpha)\gamma}} \tag{62}$$

Incumbents' output:

$$\widetilde{Y}_t^I = \widetilde{Y}_t^{I,P} + \widetilde{Y}_t^{I,NP} \tag{63}$$

Intermediate firms' output:

$$\widetilde{Y}_t^m = \widetilde{Y}_t^{NE} + \widetilde{Y}_t^I \tag{64}$$

New Entrants' demand of capital:

$$\widetilde{K}_{t-1}^{NE} = g_t\frac{\alpha\gamma p_t^m\widetilde{Y}_t^{NE}}{r_t^k - \frac{Q_t}{Q_{t-1}}\Xi_t(1-\delta)} \tag{65}$$

Aggregate demand of labor:

$$L_t = \frac{(1-\alpha)\,\gamma P_t^m \widetilde{Y}\_t^m}{\widetilde{w}_t} \tag{66}$$

Predetermined demand of loans:

$$\frac{\widetilde{b}_t^\eta}{g_t} = E_t \left\{ \frac{\alpha}{r_{t+1}^k Q_t - (1-\delta)\,\Xi_{t+1} Q_{t+1}} \frac{\left[p_t^m \gamma \frac{\xi}{\xi-1} \widetilde{\hat{A}}_{t+1}^{I,P}\right]^{\frac{1}{1-\gamma}}}{\left[\left(\frac{r_{t+1}^k Q_t - (1-\delta)\Xi_{t+1} Q_{t+1}}{\Xi_{t+1}\alpha}\right)^{\alpha} \left(\frac{\widetilde{w}_{t+1}}{1-\alpha}\right)^{1-\alpha}\right]^{\frac{\gamma}{1-\gamma}}} \right\} \tag{67}$$

Retailers.

$$\widetilde{a}_{1,t} = \widetilde{Y}_t\,(\Pi_t^*) + \beta\ \Gamma \frac{\Pi_t^*}{\Pi_{t+1}^*} \left(\frac{(\pi_t)^\mu}{(\pi_{t+1})}\right)^{1-\varepsilon} \frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t} \widetilde{a}_{1,t+1} \tag{68}$$

$$\widetilde{a}_{2,t} = p_t^m \widetilde{Y}_t + \frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t} \beta\,\Gamma \left(\frac{(\pi_t)^\mu}{(\pi_{t+1})}\right)^{(-)} \widetilde{a}_{2,t+1} \tag{69}$$

$$a_{1,t} = \frac{\varepsilon\ a_{2,t}}{\varepsilon - 1} \tag{70}$$

$$1 = (1-\Gamma)\ (\Pi_t^*)^{1-\varepsilon} + \left(\frac{(\pi_{t-1})^\mu}{(\pi_t)}\right)^{1-\varepsilon} \tag{71}$$

$$\xi_t^\pi = (1-\Gamma)\ \left(\Pi_t^{C*}\right)^{(-\ \varepsilon)} + \Gamma \left(\frac{(\pi_{t-1})^\mu}{(\pi_t)}\right)^{(-\varepsilon)} \xi_{t-1}^\pi \tag{72}$$

$$\widetilde{Y}_t = \widetilde{Y}_t^m \xi_t^\pi \tag{73}$$

Capital Market.

Capital law of motion:

$$\widetilde{K}_t = \widetilde{I}_t + \widetilde{K}_{t-1} \frac{(1-\delta)}{g_t} \tag{74}$$

Evolution of capital goods price:

$$Q_t = 1+f\left(\frac{\widetilde{I}_t}{\widetilde{I}_{t-1}}g_t\right)+f'\left(\frac{\widetilde{I}_t}{\widetilde{I}_{t-1}}g_t\right)\left(\frac{\widetilde{I}_t}{\widetilde{I}_{t-1}}g_t\right)-E_t\beta\frac{\widetilde{\lambda}_{t+1}}{g_{t+1}\widetilde{\lambda}_t}f'\left(\frac{\widetilde{I}_{t+1}}{\widetilde{I}_t}g_{t+1}\right)\left(\frac{\widetilde{I}_{t+1}}{\widetilde{I}_t}g_{t+1}\right)^2 \tag{75}$$

Capital resources constraint:

$$\widetilde{K}_{t-1} = \widetilde{K}_t^{NE} + \eta_{t-1}\frac{\widetilde{b}_{t-1}^{\eta}}{Q_{t-1}} \tag{76}$$

Financial intermediaries.

Non-Performing Incumbents transfer to financial intermediaries:

$$\widetilde{\Pi}_t^{I,NP} = P_t^m\widetilde{Y}_t^{I,NP} - \widetilde{w}_t L_t^{I,NP} + INC_t^{I,NP}\frac{\widetilde{b}_{t-1}^{\eta}}{g_t Q_{t-1}} - INC_t^{NP}\phi^I \tag{77}$$

Return on loans:

$$r_t^b\widetilde{K}_{t-1} = r_t^k\left[INC_t^P\widetilde{b}_{t-1}^{\eta} + Q_{t-1}\widetilde{K}_{t-1}^{NE}\right] + g_t\widetilde{\Pi}_t^{NP} + \left(r_t^k\,\widetilde{b}_{t-1}^{\eta} - \mu_t\right)EX_t \tag{78}$$

Leverage:

$$\Phi_t^b = \frac{\nu_t^b}{\lambda^b - \mu_t^b} \tag{79}$$

Net worth growth rate:

$$n_t = r_{t-1}^d + \left(r_t^b - r_{t-1}^d\right)\,\Phi_{t-1}^b \tag{80}$$

Gross growth rate in lending:

$$m_t = n_t\,\frac{\Phi_t^b}{\Phi_{t-1}^b} \tag{81}$$

Marginal value of assets expansion:

$$\mu_t^b = \beta\,\left(1-\theta_b\right)\,\Lambda_{t+1}\,\left(\,r_{t+1}^b - r_t^d\right) + \Lambda_{t+1}\,\beta\,\theta_b\;m_{t+1}\,\mu_{t+1}^b \tag{82}$$

Marginal value of net worth expansion:

$$\nu_t^b = r_t^d \beta \, (1-\theta_b) \; + \Lambda_{t+1} \beta \, \theta_b \, n_{t+1} \tag{83}$$

Net worth law of motion:

$$\widetilde{NW}_t g_t = n_t \, \theta_b \, \widetilde{NW}_{t-1} \varepsilon_t^{\nu} + \omega \Xi_t \widetilde{K}_{t-1} \, Q_t \tag{84}$$

Bankers premium:

$$r_t^{diff} = r_{t+1}^b - r_t^d \tag{85}$$

Leverage ratio:

$$Q_t \widetilde{K}_t = \Phi_t^b \, \widetilde{NW}_t \tag{86}$$

Market clearing and monetary policy.

Market clearing condition:

$$\widetilde{C}_t = \widetilde{Y}_t - \widetilde{I}_t - INC_t \, \phi^I - NE_t \, \phi^{NE} - EX_t \mu \tag{87}$$

Taylor rule:

$$\frac{r_t^n}{r^n} = \left(\frac{r_{t-1}^n}{r^n}\right)^{\rho_i} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\kappa^\pi} \left(\frac{Y_t}{Y}\right)^{\kappa^y}\right]^{1-\rho_i} exp(\sigma_r, \varepsilon_{r,t}) \tag{88}$$

Exogenous processes.

$$ln(g_t) = \frac{1}{(1-\alpha\gamma)} ln(g_{z,t}) \tag{89}$$

$$ln(g_{z,t}) = (1-\rho_z) ln(g_z) + \rho_z ln(g_{z,t-1}) + \sigma^z \varepsilon_t^z \tag{90}$$

$$\ln\left(\Xi_t\right) = \rho^{\Xi} \ln\left(\Xi_{t-1}\right) + \sigma^{\Xi} \varepsilon_t^{\Xi} \tag{91}$$

$$\ln \mu = (1 - \rho^{\mu}) \ln \mu^{*} + \rho^{\mu} \ln \mu + \sigma^{\mu} \varepsilon_t^{\mu} \tag{92}$$

Appendix B.

B.1. Derivation of key equations in the text.

Equation 15 - The Technology Thresholds for New Entrants.

In order to derive the technology requirement for new firms' entry we start from the NE_t profit maximization problem. New Entrants maximize their profits knowing their idiosyncratic productivity:

$$\pi_t^{j,NE} = p_t^m y_t^{j,NE} - Q_{t-1} r_t^k k_t^{j,NE} + Q_t \Xi_t k_t^{j,NE}(1-\delta) - w_t l_t^{j,NE} - \phi_t^{NE}$$

$$s.t. \quad y_t^{j,NE} = A_t^{j,NE}\left[\left(k_t^{j,NE}\right)^{\alpha}\left(l_t^{j,NE}\right)^{(1-\alpha)}\right]^{\gamma}$$

such that the focs for a generic New Entrants j are:

$$k_t^{j,NE} = \frac{\alpha\gamma p_t^m y_t^{j,NE}}{r_t^k - \frac{Q_t}{Q_{t-1}}\Xi_t(1-\delta)}$$

$$y_t^{j,NE} = \frac{(1-\alpha)\,\gamma p_t^m y_t^{j,NE}}{w_t}$$

The efficiency level of the firms able to gain zero profits defines the entry threshold $\hat{A}_{j,t}^{NE}$:

$$\hat{A}_t^{NE} = \left[\frac{\phi_t^{NE}}{(1-\gamma)}\right]^{1-\gamma} \cdot \frac{\left[\left[\frac{(Q_{t-1}r_t^k-(1-\delta)\Xi_t Q_t)}{\Xi_t\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma}}{p_t^m\gamma^{\gamma}}$$

Equation 17 - Aggregate New Entrants' Output.

Given the distribution of NEs' produuctivity:

$$f_t(A_t^{NE}) = \int_{\underline{e}_t}^{+\infty} \frac{\xi \underline{e}_t^{\xi}}{(A_t^{NE})^{\xi+1}} d(A_t^{NE}) = 1; \; \hat{A}_t^{NE} \geq \underline{e}_t$$

We can derive the total output of New Entrants:

$$
\begin{aligned}
Y_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} A_{j,t}^{NE} \left[\left(\Xi_t k_t^{j,NE} \right)^{\alpha} \left(l_t^{j,NE} \right)^{1-\alpha} \right]^{\gamma} dF(A_{j,t}^{NE}) \\
&= \left\{ \frac{p_t^m \gamma^{\gamma}}{\left[\frac{(Q_{t-1} r_t^k - (1-\delta)\Xi_t Q_t)}{\Xi_t \alpha} \right]^{\alpha\gamma} \left[\frac{w_t}{(1-\alpha)} \right]^{(1-\alpha)\gamma}} \right\}^{\frac{1}{1-\gamma}} \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}} f(A_{j,t}^{NE}) d(A_{j,t}^{NE}) \\
&= \xi \left(\underline{e}_t \right)^{\xi} \left\{ \frac{p_t^m \gamma^{\gamma}}{\left[\frac{(Q_{t-1} r_t^k - (1-\delta)\Xi_t Q_t)}{\Xi_t \alpha} \right]^{\alpha\gamma} \left[\frac{w_t}{(1-\alpha)} \right]^{(1-\alpha)\gamma}} \right\}^{\frac{1}{1-\gamma}} \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}-1-\xi} f(A_{j,t}^{NE}) d(A_{j,t}^{NE}) \\
&= \xi \left(\frac{\underline{e}_t}{\hat{A}_t^{NE}} \right)^{\xi} \left(\hat{A}_t^{NE} \right)^{\frac{1}{1-\gamma}} \left\{ \frac{p_t^m \gamma^{\gamma}}{\left[\frac{(Q_{t-1} r_t^k - (1-\delta)\Xi_t Q_t)}{\Xi_t \alpha} \right]^{\alpha\gamma} \left[\frac{w_t}{(1-\alpha)} \right]^{(1-\alpha)\gamma}} \right\}^{\frac{1}{1-\gamma}} \\
&= NE_t \frac{\xi \phi_t^{NE}}{\xi(1-\gamma) - 1}
\end{aligned}
$$

Equation 21 - Prefetermined Demand of Loans.

In order to purchase the capital stock for production in period t, η_{t-1} firms borrow the amount $\overline{b}_{t-1}^{\eta}$ from commercial banks . Their choice derives from maximization of:

$$
E_{t-1}\{\Pi_t^{j,\eta}\} = E_{t-1} \left\{ \{ p_t^m y_t^{j,\eta} - r_t^k b_{t-1}^{j,\eta} + Q_t \Xi_t k_t^{j,\eta} (1-\delta) - \{ w_t l_t^{j,\eta} - \phi_t^I \right\}
$$

$$
\begin{aligned}
s.t. \quad E_{t-1}\{y_t^{j,\eta}\} &= E_{t-1} \left\{ A_t^{j,\eta} \left[\left(\frac{\Xi_t b_{t-1}^{j,\eta}}{Q_{t-1}} \right)^{\alpha} \left(l_t^{j,\eta} \right)^{(1-\alpha)} \right]^{\gamma} \right\} \\
b_{t-1}^{j,\eta} &= Q_{t-1} k_t^{j,\eta} \\
E_{t-1}\left\{ A_t^{j,\eta} \right\} &= E_{t-1} \int_{\hat{A}_t^{I,P}}^{+\infty} \frac{\xi (\hat{A}_t^{I,P})^{\xi}}{(A_t^I)^{\xi+1}} d(A_t^I) = \frac{\xi}{\xi - 1} E_{t-1}\{\hat{A}_t^{I,P}\}
\end{aligned}
$$

So the first order conditions for potential incumbents are:

$$\frac{b_{t-1}^{j,\eta}}{Q_{t-1}} = E_{t-1}\left\{\frac{\alpha\gamma p_t^m y_t^{j,\eta}}{r_t^k - \frac{Q_t}{Q_{t-1}}\Xi_t(1-\delta)}\right\}$$

$$E_{t-1}\left\{l_t^{j,\eta}\right\} = E_{t-1}\left\{\frac{(1-\alpha)\gamma p_t^m y_t^{j,\eta}}{w_t}\right\}$$

The expected output of the potential incumbents is:

$$E_{t-1}\left\{y_t^{I,P}\right\} = E_{t-1}\left\{\frac{\xi}{\xi-1}\hat{A}_t^{I,P}\left(\frac{\gamma p_t^m}{p_t^z}\right)^{\gamma}\right\}^{\frac{1}{1-\gamma}}$$

and plugging the expected production the first order conditions we get the predetermined demand of loans:

$$\frac{\overline{b}_{t-1}^{\eta}}{Q_{t-1}} = E_{t-1}\left\{\frac{\alpha p_t^m\gamma}{r_t^k + \frac{Q_t}{Q_{t-1}}\Xi_t(1-\delta)}\left[\frac{\xi}{\xi-1}\hat{A}_t^{I,P}\left(\frac{\gamma p_t^m}{p_t^z}\right)^{\gamma}\right]^{\frac{1}{1-\gamma}}\right\}$$

Equation 27 - Aggregate Output of Profitable Incumbents.

Given the distribution of Incumbents' technology:

$$f_t(A_t^I) = \int_{\hat{A}_{t-1}^I(1-\delta^{inc})}^{+\infty}\frac{\xi(\hat{A}_{t-1}^I(1-\delta^{inc}))^{\xi}}{(A_t^I)^{\xi+1}}d(A_t^I)$$

We can derive total output of Profitable Incumbents:

$$Y_t^{I,P} = \int_{\hat{A}_t^{I,P}}^{\infty} A_t^I \left[\left(\frac{\Xi_t \overline{b}_{t-1}^{\eta}}{Q_{t-1}} \right)^{\alpha} (l_{j,t}^I)^{1-\alpha} \right]^{\gamma} dF(A_t^I) =$$

$$= \left\{ \left(\frac{\Xi_t \overline{b}_{t-1}^{\eta}}{Q_{t-1}} \right)^{\alpha\gamma} \left[\frac{(1-\alpha)\gamma p_t^m}{w_t} \right]^{(1-\alpha)\gamma} \right\}^{\frac{1}{1-(1-\alpha)\gamma}} \int_{\hat{A}_t^{I,P}}^{\infty} (A_t^I)^{\frac{1}{1-(1-\alpha)\gamma}} f(A_{j,t}^I) d(A_{j,t}^I) =$$

$$= \xi \eta_{t-1} \left((1-\delta^{inc}) \hat{A}_{t-1}^I \right)^{\xi} \left\{ \left(\frac{\Xi_t \overline{b}_{t-1}^{\eta}}{Q_{t-1}} \right)^{\alpha\gamma} \left[\frac{(1-\alpha)\gamma p_t^m}{w_t} \right]^{(1-\alpha)\gamma} \right\}^{\frac{1}{1-(1-\alpha)\gamma}} \times$$

$$\times \int_{\hat{A}_t^{I,P}}^{\infty} (A_t^I)^{\frac{1}{1-(1-\alpha)\gamma}-1-\xi} f(A_{j,t}^I) d(A_{j,t}^I) =$$

$$= \frac{\xi \left[1-(1-\alpha)\gamma\right]}{\xi \left[1-(1-\alpha)\gamma\right]-1} INC_t^P \left(\widehat{A}_t^{I,P} \right)^{\frac{1}{1-(1-\alpha)\gamma}} \left\{ \left(\frac{\Xi_t \overline{b}_{t-1}^{\eta}}{Q_{t-1}} \right)^{\alpha\gamma} \left[\frac{(1-\alpha)\gamma p_t^m}{w_t} \right]^{(1-\alpha)\gamma} \right\}^{\frac{1}{1-(1-\alpha)\gamma}}$$

Equation 28 - The Technology Thresholds for Profitable incumbents.

The productivity threshold for INC^P is derived from the zero profit condition for incumbent firms:

$$\Pi_t^{j,I} = p_t^m y_t^{j,I} - \left(Q_{t-1} r_t^k \overline{b}_{t-1}^{\eta} - Q_t \Xi_t (1-\delta) \right) \frac{\overline{b}_{t-1}^{\eta}}{Q_{t-1}} - w_t l_t^{j,I} - \phi_t^I = 0 \rightarrow$$

$$\left[1-(1-\alpha)\gamma\right] p_t^m A_t^{j,I} \left[\left(\frac{\Xi_t \overline{b}_{t-1}^{\eta}}{Q_{t-1}} \right)^{\alpha} \left(l_t^{j,I} \right)^{(1-\alpha)} \right]^{\gamma} - \left(Q_{t-1} r_t^k \overline{b}_{t-1}^{\eta} - Q_t \Xi_t (1-\delta) \right) \frac{\overline{b}_{t-1}^{\eta}}{Q_{t-1}} - \phi_t^I = 0 \rightarrow$$

$$\hat{A}_t^{I,P} = \left[\frac{\phi_t^I + \frac{\overline{b}_{t-1}^{\eta}}{Q_{t-1}} \left[Q_{t-1} r_t^k - \Xi_t Q_t (1-\delta) \right]}{1 - \left[(1-\alpha)\gamma\right]} \right]^{1-(1-\alpha)\gamma} \frac{1}{p_t^m} \left(\frac{Q_{t-1}}{\Xi_t \overline{b}_{t-1}^{\eta}} \right)^{\alpha\gamma} \left(\frac{w_t}{(1-\alpha)\gamma} \right)^{(1-\alpha)\gamma}$$

Equation 28 - The Default Cut-off.

We derive the productivity threshold for Non-Performing Incumbents from the bankers' renegotiation condition:

$$\Pi_{j,t}^{NP} = p_t^m y_{j,t}^I - \left(Q_{t-1} r_t^k \overline{b}_{t-1}^\eta - Q_t \Xi_t (1-\delta)\right) \frac{\overline{b}_{t-1}^\eta}{Q_{t-1}} - w_t l_{j,t}^I - \phi_t^I = -\mu_t \rightarrow$$

$$[1-(1-\alpha)\gamma]\, p_t^m A_{j,t}^I \left[\left(\frac{\Xi_t \overline{b}_{t-1}^\eta}{Q_{t-1}}\right)^\alpha \left(l_{j,t}^\eta\right)^{(1-\alpha)}\right]^\gamma - \left(Q_{t-1} r_t^k \overline{b}_{t-1}^\eta - Q_t \Xi_t (1-\delta)\right) \frac{\overline{b}_{t-1}^\eta}{Q_{t-1}} - \phi_t^I + \mu_t = 0 \rightarrow$$

$$\hat{A}_t^I = \left[\frac{\phi_t^I + \left[r_t^k Q_{t-1} - \Xi_t Q_t (1-\delta)\right] \frac{\overline{b}_{t-1}^\eta}{Q_{t-1}} - \mu_t}{1-[(1-\alpha)\gamma]}\right]^{1-(1-\alpha)\gamma} \frac{1}{p_t^m} \left(\frac{Q_{t-1}}{\Xi_t \overline{b}_{t-1}^\eta}\right)^{\alpha\gamma} \left(\frac{w_t}{(1-\alpha)\gamma}\right)^{(1-\alpha)\gamma}$$

Equation 30 - Aggregate Output of Non-Performing Incumbents.

Given Equation 19, we can derive the aggregate output for Non-Performing Incumbents:

$$Y_t^{I,NP} = \int_{\hat{A}_t^I}^{\hat{A}_t^{I,P}} A_t^I \left[\left(\frac{\Xi_t \overline{b}_{t-1}^I}{Q_{t-1}}\right)^\alpha \left(l_{j,t}^I\right)^{1-\alpha}\right]^\gamma dF(A_t^I) =$$

$$= \left\{\left(\frac{\Xi_t \overline{b}_{t-1}^\eta}{Q_{t-1}}\right)^{\alpha\gamma} \left[\frac{(1-\alpha)\gamma p_t^m}{w_t}\right]^{(1-\alpha)\gamma}\right\}^{\frac{1}{1-(1-\alpha)\gamma}} \int_{\hat{A}_t^I}^{\hat{A}_t^{I,P}} (A_t^I)^{\frac{1}{1-(1-\alpha)\gamma}} f(A_{j,t}^I) d(A_{j,t}^I) =$$

$$= \xi \eta_{t-1} \left((1-\delta^{inc})\hat{A}_{t-1}^I\right)^\xi \left\{\left(\frac{\Xi_t \overline{b}_{t-1}^\eta}{Q_{t-1}}\right)^{\alpha\gamma} \left[\frac{(1-\alpha)\gamma p_t^m}{w_t}\right]^{(1-\alpha)\gamma}\right\}^{\frac{1}{1-(1-\alpha)\gamma}} \times$$

$$\times \int_{\hat{A}_t^I}^{\hat{A}_t^{I,P}} (A_t^I)^{\frac{1}{1-(1-\alpha)\gamma}-1-\xi} f(A_{j,t}^I) d(A_{j,t}^I) =$$

$$= \left\{\begin{array}{c} \frac{\xi[1-(1-\alpha)\gamma]}{\xi[1-(1-\alpha)\gamma]-1} \left[INC_t \left(\widehat{A}_t^I\right)^{\frac{1}{1-(1-\alpha)\gamma}} - INC_t^P \left(\widehat{A}_t^{I,P}\right)^{\frac{1}{1-(1-\alpha)\gamma}}\right] \times \\ \times \left\{\left(\frac{\Xi_t \overline{b}_{t-1}^\eta}{Q_{t-1}}\right)^{\alpha\gamma} \left[\frac{(1-\alpha)\gamma p_t^m}{w_t}\right]^{(1-\alpha)\gamma}\right\}^{\frac{1}{1-(1-\alpha)\gamma}} \end{array}\right\}$$

Equation 35 - Financial Intermediaries.

From Equation (31), we have:

$$L_t^b = NW_t + D_t$$

$$L_t^b = \eta_{t-1}\overline{b}_{t-1}^{\eta} + Q_{t-1}K_t^{NE}$$

Conditional upon survival, the representative banker's Net Worth evolves accordingly to the fallowing equation:

$$NW_{t+1} = r_{t+1}^b L_t^b - r_t^d(L_t - NW_t) = (r_{t+1}^b - r_t^d)L_t^b + r_t^d NW_t$$

Therefore, the intermediary will keep on expanding her assets until she can gain a non negative premium exiting the market, *i.e.*,

$$E_t\beta^i \frac{\lambda_{t+1+i}}{\lambda_t}(r_{t+1+i}^b - r_{t+i}^d) \geq 0, \quad i \geq 0 \tag{93}$$

The representative intermediary objective function is:

$$V_t = E_t(1-\theta_b)\sum_{i=0}^{+\infty}\theta_b^i\beta^i\frac{\lambda_{t+1+i}}{\lambda_{t+i}}NW_{t+1+i} =$$

$$= E_t(1-\theta_b)\sum_{i=0}^{+\infty}\theta_b^i\beta^i\frac{\lambda_{t+1+i}}{\lambda_t}\left[\left(r_{t+1+i}^b - r_{t+i}^d\right)L_{t+i}^b + r_{t+i}^d NW_{t+i}\right] \tag{94}$$

In each period the bank can divert a fraction λ_b of funds and exit the market so, for lenders to be willing to supply funds to the banker, the incentive compatibility constraint must be:

$$V_t^b \geq \lambda_{b,t}L_t^b \tag{95}$$

where

$$V_t = \mu_t^b + \nu_t^b NW_t, \tag{96}$$

and μ_t^b, ν_t^b, respectively are the expected discounted marginal benefit of expanding assets by one unit and the expected discounted value of an additional unit of net worth.

$$\mu_t^b = E_t\left[(1-\theta_b)\beta\frac{\lambda_{t+1}}{\lambda_t}\left(r_{t+1}^b - r_t^d\right) + \theta_b\beta\frac{\lambda_{t+1}}{\lambda_t}m_{t+1}\mu_{t+1}^b\right],$$

$$\nu_t^b = E_t\left[(1-\theta_b)\beta\frac{\lambda_{t+1}}{\lambda_t}r_t^d + \theta_b\beta\frac{\lambda_{t+1}}{\lambda_t}n_{t+1}\nu_{t+1}^b\right],$$

$$m_t = \frac{L_t^b}{L_{t-1}^b}$$

$$n_t = \frac{NW_t}{NW_{t-1}}$$

We can rewrite (95) and (96) as:

$$L_t^b = \frac{\nu_t^b}{\lambda_b - \mu_t^b} NW_t = \Phi_t^b NW_t \tag{97}$$

where Φ_t^b is the leverage ratio in t. It follows that net worth dynamics of surviving bankers can be defined as:

$$NW_t = \left[(r_t^b - r_{t-1}^d)\Phi_{t-1}^b + r_{t-1}^d\right] NW_{t-1}. \tag{98}$$

This, in turn, implies that

$$n_t = \frac{NW_t}{NW_{t-1}} = (r_t^b - r_{t-1}^d)\Phi_{t-1}^b + r_{t-1}^d \tag{99}$$

$$m_t = \frac{\Phi_t^b}{\Phi_{t-1}^b} n_t \tag{100}$$

Finally, households finance new banks in each period, transferring a fraction $\frac{\omega}{(1-\theta_b)}$ of the value of assets that exiting bankers had intermediated in their final operating period. Accumulation of total net worth is defined by:

$$NW_t = \theta_b \left[(r_t^b - r_{t-1}^d)\Phi_{t-1}^b + r_{t-1}^d\right] NW_{t-1} + \omega Q_t K_{t-1}$$

B.2. The response of $\frac{NE_t}{\eta_{t-1}}$ to the capital quality shock in the EA Model.

IRFs functions show that the entry rate in the EA model is not affected by capital quality and monetary policy shocks. We provide a proof of this result. Productivity thresholds in the EA model are defined by the following equations:

$$\hat{A}_t^{NE} = \left[\frac{\phi_t^{NE}}{(1-\gamma)}\right]^{1-\gamma} \cdot \frac{\left[\left[\frac{(Q_{t-1}r_t^k-(1-\delta)\Xi_t Q_t)}{\Xi_t \alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma}}{p_t^m \gamma^{\gamma}}$$

$$\hat{A}_t^{I} = \left[\frac{\phi_t^{I}}{(1-\gamma)}\right]^{1-\gamma} \cdot \frac{\left[\left[\frac{(Q_{t-1}r_t^k-(1-\delta)\Xi_t Q_t)}{\Xi_t \alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma}}{p_t^m \gamma^{\gamma}}$$

The entry rate is:

$$\begin{aligned}
\frac{NE_t}{\eta_t} &= \frac{\left(\frac{e}{\widetilde{\hat{A}_t^{NE}}}\right)^{\xi}}{\left(\frac{e}{\widetilde{\hat{A}_t^{NE}}}\right)^{\xi} + \eta_{t-1}\left(\frac{\widetilde{\hat{A}_{t-1}^{I}}(1-\delta_{inc})}{\widetilde{\hat{A}_t^{I}} g_{z,t}}\right)^{\xi}} = \\
&= \frac{1}{1+\eta_{t-1}\left(\frac{\widetilde{\hat{A}_{t-1}^{I}}(1-\delta_{inc})}{z g_{z,t}}\right)^{\xi}\left(\frac{\widetilde{\hat{A}_t^{NE}}}{\widetilde{\hat{A}_t^{I}}}\right)^{\xi}} = \\
&= \frac{1}{1+\frac{\eta_{t-1}}{NE_{t-1}}\left(\frac{\widetilde{\hat{A}_{t-1}^{I}}(1-\delta_{inc})}{\widetilde{\hat{A}_{t-1}^{NE}} g_{z,t}}\right)^{\xi}\left(\frac{\phi^{NE}}{\phi^{I}}\right)^{\xi(1-\gamma)}} \\
&= \frac{1}{1+\frac{\eta_{t-1}}{NE_{t-1}}\left(\frac{(1-\delta_{inc})}{g_z}\right)^{\xi}}
\end{aligned}$$

When the shock hits ($t=1$) it must be that

$$\frac{NE_1}{\eta_1} = \frac{1}{1+\eta\left(\frac{\hat{A}^{I}(1-\delta_{inc})}{\underline{e} g_z}\right)^{\xi}\left(\frac{\phi^{NE}}{\phi^{I}}\right)^{\xi(1-\gamma)}} = \frac{NE}{\eta}.$$

Thus, iterating forward it must be that $\frac{NE_{t+j}}{\eta_{t+j}} = \frac{NE}{\eta}$.

Apprendix C. Set of steady state equations.

$$g = 1.0025 \tag{101}$$

$$g_z = g^{1-\alpha\gamma} \tag{102}$$

$$r^d = \frac{g}{\beta} \tag{103}$$

$$r^n = \frac{g}{\beta} \tag{104}$$

$$\Lambda = \frac{g}{\beta} \tag{105}$$

$$p^m = \frac{\varepsilon - 1}{\varepsilon} \tag{106}$$

$$\widetilde{a}_1 = \frac{\widetilde{Y}}{1 - \beta\,\Gamma} \tag{107}$$

$$\widetilde{a_2} = \frac{p^m\,\widetilde{Y}}{1 - \beta\,\Gamma} \tag{108}$$

$$\xi^\pi = 1 \tag{109}$$

$$\widetilde{Y} = \widetilde{Y}^m \tag{110}$$

$$Q = 1 \tag{111}$$

$$r^{diff} = 0.0025 \tag{112}$$

$$r^b = r^{diff} + r^d \tag{113}$$

$$\Phi^b = 4 \tag{114}$$

$$n = r^d + \left(r^b - r^d\right)\,\Phi^b \tag{115}$$

$$m = n \tag{116}$$

$$\mu^b = \frac{\beta\,(1-\theta_b)\,\Lambda\,\left(r^b - r^d\right)}{1 - \Lambda\beta\,\theta_b\,m} \tag{117}$$

$$\nu^b = \frac{r^d\,\beta\,(1-\theta_b)\,\Lambda}{1 - \;\Lambda\,\beta\,\theta_b\,n} \tag{118}$$

$$\lambda^b = \frac{\nu^b}{\Phi^b} + \mu^b \tag{119}$$

$$\eta = 1 \tag{120}$$

$$NE = E = H^{NE} = \left(\frac{e}{\widetilde{\hat{A}^{NE}}}\right)^{\xi} \tag{121}$$

$$INC = 1 - H^{NE} = \left(1 - \delta^{inc}\right)^{\xi} \tag{122}$$

$$INC^{NP} = 1 - NE - \left(\frac{\widetilde{\hat{A}^{I}}}{\widetilde{\hat{A}^{I,P}}}\left(1 - \delta^{inc}\right)\right)^{\xi} \tag{123}$$

$$INC^{P} = INC - INC^{NP} \tag{124}$$

To get the steady state value of r^k, $\tilde{b^{\eta}}$, μ, $\widetilde{Y^{I,NP}}$, $\widetilde{Y^{NE}}$, $\widetilde{\Pi}^{I,NP}$, $\widetilde{K}$ we solve the following system of equations:

$$r^b \widetilde{K} = r^k \left[\widetilde{K} - \left(INC^{NP} + EX\right)\widetilde{b}^{\eta}\right] + g\widetilde{\Pi}^{NP} + \left(r^k b^{\eta} - \mu\right) EX$$

$$\widetilde{b}^{\eta} = \frac{\alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\phi^I g}{\left\{1-[(1-\alpha)\gamma]-\alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\right\}\left[\left(r^k\right)-(1-\delta)\right]}$$

$$\mu = \left(r^k - 1 + \delta\right)\frac{b^{\eta}}{g} - \left[\frac{\left(INC-INC^{NP}\right)^{\frac{1}{\xi}}}{(1-\delta^{inc})}\right]^{\frac{1}{1-(1-\alpha)\gamma}}\left[\phi^I + \frac{\left[r^{k,\eta}-(1-\delta)\right]\widetilde{b}^{\eta}}{g}\right] + \phi^I$$

$$\widetilde{Y^{I,NP}} = \frac{\xi[1-(1-\alpha)\gamma]p^m}{\xi[1-(1-\alpha)\gamma]-1}\left[INC - INC^P\left(\frac{\widetilde{\hat{A}^{I,P}}}{\widetilde{\hat{A}^{I}}}\right)^{\frac{1}{1-(1-\alpha)\gamma}}\right]\left\{\frac{\phi^I + \frac{\left[r^{k,\eta}-(1-\delta)\right]\widetilde{b}^{\eta}}{g}}{[1-(1-\alpha)\gamma]}\right\}$$

$$\widetilde{Y^{NE}} = \frac{\xi}{\xi(1-\gamma)-1} NE\phi^{NE}$$

$$\widetilde{\Pi}^{I,NP} = p^m\widetilde{Y}^{I,NP} - (1-\alpha)\gamma p^m\widetilde{Y}^{I,NP} + INC^{NP}\frac{\widetilde{b}^{\eta}}{g} - INC^{NP}\phi^I$$

$$\widetilde{K} = \frac{\alpha\gamma p^m Y^{NE}}{r^k-(1-\delta)}g + INC^P\widetilde{b}^{\eta} + \left(INC^{INP} + EX\right)\widetilde{b}^{\eta}$$

Then we can easily derive the remaining steady state values:

$$\widetilde{Y^{I,P}} = INC^P\frac{\xi(1-\gamma)\phi^I}{\xi(1-\gamma)-1}\left\{\frac{1}{\left\{1-[(1-\alpha)\gamma]-\alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\right\}}\right\}\left[\left(\frac{\xi-1}{\xi}\right)^{\frac{\alpha}{1-\gamma+\alpha\gamma}}\frac{1}{\gamma}\right]^{\frac{\gamma}{1-\gamma}} \tag{125}$$

$$\widetilde{Y}^{I} = \widetilde{Y}^{I,P} + \widetilde{Y}^{I,NP} \tag{126}$$

$$\widetilde{Y}^{m} = \widetilde{Y}^{NE} + \widetilde{Y}^{I} \tag{127}$$

$$\widetilde{w} = \frac{(1-\alpha)\,\gamma p^{m}\,\widetilde{Y}^{m}}{L} \tag{128}$$

$$\widetilde{\hat{A}^{NE}} = \left[\frac{\phi^{NE}}{(1-\gamma)}\right]^{1-\gamma} (p^{m}\gamma^{\gamma})^{-1} \left[\left[\frac{r^{k}-(1+\delta)}{\alpha}\right]^{\alpha} \left[\frac{\widetilde{w}}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma} \tag{129}$$

$$\widetilde{\hat{A}^{I,P}} = \left\{\frac{\phi^{I} + \frac{\left[r^{k,\eta}-(1-\delta)\right]\widetilde{b}^{\eta}}{g}}{\left[1-(1-\alpha)\,\gamma\right]}\right\}^{1-(1-\alpha)\gamma} (p^{m})^{-1} \left[\left(\frac{g}{\widetilde{b}^{\eta}}\right)^{\alpha} \left(\frac{\widetilde{w}}{(1-\alpha)\,\gamma}\right)^{1-\alpha}\right]^{\gamma} \tag{130}$$

$$\widetilde{\widehat{A}^{I}} = \left\{\frac{\phi^{I} - \mu + \frac{\left[r^{k,\eta}-(1-\delta)\right]\widetilde{b}^{\eta}}{g}}{1-\left[(1-\alpha)\gamma\right]}\right\}^{1-(1-\alpha)\gamma} (p^{m})^{-1} \left[\left(\frac{g}{\widetilde{b}^{\eta}}\right)^{\alpha} \left(\frac{\widetilde{w}}{(1-\alpha)\,\gamma}\right)^{1-\alpha}\right]^{\gamma} \tag{131}$$

$$\widetilde{I} = \widetilde{K} - \widetilde{K}\,\frac{(1-\delta)}{g} \tag{132}$$

$$\widetilde{NW} = \frac{\widetilde{K}}{\Phi^{b}} \tag{133}$$

$$\widetilde{C} = \widetilde{Y} - \widetilde{I} - \left(INC\phi^{I} - NE\phi^{NE}\right) - EX\mu \tag{134}$$

$$\widetilde{\lambda} = \frac{g-\beta h}{(g-h)\,\widetilde{C}} \tag{135}$$

$$= \frac{\widetilde{w}}{L^{\phi}}\widetilde{\lambda} \tag{136}$$

Appendix D. Steady state comparison.

D.1. Steady state in case of optimal capital demand - EA model.

To compute the steady state of the model with optimal capital demand, we derive the new steady state value of the variables, starting from the parameter calibration of the benchmark model. To clarify notation, a generic steady state variable x has a corresponding value in $x^*$with optimal capital allocation.

We calibrate the model preserving the parameters value. For this reason, we can maintain the benchmark steady state value for the set of variables from (101) to (119) in Appendix C. Since capital is allocated optimally, we can compute the steady state solution from the calibration of r^k:

$$r^{k*} = r^b \tag{137}$$

Thus, we can solve the following system of equations:

$$NE^* = \left(\frac{e}{\widetilde{\hat{A}^{NE*}}} \right)^{\xi} \tag{138}$$

$$INC^* = \eta^* \left(1 - \delta^{inc}\right)^{\xi} \tag{139}$$

$$\eta^* = \frac{NE^*}{1 - (1 - \delta^{inc})^{\xi}} \tag{140}$$

$$INC^* = \frac{\left[(1 - \delta^{inc})^{\xi}\right] NE^*}{\left[1 - (1 - \delta^{inc})^{\xi}\right]} \tag{141}$$

$$EX^* = NE^* \tag{142}$$

$$\widetilde{Y}^{NE*} = \frac{\xi}{\xi(1-\gamma) - 1} NE^* \phi^{NE} \tag{143}$$

$$\widetilde{Y}^{I*} = \frac{\xi}{\xi(1-\gamma)-1}\, INC^{*}\phi^{I} \tag{144}$$

$$\widetilde{K^{*}} = \frac{\alpha\,\gamma p^{m}}{r^{k*}-(1-\delta)} g\left(\widetilde{Y}^{NE*} + \widetilde{Y}^{I*}\right) \tag{145}$$

$$\widetilde{w^{*}} = \frac{(1-\alpha)\,\gamma p^{m}\,\widetilde{Y^{*}}}{L^{*}} \tag{146}$$

$$\widetilde{Y^{*}} = \widetilde{Y}^{NE*} + \widetilde{Y}^{I*} \tag{147}$$

$$p^{z*} = \left[\left[\frac{r^{k*}-(1-\delta)}{\alpha}\right]^{\alpha}\left[\frac{\widetilde{w^{*}}}{(1-\alpha)}\right]^{(1-\alpha)}\right] \tag{148}$$

$$\widetilde{\hat{A}^{NE*}} = \left[\frac{\phi^{NE}}{p^{m}(1-\gamma)}\right]^{1-\gamma}\left(\frac{p^{z*}}{p^{m}\gamma^{\gamma}}\right)^{\gamma} \tag{149}$$

$$\widetilde{\hat{A}^{I*}} = \left[\frac{\phi^{I}}{p^{m}(1-\gamma)}\right]^{1-\gamma}\left(\frac{p^{z*}}{p^{m}\gamma^{\gamma}}\right)^{\gamma} \tag{150}$$

$$\widetilde{\lambda}^{*} = \frac{g-\beta h}{(g-h)\,\widetilde{C}^{*}} \tag{151}$$

$$= \frac{\widetilde{w}^{*}}{L^{*\phi}}\widetilde{\lambda}^{*} \tag{152}$$

$$\widetilde{Y^{*}} = \widetilde{C^{*}} + \widetilde{I^{*}} + \left(NE^{*}\phi^{NE} + INC^{*}\phi^{I}\right) \tag{153}$$

From the previous set of equation, we can obtain the solution for the new steady state levels for the respective variables.

D.2. Steady state in case of pre-determined demand of loans and ex-post optimal reallocation - ERA model.

To compute the steady state of the ERA model, we derive the new steady state value of the variables, starting from the parameter calibration of the benchmark model. To clarify notation, a generic steady state variable x has a corresponding value in x^{**} with ex-post optimal capital reallocation. As in the previous case, we can maintain the steady state value for the set of variables from (101) to(119) in Appendix C.

We assume that Incumbents formulate their pre-determined demand of loans maximizing on their expected productivity. For this reason, the bank will pay the monitoring cost to repossess the loan of defaulting incumbents. To obtain the steady state values, we can solve the following system of equations:

$$\widetilde{b}^{\eta **} = \frac{\alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\phi^I g}{\left\{1-[(1-\alpha)\gamma]-\alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\right\}}\frac{1}{[(r^{k**})-(1-\delta)]} \tag{154}$$

$$\widetilde{K}^{**} = \widetilde{K}^{NE**} + \widetilde{K}^{I**} + \widetilde{b}^{\eta **} EX^{**} \tag{155}$$

$$r^b \widetilde{K}^{**} = r^{k**}\left[\widetilde{K}^{NE**} + \widetilde{K}^{I**}\right] + \left(r^{k**}\widetilde{b}^{\eta **} - \mu\right) EX^{**} \tag{156}$$

$$NE^{**} = \left(\frac{e}{\widetilde{\hat{A}^{NE**}}}\right)^{\xi} \tag{157}$$

$$INC^{**} = \eta^{**}\left(1-\delta^{inc}\right)^{\xi} \tag{158}$$

$$\eta^{**} = \frac{NE^{**}}{1-(1-\delta^{inc})^{\xi}} \tag{159}$$

$$INC^{**} = \frac{\left[(1-\delta^{inc})^{\xi}\right] NE^{**}}{\left[1-(1-\delta^{inc})^{\xi}\right]} \tag{160}$$

$$EX^{**} = NE^{**} \tag{161}$$

$$\widetilde{Y}^{NE**} = \frac{\xi}{\xi(1-\gamma)-1} NE^{**} \phi^{NE} \tag{162}$$

$$\widetilde{Y}^{I**} = \frac{\xi}{\xi(1-\gamma)-1} INC^{**} \widetilde{\phi}^{I} \tag{163}$$

$$\widetilde{K^{**}} = \frac{\alpha\,\gamma p^{m}}{r^{k**}-(1-\delta)} g \left(\widetilde{Y}^{NE**} + \widetilde{Y}^{I**} \right) \tag{164}$$

$$\widetilde{w^{**}} = \frac{(1-\alpha)\,\gamma p^{m}\,\widetilde{Y^{**}}}{L^{**}} \tag{165}$$

$$\widetilde{Y^{**}} = \widetilde{Y}^{NE**} + \widetilde{Y}^{I**} \tag{166}$$

$$p^{z*} = \left[\left[\frac{r^{k**}-(1-\delta)}{\alpha} \right]^{\alpha} \left[\frac{\widetilde{w^{**}}}{(1-\alpha)} \right]^{(1-\alpha)} \right] \tag{167}$$

$$\widetilde{\hat{A}^{NE**}} = \left[\frac{\phi^{NE}}{p^{m}(1-\gamma)} \right]^{1-\gamma} \left(\frac{p^{z**}}{p^{m}\gamma^{\gamma}} \right)^{\gamma} \tag{168}$$

$$\widetilde{\hat{A}^{I**}} = \left[\frac{\phi^{I}}{p^{m}(1-\gamma)} \right]^{1-\gamma} \left(\frac{p^{z**}}{p^{m}\gamma^{\gamma}} \right)^{\gamma} \tag{169}$$

$$\widetilde{\lambda^{**}} = \frac{g-\beta h}{(g-h)\,\widetilde{C}^{**}} \tag{170}$$

$$= \frac{\widetilde{w}^{**}}{L^{*\phi}} \widetilde{\lambda}^{**} \tag{171}$$

$$\widetilde{Y^{**}} = \widetilde{C^{**}} + \widetilde{I^{**}} + \left(NE^{**}\phi^{NE} + INC^{**}\phi^{I}\right) - EX^{**}\mu \tag{172}$$

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Bianca Barbaro
University of Milano-Bicocca, Milan, Italy; email: bianca.barbaro@unimib.it

Patrizio Tirelli
University of Pavia, Pavia, Italy; Griffith University, Brisbane, Australia; email: patrizio.tirelli@unipv.it

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

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