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Demographics and inflation in the euro area: a two-sector new Keynesian perspective

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Abstract

Can the aging process affect inflation? The prolonged decline of fertility and mortality rates induces a persistent downward pressure on the natural interest rate. If this development is not internalized by the monetary policy rule, inflation can be on a downward trend. Using the structure of a two-sector overlapping generations model embedded in a New-Keynesian framework with price frictions, calibrated for the euro area, this paper shows that following a commonly specified monetary policy rule the economy features a ”disinflationary bias” since 1990, in a way that can match the downward trend of core inflation found in the data for the euro area. In this model, continuing to follow the same rule makes inflation to be on a declining pattern at least until 2030. At the same time, changing consumption patterns towards nontradable items such as health-care generate a small ”inflationary bias” a positive deviation of inflation from target of less than 0.1 percentage points between 1990 and 2030. In the model setting of this paper, this inflationary bias is not strong enough to counteract the disinflationary bias generated by the downward impact of aging on the natural interest rate.

JEL codes: E43, E52, E58, J11.

Keywords: population aging, monetary policy, inflation, euro area, consumption composition.

Non-technical summary

Europe is undergoing a demographic transition characterized by declining fertility and mortality rates which reduce the size of the cohort entering the working-age and increase the average survival probability. In parallel to this aging process, European economies have experienced persistently declining real interest rates since the 1980s and, particularly since 2012, low inflationary pressures. This paper investigates to what extent demographic change can be associated to low levels of both the real interest rate and inflation in the euro area, and does so in a context where the dynamics of prices can be influenced by the sectoral shift of demand that goes hand in hand with aging.

First, within a two-sector overlapping generations model embedded in a New-Keynesian framework with price frictions, the paper quantifies the impact on inflation of a monetary policy that does not internalize the endogenous impact of the demographic transition on the natural interest rate. As a result, a 'disinflationary bias' is generated, which stems from a decrease of the natural interest rate by about 0.85 percentage points between 1990 and 2030. When the Taylor-type monetary policy rule features a constant natural interest rate, inflation decreases persistently going from about 2% (the target) in 1990 to about 0.5% in 2030 using the UN (2017) demographic projections. This decrease seems to match the declining path of low-frequency inflation, particularly since the global financial crisis of 2007. Using a less naive monetary policy rule where the central bank updates regularly but with a delay the natural level of the economic variables the disinflationary bias is reduced, but largely remains. In both cases the central bank perceives too high a natural interest rate thus being chronically tighter than intended.

Second, the paper focuses on the sectoral analysis in order to capture the impact of the evolving structure of consumption brought about by aging with an ensuing structural change in production. It quantifies to what extent the change in demand composition due to aging can affect the dynamics of inflation. It shows that due to the old-age consumption propensity towards nontradable items and sectoral dynamics associated with imperfect labor mobility, prices in the nontradable sector (e.g. health sector) grow more than in the tradable sector. The failure to internalize these sectoral developments, generates a small "inflationary bias" of less than 0.1 percentage points between 1990 and 2030 from target inflation Hence, this inflationary bias associated with the old-age consumption propensity towards nontradable goods and services such as healthcare expenditures is not able to counterbalance the disinflationary bias stemming from the impact of aging on the natural rate.

In conclusion and based on our model analysis, the ongoing demographic transition can carry important implications for monetary policy. Even though the impact of demographic transition on inflation in the short- to medium-term horizon, which is relevant for monetary policy-making, is not bearing, this slow-moving force seems to have contributed to a disinflationary bias in the euro area in the last two decades. Monetary policy rules that do not internalize the downward impact of aging on the natural interest rate are likely to be tighter than intended thus generating a potential downward trend of inflation.

Looking ahead, aging and the declining working-age population are expected to continue in those American and European countries. I cannot entirely rule out the looming menace that may unveil itself into downward pressure on inflation rates [...]. (Shirakawa (2012), Demographic Changes and Macroeconomic Performance: Japanese Experiences)

An excess of savings would simply mean that the equilibrium real interest rate required to deliver price stability would be lower, and the central bank would have to factor that into its monetary policy. Put another way, the effects of ageing would call for us to adjust our instruments, but not our objectives. (Draghi (2016), How central banks meet the challenge of low inflation)

1 Introduction

Europe is undergoing a demographic transition. Though the extent and timing can vary substantially across EU countries, fertility and mortality rates are decreasing thus reducing the size of the cohort entering the working-age and increasing the average survival probability (see Figure 6). The result is that the relative number of elderly is dramatically increasing. UN (2017) population statistics highlight that before the 1980s the ratio of the elderly (aged 65 and above) to working-age (aged 15-64) was less than 2 to 10, while in 2050 the proportion will be more than 5 to 10 in Europe (see Figure 7). Only unimaginable inflows of young migrants could overturn this trend (Boersch-Supan et al., 2019). Similar developments can be observed not only in Europe but also in other advanced economies.

In parallel to the demographic transition, advanced economies have experienced persistently declining real interest rates since the 1980s (Rachel and Smith (2015), Rachel and Summers (2019)) and at the same time there has been a prolonged period of low inflation, particularly since the global financial crisis (Draghi (2016), Yellen (2017)). Both observations have led to a search for "slow-moving secular forces" (Eggertsson et al., 2019) such as demographic change that might connect the two occurrences, questioning whether inflation is purely a business cycle phenomenon.

Can the aging process affect inflation? In other words, can aging affect the central bank's ability to attain its inflation objective? In the context of the "post-monetary world" (Woodford, 1998) featured in the New-Keynesian theoretical setting, adopted in this paper and widely used in central banking, the immediate answer would be 'no'. In such an environment, in principle, the central bank can attain target inflation in each period. If it is so, then the nominal interest rate is equal to its natural counterpart, i.e. the rate of interest that, following Wicksell (1898)'s intuitions, would prevail in a counterfactual economy absent nominal rigidities bringing output in line with its potential or natural level. The policy implication would be to monitor the natural interest rate when setting the nominal interest rate.

The practical shortcoming is that, even assuming to have the "true model" of the economic reality and knowing all its parameters, one would need to observe in real time the realized values of all the shocks impinging on the natural interest rate. Instead, in most applications the central bank is assumed to set the nominal interest rate following a policy rule based on observable variables where the natural interest rate is often constant (see e.g. Gomes et al. (2012), a model of policy analysis for the euro area), while considerable effort is devoted in estimating the natural interest rate trying to assess the stance of monetary policy (see WGEM (2018)). The demographic transition can essentially be thought as one of those shocks to the natural interest rate characterized by being slow-moving and persistent.

It is widely recognized that aging leads to a progressive decrease of the natural real interest rate (recently see e.g. Krueger and Ludwig (2007), Gagnon et al. (2016), Carvalho et al. (2016), Lisack et al. (2017), Jones (2018), Bielecki et al. (2018), Papetti (2019)). Labor-input becomes scarcer and individuals are willing to save more in expectation of higher survival probabilities. Both factors are the dominant forces in most general equilibrium overlapping-generations (OLG) models and contribute to increase the capital-labor ratio, thus decreasing the marginal product of capital which equals the natural real interest rate. Generally this happens despite the aggregate saving rate decreases as the share of elderly (dissavers, according to the underlying life-cycle structure) increases. Central bankers have commonly considered this development induced by aging as negligible and too slow-moving to call for a monetary policy reaction.[1] Therefore, it becomes legitimate to ask whether neglecting for long the 'glacial' yet continuous developments induced by aging might bring about significant misalignments of inflation from its target.

The main contribution of this paper is to investigate whether the demographic transition can add to explaining the recent low levels of inflation in the euro area. It does so in a threefold way. First, within the New-Keynesian framework, it quantifies the impact on inflation of a monetary policy that does not internalize the endogenous impact of the demographic transition on the natural interest rate. The 'disinflationary bias' so generated can be conspicuous, in presence of a natural interest rate projected to decreases about 0.85 percentage points between 1990 and 2030. When the Taylor-type monetary policy rule features a constant natural interest rate, inflation decreases persistently going from about 2% (the target) in 1990 to about 0.5% in 2030 using the UN (2017) demographic projections. This decrease seems to match the declining path of low-frequency inflation, particularly since the global financial crisis of 2007. Using a less naive monetary policy rule where the central bank updates regularly but with a delay the natural level of the economic variables, the disinflationary bias is reduced, but largely remains. In both cases the central bank perceives too high a natural interest rate thus being chronically tighter than intended. The key exogenous variable to understand the impact of the demographic transition on the natural interest rate, hence on inflation, is the growth rate of the effective labor-population ratio. It depends not only on the number of workers but also on their age-dependent productivity, as compared to the number of people in the whole economy.

The second contribution is to focus the analysis on a two-sector model in order to capture the impact of the evolving structure of consumption brought about by aging with ensuing structural change in production. As analyzed by Boersch-Supan (2001), demand will shift towards more services and products for older members of the society. Groneck and Kaufmann (2017) and Giagheddu and Papetti (2017) find that this shift occurs particularly towards consumption items classifiable as nontradable with a consequent increase of the relative price of nontradables in presence of imperfect mobility of labor between sectors.[2] Keeping their same tradable vs non-tradable classification and theoretical structure, despite considering the euro area as a closed economy, we quantify to what extent the change in demand composition due to aging can affect the dynamics of inflation. We find that, due to the old-age consumption propensity towards nontradable items and sectoral dynamics associated with imperfect labor mobility, prices in the nontradable sector grow more than in the tradable sector. When the central bank

[1] The observation by Bean (2004): "*the 'slow burn' nature of demographic change suggests that the immediate implications for monetary policy could be modest*" has generally found support among central bankers. See e.g. Trichet (2007): "the demographic developments [...] are not the types of shock that may trigger significant and immediate monetary policy responses" or Papademos (2007). Among academics, see e.g. Mojon (2002) commenting on Miles (2002); Kara and von Thadden (2010) conclude: "*The main finding is that demographic changes, while contributing slowly over time to a decline in the equilibrium interest rate, are not visible enough within the shorter time horizon relevant for monetary policy-making to require monetary policy reactions*".

[2] Notice that in the absence of frictions such as imperfect mobility of labor, a change in the relative demand would be simultaneously matched by a change in relative supply with no change in relative prices.

internalizes the downward impact on the natural *real* interest rate throughout the demographic transition, implicitly targeting only inflation in the tradable sector, which serves as the numerarire, generates an "inflationary bias" which is, however, small in magnitude: a deviation of inflation from target of less than 0.1 percentage points between 1990 and 2030.

The third contribution is methodological. Kara and von Thadden (2016) notice that monetary policy is typically addressed using New Keynesian dynamic stochastic general equilibrium (DSGE) frameworks in which demographic changes are not explicitly modelled. Trying to obviate this shortcoming, we provide a New Keynesian framework that can be used to characterize the response of macroeconomic variables to demographic shocks. The model we use is a version with nominal rigidities of Papetti (2019) who in turn builds on Jones (2018), extending the model to two sectors with age-varying sectoral consumption shares and imperfect labor mobility. Relying on his methodology allows to approximate the equilibrium of a fully-fledged OLG model solving only for the aggregates. While we use it for the purpose of perfect-foresight simulations where demographic change is the unique driver, the structure of this model could be introduced into canonical DSGE frameworks, thus taking into account demographic change via analytic and computational tools commonly used in policy making under the representative agent paradigm. The drawback is that it does not offer an exact equilibrium characterization of the underlying OLG model.

So far, the academic literature on the link between demographics and inflation is scarce and has focused mainly to establish an empirical link not finding a consensus. Bobeica et al. (2017) find a positive long-run relationship between euro area inflation and the growth rate of the working-age population ratio, consistently with our theoretical prediction and with the empirical work of e.g. Gajewski (2015) and Yoon et al. (2018). Opposing results are found by Juselius and Takats (2018) where increases of the share of relatively older cohorts seem to be inflationary; and also by Aksoy et al. (2019) who include inflation in an empirical long-run model and build a non-monetary theoretical model where the effect of demographics on innovation is key.[3]

While it is common to evaluate the implications of demographic change using real macroeconomic theoretical models, there exists only a limited set of papers that evaluate the nominal implications. Within this set, the works mostly related to our contributions are Carvalho and Ferrero (2014), Kara and von Thadden (2016), Bielecki et al. (2018), Jones (2018). Focusing on Japan, Carvalho and Ferrero (2014) use a two-age groups model based on Gertler (1999), augmented with nominal rigidities in a New-Keynesian framework, clarifying to what extent deflation might arise in equilibrium as a consequence of a nominal interest rate that does not adjust to the low-frequency movements of the natural real interest rate induced by demographic change. A similar setting is used by Kara and von Thadden (2016) who find below-target inflation persistence in the euro area due to aging, but only to a limited extent. Bielecki et al. (2018) emphasize that setups *à la* Gertler (1999) might give results too sensitive to the simplifying 'Blanchard-Yaari' assumptions (which involve an age-independent mortality risk during retirement). Conventionally solving for the transition dynamics of a fully-fledged OLG model within a New-Keynesian framework calibrated on the euro area, they estimate a prolonged period of

[3] See Boersch-Supan et al. (2019), section 7, for a short review of the literature.

non-negligible below-target inflation due to a monetary authority that learns slowly over time the impact of the demographic processes on the natural interest rate and potential output. A magnitude which is in line with our estimates. Jones (2018), as explained above, offers the theoretical framework on which we built our model, while his contribution is more focused on the deviation of output from its long-run trend in the US, including business cycle fluctuations and the zero lower bound (ZLB) for monetary policy. Härlt and Leite (2018) analyze the interplay between aging and inflation in a OLG model with money in the utility function. They find that decreases in the population size are the main drivers of disinflationary pressures while changes in the population structure matter little. Their approach differs essentially from ours as it abstracts from a bond interest rate that equates aggregate money demand and supply which instead is key for our New Keynesian perspective.

To our knowledge, Katagiri (2012) is the only paper that tries to quantify the impact of changes in the demand structure induced by aging for monetary policy. However, there the main mechanism is based on the assumption that aggregate productivity decreases (with a sequence of unexpected revisions of population aging forecasts in Japan) because the change in demand structure brings about a resource reallocation away from the sector with relatively higher productivity (manufacturing). Again, the implied decrease of the natural interest rate not internalized by the central bank leads to deflation. We focus instead on a mechanism purely based on relative prices under imperfect labor mobility.

Finally, the presence of a declining natural interest rate carries an important challenge for monetary policy that deserves further investigation. If the central bank is successful in keeping inflation at target, it must be the case that the policy rate is declining too. It is 'physiological'. Therefore, it is obvious that the demographic transition *per se* increases the probability of hitting the ZLB.[4] As a consequence, the central bank might be forced to reconsider the inflation target or perhaps adjust policies with exceptional instruments in order to have sufficient ammunition against recessionary shocks (see Ball (2014), Williams et al. (2016), Kiley and Roberts (2017), Rogoff et al. (2017)). Andrade et al. (2018) have studied the relationship between the optimal inflation target and the natural interest rate in a full-blown New-Keynesian DSGE model that incorporates the potential non-linearities due to the ZLB. How this relationship might be affected by demographic change is an exercise that still needs to be done in the literature. We hope that our framework can provide help in this regard for future research.

The rest of the paper proceeds as follows. Section 2 presents the two-sector OLG model for a closed-economy with "New Keynesian" frictions in price setting and how it can be approximated with an aggregate representation. Section 3 provides the quantitative analysis by setting the calibration strategy, discussing the effect of demographic transition on the real interest rate and inflation and inspecting the mechanism through which the demographic transition can impact these two. It also examines the impact of the demographic transition on sectoral developments and its implications for inflation. Section 4 concludes. Technical issues are relegated to the Appendices at the end of the paper.

[4] Quantification of this probability is provided by Bielecki et al. (2018) for the euro area and Jones (2018) for US interacting demographic trends with the estimation of standard temporary shocks. We do not do this exercise, focusing uniquely on demographic forces.

2 Model

This section presents a two-sector overlapping-generation (OLG) model for a closed-economy with "New Keynesian" frictions in price setting. The exogenous demographic change is the only source of change in the model (together with exogenous technological change in the production functions, but the baseline analysis will abstract from it). Two sectors produce two different goods: tradable (T) and nontradable (N).

2.1 OLG part

POPULATION. For age $j = 0, 1, \cdots J$, the size of the population in period t, $N_{t,j}$, is given recursively by:[5]

$$N_{t,j} = N_{t-1,j-1} s_{t,j}$$

where $s_{t,j}$ is the conditional survival probability. Given that a person is aged $j-1$ at time $t-1$, $s_{t,j}$ is the probability to be alive at age j in period t.

HOUSEHOLD. The representative household at time t chooses: consumption in each sector $c^N_{t+j,j}$, $c^T_{t+j,j}$ and the amount of assets to hold the sequent period $a_{t+j+1,j+1}$ under the assumption of a perfect domestic annuities market[6] for each age $j \in \{0, 1, 2, \cdots J\}$; how to allocate in each sector $h^N_{t+j,j}$, $h^T_{t+j,j}$ an exogenously given amount of hours to work $h_{t+j,j}$ for each age $j \in \{0, 1, 2, \cdots jr\}$, with income $y_{t+j,j}$ composed by net of tax labor-income $(1 - \tau_{t+j})(w^N_{t+j} h^N_{t+j,j} + w^T_{t+j} h^T_{t+j,j})$, pension transfer from the government $d_{t+j,j}$ and a share of the real profits from the firms Ω_{t+j}. The maximization problem is written in real terms where the price of T-goods is the numeraire. Hence, given sectoral real hourly wages and the relative price of N-goods:

$$w^T_{t+j} \equiv \frac{W^T_{t+j}}{P^T_{t+j}}; \quad w^N_{t+j} \equiv \frac{W^N_{t+j}}{P^T_{t+j}}; \quad Z_{t+j} \equiv \frac{P^N_{t+j}}{P^T_{t+j}}$$

where W^s_{t+j}, P^s_{t+j} for $s \in \{T, N\}$ identify the sectoral nominal hourly wage and good price respectively,

[5] Following Domeij and Floden (2006), the the survival probabilities $s_{t,j}$ are backed up using data on $N_{t,j}$ for all t, j. Therefore, due to migration, the survival probabilities can exceed 1.

[6] The assumption of "perfect annuities market" means that the agents within each age group j agree to share the assets of the dying members of their age group among the surviving members. Using the notation just introduced, consider those that at time t are aged j. The total amount of assets of the dying members is: $a_{t,j}(1 - s_{t,j})N_{t-1,j-1}$, while the number of surviving members is: $N_{t,j} = N_{t-1,j-1}s_{t,j}$. Hence, in the budget constraint the asset holding in period $t+1$ will depend on what as been accumulated plus this sort of 'equal gift' from the dying members given the real interest rate (r_t) at which these assets can be invested (minus consumption plus income):

$$\begin{aligned} a_{t+1,j+1} &= a_{t,j}(1 + r_t) + \frac{a_{t,j}(1 + r_t)(1 - s_{t,j})N_{t-1,j-1}}{N_{t-1,j-1}s_{t,j}} - c_{t,j} + y_{t,j} \\ &= \frac{a_{t,j}(1 + r_t)}{s_{t,j}} - c_{t,j} + y_{t,j} \end{aligned}$$

which is the budget constraint written in the main text.

the problem to solve from period t to infinity is the following:

$$\max_{c^N_{t+j,j}, c^T_{t+j,j}, h^N_{t+j,j}, h^T_{t+j,j}, a_{t+j+1,j+1}} \left\{ \sum_{j=0}^{J} \beta^j \pi_{t+j,j} \frac{(c_{t+j,j})^{1-\sigma}}{1-\sigma} \right\}$$

subject to

$$\begin{aligned}
c_{t+j,j} &= (c^T_{t+j,j})^{\alpha_j} (c^N_{t+j,j})^{1-\alpha_j} \\
h_{t+j,j} &= \left[\chi^{-\frac{1}{\varepsilon}} (h^T_{t+j,j})^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (h^N_{t+j,j})^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}} \\
a_{t+j+1,j+1} &= \frac{a_{t+j,j}(1+r_{t+j})}{s_{t+j,j}} - c^T_{t+j,j} - Z_{t+j} c^N_{t+j,j} + y_{t+j,j} \\
y_{t+j,j} &= (1-\tau_{t+j})(w^N_{t+j} h^N_{t+j,j} + w^T_{t+j} h^T_{t+j,j}) I(j \leq jr) + d_{t+j,j} I(j > jr) + \Omega_{t+j} \\
a_{t+J+1,J+1} &= 0 \\
a_{t,0} &= 0
\end{aligned}$$

where $\pi_{t+j,j} = \prod_{k=0}^{j} s_{t+k,k}$ represents the unconditional survival probability with $s_{t,0} = 1$; β is the discount factor; r_{t+j} is the real interest rate; $0 < \alpha_j < 1 \; \forall j$ are the age-dependent consumption shares on T-goods; in the CES function for aggregate hours: $0 < \chi < 1$ and $\varepsilon > 0$; τ_{t+j} is a tax rate identified below; $I(\cdot)$ is an indicator function; jr denotes the last working age (so that $jr+1$ is the first period of retirement) which is exogenously imposed.[7] The household's labor supply in efficiency units, $h_{t+j,j} = h_j$ for all t, is exogenous and depends on age but is constant over time. Particularly, it varies because of changes in productivity and labor market participation similarly to Domeij and Floden (2006). The parameter $\sigma > 0$ is the the coefficient of risk-aversion.

GOVERNMENT. Given a certain level of generosity of the PAYGO pension system, i.e. the replacement rate $\bar{d}$ defined as the pension benefit d_t received by each household per unit of the average labor income $w_t(1-\tau_t)\bar{h}$, the government sets a tax rate τ_t such that its budget is balanced in each period:

$$d_t = \bar{d} w_t (1-\tau_t) \bar{h} \tag{2.1}$$

$$\tau_t = \frac{d_t \sum_{j=jr+1}^{J} N_{t,j}}{w_t L_t} \tag{2.2}$$

where w_t is the economy-wide hourly wage (whose expression is implied by the functional form with which sectoral hours are aggregated), $w_t \equiv \frac{W_t}{P^T_t}$:

$$w_t = \left[\chi (w^T_t)^{\varepsilon+1} + (1-\chi)(w^N_t)^{\varepsilon+1} \right]^{\frac{1}{\varepsilon+1}}$$

[7]The two main non-standard features of the model are on the household's side: age-varying sectoral consumption shares, α_j, justified by empirical findings; CES aggregator for sectoral hours, thus assuming that the representative household has a preference to work in both sectors (for any positive sectoral wage) and that hours are partially substitutable between the two sectors (as long as $\varepsilon < \infty$). Particularly, notice that as $\varepsilon \to \infty$, sectoral hours tends to be perfectly substitutable, i.e. wages are equalized between sectors and it results $h_{t+j,j} = h^T_{t+j,j} + h^N_{t+j,j}$. On the contrary, as $\varepsilon \to 0$, the relative supply of sectoral hours tends to be perfectly rigid. A motivation for the use of this CES short-cut is provided in Cardi and Restout (2015) who also provide country-specific empirical estimates of ε. It was first used by Horvath (2000).

L_t represents the (exogenously given) aggregate hours worked:

$$L_t = \sum_{j=0}^{J} h_j N_{t,j}$$

and

$$\bar{h} = \frac{\sum_{j=0}^{jr} h_j}{jr+1}$$

is the average efficiency-hours worked in the working life-periods.

2.2 New Keynesian part

FIRMS. On the firms side, elements of the standard New Keynesian framework are introduced symmetrically in both sectors. The combination of the OLG part of the model with the New Keynesian one will depend on the representation of the OLG household problem with wedges which means solving a problem of an infinitely lived agent like in Jones (2018).

Final goods producers. For each sector $s \in \{T, N\}$ the final good is produced under perfect competition using intermediate goods indexed by $i \in [0,1]$ with a constant-returns-to-scale technology, solving the profit-maximization problem (taking as given all intermediate goods prices P^s_{it} and the final good price P^s_t):

$$\max_{Y^s_{it}} \quad \left\{ P^s_t Y^s_t - \int_0^1 P^s_{it} Y^s_{it} di \right\} \tag{2.3}$$

$$\text{s.t} \qquad Y^s_t = \left(\int_0^1 (Y^s_{it})^{\frac{\eta_s - 1}{\eta_s}} di \right)^{\frac{\eta_s}{\eta_s - 1}} \tag{2.4}$$

whose solution gives the demand function of input i for the production of final good s: $Y^s_{it} = \left(\frac{P^s_{it}}{P^s_t}\right)^{-\eta_s} Y^s_t$ for all $i \in [0,1]$. Thus, η^s measures the constant elasticity of demand for each intermediate good.

Intermediate goods producers. The problem for each monopolistically competitive intermediate good producer $i \in [0,1]$ is divided in two stages. In the first stage, taking nominal input prices $W^s_t, P^T_t r_t$ as given, firm i in each sector $s \in \{T, N\}$ solves the following cost minimization problem, choosing sectoral labor (L^s_{it}) and capital (K^s_{it}) inputs:

$$\min_{L^s_{it}, K^s_{it}} W^s_t L^s_{it} + P^T_t r_t K^s_{it} \tag{2.5}$$

$$\text{s.t.} \qquad Y^s_{it} = (K^s_{it})^{\psi} (A^s_t L^s_{it})^{1-\psi} \tag{2.6}$$

where the last expression is the supply of inputs for the final good s producer. In the second stage, each firm $i \in [0,1]$ in each sector $s \in \{T, N\}$ chooses the price that maximizes the discounted real profits, taking as given the demand for their differentiated product, with quadratic cost of changing prices *à la*

Rotemberg (1982):[8]

$$\max_{P_{it}^s} \left\{ E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[(P_{it}^s - MC_t^s) \frac{Y_{it}^s}{P_t^T} - \frac{\theta_s}{2} \left(\frac{P_{it}^s}{\Pi^s P_{it-1}^s} - 1 \right)^2 \frac{P_t^s Y_t^s}{P_t^T} \right] \right\} \tag{2.7}$$

$$\text{s.t.} \qquad Y_{it}^s = \left(\frac{P_{it}^s}{P_t^s} \right)^{-\eta^s} Y_t^s \tag{2.8}$$

with $\Lambda_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$, where λ_t is the Lagrangian multiplier in the economy's resource constraint whose existence owes to the representation of the household's problem in terms of aggregate wedges (or "demographic adjustment factors", see Jones (2018)) that allow to transform the OLG structure of the problem into a standard infinitely lived representative agent problem; MC_t^s identifies the nominal marginal cost in sector s which is the Lagrangian multiplier in the cost minimization problem in the first stage above.

MONETARY POLICY. The central bank follows the following simple Taylor-type rule with reaction parameter $\phi_\pi > 1$:[9]

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \tag{2.9}$$

where $R_t \equiv (1 + i_t)$ is the gross nominal interest rate (R its steady state vale), $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross rate of aggregate inflation (Π its steady state value) and can be identified by using the following identity:

$$P_t C_t = P_t^T C_t^T + P_t^N C_t^N$$

where $C_t = \sum_{j=0}^{J} N_{t,j} c_{t,j}$, $C_t^T = \sum_{j=0}^{J} N_{t,j} c_{t,j}^T$, $C_t^N = \sum_{j=0}^{J} N_{t,j} c_{t,j}^N$.

Once the OLG part of the model is represented as an infinite lived representative household problem, it will be necessary to add a nominal bond as a choice variable for the representative household in order to make monetary policy implementable.

CLEARING. With pricing cost *à la* Rotemberg (1982), aggregation on the firms side leads to have simply: $K_{it}^s = K_t^s$, $L_{it}^s = L_t^s$ for each intermediate good firm $i \in [0, 1]$, for each sector $s \in \{T, N\}$. On the household side one needs to add up over each age class j.

Hence, the expression for labor market clearing in each sector $s \in \{T, N\}$ for each period t is:

$$L_t^s = \sum_{j=0}^{J} h_{t,j}^s N_{t,j}$$

[8] Similarly to Keen and Wang (2007) it is assumed that each intermediate goods firm, in each sector s, pays a quadratic cost of nominal price adjustment when the size of its price increase deviates from steady state inflation, Π^s. The fact that it is measured in deviation from steady state inflation will simplify the first order conditions.

[9] The assumed simplicity of this policy rule serves to isolate the channel of interest and will be relaxed later, see section 3.4.

The capital market also needs to clear:

$$K_t^T + K_t^N = \sum_{j=0}^{J} a_{t+1,j+1} N_{t,j}$$

The market for both goods needs to clear. It is assumed that only T-goods can be used for the purpose of capital investment. Hence:

$$\begin{aligned} Y_t^N &= \sum_{j=0}^{J} c_{t,j}^N N_{t,j} + \frac{\theta_N}{2}\left(\frac{\Pi_t^N}{\Pi^N} - 1\right)^2 Y_t^N \\ Y_t^T &= \sum_{j=0}^{J} c_{t,j}^T N_{t,j} + \frac{\theta_T}{2}\left(\frac{\Pi_t^T}{\Pi^T} - 1\right)^2 Y_t^T + K_{t+1}^T + K_{t+1}^N - (1-\delta)(K_t^T + K_t^N) \end{aligned}$$

Finally, one needs an expression to distribute the profits generated by the monopolistically competitive firms to the household as an age-independent transfer Ω_t.

2.3 Aggregate representation with demographic wedges

Appendix A shows that the model can be approximated in terms of an infinitely lived representative agent using demographic wedges, i.e. exogenous time-varying parameters that capture the evolving age-structure in the economy (that directly affects the preferences of the representative agent and the efficiency units of labor used in production).[10] The special case of logarithmic preferences (i.e. $\sigma = 1$) is used. Again, all real variables are meant to be evaluated in terms of the price of T-goods.

HOUSEHOLD. On the household side the age-heterogeneity is now represented by the demographic wedges γ_t^T, γ_t^N:

$$\gamma_t^T = \sum_{j=0}^{J} \lambda^{i,j} \phi_t^{i,j} N_{t,j} \alpha_j, \quad \gamma_t^N = \sum_{j=0}^{J} \lambda^{i,j} \phi_t^{i,j} N_{t,j} (1-\alpha_j) \tag{2.10}$$

where, following Jones (2018), the welfare weights $\lambda^{i,j}$ are set to be equal across all j and $\phi_t^{i,j}$ is always 1. The representative household now chooses aggregate sectoral consumptions (C_t^T, C_t^N), aggregate sectoral labor supplies (L_t^T, L_t^N), savings in the form of claims on aggregate capital (K_t) and of nominal

[10]The methodology of approximating the model has been first developed by Jones (2018) who shows (see his Appendix) how close the approximate solution is to the actual solution obtained by solving the OLG model in a standard way (i.e. by keeping track of the dynamics of each generation's variables, not only of the aggregate variables). The aggregate representation presented here is based on Papetti (2019) and differs from Jones (2018) in two main ways: (*i*) instead of one sector, here there are two sectors with imperfect substitutability of sectoral hours worked and with age-specific sectoral consumption shares; (*ii*) instead of endogenous aggregate labor choice, here the supply of labor is fully exogenous (and depends, as in Jones (2018), on age-specific productivity in addition to the number of people in the labor force with exogenous retirement age).

bonds (B_t), solving the problem:

$$\max_{C_t^T, C_t^N, L_t^T, L_t^T, K_t, B_t} E_0 \sum_{t=0}^{\infty} \beta^t \log \left[\left(C_t^T\right)^{\gamma_t^T} \left(C_t^N\right)^{\gamma_t^N} \right] \tag{2.11}$$

$$\text{s.t.} \quad C_t^T + Z_t C_t^N + \frac{B_t}{P_t^T} + I_t = (1-\tau_t)(w_t^T L_t^T + w_t^N L_t^N) + r_t K_{t-1} + R_{t-1}\frac{B_{t-1}}{P_t^T} + T_t \tag{2.12}$$

$$K_t = (1-\delta)K_{t-1} + I_t \tag{2.13}$$

$$L_t = \left[\chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{\varepsilon+1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon+1}} \tag{2.14}$$

where R_t is the gross nominal interest rate, T_t denotes transfers (pension from the government, profits from firms), the tax rate τ_t is the same as the one identified in the previous section.

FIRMS. On the firms side the problem is identical to the one presented in section 2.2.

MONETARY POLICY. The Taylor rule of section 2.2 holds, where the gross inflation rate is identified by the identity for aggregate consumption ($P_t C_t = P_t^T C_t^T + P_t^N C_t^N$) which gives (see Appendix A):

$$\overline{P}_t \equiv \frac{P_t}{P_t^T} = \frac{C_t^T + Z_t C_t^N}{C_t} = \frac{\sum_{j=0}^{J} \lambda^{i,j} \phi_t^{i,j} N_{t,j}}{\sum_{j=0}^{J} \lambda^{i,j} \phi_t^{i,j} N_{t,j} \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z_t}\right)^{1-\alpha_j}} \tag{2.15}$$

Have $\overline{\Pi}_t \equiv \frac{\overline{P}_t}{\overline{P}_{t-1}}$, it follows:[11]

$$\Pi_t = \overline{\Pi}_t \Pi_t^T$$

Finally, to identify inflations one needs to use the following identity:

$$\Pi_t^N = \frac{Z_t}{Z_{t-1}} \Pi_t^T$$

CLEARING. Aggregate labor (in efficiency units) and capital markets clear:[12]

$$L_t = \sum_{j=0}^{J} h_j N_{t,j} \tag{2.16}$$

$$K_{t-1} = K_t^N + K_t^T \tag{2.17}$$

[11] Consider the definition of gross inflation in the T-sector:

$$\Pi_t^T \equiv \frac{P_t^T}{P_{t-1}^T} = \frac{P_t^T}{P_{t-1}^T}\frac{P_t}{P_t}\frac{P_{t-1}}{P_{t-1}} = \frac{\overline{P}_{t-1}}{\overline{P}_t}\Pi_t$$

[12] Consider the underlying timing and dynamics. Throughout period $t-1$ the representative household saves K_{t-1}. At the end of period $t-1$, a financial intermediary stores the household's savings K_{t-1} with a costless technology. In period t this intermediary transforms savings into capital: $K_{t-1} = K_t^N + K_t^T$. How? Capital K_{t-1} is rented to the firms which pay rental rate $r_t K_t$ and return undepreciated capital $(1-\delta)K_t$ to the intermediary. This financial intermediary pays interest, define it r_t^F, to the household $(1+r_t^f)K_{t-1}$ making zero profit so that $r_t K_{t-1} + (1-\delta)K_{t-1} - (1+r_t^F)K_{t-1} = 0$. Hence the household's savings K_{t-1} give a return: $1 + r_t^F = 1 + r_t - \delta$, which justifies the expression in the representative household's budget constraint above.

The two goods markets clear:

$$C_t^N = Y_t^N - \frac{\theta_N}{2}\left(\frac{\Pi_t^N}{\Pi^N} - 1\right)^2 Y_t^N \tag{2.18}$$

$$C_t^T + K_t = (1-\delta)K_{t-1} + Y_t^T - \frac{\theta_T}{2}\left(\frac{\Pi_t^T}{\Pi^T} - 1\right)^2 Y_t^T \tag{2.19}$$

Bonds are in zero net supply, hence $B_t = 0$.

SHOCKS. Aggregate uncertainty is not considered in this model. Instead, the model has a perfect-foresight set-up: there is a one-time shock, that moves the system outside the initial steady state, where the time-path of all exogenous demographic variables is revealed; the initial shock is unanticipated but agents are perfectly aware of the entire path revealed, including the fact that at some point in the future demographic variables will remain at the given constant level forever. Essentially, the exogenous variation in the number of people $N_{t,j}$ in all periods t and in all cohorts j is the only shock in the model. It gives rise to three exogenous variables, γ_t^T, γ_t^N, L_t, and it appears in the auxiliary price index $\overline{P}_t$ whose dynamics is not exogenous as it depends on the endogenous relative price of N-goods, Z_t.

EQUILIBRIUM. Given the dynamics of the exogenous number of people $N_{t,j}$ in all periods t and in all cohorts j (which leads to the exogenous dynamics of the parameters: γ_t^T, γ_t^N, L_t, according to (2.10) and (2.16)) and sectoral production technologies A_t^T, A_t^N, equilibrium for this closed economy is a sequence of prices $\{w_t^T, w_t^N, w_t, r_t, mc_t^T, mc_t^N, Z_t, R_t, \Pi_t^T, \Pi_t^N, \Pi_t, \overline{\Pi}_t\}_{t=0}^{\infty}$ and quantities $\{L_t^T, L_t^N, K_t^T, K_t^N, K_t, Y_t^T, Y_t^N, C_t^N, C_t^T\}_{t=0}^{\infty}$, such that:

1. The representative household solves (2.11), maximizing expected utility function subject to the budget constraint (2.12), the law of motion of capital (2.13) and the preference to work in either sector (2.14);

2. For each sector $s \in \{T, N\}$ final goods producers solve (2.3) maximizing profits subject to their technology constraint (2.4); intermediate goods producers solve (2.5) and (2.7) maximizing profits subject to their technology constraint (2.6) and the demand for their differentiated product (2.8);

3. the fiscal authority sets a tax rate (2.2) such that its budget is balanced in each period given a certain individual pension transfer (2.1); the monetary authority sets the nominal interest rate according to the policy rule (2.9);

4. The markets for capital (2.17) and for goods (2.19), (2.18) clear.

Appendix B shows the equilibrium conditions. They are derived in section B.1 and reported with quantities expressed in efficiency units (divided by the exogenous L_t) in section B.2. The initial steady state is analytically derived and discussed in comparison to the final steady state in section B.3. The version of the model log-linearized around the initial steady state (see section B.4) is used to study the transition dynamics presented in the next session. The focus is on an equilibrium with no exogenous sectoral technology growth, i.e. $A_t^T = A^T$, $A_t^N = A^N$ for all periods t. Notice that since labor supply is exogenous, the fiscal authority is uninfluential in this representative agent's economy so that equations (2.2), (2.1) do not enter in the equilibrium conditions necessary to pin down prices and quantities.

3 Quantitative analysis

The goal of the quantitative analysis is to study the transition dynamics of the macroeconomic system from an initial to a final steady state, where the unique exogenous driving process is the time-varying demographic structure. The focus is on the euro area comprised of twelve countries (EA12 henceforth) modelled as a closed economy.[13] The initial steady state is assumed to be year 1950 and agents learn about the future demographic development at the beginning of the following year. The demographic structure varies over the period 1950-2100 as provided in the data, remaining at the reached level forever after 2100. Thus, the final steady state is reached at some point in time after 2100. The model is set at the yearly frequency.

3.1 Calibration and time-varying parameters

Table 1 summarizes the values of the parameters in the baseline calibration. The initial steady state is such that the capital- and investment-output ratios match the empirical average values for EA12 which are found to be: $K/Y = 2.76$, $I/Y = .21$.[14] The annual depreciation rate of the capital stock, δ, follows immediately from the law-of-motion of capital (2.13) evaluated in steady state: $\delta = .21/2.76 = .0761$. The capital elasticity of output in the Cobb-Douglas production function, ψ, is set to the standard value of .33. The degree of labor mobility between the two sector is captured by the parameter ε in the CES aggregator (2.14) whose chosen value, .895 , is the GDP-weighted average on the country-based estimates provided by Cardi and Restout (2015).[15] To fully isolate the effect of demographic change, differences in sectoral labor-technology parameters and in their growth are not considered. Hence $A^T = A^N = 1$ for all periods. Following Gomes et al. (2012), the constant elasticity of demand for intermediate

[13] EA12 consists of the following countries: Austria (AT), Belgium (BE), Finland (FI), France (FR), Germany (DE), Greece (EL), Ireland (IE), Italy (IT), Luxembourg (LU), Netherlands (NL), Portugal (PT), Spain (ES).

[14] The series used are: "Gross capital formation (constant LCU)", "Gross fixed capital formation (constant LCU)" and "GDP (constant LCU)", data source: World Development Indicators (WDI) by the World Bank (update: January 2018). The capital stock is estimated by applying the perpetual inventory method (see Technical Appendix of Cardi and Restout (2015)). The initial capital stock (the base year is 1970, the first year data are available for all EA12 countries) is computed using the formula:

$$K_{1970} = \frac{I_{1970}}{g_I + \delta_K}$$

where I_{1970} corresponds to the gross capital formation in 1970. g_I is the average growth rate, while δ_K is set to 6% (see McQuinn and Whelan (2016)). The capital stock is obtained via the neoclassical law-of-motion: $K_{t+1} = (1-\delta)K_t + I_t$. Averages are taken over the time range available, 1970-2016. The series for EA12 are obtained by weighting each country with its real GDP share in year 2000.

[15] They provide estimates for 14 OECD countries, of which only 8 are EA12 members (specifically: Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Spain). Hence, the final value of ε is the GDP-weighted average on these 8 countries. Weights are obtained from the 2000 "Gross domestic product at market prices, chain linked volumes (2005), million euro" provided by EUROSTAT. The comparison between the value obatained for EA12, 0.895, and the one of Cardi and Restout (2015) reported for the United States, 1.8, reveals that the United States has more labor mobility between sectors than the euro area.

goods is set to $\eta^T = 6$ in the T-sector and to $\eta^N = 3$ in the N-sector, which implies a steady state mark-up of price over marginal cost in the two sectors $(\eta^T/(\eta^T-1), \eta^N/(\eta^N-1))$ of 1.2 and 1.5, respectively.

Table 1: Baseline calibration: parameter values

Parameter	Value	Note
δ	0.0761	depreciation rate of capital (target:$K/Y = 2.76$, $I/Y = .21$, source: WDI)
ψ	0.33	capital elasticity of output (in Cobb-Douglas production function, both sectors)
ε	0.895	degree of sectoral labor mobility (immobility: $\varepsilon = 0$). Source: Cardi and Restout (2015)
A^T	1	labor-technology level in the T-sector
A^N	1	labor-technology level in the N-sector
η^T	6	elasticity of demand for intermediate goods of the T-sector. Source: Gomes et al. (2012)
η^N	3	elasticity of demand for intermediate goods of the N-sector Source: Gomes et al. (2012)
J	86	terminal life-age (100). Death with certainty at age 101
jr	50	terminal working-age (64), see Kara and von Thadden (2016), Bielecki et al. (2018)
α_j	Figure 8	share of private consumption devoted to T-goods. Source: EUROSTAT 2010
h_j	Figure 9	individual life-cycle labor supply in efficiency units. Source: Domeij and Floden (2006)
γ_t^N/γ_t^T	Figure 10	relative wedge on the representative household preferences. Source: UN, see α_j
L_t	Figure 11	aggregate labor supply in efficiency units. Source: UN, see h_j
χ	0.6511	bias towards the T-sector for labor supply choice, see Cantelmo and Melina (2017)
β	0.9847	individual discount factor (endogenous in the initial steady state)
θ^T	34.135	Rotemberg (1982) price adjustment cost in the T-sector, based on Gomes et al. (2012)
θ^N	13.654	Rotemberg (1982) price adjustment cost in the N-sector, based on Gomes et al. (2012)
Π	1.02	inflation target. Source: Gomes et al. (2012)
ϕ_π	1.5	inflation coefficient in the Taylor rule. Source: Kara and von Thadden (2016)

There are two age-variant (time-invariant) parameters. (i) The share of private consumption devoted to T-goods, α_j, which is obtained using a cubic interpolation from EUROSTAT data (see Figure 8 where the blue-thick line representing EA12 is the GDP-weighted average of the single countries series (thin lines)). The same is used in Giagheddu and Papetti (2017). It can be seen that the consumption share on T-goods is stable at about 50% till age 50, then it smoothly declines reflecting increased expenditure in items such as health care. (ii) The individul labor supply in efficiency units, h_j, which is interpolated using the data points provided by Domeij and Floden (2006) (see Figure 9). It is assumed that individuals enter the world as workers at age 15, all retiring at age $jr + 1 = 65$ (this is why h_j drops abruptly to zero at age 65). An assumption which is standard, see Kara and von Thadden (2016), Bielecki et al. (2018) for the euro area.

The empirical number of people, $N_{t,j}$, by single age groups (ages 15, 16, ..., 100+) in the time-range 1950-2100 is taken from UN (2017): *World Population Prospects: The 2017 Revision*.[16] These data multiplied with α_j and h_j by age j for each year t allow to identify the three exogenous time-varying parameters in the model: γ_t^T, γ_t^N, L_t. The first two are depicted in Figure 10 and only their ratio (red line, right-hand-side scale) matters to identify prices and quantities in steady state. In the initial steady state the value of this ratio, γ^N/γ^T, is 1.0316, the empirical value in 1950. The third one instead, the exogenous aggregate labor supply in efficiency units L_t, is not strictly necessary to characterize the steady state once variables are considered in units of labor efficiency (see Appendix B.3). Following Cantelmo and Melina (2017), the representative household's bias towards the T-sector in the choice of

[16]Before year 1990, the number of people aged more than 80 are grouped together in the set 80+ for all countries. Therefore, as a strategy to identify the number of people in each single age group after age 80 for years 1950-1989, the implied survival probabilities of 1990 for those aged more than 80 have been applied backwards.

sectoral labor supply (see (2.14)), χ, is calibrated such that in the initial steady state wage equalization (i.e. $w^N = w^T$) is given. This requires to determine χ endogenously in the initial steady state (see Appendix B.3.1). Given the values of the parameters mentioned above, the initial steady state is solved numerically in order to obtain the value of the individual discount factor β that allows to match the targeted capital-output ratio. The obtained value of β is 0.9847 which in turn implies $\chi = 0.6511$ and a steady state real interest rate, $1 + r - \delta$, of about 1.56%.[17] As shown in Appendix B.3.2, the final steady differs from the initial one as χ remains fixed at its initial level so that any change induced by a permanent shift of γ^N/γ^T has to be compensated by changes in relative prices.

On the nominal side of the model, the sectoral Rotemberg's price adjustment costs, θ^T, θ^N, are set to match a probability of optimally resetting prices *à la* Calvo (1983) at the quarterly frequency of (1-0.92) in both sectors, as estimated by both Smets and Wouters (2003) and Christoffel et al. (2008) for the euro area, used by Gomes et al. (2012), in order to have the same slope of the "New-Keynesian Phillips Curve" in both sectors (see equations (B.25), (B.26)) given the values of η^T, η^N. This procedure gives $\theta^T = 34.135, \theta^N = 13.654$.[18] The steady state target inflation is assumed to be at the yearly 2%, i.e. $\Pi = 1.02$, as in Gomes et al. (2012).[19] The inflation coefficient in the Taylor rule (2.9), ϕ_π, has the standard value of 1.5 as used by Kara and von Thadden (2016) to study the impact of demographic shocks in the euro area. Contrary to them, the inertial parameter of the nominal interest rate is set zero in the baseline specification, in the spirit of Carvalho and Ferrero (2014).

3.2 The impact of the demographic transition on interest rates and inflation

Figure 1 and 2 summarize the main results of the transition dynamics stemming from the log-linearised system in deviations from the initial steady state (see system of equations (B.1) – (B.26) in Appendix B.4), where the time-varying parameters (guided by the change in the demographic age-structure, the

[17] The value of 1.56% for the steady state real interest rate $1 + r - \delta = 1/\beta$ is smaller than what is assumed by Christoffel et al. (2008), Gomes et al. (2012), Kara and von Thadden (2016), who have values of 2.5%, 3%, 3.9% respectively, but higher than what assumed by Bielecki et al. (2018) who target an average real interest rate of 1.2% observed in the euro area over the period 1999-2008. Hence, the value found is broadly in line with the literature, providing model's predictions that are consistent with the levels of the empirical variables the model is compared to, see section 3.2. For the calibration of χ the closest comparison can be made in the literature is Cardi and Restout (2015), who set it to 0.4 to match a tradable content of labor compensation of 35%. However, they do not measure labor in units of efficiency, while they assume that firms in the T-sector are 50% more productive than in the N-sector. Hence the higher value for χ here compared to Cardi and Restout (2015) can be connected to the fact that here labor efficiency is considered (notice that considering a relatively higher productivity in the T-sector of 50% would give a $\chi = 0.4 \times 1.5 = 0.6$ which is roughly what calibrated here).

[18] Gomes et al. (2012) estimate for the euro-zone a Calvo parameter at the quarterly frequency for prices of both domestic tradables and nontradables of $\xi_H = 0.92$. To get the value at the annual frequency, consider that the Calvo probability of resetting prices within two periods of different lengths $h_i \neq h_j$ must satisfy $(1 - \xi_H(h_i))/h_i = (1 - \xi_H(h_j))/h_j$, see Ahrens and Sacht (2014). Hence, if the base period is one quarter, then $h_j = 1, \xi_H(h_j) = \xi_H$ and one year period in quarters is $h_i = 4$. Hence, the Calvo parameter at the annual frequency obtained from the quarterly frequency is $\xi \equiv \xi(h_i) = 1 - (h_i/h_j)(1 - \xi_H(h_j)) = 1 - 4(1 - 0.92) = 0.68$. The slope of the log-linearized New Keynesian Phillips Curve for nontradables in Gomes et al. (2012) is given by $(1 - \beta\xi_H)(1 - \xi_H)/(\xi_H(1 + \beta\chi_H))$, where $\chi_H = 0.5$ is the indexation to previous quarter's inflation. Hence, by equating this to the corresponding slope in (B.26), i.e. $(\eta^N - 1)/\theta^N$, using yearly values one gets $\theta^N = (\eta^N - 1)\xi(1 + \beta\chi_H^4)/((1 - \beta\xi)(1 - \xi)) = 13.654$. A specular procedure is applied to identify θ^T given η^T.

[19] This assumption might be improper given that the initial steady state is calibrated with the demographics of year 1950, but it is coherent with the focus after the 80s of the subsequent analysis.

unique source of variation in the model, see equations (B.1) – (B.5)) have been conveniently smoothed. Two monetary policy rules are compared.[20] A policy of strict inflation targeting for each period t:

$$\Pi_t = \Pi \tag{3.1}$$

and a simplified Taylor rule:

$$R_t = R \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \tag{3.2}$$

where ϕ_π is set to the standard value of 1.5.

Recall that in the standard New-Keynesian framework adopted here the central bank can stay at target inflation in each period. Essentially, this is because it is assumed that inflation is a monetary phenomenon such that the central bank, by changing the money supply given the money demand, can attain whatever level of inflation (under normal circumstances, e.g. the zero lower bound on the nominal interest rate is not binding). Thus, the equilibrium in the money market leads to a nominal interest rate which is the only montetary policy instrument in the baseline New Keynesian framework.[21] Monetary policy can have an impact on real macroeconomic variables only if, by changing the nominal interest rate, it brings about a change in the real interest rate. At least this is the conventional view.[22] In this framework, a change in the nominal rate implies a non-zero change in the real interest rate only if there is price rigidity. However, if the central bank decides to stay at target inflation in each period, the implicit assumption is that there is no price rigidity, because a non-zero cost of changing prices arises only if inflation deviates from its target. Therefore, by following a policy of strict inflation targeting the central bank can replicate the equilibrium allocation that would prevail with flexible prices, i.e. the equilibrium where real macro-variables are at their "natural" level that one could attain by setting the price rigidity parameters, θ^T and θ^N, equal to zero.[23] A corollary is that under rule (3.1) the central bank sets a nominal interest rate that allows the real interest rate to be equal to its natural level in each period.

On the contrary, suppose that inflation happens to be at target under rule (3.2). In this case, the rule says to set the nominal interest rate equal to its steady state value. But this value might be too high or too

[20] See Carvalho and Ferrero (2014) for a similar comparison.

[21] Notice that the model is solved without any equation keeping track of the monetary developments. The underlying assumption is that the money supply, set by the central bank, adjusts endogenously in order to bring about the desired change in the nominal interest rate given a certain money demand, so that the money market clears. But the model can be solved with no reference to the money market, see Gali (2015).

[22] For a critical assessment of the monetary transmission mechanism in New-Keynesian models see Rupert and Sustek (2019).

[23] Precisely, consider the standard New Keynesian Phillips curve with only one sector in log-deviations from steady state: $(\theta/(1-\eta))\widehat{\pi}_t = \widehat{mc}_t + (\theta/(1-\eta))\beta E_t\widehat{\pi}_{t+1}$, where θ is the Rotemberg's price adjustment cost, and η is the intermediate goods elasticity of demand. If the central bank decides to keep inflation at target in each period, $\widehat{\pi}_t = 0\ \forall t$, there is no shock that creates a trade-off between inflation and marginal costs, so that marginal costs are always at steady state, $\widehat{mc}_t = 0$ $\forall t$. That is, a policy of strict inflation targeting attains the same allocation that would prevail with $\theta = 0$. In the case of two sectors analysed here, when the central bank sets the aggregate inflation (i.e. the composite of the inflations in the two sectors) at target in each period, what happens is that any change in the inflation of one sector induced by a demographic shock is compensated by a change in the inflation of the other sector for a magnitude such that aggregate inflation is fixed at target. In this way, despite there being changes at the sectoral level, there are no changes at the aggregate level so that the macro-equilibrium allocation with strict inflation targeting is the same as the one with $\theta^T = \theta^N = 0$.

low for the effective objective of reaching target inflation in the medium-term depending on whether the natural rate is lower or higher than its steady state value due to structural forces in the economy such as demographics.

The line with '+' in Figure 1a plots the real interest rate, $1 + r_t - \delta$, that prevails under strict inflation targeting, namely the natural real interest rate, as a consequence of the exogenous demographic transition in the model. In the period 1990-2030 it is projected to decrease about 0.72 percentage points (pp) going from 1.57% in 1990 to 0.85% in 2030. The violet-shaded area highlights the discrepancy between the steady state and the natural values. It is labelled as a "physiological discrepancy" because the model predicts that the forces of the demographic transition lead the real economy there, unavoidably. The line with '+' in Figure 2a shows the corresponding natural nominal interest rate, which goes from 3.59% in 1990 to 2.75% in 2030, a decrease of 0.84pp.[24] Again, this decrease is physiological in the sense that it is unavoidable: that dashed line is the path of the nominal interest rate that the central bank should follow to have inflation at target in each period given the demographic transition.

What happens if instead the central bank follows the Taylor rule? The continuous line in Figure 2a shows the nominal interest rate under the Taylor rule specified in (3.2). It falls considerably more than in the case of strict inflation targeting, decreasing overall by about 2.5pp from 1990 to 2030. Furthermore, as shown in Figure 2b (continuous line), the inflation rate continuously decreases going from about the target level of 2% in 1990 to a level as low as 0.37% in 2030. As also noted by Carvalho and Ferrero (2014), the failure in the conduct of monetary policy to internalize the consequences of the demographic transition on the natural real interest rate generates a sort of "perverse general equilibrium effect": by following the Taylor rule the central bank ends up in an equilibrium with systematically lower nominal interest rates and inflation than in the case of strict inflation targeting. In other terms, by not accounting in the monetary policy rule for a natural interest rate that declines over time due to the demographic transition (always below its steady state value), the central bank generates a "disinflationary bias" which is highlighted by the lightblue shaded–area in Figure 2. By following rule (3.2) it is as if the central bank generates a contractionary monetary policy shock in each period, in partial equilibrium.[25] Such a shock generates disinflation which leads the central bank to reduce the nominal interest rate. In the model, in general equilibrium, the nominal interest rate decreases meaning that the initial contractionary shock is more than compensated. But not sufficiently given that disinflation is not avoided. Indeed, in this context, disinflation is generated because the central bank does not react enough (i.e. ϕ_π is not high enough), as shown in the next section.

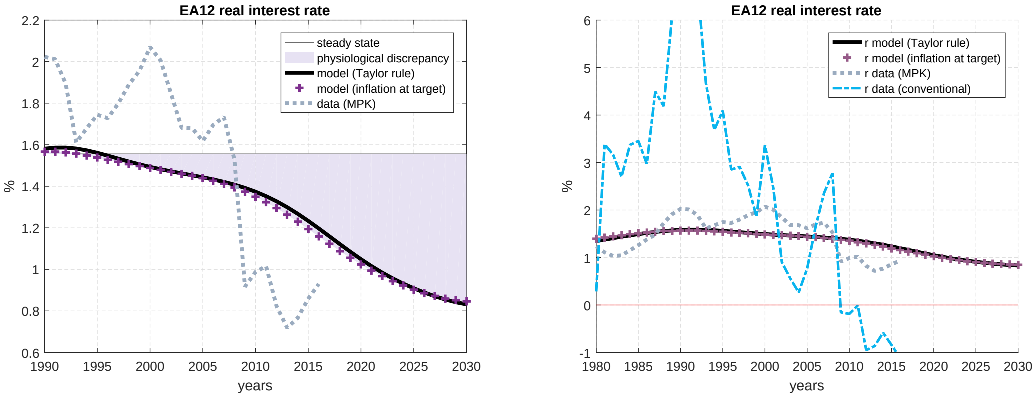
It is an interesting feature of the model that such dynamics of the nominal interest rate and inflation occurs with almost no discrepancy between the natural and the actual real interest rate. The continuous line in Figure 1a shows that the real interest rate generated by the model with the Taylor rule under price rigidity, the 'actual' one, is almost identical to the one under strict inflation targeting (line with '+'),

[24] Notice that the nominal interest rate is $R_t = (1 + r_t - \delta)\Pi^T_{t+1}$, given that the good in the T-sector is the numeraire. The reason why the natural R_t decreases more than the natural r_t is that inflation in the T-sector under flexible prices decreases slightly as a consequence of demographic change.

[25] Recall that in the standard three-equations Gali (2015)'s linear model, a negative shock to the natural real interest rate enter the equilibrium conditions in a way equivalent to a positive (i.e. contractionary) monetary policy shock. See section 3.3.

the ‘natural’ one, even though the composition is very different: much lower nominal interest rate and inflation in the case of the Taylor rule. The next section will keep under scrutiny this result which is also shared by Carvalho and Ferrero (2014) in a similar setting and points to a limited role of price rigidity in this context.

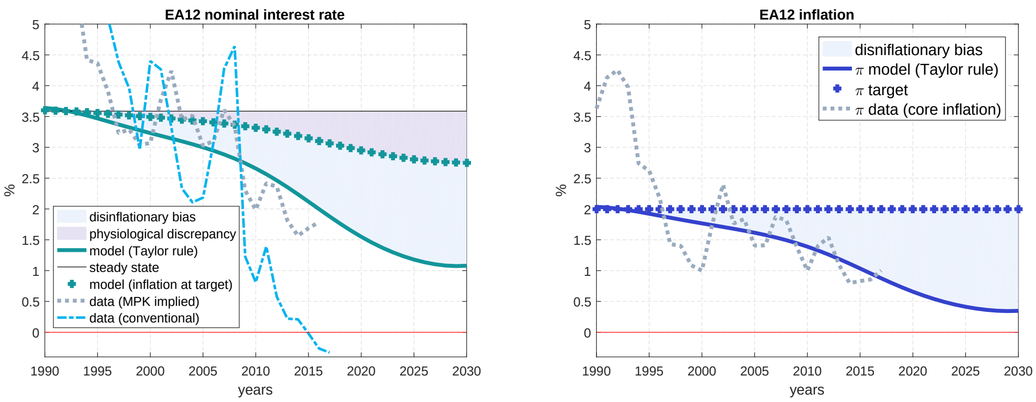
Finally, consider the data used to have an empirical reference for the model. First, because of no arbitrage, the model does not distinguish between risk-free rate and net return on capital which, by optimal inputs utilization, is equal to the net marginal product of capital (MPK). Hence the real interest rate generated by the demographic transition in the model is compared with the empirical net MPK. Figure 1 shows that the model (continuous line) captures some low-frequency movement of the empirical net MPK (dotted line). Second, given the slow-moving nature of the demographic transition, inflation generated by the model under rule (3.2) is compared with “core inflation”, namely a measure of underlying inflation that tends to exclude short-term volatility (it is obtained from the Harmonized Index of Consumer Prices (HICP) excluding the items that pertain food and energy). Figure 2b shows that the model (continuous line) captures the downward trend of core inflation found in the data since the 1990s (dotted line)). Consistently, the reference empirical nominal interest rate is computed as the sum of the net MPK and core inflation (see dotted line in Figure 2a). Figure 2a plots also a more conventional measure of the nominal interest rate based on the Euribor 3-months (used e.g. in the Area-Wide model database). One can see (dashed-dotted line) that it varies more, turning negative after year 2015, while it co-moves with the lower-frequency measure based on the net MPK. The corresponding real interest rate (dashed-dotted line in Figure 1b), obtained by subtracting core inflation, makes clear to what extent the MPK compares with a more conventional measure of the real interest rate, hence the type of low-frequency variation that the slow demographic change can capture. Clearly, according to the model, demographics alone cannot account for the wide swings observed in the data using a measure that is not strictly related to (physical) capital. Nonetheless, one can interpret the observed decline of core inflation as a situation where the central bank, by following a standard Taylor rule, fails to recognize that the natural real interest rate is time-varying because of demographic change. The demographic transition features as a slow-moving process almost uninfluential on a year to year basis but whose continuous “glacial” movement, if failed to be recognized, leads the economy on a disinflationary path.



(a) (b)

Figure 1: Real interest rate in EA12, $r_t - \delta$

Note. The Taylor rule applied is $R_t = R(\Pi_t/\Pi)^{\phi_\pi}$, with $\phi_\pi = 1.5$; under (strict) inflation targeting the central bank follows the rule $\Pi_t = \Pi$ for all t. *Data*: (*i*) "data (MPK)" (black thin dashed-dotted line) refers to the net marginal product of capital computed applying the first order condition (that in the model in this paper applies at the sectoral level) net of capital depreciation δ: $r_t = mc(Y_t/K_t)\psi - \delta$, where mc is computed as a weighted average of the steady state sectoral marginal costs, i.e. $mc = (\eta - 1)/\eta$ with $\eta = .3\eta^T + (1 - .3)\eta^N$ reflecting the higher value added share that the N-sector has in the data, while values of η^T, η^N, ψ, δ are taken from Table 1. Data for aggregate output Y_t and capital stock K_t are the same used for calibration, particularly they are "GDP (constant LCU)" for output and "Gross fixed capital formation (constant LCU)" for investment both sourced from WDI 2017, where capital is computed as explained in footnote 14. (*ii*) "r data (conventional)" (lightblue thin dotted line) refers to the difference between the nominal short term interest rate and core inflation both plotted and explained in Figure 2.



(a) Nominal interest rate, $R_t - 1$ (b) Inflation, $\Pi_t - 1$

Figure 2: Inflation and nominal interest rate in EA12

Note. The Taylor rule applied is $R_t = R(\Pi_t/\Pi)^{\phi_\pi}$, with $\phi_\pi = 1.5$; under (strict) inflation targeting the central bank follows the rule $\Pi_t = \Pi$ for all t. *Data*: (*i*) "core inflation" (blue thin dashed-dotted line), i.e. all–items of HICP excluding energy and food; (*ii*) "conventional" (grey thin dotted line): following the convention for the "Area Wide Model" (see `https://eabcn.org/page/area-wide-model`) the nominal short term interest rate is measured with the Euribor 3-months. Both data series are sourced from `sdw.ecb.europa.eu` and are annual averages on the year to year percentage change at the monthly frequency. (*iii*) "MPK implied" (green thin dashed-dotted line) is obtained as the sum of core inflation and the real interest rate computed on the basis of the marginal product of capital, see Figure 1.

3.3 Inspecting the mechanism

To understand the mechanism through which the demographic transition impacts interest rates and inflation, the full model with two sectors (see (B.1)–(B.26)) is simplified to a basic one sector model with endogenous capital and compared to the textbook three equations model by Gali (2015). In all models, the demographic transition enters as an exogenous path of the difference between the growth rate of the labor force and population (see the aggregate approximation of the OLG model in Appendix B) which has the interpretation of being the natural real interest rate for the case of the Gali (2015)'s textbook three equations model.

First consider the Gali (2015)'s textbook model. It is a log-linearised system of three equations where variables are evaluated in log-deviations from the initial steady state:

$$\frac{\theta}{(1-\theta)(1-\beta\theta)}\widehat{\Pi}_t = \frac{\beta\theta}{(1-\theta)(1-\beta\theta)}E_t\widehat{\Pi}_{t+1} + \widehat{\widetilde{y}}_t \tag{3.3}$$

$$\widehat{\widetilde{y}}_t = E_t\widehat{\widetilde{y}}_{t+1} - \left[\widehat{\mathrm{R}}_t - E_t\widehat{\Pi}_{t+1} - \widehat{r}_t^n\right] \tag{3.4}$$

$$\widehat{\mathrm{R}}_t = \phi_\pi\widehat{\Pi}_t \tag{3.5}$$

Parameters are standard: θ is the Calvo's probability for a firm of not being able to reset prices within a reference period, set to 0.68 to have the same slope of the Phillips curve used in the full model (see footnote 18); β and ϕ_π are the same as in Table 1. The variable $\widehat{\widetilde{y}}_t$ denotes the "output gap", i.e. deviations of output from its natural level which is not specified in this context. The natural real interest rate is defined as the ex-ante real rate that prevails in a counterfactual economy with no price-rigidity (i.e. $\theta = 0$), namely when the output gap is zero. With no price rigidity, from (3.4) the ex-ante real interest rate, $\widehat{\mathrm{R}}_t - E_t\widehat{\Pi}_{t+1}$ is equal to $\widehat{r}_t^n$ which therefore denotes the natural real interest rate and features as a perfectly anticipated stand-in shock that captures the demographic transition:

$$\widehat{r}_t^n = \widehat{l}_{t+1}^g - \widehat{\varphi}_{t+1}^g \tag{3.6}$$

In the case of one sector (see Appendix B and Jones (2018)), the demographic wedges attached to the representative households drop to be simply the size of the population in each period (with logarithmic preferences). Then, from the Euler equation, the population growth rate (denoted by $\widehat{\varphi}_{t+1}^g$ in log-deviation from steady state) enters into the determination of the natural real interest rate. Furthermore, since what matters for the determination of factor prices is the efficiency unit of each factor, variables are divided by the exogenous labor in efficiency units (there is no other exogenous 'technology'). In this way, from the Euler equation, also the growth rate of labor in efficiency units (denoted by $\widehat{l}_{t+1}^g$ in log-deviation from steady state) enters into the determination of the natural real interest rate. Hence, the deviations of the natural real interest rate from steady state are captured by the exogenous variation of the "working-age population ratio", where the number of people in the working age is evaluated in efficiency units (i.e. correcting for the age-varying labor productivity h_j). Specifically, the two exogenous variables are (see

Appendix B.4.1):

$$\widehat{l}^g_{t+1} = \frac{\sum_{j=0}^{jr} h_j N_j (\widehat{n}_{t+1,j} - \widehat{n}_{t,j})}{\sum_{j=0}^{jr} h_j N_j}, \quad \widehat{\varphi}^g_{t+1} = \frac{\sum_{j=0}^{J} N_j (\widehat{n}_{t+1,j} - \widehat{n}_{t,j})}{\sum_{j=0}^{J} N_j} \tag{3.7}$$

As explained in Papetti (2019), one can interpret the prolonged decrease of $\widehat{r}^n_t$ as a negative shock to the growth rate of total factor productivity for output per capita growth which makes the representative agent more patient (i.e. akin to a positive discount factor shock), willing to save more and consume less, as the number of effective workers in support of the number of total consumers (the population size) shrinks.[26] In the data, over the transition period considered, both the growth rate of the labor force and the growth rate of population decrease but the former decreases more, so that the natural real interest rate is on a declining pattern. The green line in Figure 3 shows the log-deviations $\widehat{r}^n_t$, while the green line in the bottom-right panel of Figure 12 shows the level of the natural interest rate implied by those log-deviations.[27]

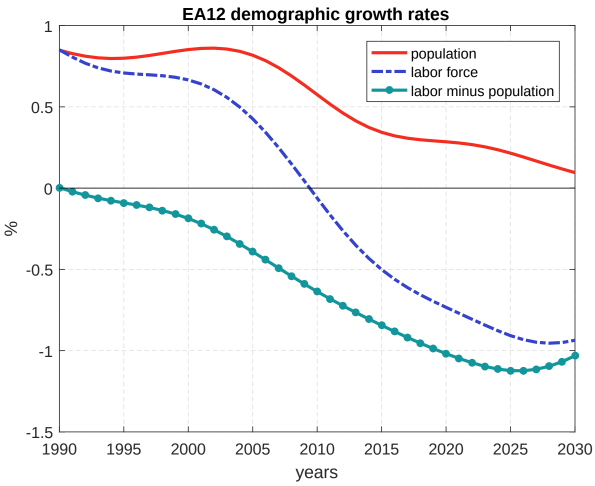


Figure 3: Demographic growth rates in EA12

Note. The number of people $N_{t,j}$ is taken from data provided by UN (2017) *World Population Prospects: The 2017 Revision* for year $t \in [1950, 2100]$, medium variant after year 2016 (see footnote 16). The series plotted are $\widehat{l}^g_{t+1}$ and $\widehat{\varphi}^g_{t+1}$ in (3.7), multiplied by 100, where the log-deviations of the number of people from the initial steady state in each age-bin j for all years t, $\widehat{n}_{t,j}$, has been conveniently smoothed using a "loess" method (local regression using weighted linear least squares and a 2nd degree polynomial model) of the smooth function in Matlab with a span of .08, a low value to preserve the actual data but at the same time avoid kinks. Recall that $\widehat{l}^g_{t+1} = l^g_{t+1}/l^g - 1$, $\widehat{\varphi}^g_{t+1} = \varphi^g_{t+1}/l^g - 1$ where the steady state values l^g, φ^g are both equal to 1, so that $\widehat{l}^g_{t+1}$, $\widehat{\varphi}^g_{t+1}$ are the net growth rates of the labor force (in efficiency units) and population, respectively.

Approximately, in year 1990 both labor force and population are growing at the rate of 0.85% a year.

[26]Compare these considerations with equation (23) of Gali (2015)'s chapter 3.

[27]For the sake of comparison with the full model with endogenous capital, see later, the log-deviations are evaluated starting from the same initial steady state. Hence, the rate plotted in Figure 12 is: $r^n_t = (1 + r - \delta)\exp\{\widehat{r}^n_t\} - 1$ where the depreciation rate δ and the steady state rental rate r are the one implied by the calibration, see Table 1. Notice that $\widehat{r}^n_t$ denotes log percentage deviations of the *gross* rate.

However, in year 2030 the annual growth rate of the population is about zero while that of the labor force -1%. In levels, this discrepancy generates a natural interest rate that goes from about 1.6% in 1990 to 0.5% in 2030, a decrease of 1.1 percentage points.

With price stickiness the ex-ante real interest rate has periods in which it is either above or below its natural counterpart (see yellow line in bottom-right panel of Figure 12). The somewhat small discrepancy between the two is sufficient to generate a persistent non-zero output gap (see yellow line in the top-right panel) which has a peak in year 2009, at about 0.6%, and a trough in year 2030 at about -0.9%. Indeed, by solving forward (3.4) the output gap is proportional to the sum of current and anticipated deviations between the ex-ante real interest rate and its natural counterpart:[28]

$$\widehat{\widehat{y}}_t = -\sum_{k=0}^{\infty} \left(\widehat{\mathrm{R}}_{t+k} - E_t \widehat{\Pi}_{t+k+1} - \widehat{r}_t^n\right)$$

It is interesting that the dynamics of inflation and the nominal interest rate do not differ much in the two cases, sticky vs flexible prices, when the central bank follows rule (3.5) in both cases (compare the green and the yellow lines in Figure 12). Notice that under flexible prices there is no trade-off between output and inflation gaps, so that the system has a "classical dichotomy". That is, with $\widehat{\widehat{y}}_t = 0$ for all t, from (3.4) it follows $\widehat{\mathrm{R}}_t - E_t\widehat{\Pi}_{t+1} = \widehat{r}_t^n$ for all t so that any pair of real numbers for current nominal interest rate $\widehat{i}_t$ and expected inflation $E_t\widehat{\pi}_{t+1}$ is an equilibrium of the system. It is common practice to pick one special equilibrium of the infinitely many possible under flexible prices, namely the one where inflation is always at target, i.e. $\widehat{\Pi}_t = 0$ for all t which gives the implicit monetary policy rule $\widehat{\mathrm{R}}_t = \widehat{r}_t^n$. In this case, the nominal interest rate would feature only what has been labeled before as "physiological discrepancy" from the steady state since it would map one to one with the exogenous changes of the natural real interest rate. Nonetheless, the paths of inflation and nominal rate are quite different when the central bank follows rule (3.5) under flexible prices. To understand the monetary policy mechanism, suppose in rule (3.5) there is also a monetary policy shock ν_t:

$$\widehat{\mathrm{R}}_t \quad = \quad \phi_\pi \widehat{\Pi}_t + \nu_t \tag{3.8}$$

Plug this into (3.4) with $\widehat{\widehat{y}}_t = 0$, and solve forward to have:[29]

$$\widehat{\Pi}_t = \frac{1}{\phi_\pi}\sum_{k=0}^{\infty}\left(\frac{1}{\phi_\pi}\right)^k \widehat{r}_{t+k}^n - \frac{1}{\phi_\pi}\sum_{k=0}^{\infty}\left(\frac{1}{\phi_\pi}\right)^k \nu_{t+k}$$

This expression makes clear that few mechanisms are involved when the central bank follows the type of Taylor rule (3.8) under flexible prices. First, absent monetary policy shocks (i.e. $\nu_t = 0$ for all t), inflation is determined uniquely by the path of the natural real interest rate discounted by the inverse of the central bank's reaction parameter $\phi_\pi > 1$. Second, the effect on inflation of a given path of the natural real interest rate can be perfectly mimicked by a path of monetary policy shocks equal in magnitude and

[28]The result relies also on: $\lim_{k\to\infty} \widehat{\widehat{y}}_{t+k+1} = 0$.
[29]The result relies also on $\lim_{k\to\infty}(1/\phi_\pi)^k \widehat{\pi}_{t+k} = 0$, with the Taylor principle $\phi_\pi > 1$.

opposite in direction. That is, a prolonged decline in the natural real interest rate acts like a prolonged tightening of monetary policy. Third, by strongly reacting to any deviation of inflation from its target the central bank can nullify the dampening effect on inflation of a declining natural real interest rate. In the limit ($\phi_\pi \to \infty$), inflation remains always at target while the nominal interest rate is always close to its natural counterpart. In this case, the only problem for the central bank to maintain inflation at target in each period would be that the prolonged exogenous decline of the natural real interest rate leads the nominal interest rate to hit its lower bound. While these mechanisms hold exactly only in the flexible price case, Figure 12 – compare the yellow line with the green one – suggests that the same mechanisms are at play in the model with sticky prices.

Next consider the Gali (2015)'s textbook three equations augmented to account for endogenous physical capital (and exogenous labor supply). This model corresponds to the one-sector version of the full two-sector model (see (B.1)–(B.26)). The system of recursive equations is:

$$\frac{\theta}{(1-\theta)(1-\beta\theta)}\widehat{\Pi}_t = \frac{\beta\theta}{(1-\theta)(1-\beta\theta)}E_t\widehat{\Pi}_{t+1} + \widehat{mc}_t \tag{3.9}$$

$$\widehat{\widetilde{c}}_t = E_t\widehat{\widetilde{c}}_{t+1} - \left[\widehat{\mathrm{R}}_t - E_t\widehat{\Pi}_{t+1} - \widehat{r}_t^n\right] \tag{3.10}$$

$$\widehat{\mathrm{R}}_t = \phi_\pi\widehat{\Pi}_t \tag{3.11}$$

$$\widehat{\widetilde{c}}_t = E_t\widehat{\widetilde{c}}_{t+1} - \left[\frac{r}{1+r-\delta}\widehat{r}_{t+1} - \widehat{r}_t^n\right] \tag{3.12}$$

$$\widehat{\widetilde{y}}_t = \psi\widehat{\widetilde{k}}_{t-1} \tag{3.13}$$

$$\widehat{\widetilde{k}}_{t-1} = \widehat{w}_t - \widehat{r}_t \tag{3.14}$$

$$\widehat{mc}_t = \psi\widehat{r}_t + (1-\psi)\widehat{w}_t \tag{3.15}$$

$$\widehat{\widetilde{y}}_t = \frac{\widetilde{C}}{\widetilde{Y}}\widehat{\widetilde{c}}_t + \frac{\widetilde{K}}{\widetilde{Y}}\left[\widehat{l}^g_{t+1} + \widehat{\widetilde{k}}_t - (1-\delta)\widehat{\widetilde{k}}_{t-1}\right] \tag{3.16}$$

Given that now there is also capital as input, not only labor as in the three equations model, there is the additional parameter ψ whose value is the same as in Table 1. Furthermore, $\widetilde{C}/\widetilde{Y}$ and $\widetilde{K}/\widetilde{Y}$ have the same values as in the full model for the sake of comparison. Notice that this model is shocked with the exogenous path of $\widehat{r}_t^n$, the same used for the three equations model above, which enters both the Euler equation for consumption (3.10) and the Euler equation for capital (3.12). However, it does not represent the natural real interest rate. The reason is that consumption is not necessarily equal to output because of the presence of investment in capital so that equation (3.10) differs from (3.4). Consequently, in case of no price rigidity ($\theta = 0$) the ex ante real interest rate is not necessarily equal to $\widehat{r}_t^n$ which therefore cannot be qualified as natural real interest rate. Generally, the real interest rate is established in a more complex system of equations compared to the three equations system above. Finally, notice that since all variables are evaluated in units of labor efficiency, in the clearing condition (3.16) the exogenous growth of labor in efficiency units $\widehat{l}^g_{t+1}$ appears. So, $\widehat{r}_t^n = \widehat{l}^g_{t+1} - \widehat{\varphi}^g_{t+1}$ and $\widehat{l}^g_{t+1}$ itself are the exogenous driving forces.

The blue lines in Figure 12 reports the results of the full two-sector model presented already in

Figures 1 and 2. The light-blue and violet lines show results of the one-sector model with endogenous capital with and without price rigidity, respectively. Clearly the one-sector model gives results that are strikingly close to the ones of the two-sector model and that bear basically no visual difference between the two cases (with and without price rigidity). In comparison to the model with no capital, inflation and interest rates remain always at a higher level. This is because a given negative shock in $\widehat{r}_t^n$ (which, as seen above, can be compared to a tightening of monetary policy) does not have to be borne entirely by consumption. Consumption can be kept relatively smoothed by adjusting investment which makes up only a small fraction of the capital stock and therefore by inducing a relatively small effect on the return on capital and thus, by no arbitrage condition, on the ex-ante real interest rate.

Some conclusions can be drawn. First, if one is interested only in the aggregates, the sectoral allocations can be basically overlooked, at least with the currently specified model. Second, in comparison to the three equations model, the model with endogenous capital produces less dramatic dynamics of interest rates and inflation and has smaller discrepancies between the two cases, flexible versus sticky prices, because the presence of investment allows for smoother responses. Still, the qualitative implications of the demographic transition are not changed. Third, given that when the central bank follows the Taylor-type rule there are no significant differences in the paths of inflation and interest rates between the two cases, flexible versus sticky prices – more so with endogenous capital – there is support for the recent claim on New-Keynesian models by Rupert and Sustek (2019) who "demonstrate that equilibrium inflation is approximately determined as in a flexible-price model". In turn, this implies that an equilibrium where the ex-ante real interest rate is always close to its natural counterpart might not be an equilibrium where inflation is stable, depending on the monetary policy rule in place. In other terms, observing a positive deviation of the ex-ante real interest rate from its natural counterpart is not a necessary condition for observing a negative deviation of inflation from its target.

3.4 Different Taylor-type rules

As shown in the above sections, when the central bank follows a "strict inflation targeting" rule, the nominal interest rate is always equal to its natural counterpart. In this case inflation is always at target and output is always equal to its natural counterpart (i.e. output gap is zero). Can the central bank move close to this equilibrium by being more aggressive on inflation deviations from its target uniquely, while still not internalizing the effect of the demographic transition on the natural real interest rate (i.e. by using the same Taylor rule with fixed intercept)? The answer is yes. Figure 4 shows to what extent, by plotting results under rule (3.2) (i.e. rule (3.5) in log-deviations) for different values of the reaction parameter ϕ_π, respecting the Taylor principle of $\phi_\pi > 1$. It shows that by increasing ϕ_π the system moves closer to the natural allocation.

What happens if the central bank has also some inertia of the nominal interest rate in his Taylor-type rule? Figure 13 shows that as the inertia of the nominal interest rate to its previous period value increases (i.e. ϕ_R increases) the system gets closer to the natural allocation. This is unsurprising given that the discrepancies between the real interest rate and its natural counterpart are tiny (so that having inertia is like partially internalizing the downward impact of demographic change on the natural interest rate). In

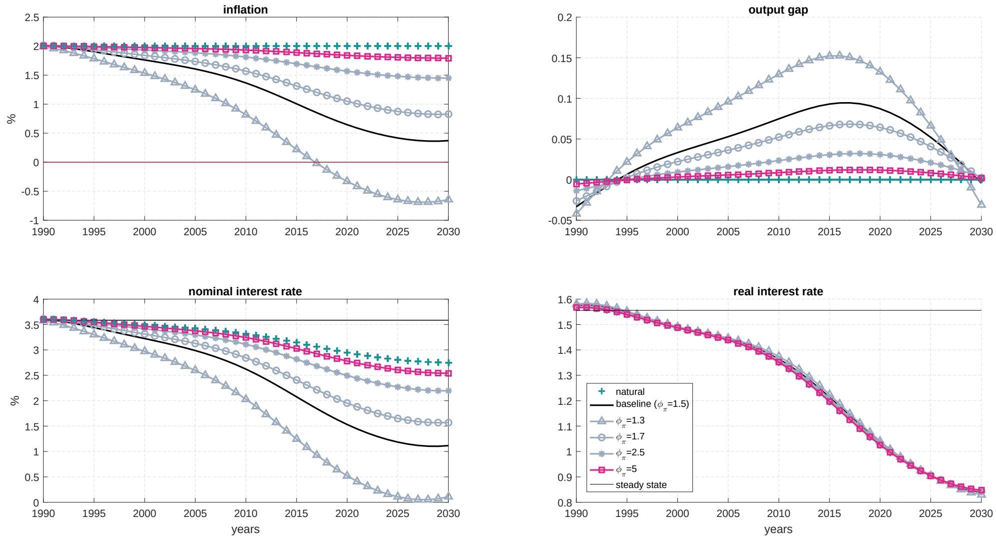


Figure 4: Demographic transition: different reactions to inflation deviations from target (ϕ_π), Taylor rule in log-deviations: $\widehat{\mathtt{R}}_t = \phi_\pi \widehat{\Pi}_t$

Note. The levels of the variables are obtained from the linear system in log-deviations from the initial steady state (B.1)–(B.26). The output gap is defined as percentage deviation of output from its natural level, i.e. $\widehat{\widetilde{y}}_t - \widehat{\widetilde{y}}^n_t = \log\{\widetilde{Y}_t/\widetilde{Y}^n_t\}$ where $\widetilde{Y}_t = \widetilde{Y}^T_t + Z_t\widetilde{Y}^N_t$ is aggregate output per unit of labor efficiency, L_t, and $\widetilde{Y}^n_t$ is its natural counterpart (output prevailing under flexible prices). Recall, see *Note* in Figure 12, that the real interest rate is obtained applying: $r^f_t = (1 + r - \delta)\exp\{(r/(1 + r - \delta))\widehat{r}_t\} - 1$.

this context, an important policy prescription is that the central bank can get close to the natural allocation even when it does not observe the natural real interest rate by simply setting the first difference of the policy rate close to the deviation of inflation from target in each period. Notice that the reference period in this setting is one year. It is common in the literature to set the inertia parameter at the quarterly frequency around 0.8. This value implies a speed of adjustment of the policy rate of about 20% per quarter which means that over a year the speed of adjustment is about 80%. Therefore, at the annual frequency the inertia parameter corresponding to what is common in the literature is $\phi_R = 0.2$.

What if the central bank responds not only to inflation gaps but also to output gaps? To answer this question one needs to think which output gaps to consider. In the same way as it can be argued that the central bank does not observe the time-varying natural real interest rate, thus reacting only to deviations of the policy rate to its steady state value, it can be argued that also the natural level of output is an unobservable for the central bank. Hence, as a first exercise, suppose that the central bank reacts to deviations of the output in units of labor efficiency, $\widetilde{Y}_t$, from its initial steady state value $\widetilde{Y}$ instead of its natural level $\widetilde{Y}^n_t$. Notice that $\widetilde{Y}$ would be the correct value to which output reverted in the long-run if there were no age-varying sectoral preferences in presence of imperfect mobility of labor (see section B.4.2). Figure 14 shows that when the central bank reacts to deviations of output from its steady state value, it ends up being much more disinflationary than in the baseline case where it reacts only to deviations of inflation from its target. Both inflation and the nominal rate go in negative territory. Demographic change leads capital per unit of labor efficiency and therefore output per unit of labor efficiency to increase with respect to its steady state value – this impact is directly related to the decrease of the real interest rate induced by demographic change. Hence, facing a positive deviation of output from steady state in the period considered, the central bank tends to be tighter than in the case where it only reacts to inflation gaps. This results in bigger discrepancies between the real interest rate and its natural counterpart which are associated to wider output gaps. Figure 14 shows also that the central bank would get closer to the baseline results if instead of reacting to deviations of the *level* of output from steady state it reacted to deviations of output *growth* from steady state (i.e. from zero). The reason is that the growth rate of output per unit of labor efficiency on a annual basis induced by demographic change is relatively small. Nonetheless, inflation is further apart from target as compared to the baseline case. Overall, the central bank does a worse job in terms of inflation gaps minimization when it reacts not only to inflation gaps but also to output gaps in the period considered.

As a final exercise, assume a slightly more complex Taylor rule which entails a learning process featured by Bielecki et al. (2018) in a similar context, in the spirit of Evans and Honkapohja (2001). Under this learning process, the Taylor rule in log-deviations from the initial steady state is assumed to be:

$$\widehat{\mathrm{R}}_t = \phi_R \widehat{\mathrm{R}}_{t-1} + (1-\phi_R)\left\{\widehat{\mathrm{R}}_t^e + \phi_\pi \widehat{\Pi}_t + \phi_{\Delta y}\left[(\widehat{y}_t - \widehat{y}_{t-1}) - (\widehat{y}_t^e - \widehat{y}_{t-1}^e)\right]\right\} \tag{3.17}$$

In case the central bank has perfect knowledge, the perceived values $(\widehat{\mathrm{R}}_t^e, \widehat{y}_t^e)$ are equal to their natural values,

$$\begin{aligned} \widehat{\mathrm{R}}_t^e &= \widehat{\mathrm{R}}_t^n = (r/(1+r-\delta))\widehat{r}_{t+1}^n + \widehat{\Pi}_{t+1}^{T,n} \\ \widehat{y}_t^e &= \widehat{y}_t^n \end{aligned} \tag{3.18}$$

Notice two features. First, as long as the Taylor rule has output *growth* gaps, obviously it does not matter whether output is defined as total, per capita, per worker or per unit of labor efficiency. So here $\widehat{y}_t$ denotes a generic output in log-deviations from its initial steady state. Second, the natural *nominal* interest rate $\widehat{\mathrm{R}}_t^n$ is the sum of the deviations of the return on capital and of expected inflation in the T-sector under flexible prices. Usually, in a one-sector model, the natural nominal interest rate would be equal to net return on capital multiplied by the inflation target. But in the current model there are sectoral variations that can impact inflation and the overall aggregate allocations even in the absence of price distortions. This is why $\widehat{\Pi}_{t+1}^{T,n}$ appears in the expression for the natural nominal interest rate denoting the level of expected inflation in the T-sector when prices are flexible (or, which is the same, when the central bank is at the target aggregate inflation in each period).

In the case of imperfect knowledge the perceived values are:

$$\widehat{\mathtt{R}}^e_t = \widehat{\mathtt{R}}^e_{t-1} + \lambda(\widehat{\mathtt{R}}^n_{t-1} - \widehat{\mathtt{R}}^e_{t-1}) \quad (3.19)$$

$$\widehat{\mathtt{y}}^e_t = \widehat{\mathtt{y}}^e_{t-1} + \lambda(\widehat{\mathtt{y}}^n_{t-1} - \widehat{\mathtt{y}}^e_{t-1}) \quad (3.20)$$

where the central bank observes the true values of the natural variables only with a lag and updates the current guess with a fraction λ of the previous forecast error. The parameter λ captures the speed of learning by the central bank. Following the parametrization by Bielecki et al. (2018), the speed of learning λ is in the range of an annual rate of 8% and 20%.

Figure 15 shows that under the learning scenario, the disinflationary bias is smaller throughout the whole period than in the baseline case when the central bank does not internalize at all the impact of the demographic transition (Taylor rule with fixed intercept). Furthermore, as the speed of learning increases (λ goes from 0.08 to 0.2), inflation gets closer to its target, the nominal interest rate gets closer to its natural counterpart and the output gap is more in the proximity of zero. Furthermore, Figure 15 shows that the disinflationary bias is mostly due to the misperception of the natural nominal rate $\widehat{\mathtt{R}}^n$: when only natural output $\widehat{\mathtt{y}}_t$ is misperceived, the deflationary bias (see shaded area in the top-left panel) is relatively small so that the remaining bigger part is explained by the misperception of $\widehat{\mathtt{R}}^n$ when rule (3.17) is applied with $\lambda = 0.08$. In both cases monetary policy ends up being too tight thus generating a disinflationary bias. Notice that under the learning processes (3.19) and (3.20) the central bank overestimates the natural rate and underestimates the natural output.

3.5 Demographic transition and sectoral developments: impact on inflation

To understand the sectoral dynamics first consider the model with flexible prices. Figure 16 shows what happens to the relative price of N-goods and to sectoral shares over the demographic transition. As population ages the aggregate share of consumption devoted to N-goods is projected to increase about 2 percentage points (going from about 51% to about 53%) given that older people consume relatively more N-goods (see Figure 8). This is accompanied by a steady increase of the relative price of N-goods which in the long run has an increase of about 4% (going from 1.237 to 1.288). This process is associated with a reallocation of both labor and capital from the T-sector to the N-sector, with a corresponding reallocation of production.

The dashed lines of Figure 16 show what happens when the sectoral consumption shares are constant across ages. In this case the share of consumption devoted to N-goods remains always constant at its steady state value (50.8%) while the impact on the relative price and sectoral shares is much smaller, approximately halved, suggesting that about half of the long-run increase in the relative price is due to the bias in consumption towards N-goods of relatively older cohorts; the remaining half is due to general equilibrium effects mostly determined by the preference parameter χ which governs the optimal sectoral mix of labor and capital given factor prices. In particular, aging leads to a progressive decrease of the aggregate investment rate, going from about 23% of output in 1980 to 19% in 2050: as the labor force declines it is optimal to reduce the capital stock with which workers are complemented in production.

Since capital can be only produced using T-goods, less investment in capital means less need of labor force employed in the T-sector. Therefore, because of full-employment, a decrease of the investment rate goes hand in hand with a reallocation of the effective labor force from the T-sector to the N-sector *independently* of whether there is a bias in consumption towards the goods of a certain sector due to aging. Due to the assumption of imperfect labor mobility, the only way this sectoral reallocation can happen is by offering to workers an higher relative wage of the N-sector which therefore increases along the transition, and so does the relative price of N-goods (as workers are paid their marginal product).

Finally, Figure 16 shows the dynamics resulting from the model when additionally there is perfect labor mobility between sectors (i.e. when the parameter ε is set at a value which proxies infinity and still the consumption shares are constant over the ages). Unsurprisingly, there is no effect on the relative price which stays always at its steady state value (1.25). Indeed, the way the model generates an impact of the evolving demand composition on the relative price is via imperfect labor mobility. As more N-goods are demanded firms need to attract more labor-input in the N-sector. The only way to do so is to increase the real wage in the N-sector. If there was perfect labor mobility the real wage would increase 1 to 1 so to have wage equalization between sectors. Instead, with imperfect mobility, the *relative* wage of the N-sector needs to increase accommodating the permanent preference of the representative household to work in either sector. The increase in the relative wage is directly related to the increase of the relative price (see section B.4.2). In Figure 16 it can be seen (red dotted line) that with perfect labor mobility the share of labor employed in the N-sector is higher or lower than in the case with imperfect labor mobility depending on whether the relative price was higher or lower than its steady state value. That is, with imperfect labor mobility only part of the level of labor in the N-sector that would prevail with perfect labor mobility is attained. This discrepancy is a mirror image of the deviation of the relative price from its steady state value. The figure also shows that whether labor can perfectly move or not between sectors has only a limited impact on the evolution of the share of capital employed in the two sectors.

To understand how the sectoral developments impact monetary policy and its objective of staying at target inflation, consider the case when the central bank sets the deviation from steady state of the nominal interest rate equal to the deviations of the natural *real* interest rate:

$$\widehat{\mathrm{R}}_t = (r/(1+r-\delta))\widehat{r}^n_{t+1} \tag{3.21}$$

This rule would allow to attain target inflation in each period if it was a one-sector model. However, there are sectoral developments induced by the demographic transition that can lead inflation in the two sectors to diverge. As shown in the previous section, the central bank would be at target inflation by following rule (3.18), i.e. by setting $\widehat{\mathrm{R}}_t = \widehat{\mathrm{R}}^n_t = (r/(1+r-\delta))\widehat{r}^n_{t+1} + \widehat{\Pi}^{T,n}_{t+1}$. In other terms, by following (3.21) thus omitting developments of inflation in the T-sector $\widehat{\Pi}^{T,n}_{t+1}$, the central bank is not internalizing the sectoral developments induced by the demographic transition which, as discussed just above, are in part due to the consumption bias towards N-goods of the elderly and in part due to general equilibrium effects in presence of imperfect labor mobility between sectors. Figure 5 shows that when the central bank does not internalize the sectoral developments there is an inflationary bias.[30] By following rule (3.21) the central bank is closely targeting inflation in the T-sector (see dotted line) disregarding the fact that due to aging there is a sectoral reallocation that leads the relative price of N-goods on an increasing path. Inflation in the N-sector increases accordingly (see starred line). Aggregate inflation, given that it is a convex combination of sectoral inflations, is on an intermediate path between the two inflations (see continuous line). However, the magnitude of its increase is small: between 1990 and 2030 inflation increases less than 0.1 percentage points from target. The associated output gap is small too.

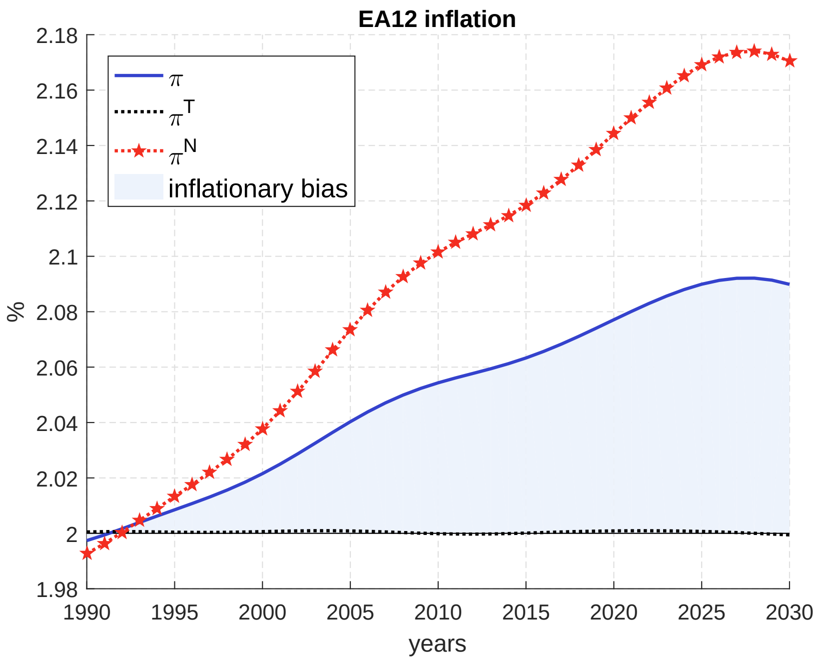


Figure 5: Demographic transition: impact on inflation when the central bank does not internalize sectoral developments, i.e. sets $\widehat{\mathrm{R}}_t = (r/(1+r-\delta))\widehat{r}^n_{t+1}$.

Note. Results from the baseline model when the central bank follows rule (3.21) instead of the Taylor rule (3.2).

4 Conclusion

The ongoing demographic transition – in Europe as well as in most advanced economies – carries important implications for inflation and possible challenges for monetary policy. Even though the impact of the demographic transition on inflation seems to be negligible for the short- to medium-term horizon, which is relevant for monetary policy-making, its slow-moving nature seems to have been prone to generating a disinflationary bias in the euro area in the last two decades at least in the model set-up brought forward in this paper. Based on our analysis, monetary policy rules that do not internalize the downward impact of aging on the natural interest rate may end up being tighter than intended thus generating a downward trend of inflation. For the euro area and using the structure of a two-sector overlapping generations model

[30]see Figure 17 for the corresponding dynamics of output gap, nominal and real interest rate.

embedded in a New-Keynesian framework with price frictions we have quantified a sizable “disinflationary bias” since 1990 explained by this channel. This bias occurs in our model when the central bank follows a monetary policy rule setting the policy rate only in reaction to inflation deviation from target with a fixed natural interest rate. This bias may be fortified if the central bank also reacts to deviations of the output level from its steady state value in addition to the inflation gap. In this model the bias can be reduced applying a less naive monetary policy rule where the central bank adjusts regularly but with a delay to the natural level of the real economic variables. Finally, the disinflationary bias is not significantly counteracted by the inflationary pressures associated with the old-age consumption propensity towards non-tradable goods and services such as healthcare expenditures.

Based on our analysis, the presence of a persistently declining natural interest rate, even if perfectly internalized in the conduct of monetary policy, would require the policy rate to move downwards too, to be consistent with the inflation objective. Therefore, the demographic transition – projected to intensify in the incoming years till 2030 – is likely to increase the probability of hitting the zero-lower bound (ZLB) for the policy rate.

Of course, important research questions that connect demographic change to monetary policy remain open and still need to be addressed. One of such questions is how the relationship between the optimal inflation target and the natural interest rate might be affected by demographic change.

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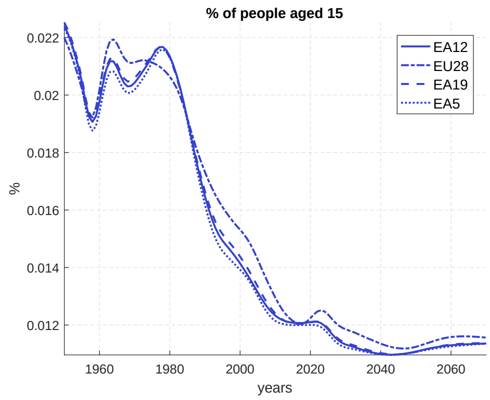
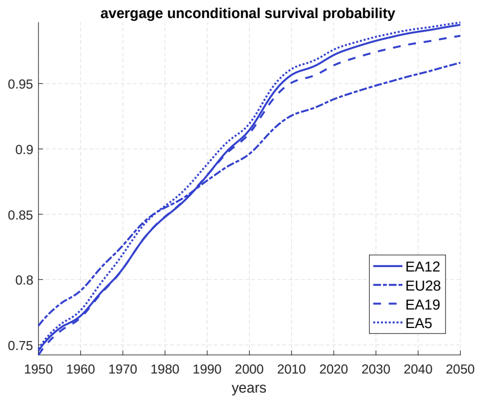
5 Additional figures



(a) Fraction of people aged 15



(b) Average unconditional survival probability

Figure 6: Demographic transition in Europe

Note. The indicator in panel 6a is the number of people aged 15 over the number of people aged more than 15. The indicator in panel 6b is computed by first retrieving the implied unconditional survival probabilities $\pi_{t,j}$ applying the recursive formula $N_{t+j,j} = \pi_{t+j,j} N_{t,0}$ using data for the cohort size $N_{t,j}$ for each year t and age-bin j (with $N_{t,0}$ corresponding to the incoming cohort size, those aged 15); then, by averaging across cohorts for each year so that the indicator is $\zeta_t = \sum_j \pi_{t,j}(N_{t,j}/N_t)$ with population size $N_t = \sum_j N_{t,j}$. Data from UN (2017) *World Population Prospects: The 2017 Revision*, medium variant after year 2016 (see footnote 16). The following groups of countries hold. EA19: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherland, Portugal, Slovakia, Slovenia, Spain; EA12 is EA19 excluding Cyprus, Estonia, Latvia, Lithuania, Malta, Slovakia, Slovenia; EA5: France, Germany, Italy, Netherlands, Spain; EU28 comprises EA19 and the following non-EA members: Bulgaria, Croatia, Czech Republic, Hungary, Poland, Romania, Sweden, Denmark and United Kingdom.

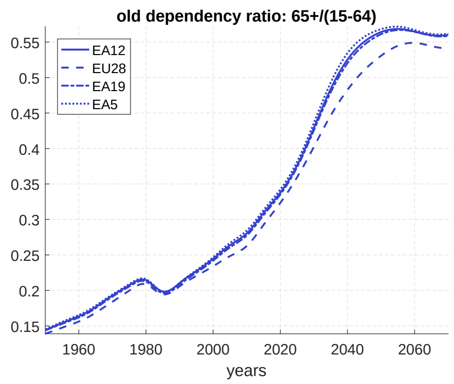


Figure 7: Old dependency ratio in Europe

Note. The indicator in the figure is the the number of people aged more than 64 over the number of people aged between 15 and 64. Data source and groups as in Figure 6a.

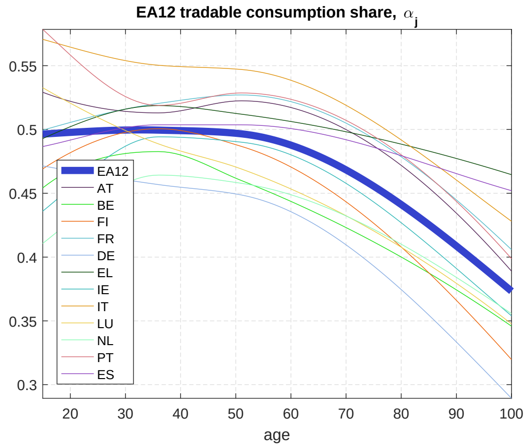


Figure 8: Age dependent tradable shares of (private) consumption expenditure in EA12, α_j

Note. *Data source*: EUROSTAT, 2010 series name: "Structure of consumption expenditure by age of the reference person (COICOP level 2) (1 000) [hbs_str_t225]" in year 2010, which is the (average) private consumption expenditure (measured in euro/PPS). The following sectors are categorized as tradable: food, clothing, furniture and equipment, transports, communications; as nontradable: housing, health, culture and entertainment, education, restaurants and accommodation. The age classes available from which a (cubic) interpolation is obtained are: 0-29, 30-44, 45-59, 60+. The EA12 profile (blue thick line) is obtained as weighted average of the of single countries' profiles, using as weights the shares of GDP (Source: EUROSTAT, Gross domestic product at market prices, Chain linked volumes (2005), million euro). See Giagheddu and Papetti (2017).

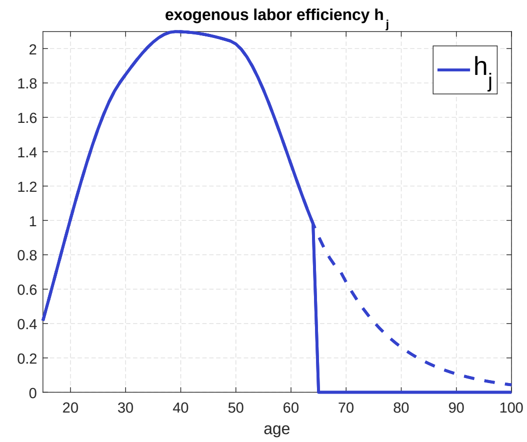


Figure 9: Age dependent labor supply in efficiency units, h_j

Note. The profile is obtained with a cubic interpolation (for age 15 to 70) on the data points provided in Domeij and Floden (2006). These data points are the product of participation rates provided by Fullerton (1999) and productivity provided by Hansen (1993). Lacking data, for $j \geq 70$ the profile is obtained from the following logistic function: $C/(1 + Ae^{-Bj})$, with $A = .49$, $C = 50$, $B = (1/70) \log [h_{70}A/(C - h_{70})]$. The blue continuous line denotes the baseline profile with exogenous retirement age at $jr + 1 = 65$. The dashed line denotes the part of the profile which is not binding in the baseline.

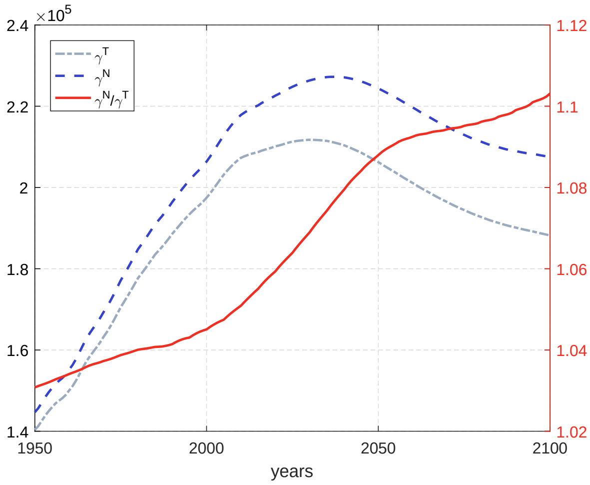


Figure 10: Time-varying parameters (EA12): γ_t^T,γ_t^N

Note. The time-varying parameters γ_t^T, γ_t^N are the exogenous wedges on the representative household's preferences, see (2.11). They are: $\gamma_t^T = \sum_{j=0}^{J} \alpha_j N_{t,j}, \gamma_t^N = \sum_{j=0}^{J}(1-\alpha_j)N_{t,j}$, where the number of people $N_{t,j}$ is taken from data provided by the United Nations *World Population Prospects: The 2017 Revision* for year $t \in [1950, 2100]$, medium variant after year 2016 (see footnote 16), and α_j is in Figure 8. Notice that $\gamma_t^T + \gamma^N$ identifies the population size in the economy for each year t, where population in the model refers to those aged more than 15, given that $j = 0$ corresponds to age 15.

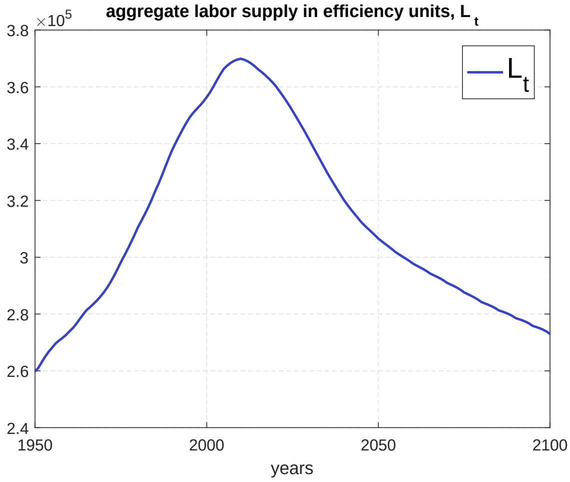


Figure 11: Aggregate labor supply in efficiency units in EA12, L_t

Note. The exogenous time-varying aggregate labor supply in efficiency units is: $L_t = \sum_{j=0}^{jr} h_j N_{t,j}$ with $jr = 50$ which corresponds to age 64. Individuals enter the world as workers at age $j = 0$ which corresponds to age 15. The number of people $N_{t,j}$ is taken from data provided by the United Nations *World Population Prospects: The 2017 Revision* for year $t \in [1950, 2100]$, medium variant after year 2016 (see footnote 16), h_j is in Figure 9.

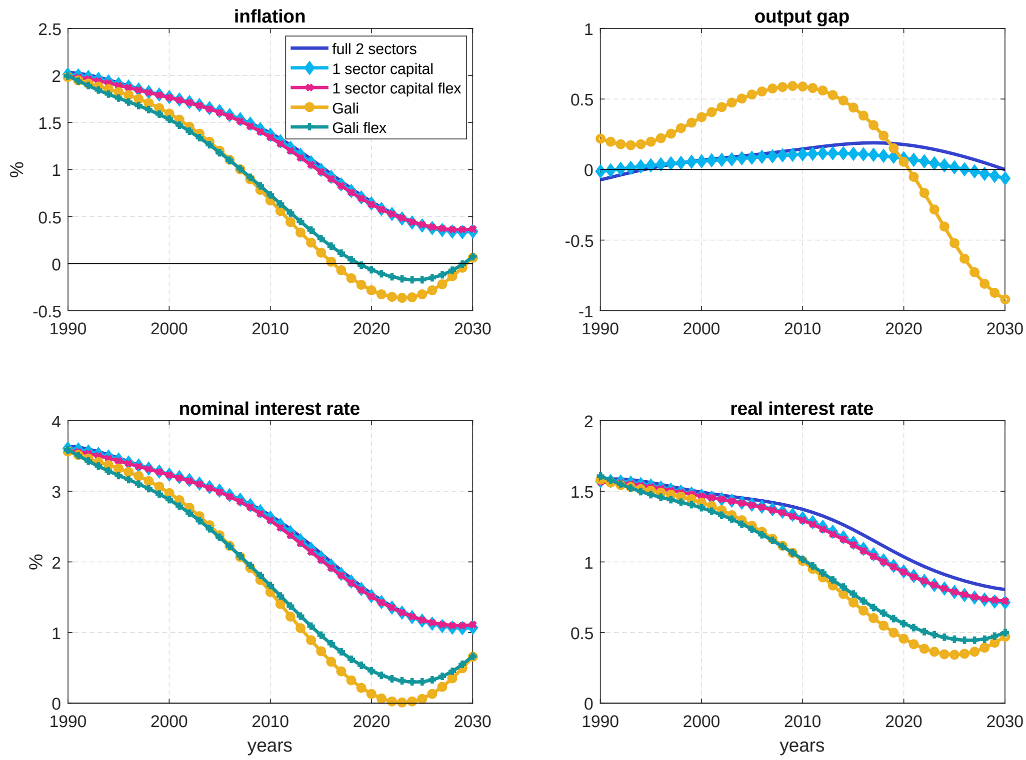


Figure 12: Demographic transition: comparison of three models, flexible *vs* sticky prices

Note. The figure shows the results from three models where the sole exogenous change is demographics over time as captured by $\widehat{l}^g_{t+1}$ and $\widehat{\varphi}^g_{t+1}$ with $\widehat{r}^n_t = \widehat{l}^g_{t+1} - \widehat{\varphi}^g_{t+1}$. "Full 2 sectors" is the system (B.1)–(B.26); "1 sector capital" is the system (3.9)–(3.16); "Gali" is the system (3.3)–(3.5). "Flex" identifies the model with flexible prices obtained by setting θ to zero. Otherwise $\theta = .68$. Notice that the systems have log-linearized variables in deviations from the initial steady state. To make the comparison in levels, it is assumed that all variables start at the same initial value across models. What it is plot in the bottom-down panel is the risk-free real interest rate, denote it by r^f_t, equal to $r_t - \delta$ by no arbitrage condition between return on capital and bonds. By equating (3.10) with (3.12), it results: $\widehat{\mathtt{R}}_t - E_t\widehat{\Pi}_{t+1} = (r/(1+r-\delta))\widehat{r}_t$. It follows that to get the level of the real risk-free rate, whose gross value in steady state is $(1+r-\delta)$, from the log-deviations of the return on capital $\widehat{r}_t$ one needs to apply: $r^f_t = (1+r-\delta)\exp\{(r/(1+r-\delta))\widehat{r}_t\} - 1$.

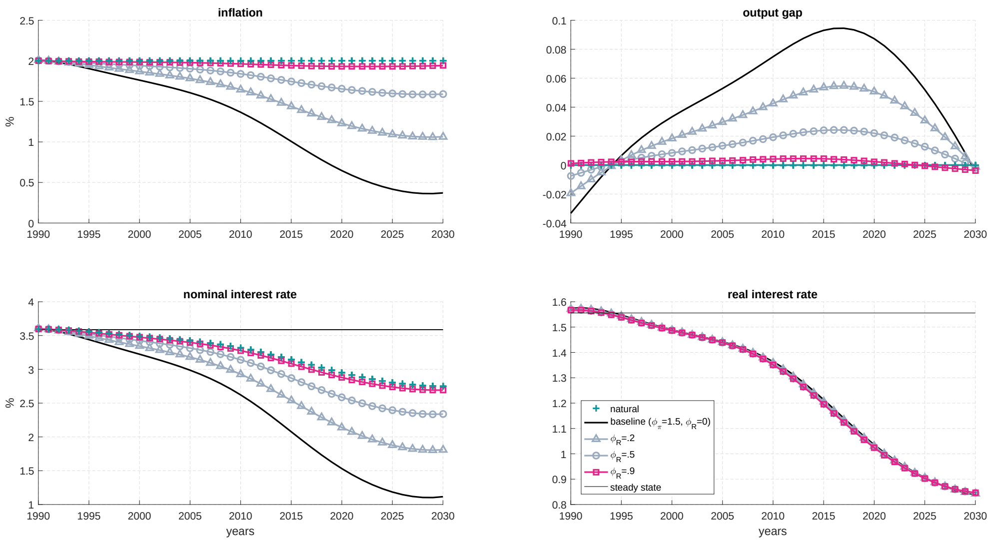


Figure 13: Demographic transition: different interest rate inertia (ϕ_R), Taylor rule in log-deviations: $\widehat{\mathtt{R}}_t = \phi_R \widehat{\mathtt{R}}_{t-1} + \phi_\pi \widehat{\Pi}_t$

Note. See *Note* of Figure 4.

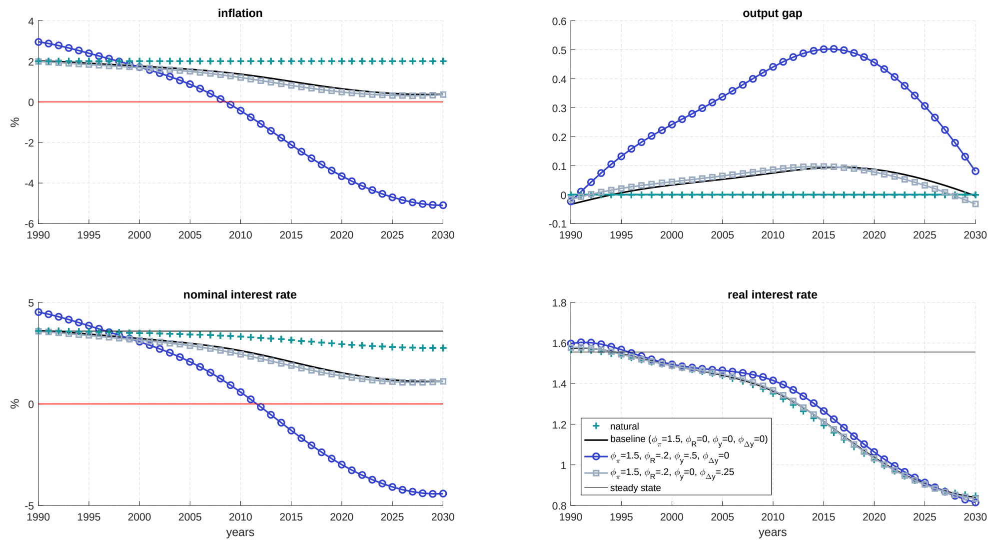


Figure 14: Demographic transition: central bank responding to output gap ($\phi_y > 0$), Taylor rule in log-deviations: $\widehat{\mathtt{R}}_t = \phi_R \widehat{\mathtt{R}}_{t-1} + (1 - \phi_R)(\phi_\pi \widehat{\Pi}_t + \phi_y \widehat{\widetilde{y}}_t + \phi_{\Delta y}(\widehat{\widetilde{y}}_t - \widehat{\widetilde{y}}_{t-1}))$

Note. See *Note* of Figure 4.

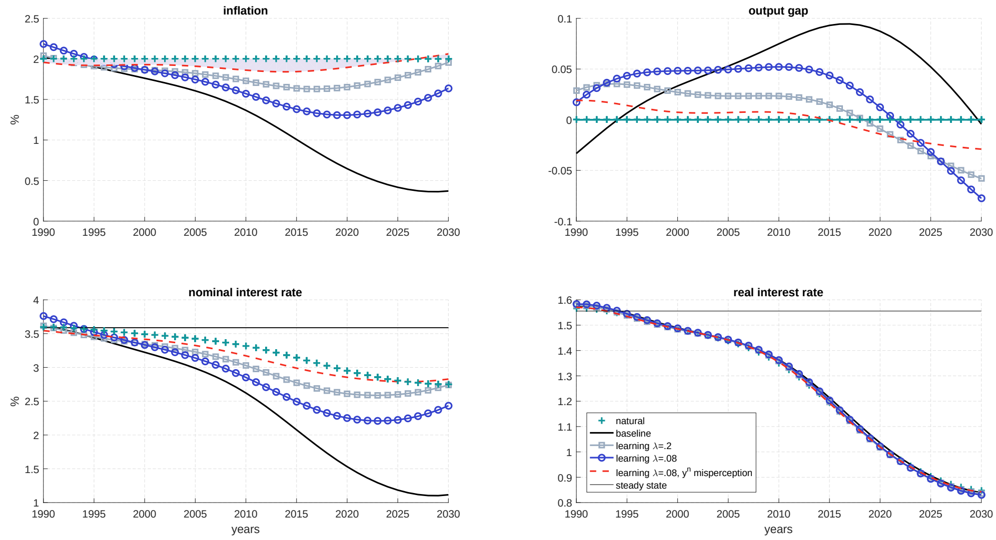


Figure 15: Demographic transition: Taylor rule with learning process

Note. See *Note* of Figure 4. See section 3.4 for the specification of the Taylor rule.

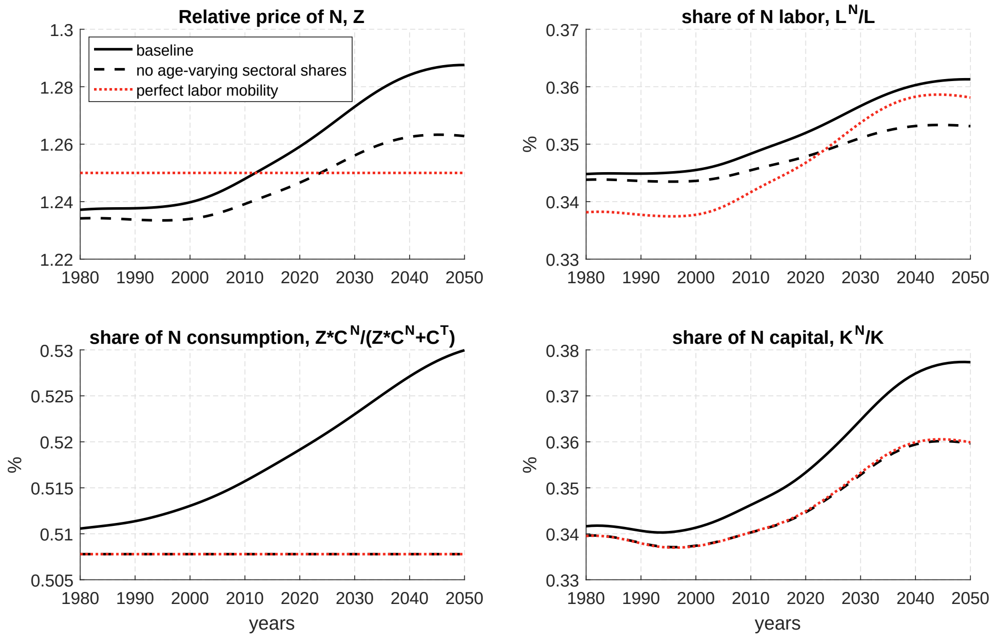


Figure 16: Demographic transition: sectoral developments

Note. Results from the baseline model (B.1)–(B.26) with flexible prices. The scenario *no age-varying sectoral shares* assumes that $\alpha_j = \alpha$ for all j, which implies the following new exogenous inputs for the log-linearized model: $\widehat{\gamma}_t^N - \widehat{\gamma}_t^T = 0$, $A_t = 0$, $\widehat{\gamma}_{t+1}^{T,g} = \widehat{\varphi}_{t+1}^g$ for all t, where the last term denotes that the consumption wedge in the Euler equation for the representative household is the growth rate of the population (people between age 15 and 100+), see section B.4. The scenario *perfect labor mobility* is obtained by setting the parameter ε at a value that proxies infinity given the *no age-varying sectoral shares* parametrization.

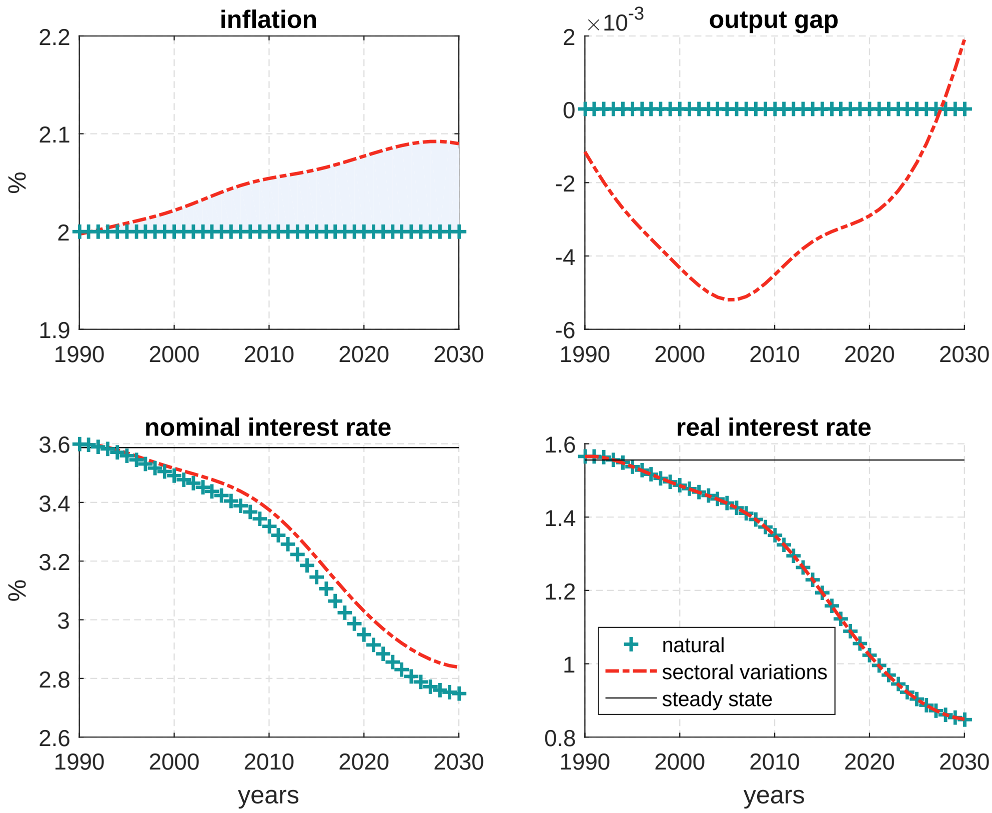


Figure 17: Demographic transition: impact when the central bank does not internalize sectoral developments, i.e. sets $\widehat{\mathrm{R}}_t = (r/(1 + r - \delta))\widehat{r}^n_{t+1}$.

Note. Results from the baseline model when the central bank follows rule (3.21) instead of the Taylor rule (3.2).

Appendix

A Aggregate representation of the OLG model with demographic wedges

A.1 Consumption

Following Jones (2018), the derivation of the aggregate wedges that allow to represent the life-cycle (finite-life) individuals' problem in the OLG model (i.e. a problem where the heterogeneity across individuals is given by the age) of section 2.1 as a problem of an infinitely-lived representative agent problem proceeds in three steps. First, rewriting the individual's life-cycle problem as an infinite horizon one, it is shown that under complete markets (namely, under the assumption of "perfect annuities market" for unintentional bequest, see footnote 6) there exist welfare weights attached to each individual utility function in the social planner's problem that allow to equate the planner's solution to the decentralized equilibrium. Second, by solving the social planner's dynamic problem of optimizing aggregate sectoral consumption and savings over time, the decentralized equilibrium is related to the planner's solution. Third, by solving the social planner's static problem of choosing sectoral consumption for each individual in each cohort to maximize the sum of individuals utilities weighted by the welfare weights, an expression for the aggregate demographic wedge attached to aggregate consumption is derived.

1. Consider in the OLG model of section 2.1 an individual i belonging to the cohort born in period s. Rewrite his life-cycle problem as an infinite-horizon problem, to solve for each time-period t from the period he is born in s onwards (till infinity), in the following way:[31]

$$\max_{\{c_t^{T,i,s}, c_t^{N,i,s}, a_{t+1}^{i,s}\}} \sum_{t=s}^{\infty} \beta^t \pi_{t,s} \phi_t^{i,s} u_s(c_t^{T,i,s}, c_t^{N,i,s})$$

$$\text{s.t.} \qquad a_{t+1}^{i,s} = \frac{a_t^{i,s}(1+r_t)}{s_t^{i,s}} - c_t^{T,i,s} - Z_t c_t^{N,i,s} + y_t^{i,s}$$

where $\pi_{t,s}$ denotes the unconditional survival probability in period t for an individual born in period s; as in Jones (2018), $\phi_t^{i,s}$ is a "preference process that proxies for the life-cycle" which takes value "one when the individual is alive and zero otherwise".[32] Write the Lagrangian:

$$\mathcal{L} = \sum_{t=s}^{\infty} \beta^t \pi_{t,s} \left\{ \phi_t^{i,s} u_s(c_t^{T,i,s}, c_t^{N,i,s}) - \lambda_t^{i,s} \left[\frac{a_t^{i,s}(1+r_t)}{s_t^s} - c_t^{T,i,s} - Z_t c_t^{N,i,s} + y_t^{i,s} - a_{t+1}^{i,s} \right] \right\}$$

where the individual's Lagrangian multiplier $\lambda_t^{i,s}$ has been conveniently multiplied by $\pi_{t,s}$. The first

[31] The problem of choosing sectoral hours to work is omitted since in the setting of section 2.1 the total hours to work are exogenous, thus not entering the utility function.

[32] In other terms, the problem is still a finite life one, because for each t greater than the terminal life period (J in the notation of section 2.1) $\phi_t^{i,s}$ is equal to zero in the problem above. But this 'fiction' serves to derive the result later.

order conditions are:

$$\begin{aligned}
\phi_t^{i,s} u_{s,1}(c_t^{T,i,s}, c_t^{N,i,s}) &= \lambda_t^{i,s} \\
\phi_t^{i,s} u_{s,2}(c_t^{T,i,s}, c_t^{N,i,s}) &= Z_t \lambda_t^{i,s} \\
\lambda_t^{i,s} &= \beta(1 + r_{t+1})\lambda_{t+1}^{i,s}
\end{aligned}$$

where the assumption of perfect annuities market gives the last equation, i.e. the standard Euler equation (independent of survival probabilities), given that $\frac{\pi_{t+1,s}}{\pi_{t,s}} = s_{t+1}^s$. Consider a different individual i' born in a different period s'. It follows for all i, i', $i \neq i'$:

$$\frac{\lambda_t^{i,s}}{\lambda_t^{i',s'}} = \frac{\lambda_{t+1}^{i,s}}{\lambda_{t+1}^{i',s'}} = \frac{\lambda_{t+2}^{i,s}}{\lambda_{t+2}^{i',s'}} = \cdots = \frac{\lambda^{i,s}}{\lambda^{i',s'}} \quad \text{for all } t$$

that is, the ratio of the marginal utilities of any two consumers is constant over time (which is a standard result under complete markets). This allows to represent the individual Lagrangian multipliers in the form $\lambda_t^{i,s} = \frac{\lambda_t}{\lambda^{i,s}}$, where λ_t is the Lagrangian multiplier on the aggregate budget constraint (which is identified later) and thus to map the social planner's solution to the decentralized equilibrium. Hence:

$$\begin{aligned}
\lambda^{i,s}\phi_t^{i,s} u_1(c_t^{T,i,s}, c_t^{N,i,s}) &= \lambda_t \\
\lambda^{i,s}\phi_t^{i,s} u_2(c_t^{T,i,s}, c_t^{N,i,s}) &= Z_t \lambda_t
\end{aligned}$$

These two equations together with the each individual's budget constraint and aggregate definitions characterize the decentralized equilibrium.

2. Consider the social planner's dynamic problem of optimizing aggregate sectoral consumption and savings over time:

$$\begin{aligned}
&\max_{C_t^T, C_t^N, K_t} \sum_{t=0}^{\infty} \beta^t U(C_t^T, C_t^N) \\
\text{s.t} \quad &C_t^T + Z_t C_t^N = \cdots + (1 + r_t)K_{t-1} - K_t
\end{aligned}$$

Letting λ_t denoting the Lagrangian multiplier on the aggregate budget constraint, the first order conditions of this problem are the aggregate equivalent of those in the decentralised equilibrium:

$$\begin{aligned}
U_1(C_t^T, C_t^N) &= \lambda_t \\
U_2(C_t^T, C_t^N) &= Z_t \lambda_t \\
\lambda_t &= \beta(1 + r_t)\lambda_{t+1}
\end{aligned}$$

3. Consider the social planner's static problem of choosing sectoral consumption for each individual in

each cohort to maximize the sum of individuals utilities weighted by the welfare weights.

$$\begin{aligned} U(C_t^T, C_t^N) &= \max_{c_{t,s}^T, c_{t,s}^N} \left\{ \sum_s \int \lambda^{i,s} \phi_t^{i,s} u(c_t^{T,i,s}, c_t^{N,i,s}) di \right\} \\ \text{s.t.} & \quad \sum_s \int c_t^{T,i,s} di + Z_t \sum_s \int c_t^{N,i,s} di = C_t^T + Z_t C_t^N \end{aligned}$$

Recall that in the OLG model individuals within each cohort are identical. Moreover, within each cohort it is assumed that the mass of identical individuals is $N_{t,s}$ which denotes the (exogenous) number of people of age s at time t. It follows that each individual chooses $c_t^{T,i,s} \equiv c_{t,s}^T$, $c_t^{N,i,s} \equiv c_{t,s}^N$ for all i. Hence, the social planner's problem becomes:

$$\begin{aligned} U(C_t^T, C_t^N) &= \max_{c_{t,s}^T, c_{t,s}^N} \left\{ \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} u(c_{t,s}^T, c_{t,s}^N) \right\} \\ \text{s.t.} & \quad \sum_s N_{t,s} c_{t,s}^T + Z_t \sum_s N_{t,s} c_{t,s}^N = C_t^T + Z_t C_t^N \end{aligned}$$

With the functional forms assumed in the model presented in section 2:

$$u_s(c_{t,s}^T, c_{t,s}^N) = \frac{\left((c_{t,s}^T)^{\alpha_s} (c_{t,s}^N)^{1-\alpha_s}\right)^{1-\sigma}}{1-\sigma}$$

the Lagrangian for this static problem (with Lagrangian multiplier μ_t) is:

$$\mathcal{L} = \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} \frac{\left((c_{t,s}^T)^{\alpha_s} (c_{t,s}^N)^{1-\alpha_s}\right)^{1-\sigma}}{1-\sigma} + \mu_t \left[C_t^T + Z_t C_t^N - \sum_s N_{t,s} c_{t,s}^T - Z_t \sum_s c_{t,s}^N \right]$$

The optimal choice of both $c_{t,s}^T$ and $c_{t,s}^N$ leads to the first order conditions:

$$\begin{aligned} \lambda^{i,s} \phi_t^{i,s} \left((c_{t,s}^T)^{\alpha_s} (c_{t,s}^N)^{1-\alpha_s}\right)^{1-\sigma} \frac{\alpha_s}{c_{t,s}^T} &= \mu_t \\ \lambda^{i,s} \phi_t^{i,s} \left((c_{t,s}^T)^{\alpha_s} (c_{t,s}^N)^{1-\alpha_s}\right)^{1-\sigma} \frac{(1-\alpha_s)}{c_{t,s}^N} &= Z_t \mu_t \end{aligned}$$

while the envelope conditions are: $U_1(C_t^T, C_t^N) = \mu_t; U_2(C_t^T, C_t^N) = Z_t \mu_t$. Combining the two expressions above, the system becomes:

$$c_{t,s}^T = \frac{\alpha_s}{1-\alpha_s} Z_t c_{t,s}^N \tag{A.1}$$

$$\mu_t = \lambda^{i,s} \phi_t^{i,s} \left((c_{t,s}^T)^{\alpha_s} (c_{t,s}^N)^{1-\alpha_s}\right)^{1-\sigma} \frac{\alpha_s}{c_{t,s}^T} \tag{A.2}$$

Managing (A.2) it results:

$$c_{t,s}^{N} \;=\; \left[\frac{\mu_t}{\alpha_s \lambda^{i,s}\phi_t^{i,s}(c_{i,s}^{T})^{\alpha_s(1-\sigma)-1}}\right]^{\frac{1}{(1-\alpha_s)(1-\sigma)}}$$

Plug the last expression into (A.1) to have:

$$c_{t,s}^{T} \;=\; \frac{\alpha_s}{1-\alpha_s} Z_t \left[\frac{\mu_t}{\alpha_s \lambda^{i,s}\phi_t^{i,s}(c_{i,s}^{T})^{\alpha_s(1-\sigma)-1}}\right]^{\frac{1}{(1-\alpha_s)(1-\sigma)}}$$

i.e.

$$c_{t,s}^{T} \;=\; \left(\frac{(1-\alpha_s)}{Z_t \alpha_s}\right)^{\frac{(1-\alpha_s)(1-\sigma)}{\sigma}} \left(\frac{\alpha_s \lambda^{i,s}\phi_t^{i,s}}{\mu_t}\right)^{\frac{1}{\sigma}}$$

The goal now is to find a utility function for the representative agent to let him choose only aggregate sectoral consumptions. The goal is thus to find those (time-varying exogenous) parameters attached to each sectoral aggregate consumption that captures the change of the age structure in the economy. To this end, first consider aggregate consumption in the T-sector:

$$C_t^T = \sum_s N_{t,s} c_{t,s}^{T} = \mu_t^{-\frac{1}{\sigma}} \sum_s N_{t,s} \left(\frac{(1-\alpha_s)}{Z_t \alpha_s}\right)^{\frac{(1-\alpha_s)(1-\sigma)}{\sigma}} (\alpha_s \lambda^{i,s}\phi_t^{i,s})^{\frac{1}{\sigma}}$$

By plugging (A.2) into this expression one gets:

$$C_t^T = \left[\lambda^{i,s}\phi_t^{i,s}\left((c_{t,s}^{T})^{\alpha_s}(c_{t,s}^{N})^{1-\alpha_s}\right)^{1-\sigma}\frac{\alpha_s}{c_{t,s}^{T}}\right]^{-\frac{1}{\sigma}} \sum_s N_{t,s} \left(\frac{(1-\alpha_s)}{Z_t \alpha_s}\right)^{\frac{(1-\alpha_s)(1-\sigma)}{\sigma}} (\alpha_s \lambda^{i,s}\phi_t^{i,s})^{\frac{1}{\sigma}}$$

i.e.

$$C_t^T = \frac{(c_{t,s}^{T})^{\frac{1}{\sigma}}}{\left[\alpha_s \lambda^{i,s}\phi_t^{i,s}\left((c_{t,s}^{T})^{\alpha_s}(c_{t,s}^{N})^{1-\alpha_s}\right)^{1-\sigma}\right]^{\frac{1}{\sigma}}} \sum_s N_{t,s} \left(\frac{(1-\alpha_s)}{Z_t \alpha_s}\right)^{\frac{(1-\alpha_s)(1-\sigma)}{\sigma}} (\alpha_s \lambda^{i,s}\phi_t^{i,s})^{\frac{1}{\sigma}} \quad \text{(A.3)}$$

Similarly, the aggregate consumption of N-goods (recall (A.1)) reads:

$$C_t^N = \sum_s N_{t,s} c_{t,s}^{N} = \frac{1}{Z_t}\sum_s N_{t,s}\frac{(1-\alpha_s)}{\alpha_s} c_{t,s}^{T} = \frac{\mu_t^{-\frac{1}{\sigma}}}{Z_t}\sum_s N_{t,s}\frac{(1-\alpha_s)}{\alpha_s}\left(\frac{(1-\alpha_s)}{Z_t \alpha_s}\right)^{\frac{(1-\alpha_s)(1-\sigma)}{\sigma}} (\alpha_s \lambda^{i,s}\phi_t^{i,s})^{\frac{1}{\sigma}}$$

with (A.2) into this expression and re-managing, it results:

$$C_t^N = \frac{(c_{t,s}^{T})^{\frac{1}{\sigma}}}{\left[\alpha_s \lambda^{i,s}\phi_t^{i,s}\left((c_{t,s}^{T})^{\alpha_s}(c_{t,s}^{N})^{1-\alpha_s}\right)^{1-\sigma}\right]^{\frac{1}{\sigma}}} \sum_s N_{t,s} \left(\frac{(1-\alpha_s)}{Z_t \alpha_s}\right)^{1+\frac{(1-\alpha_s)(1-\sigma)}{\sigma}} (\alpha_s \lambda^{i,s}\phi_t^{i,s})^{\frac{1}{\sigma}} \quad \text{(A.4)}$$

Consider the special case of *logarithmic preferences*, namely $\boxed{\sigma = 1}$. In this case, (A.3) and (A.4) reduce to:

$$
\begin{aligned}
c_{t,s}^{T} &= \frac{\alpha_s \lambda^{i,s} \phi_t^{i,s}}{\sum_s N_{t,s} \alpha_s \lambda^{i,s} \phi_t^{i,s}} C_t^{T} \\
\frac{c_{t,s}^{T}}{Z_t} &= \frac{\alpha_s \lambda^{i,s} \phi_t^{i,s}}{\sum_s N_{t,s} (1-\alpha_s) \lambda^{i,s} \phi_t^{i,s}} C_t^{N}
\end{aligned}
$$

that, inserted into the objective function of the representative agent under logarithmic preferences, gives:

$$
\begin{aligned}
\sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} u_s(c_{t,s}^{T}, c_{t,s}^{N}) &= \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} \log\left[(c_{t,s}^{T})^{\alpha_s} (c_{t,s}^{N})^{1-\alpha_s}\right] \\
&= \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} \log\left[(c_{t,s}^{T})^{\alpha_s} \left(\frac{c_{t,s}^{T}}{Z_t} \frac{1-\alpha_s}{\alpha_s}\right)^{1-\alpha_s}\right] \\
&= \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} \log\left[\left(\frac{\alpha_s \lambda^{i,s} \phi_t^{i,s}}{\sum_s N_{t,s} \alpha_s \lambda^{i,s} \phi_t^{i,s}} C_t^{T}\right)^{\alpha_s} \left(\frac{\alpha_s \lambda^{i,s} \phi_t^{i,s}}{\sum_s N_{t,s} (1-\alpha_s) \lambda^{i,s} \phi_t^{i,s}} C_t^{N} \frac{1-\alpha_s}{\alpha_s}\right)^{1-\alpha_s}\right] \\
&= \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} \log\left[(C_t^{T})^{\alpha_s} (C_t^{N})^{1-\alpha_s} \frac{\left(\frac{1-\alpha_s}{\alpha_s}\right)^{1-\alpha_s} \alpha_s \lambda^{i,s} \phi_t^{i,s}}{\left(\sum_s N_{t,s} \alpha_s \lambda^{i,s} \phi_t^{i,s}\right)^{\alpha_s} \left(\sum_s N_{t,s} (1-\alpha_s) \lambda^{i,s} \phi_t^{i,s}\right)^{1-\alpha_s}}\right] \\
&= \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} \left\{\log\left[(C_t^{T})^{\alpha_s} (C_t^{N})^{1-\alpha_s}\right] + v_{t,s}\right\} \\
&= \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} \log\left[(C_t^{T})^{\alpha_s} (C_t^{N})^{1-\alpha_s}\right] + V_t \\
&= \log\left[(C_t^{T})^{\sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} \alpha_s} (C_t^{N})^{\sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} (1-\alpha_s)}\right] + V_t
\end{aligned}
$$

where $v_{t,s} = \log\left[\frac{\left(\frac{1-\alpha_s}{\alpha_s}\right)^{1-\alpha_s} \alpha_s \lambda^{i,s} \phi_t^{i,s}}{\left(\sum_s N_{t,s} \alpha_s \lambda^{i,s} \phi_t^{i,s}\right)^{\alpha_s} \left(\sum_s N_{t,s} (1-\alpha_s) \lambda^{i,s} \phi_t^{i,s}\right)^{1-\alpha_s}}\right]$ and $V_t = \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} v_{t,s}$. The latter is a time-varying exogenous intercept in the objective function, as such it will not enter the first order conditions of the representative agent's maximization problem. Following Jones (2018), $\phi_t^{i,s}$ is always equal to 1 and the Pareto weights attached to each individual, $\lambda^{i,s}$, are the same for each individual. Without loss of generality, I will assume that $\lambda^{i,s}$ is equal to 1 for all individuals. Hence the final representation is:[33]

$$
\boxed{\sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} u_s(c_{t,s}^{T}, c_{t,s}^{N}) \quad = \quad \log\left[(C_t^{T})^{\gamma_t^{T}} (C_t^{N})^{\gamma_t^{N}}\right] + V_t}
$$

[33] Notice that this case with two types of goods encompasses easily the case of one composite good considered by Jones (2018). Suppose there are only T-goods, hence set $\alpha_s = 1\ \forall s$ (the term V_t disappears with a unique first order condition). Then:

$$
\sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} u_s(c_{t,s}^{T}) = \sum_s N_{t,s} \log C_t^{T}
$$

which is the case considered by Jones (2018) when $\sigma = 1$.

where

$$\gamma_t^T = \sum_s N_{t,s}\alpha_s$$
$$\gamma_t^N = \sum_s N_{t,s}(1-\alpha_s)$$

Hence, using the tractable case of logarithmic preferences ($\sigma = 1$), in the aggregate representation of the model the representative household subject to the aggregate resource constraint solves, with respect to consumption:

$$\max_{\{C_t^T, C_t^N\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left[(C_t^T)^{\gamma_t^T} (C_t^N)^{\gamma_t^N} \right] + V_t \right\}$$
$$\text{s.t.} \quad C_t^T + Z_t C_t^N = \cdots$$

Finally, it is possible to back up the consumption aggregator, C_t (to which a proper price index must be associated).[34] Consider the individual's problem seen above (point 1.) with logarithmic utility $u_s(c_{t,s}^T, c_{t,s}^N) = \log[(c_{t,s}^T)^{\alpha_s}(c_{t,s}^N)^{1-\alpha_s}]$. Then, from the first order conditions it results:

$$c_{t,s}^T = \frac{\lambda^{i,s}\phi_t^{i,s}\alpha_s}{\lambda_t}$$
$$c_{t,s}^N = \frac{\lambda^{i,s}\phi_t^{i,s}(1-\alpha_s)}{Z_t\lambda_t}$$

Hence:

$$C_t = \sum_s N_{t,s}\left(c_{t,s}^T\right)^{\alpha_s}\left(c_{t,s}^N\right)^{1-\alpha_s} = \sum_s N_{t,s}\left(\frac{\lambda^{i,s}\phi_t^{i,s}\alpha_s}{\lambda_t}\right)^{\alpha_s}\left(\frac{\lambda^{i,s}\phi_t^{i,s}(1-\alpha_s)}{Z_t\lambda_t}\right)^{1-\alpha_s}$$
$$= \frac{1}{\lambda_t}\sum_s N_{t,s}\lambda^{i,s}\phi_t^{i,s}\alpha_s^{\alpha_s}\left(\frac{1-\alpha_s}{Z_t}\right)^{1-\alpha_s}$$

Using the consumption aggregator inflation can be identified from the identity:

$$P_t C_t = P_t^T C_t^T + P_t^N C_t^N$$

from which it follows:

$$\overline{P}_t \equiv \frac{P_t}{P_t^T} = \frac{C_t^T + Z_t C_t^N}{C_t}$$

[34] This is necessary in order to identify the price index $\frac{P_t}{P_t^T}$ in the model and hence inflation: $\Pi_t = \frac{P_t}{P_{t-1}}$ as it is explained in the main text.

Now consider the definition of gross inflation in the T-sector:

$$\Pi_t^T \equiv \frac{P_t^T}{P_{t-1}^T} = \frac{P_t^T}{P_{t-1}^T}\frac{P_t}{P_t}\frac{P_{t-1}}{P_{t-1}} = \frac{\overline{P}_{t-1}}{\overline{P}_t}\Pi_t$$

Hence, the gross inflation relevant for monetary policy is:

$$\Pi_t = \frac{\overline{P}_t}{\overline{P}_{t-1}}\Pi_t^T$$

Since the first order conditions for the aggregate problem gives:

$$\begin{aligned} C_t^T = \sum_s N_{t,s}c_{t,s}^T &= \frac{\sum_s N_{t,s}\alpha_s\lambda^{i,s}\phi_t^{i,s}}{\lambda_t} \\ C_t^N = \sum_s N_{t,s}c_{t,s}^N &= \frac{\sum_s N_{t,s}(1-\alpha_s)\lambda^{i,s}\phi_t^{i,s}}{Z_t\lambda_t} \end{aligned}$$

Hence:

$$\overline{P}_t = \frac{\frac{\sum_s N_{t,s}\alpha_s\lambda^{i,s}\phi_t^{i,s}}{\lambda_t} + Z_t\frac{\sum_s N_{t,s}(1-\alpha_s)\lambda^{i,s}\phi_t^{i,s}}{Z_t\lambda_t}}{\frac{1}{\lambda_t}\sum_s N_{t,s}\lambda^{i,s}\phi_t^{i,s}\alpha_s^{\alpha_s}\left(\frac{1-\alpha_s}{Z_t}\right)^{1-\alpha_s}} \overset{\lambda^{i,s}=\phi_t^{i,s}=1}{=} \frac{\sum_s N_{t,s}}{\sum_s N_{t,s}\alpha_s^{\alpha_s}\left(\frac{1-\alpha_s}{Z_t}\right)^{1-\alpha_s}}$$

which gives:[35]

$$\overline{\Pi}_t \equiv \frac{\overline{P}_t}{\overline{P}_{t-1}} = \frac{\sum_s N_{t,s}}{\sum_s N_{t-1,s}}\frac{\sum_s N_{t-1,s}\alpha_s^{\alpha_s}\left(\frac{1-\alpha_s}{Z_{t-1}}\right)^{1-\alpha_s}}{\sum_s N_{t,s}\alpha_s^{\alpha_s}\left(\frac{1-\alpha_s}{Z_t}\right)^{1-\alpha_s}}$$

A.2 Labor

The social planner (i.e. the representative infinitely lived agent) needs to choose also the aggregate sectoral hours to work in the economy. The static optimal problem considering also the choice of sectoral

[35] Notice that in the special case of age-invariant consumption shares $\alpha_j = \alpha \; \forall j$ it results:

$$\frac{\overline{P}_t}{\overline{P}_{t-1}} = \frac{\sum_s N_{t,s}}{\alpha^\alpha(1-\alpha)^{1-\alpha}\left(\frac{1}{Z_t}\right)^{1-\alpha}\sum_s N_{t,s}}\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}\left(\frac{1}{Z_{t-1}}\right)^{1-\alpha}\sum_s N_{t-1,s}}{\sum_s N_{t-1,s}} = \left(\frac{Z_t}{Z_{t-1}}\right)^{1-\alpha}$$

working hours is:

$$
\begin{aligned}
U(C_t^T, C_t^N, L_t^T, L_t^N) &= \max_{c_{t,s}^T, c_{t,s}^N, h_{t,s}^T, h_{t,s}^N} \left\{ \sum_s N_{t,s} \lambda^{i,s} \phi_t^{i,s} u(c_{t,s}^T, c_{t,s}^N) \right\} \\
\text{s.t.} \qquad & \sum_s N_{t,s} c_{t,s}^T + Z_t \sum_s N_{t,s} c_{t,s}^N = (1-\tau_t)[w_t^T L_t^T + w_t^N L_t^N] + \cdots \\
& \sum_s \underbrace{\left[\chi^{-\frac{1}{\varepsilon}} (h_{t,s}^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (h_{t,s}^N)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}}}_{h_s} N_{t,s} = L_t \\
& L_t^T = \sum_s h_{t,s}^T N_{t,s} \\
& L_t^N = \sum_s h_{t,s}^N N_{t,s}
\end{aligned}
$$

The first order conditions with respect to $h_{t,s}^T$, $h_{t,s}^N$ are (exactly equal to those in the decentralized equilibrium):

$$
\begin{aligned}
h_{t,s}^T &= \chi h_s \left(\frac{w_t^T}{w_t} \right)^{\varepsilon} \\
h_{t,s}^N &= (1-\chi) h_s \left(\frac{w_t^N}{w_t} \right)^{\varepsilon}
\end{aligned}
$$

where

$$
w_t = \left[\chi (w_t^T)^{1+\varepsilon} + (1-\chi)(w_t^N)^{1+\varepsilon} \right]^{\frac{1}{1+\varepsilon}}
$$

The goal now is to prove that the following holds:

$$
L_t = \sum_s \left[\chi^{-\frac{1}{\varepsilon}} (h_{t,s}^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (h_{t,s}^N)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}} N_{t,s} = \left[\chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}}
$$

where $L_t^T = \sum_s h_{t,s}^T N_{t,s}$, $L_t^N = \sum_s h_{t,s}^N N_{t,s}$, namely to prove that choosing the individual sectoral hours $(h_{t,s}^T, h_{t,s}^N)$ is equivalent to choose the aggregate hours (L_t^T, L_t^N) under that CES aggregator for the social planner. To prove it, start from the definition of aggregate hours, plugging into the individual's first order conditions:

$$
\begin{aligned}
L_t^T &= \sum_s h_{t,s}^T N_{t,s} = \chi \left(\frac{w_t^T}{w_t} \right)^{\varepsilon} \sum_s h_s N_{t,s} \\
L_t^N &= \sum_s h_{t,s}^N N_{t,s} = (1-\chi) \left(\frac{w_t^N}{w_t} \right)^{\varepsilon} \sum_s h_s N_{t,s}
\end{aligned}
$$

that imply:

$$w_t^T = \left(\frac{L_t^T}{\chi \sum_s h_s N_{t,s}}\right)^{\frac{1}{\varepsilon}}$$

$$w_t^N = \left(\frac{L_t^N}{(1-\chi) \sum_s h_s N_{t,s}}\right)^{\frac{1}{\varepsilon}}$$

plug the last two expressions into the expression for the wage (which is implied by the individual's problem), $w_t = \left[\chi (w_t^T)^{1+\varepsilon} + (1-\chi)(w_t^N)^{1+\varepsilon}\right]^{\frac{1}{1+\varepsilon}}$, to have:

$$L_t = \sum_s h_s N_{t,s} = \left[\chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{\varepsilon+1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon+1}}$$

□

B Solving the model with demographic wedges

B.1 Optimal conditions

In this section it is presented the derivation of the optimal conditions for the model of section 2.3.

Household. Have, as in the previous section, : $\gamma_t^T = \sum_{j=0}^J N_{t,j}\alpha_j, \gamma_t^N = \sum_{j=0}^J N_{t,j}(1-\alpha_j)$ The Lagrangian for the representative household's problem is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \log \left[(C_t^T)^{\gamma_t^T} (C_t^N)^{\gamma_t^N}\right] +$$

$$+\sum_{t=0}^{\infty} \beta^t \lambda_t \left[(1-\tau_t)(w_t^T L_t^T + w_t^N L_t^N) + r_t K_{t-1} + R_{t-1}\frac{B_{t-1}}{P_t^T} - K_t + (1-\delta)K_{t-1} + T_t - C_t^T - Z_t C_t^N - \frac{B_t}{P_t^T}\right] +$$

$$+\sum_{t=0}^{\infty} \beta^t \lambda_t \nu_t \left\{L_t - \left[\chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{\varepsilon+1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon+1}}\right\}$$

where λ_t and $\lambda_t \nu_t$ are the two Lagrangian multipliers associated with the two constraints. The first order conditions for sectoral consumptions, capital and nominal bonds are:

$$
\begin{aligned}
C_t^T &: \frac{\gamma_t^T}{C_t^T} = \lambda_t \\
C_t^N &: \frac{\gamma_t^N}{C_t^N} = Z_t \lambda_t \\
K_t &: \beta(1-\delta+r_{t+1})\lambda_{t+1} = \lambda_t \\
B_t &: \beta E_t \left[\frac{R_t}{\Pi_{t+1}^T} \lambda_{t+1} \right] = \lambda_t
\end{aligned}
$$

The first order conditions for sectoral hours worked read:

$$
\begin{aligned}
L_t^T &: (1-\tau_t) w_t^T = \nu_t L_t^{-\frac{1}{\varepsilon}} \chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{1}{\varepsilon}} \\
L_t^N &: (1-\tau_t) w_t^N = \nu_t L_t^{-\frac{1}{\varepsilon}} (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{1}{\varepsilon}} \\
\nu_t &: L_t = \left[\chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}}
\end{aligned}
$$

where the following result has been used:

$$
\begin{aligned}
\left[\chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}-1} &= \left[\chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}} \left[\chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}\frac{-(\varepsilon+1)}{\varepsilon}} \\
&= L_t L_t^{-\frac{\varepsilon+1}{\varepsilon}} = L_t^{-\frac{1}{\varepsilon}}
\end{aligned}
$$

From the ratio of the first two conditions:

$$
\frac{L_t^N}{L_t^T} = \frac{1-\chi}{\chi} \left(\frac{w_t^N}{w_t^T} \right)^{\varepsilon}
$$

By using this condition one can derive an expression for the aggregate real wage (w_t) as a composite of the two sectors real wage by imposing the identity:

$$
w_t^T L_t^T + w_t^N L_t^N = w_t \left[\chi^{-\frac{1}{\varepsilon}} (L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} (L_t^N)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}}
$$

$$
\text{i.e.} \qquad w_t^T + w_t^N \frac{L_t^N}{L_t^T} = w_t \left[\chi^{-\frac{1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} \left(\frac{L_t^N}{L_t^T} \right)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}}
$$

By plugging into the relationship between the hours worked in the two sectors found above, it results:

$$
w_t^T + w_t^N \frac{1-\chi}{\chi} \left(\frac{w_t^N}{w_t^T} \right)^{\varepsilon} = w_t \left[\chi^{-\frac{1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}} \left(\frac{1-\chi}{\chi} \left(\frac{w_t^N}{w_t^T} \right)^{\varepsilon} \right)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}}
$$

simplifying:

$$1 + \frac{1-\chi}{\chi}\left(\frac{w_t^N}{w_t^T}\right)^{\varepsilon+1} = \left(\frac{w_t}{w_t^T}\right)^{\varepsilon+1}$$

i.e.

$$w_t = \left[\chi(w_t^T)^{\varepsilon+1} + (1-\chi)(w_t^N)^{\varepsilon+1}\right]^{\frac{1}{\varepsilon+1}}$$

Given this representation, that is: $w_t^T L_t^T + w_t^N L_t^N = w_t L_t$ with $w_t = \left[\chi(w_t^T)^{\varepsilon+1} + (1-\chi)(w_t^N)^{\varepsilon+1}\right]^{\frac{1}{\varepsilon+1}}$, from the Lagrangian above it must be:

$$\nu_t = (1-\tau_t)w_t$$

Hence, from the first order conditions above:

$$\boxed{\begin{aligned} L_t^T &= \chi L_t \left(\frac{w_t^T}{w_t}\right)^{\varepsilon} \\ L_t^N &= (1-\chi) L_t \left(\frac{w_t^N}{w_t}\right)^{\varepsilon} \\ w_t &= \left[\chi(w_t^T)^{\varepsilon+1} + (1-\chi)(w_t^N)^{\varepsilon+1})\right]^{\frac{1}{\varepsilon+1}} \end{aligned}}$$

where L_t is exogenous and given the choice of sectoral hours worked above is such that:

$$L_t = \left[\chi^{-\frac{1}{\varepsilon}}(L_t^T)^{\frac{\varepsilon+1}{\varepsilon}} + (1-\chi)^{-\frac{1}{\varepsilon}}(L_t^N)^{\frac{\varepsilon+1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon+1}}$$

Firms. Under monopolistic competition, each intermediate good producer $i \in [0,1]$ for each sector $s \in \{T, N\}$ chooses the price level P_{it}^s in order to maximize expected discounted nominal profits:

$$\begin{aligned} &\max_{P_{it}^s} \left\{ E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[(P_{it}^s - MC_t^s) Y_{it}^s - \frac{\theta_s}{2}\left(\frac{P_{it}^s}{\Pi^s P_{it-1}^s} - 1\right)^2 P_t^s Y_t^s \right] \right\} \\ &\text{s.t.} \quad Y_{it}^s = \left(\frac{P_{it}^s}{P_t^s}\right)^{-\eta^s} Y_t^s \end{aligned}$$

Namely:

$$\max_{P_{it}^s} \left\{ E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[(P_{it}^s - MC_t^s) \left(\frac{P_{it}^s}{P_t^s}\right)^{-\eta^s} Y_t^s - \frac{\theta_s}{2}\left(\frac{P_{it}^s}{\Pi^s P_{it-1}^s} - 1\right)^2 P_t^s Y_t^s \right] \right\}$$

In real terms, given that it has been chosen to express all real variables in terms of the price of T-goods:

$$\max_{P_{it}^s} \left\{ E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[(P_{it}^s - MC_t^s) \left(\frac{P_{it}^s}{P_t^s} \right)^{-\eta^s} \frac{Y_t^s}{P_t^T} - \frac{\theta_s}{2} \left(\frac{P_{it}^s}{\Pi^s P_{it-1}^s} - 1 \right)^2 P_t^s \frac{Y_t^s}{P_t^T} \right] \right\}$$

The first order condition of this problem reads:

$$\Lambda_{t,t}(1-\eta^s)\frac{(P_{it}^s)^{-\eta^s}}{(P_t^s)^{-\eta^s}}\frac{Y_t^s}{P_t^T} - \Lambda_{t,t}(-\eta^s)MC_t^s\frac{(P_{it}^s)^{-\eta^s-1}}{(P_t^s)^{-\eta^s}}\frac{Y_t^s}{P_t^T} - \theta_s\Lambda_{t,t}\left(\frac{P_{it}^s}{\Pi^s P_{it-1}^s}-1\right)\frac{P_t^s}{\Pi^s P_{it-1}^s}\frac{Y_t^s}{P_t^T}$$
$$-\theta_s E_t\Lambda_{t,t+1}\left(\frac{P_{it+1}^s}{\Pi^s P_{it}^s}-1\right)(-1)\frac{P_{it+1}^s P_{t+1}^s Y_{t+1}^s}{\Pi^s P_{it}^s P_{it}^s P_{t+1}^T} = 0$$

The contemporaneous discount factor $\Lambda_{t,t} = 1$, while $\Lambda_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$. Considering the symmetric equilibrium where $P_{it}^s = P_t^s$ for all i, the above first order condition reads:

$$(1-\eta^s)\frac{Y_t^s}{P_t^T} + \eta^s\frac{MC_t^s}{P_t^s}\frac{Y_t^s}{P_t^T} - \theta_s\left(\frac{P_t^s}{\Pi^s P_{t-1}^s}-1\right)\frac{P_t^s}{\Pi^s P_{t-1}^s}\frac{Y_t^s}{P_t^T} - \theta_s E_t\Lambda_{t,t+1}\left(\frac{P_{t+1}^s}{\Pi^s P_t^s}-1\right)(-1)\frac{P_{t+1}^s P_{t+1}^s Y_{t+1}^s}{\Pi^s P_t^s P_t^s P_{t+1}^T} = 0$$

or

$$(1-\eta^s) + \eta^s mc_t^s - \theta_s\left(\frac{P_t^s}{\Pi^s P_{t-1}^s}-1\right)\frac{P_t^s}{\Pi^s P_{t-1}^s}\frac{Y_t^s}{P_t^T} + \theta_s E_t\Lambda_{t,t+1}\left(\frac{P_{t+1}^s}{\Pi^s P_t^s}-1\right)\frac{P_{t+1}^s P_{t+1}^s P_t^T Y_{t+1}^s}{\Pi^s P_t^s P_t^s P_{t+1}^T Y_t^s} = 0$$

where $mc_t^s \equiv \frac{MC_t^s}{P_t^s}$. Using the gross inflation definition: $\Pi_t^s \equiv \frac{P_t^s}{P_{t-1}^s}$ it results:

$$(1-\eta^s) + \eta^s mc_t^s - \theta_s\left(\frac{\Pi_t^s}{\Pi^s}-1\right)\frac{\Pi_t^s}{\Pi^s} + \theta_s E_t\Lambda_{t,t+1}\left(\frac{\Pi_{t+1}^s}{\Pi^s}-1\right)\frac{\Pi_{t+1}^s}{\Pi^s}\frac{P_{t+1}^s P_t^T Y_{t+1}^s}{P_t^s P_{t+1}^T Y_t^s} = 0$$

Hence, in the two sectors the first order conditions read:

$$\boxed{(1-\eta^T) + \eta^T mc_t^T - \theta_T\left(\frac{\Pi_t^T}{\Pi^T}-1\right)\frac{\Pi_t^T}{\Pi^T} + \theta_T E_t\Lambda_{t,t+1}\left(\frac{\Pi_{t+1}^T}{\Pi^T}-1\right)\frac{\Pi_{t+1}^T}{\Pi^T}\frac{Y_{t+1}^T}{Y_t^T} = 0}$$

$$\boxed{(1-\eta^N) + \eta^N mc_t^N - \theta_N\left(\frac{\Pi_t^N}{\Pi^N}-1\right)\frac{\Pi_t^N}{\Pi^N} + \theta_N E_t\Lambda_{t,t+1}\left(\frac{\Pi_{t+1}^N}{\Pi^N}-1\right)\frac{\Pi_{t+1}^N}{\Pi^N}\frac{Z_{t+1}Y_{t+1}^N}{Z_t Y_t^N} = 0}$$

where $Z_t \equiv \frac{P_t^N}{P_t^T}$, $\lambda_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$ and $mc_s^s = \frac{MC_t^s}{P_t^s}$ is identified immediately below.

Each monopolistically competitive intermediate firm i in each sector s needs to minimise total nominal cost subject to the production function taking prices as given:

$$\max_{L_{it}^s, K_{it}^s} -W_t^s L_{it}^s - P_t^T r_t K_{it}^s + MC_{it}^s[(K_{it}^s)^\psi (A_t^s L_{it}^s)^{1-\psi} - Y_{it}^s]$$

where r_t is the real capital rental rate in T-goods unit and MC_{it}^s is the Lagrangian multiplier which coincides with the nominal marginal cost. The first order conditions of this problem read:

$$
\begin{aligned}
W_t^s &= MC_{it}^s(1-\psi)\left(\frac{K_{it}^s}{L_{it}^s}\right)^{\psi}(A_t^s)^{1-\psi} \\
P_t^T r_t &= MC_{it}^s\psi\left(\frac{L_{it}^s}{K_{it}^s}\right)^{1-\psi}(A_t^s)^{1-\psi} \\
Y_{it}^s &= (K_{it}^s)^{\psi}(A_t^s L_{it}^s)^{1-\psi}
\end{aligned}
$$

i.e.

$$
\begin{aligned}
MC_{it}^s &= \frac{W_t^s}{(1-\psi)(A_t^s)^{1-\psi}}\left(\frac{L_{it}^s}{K_{it}^s}\right)^{\psi} \\
P_t^T r_t &= \frac{W_t^s\psi}{(1-\psi)}\left(\frac{L_{it}^s}{K_{it}^s}\right) \\
Y_{it}^s &= (K_{it}^s)^{\psi}(A_t^s L_{it}^s)^{1-\psi}
\end{aligned}
$$

i.e.

$$
\begin{aligned}
MC_{it}^s &= \frac{W_t^s}{(1-\psi)(A_t^s)^{1-\psi}}\left(\frac{P_t^T r_t(1-\psi)}{W_t^s\psi}\right)^{\psi} \\
\frac{L_{it}^s}{K_{it}^s} &= \frac{P_t^T r_t(1-\psi)}{W_t^s\psi} \\
Y_{it}^s &= (K_{it}^s)^{\psi}(A_t^s L_{it}^s)^{1-\psi}
\end{aligned}
$$

Notice that all firms i face the same nominal marginal cost since labor and capital inputs are supplied by homogeneous factor markets. Notice also that all firms use the same capital-output ratio. Hence it can be written: $MC_t^s \equiv MC_{it}^s$, $L_{it}^s \equiv L_t^s$, $K_t^s \equiv K_{it}^s$, $Y_t^s \equiv Y_{it}^s$. By further defining the sectoral real wage as $w_t^s \equiv \frac{W_t^s}{P_t^T}$ the above conditions can be rewritten as:

$$
\begin{aligned}
\frac{MC_{it}^s}{P_t^T} &= \frac{\frac{W_t^s}{P_t^T}}{(1-\psi)(A_t^s)^{1-\psi}}\left(\frac{r_t(1-\psi)}{\frac{W_t^s}{P_t^T}\psi}\right)^{\psi} \\
\frac{L_t^s}{K_t^s} &= \frac{r_t(1-\psi)}{\frac{W_t^s}{P_t^T}\psi} \\
Y_t^s &= (K_t^s)^{\psi}(A_t^s L_t^s)^{1-\psi}
\end{aligned}
$$

i.e

$$
\begin{aligned}
MC_t^s &= P_t^T \left(\frac{w_t^s}{(1-\psi)A_t^s}\right)^{1-\psi} \left(\frac{r_t}{\psi}\right)^{\psi} \\
K_{it}^s &= \frac{w_t^s \psi}{r_t(1-\psi)} L_{it}^s \\
Y_t^s &= (K_t^s)^{\psi} (A_t^s L_t^s)^{1-\psi}
\end{aligned}
$$

Now, the relevant real marginal cost entering the boxed first order conditions above for each sector s is:

$$
mc_t^s \equiv \frac{MC_t^s}{P_t^s}
$$

It follows that the first order conditions in the two sectors are:

$$
\boxed{
\begin{aligned}
mc_t^T &\equiv \frac{MC_t^T}{P_t^T} = \left(\frac{w_t^T}{(1-\psi)A_t^T}\right)^{1-\psi} \left(\frac{r_t}{\psi}\right)^{\psi} \\
mc_t^N &\equiv \frac{MC_t^N}{P_t^N} = \frac{1}{Z_t}\left(\frac{w_t^N}{(1-\psi)A_t^N}\right)^{1-\psi} \left(\frac{r_t}{\psi}\right)^{\psi} \\
K_t^T &= \frac{w_t^T \psi}{r_t(1-\psi)} L_t^T \\
K_t^N &= \frac{w_t^N \psi}{r_t(1-\psi)} L_t^N \\
Y_t^T &= (K_t^T)^{\psi} (A_t^T L_t^T)^{1-\psi} \\
Y_t^N &= (K_t^N)^{\psi} (A_t^N L_t^N)^{1-\psi}
\end{aligned}
}
$$

B.2 Optimal conditions in units of labor efficiency

Recall that the the exogenous labor supply in units of efficiency is:

$$
L_t = \sum_{j=0}^{J} h_j N_{t,j}
$$

where the individual's households labor supply in efficiency units h_j varies with age but is constant over time. Since the size of each demographic cohort varies with time, so that in the new long-run steady state the labor-supply is different from the one in the initial steady state, it is convenient to measure all variables relative to L_t for each t. For each variable X_t, have $\widetilde{X}_t \equiv \frac{X_t}{L_t}$. The exception is capital, because of its predetermined nature it will be $\widetilde{K}_{t-1} = \frac{K_{t-1}}{L_t}$. The representative household's first order conditions with respect to sectoral consumptions, capital and bond holdings result:

$$\begin{aligned}
\frac{\gamma_t^T}{\widetilde{C}_t^T} &= L_t \lambda_t \\
\frac{\gamma_t^N}{\widetilde{C}_t^N} &= Z_t L_t \lambda_t \\
\beta(1-\delta+r_{t+1})\lambda_{t+1} &= \lambda_t \\
\beta E_t \left[\frac{R_t}{\Pi_{t+1}^T} \lambda_{t+1} \right] &= \lambda_t
\end{aligned}$$

By subsequent substitutions:

$$\begin{aligned}
\frac{\gamma^T}{\widetilde{C}_t^T} &= L_t \lambda_t \\
\widetilde{C}_t^N &= \frac{\gamma_t^N \widetilde{C}_t^T}{\gamma_t^T Z_t} \\
\beta E_t \left[(1-\delta+r_{t+1}) \frac{\gamma_{t+1}^T}{L_{t+1} \widetilde{C}_{t+1}^T} \right] &= \frac{\gamma_t^T}{L_t \widetilde{C}_t^T} \\
E_t \left[\frac{R_t}{\Pi_{t+1}^T} \right] &= 1-\delta+r_{t+1}
\end{aligned}$$

The Euler equation can be also written as:

$$1 = \beta E_t \left[\frac{\gamma_{t+1}^{T,g}}{L_{t+1}^g} (1-\delta+r_{t+1}) \frac{\widetilde{C}_t^T}{\widetilde{C}_{t+1}^T} \right]$$

where:

$$\begin{aligned}
\gamma_{t+1}^{T,g} &\equiv \frac{\gamma_{t+1}^T}{\gamma_t^T} \\
L_{t+1}^g &\equiv \frac{L_{t+1}}{L_t}
\end{aligned}$$

Hence, the first order conditions result:

$$
\begin{aligned}
\frac{\gamma_t^T}{\widetilde{C}_t^T} &= L_t \lambda_t \\
\widetilde{C}_t^N &= \frac{\gamma_t^N \widetilde{C}_t^T}{\gamma_t^T Z_t} \\
1 &= \beta E_t \left[\frac{\gamma_{t+1}^{T,g}}{L_{t+1}^g} (1-\delta+r_{t+1}) \frac{\widetilde{C}_t^T}{\widetilde{C}_{t+1}^T} \right] \\
E_t \left[\frac{R_t}{\Pi_{t+1}^T} \right] &= 1-\delta+r_{t+1} \\
\widetilde{L}_t^T &= \chi \left(\frac{w_t^T}{w_t} \right)^{\varepsilon} \\
\widetilde{L}_t^N &= (1-\chi) \left(\frac{w_t^N}{w_t} \right)^{\varepsilon} \\
w_t &= \left[\chi (w_t^T)^{\varepsilon+1} + (1-\chi)(w_t^N)^{\varepsilon+1}) \right]^{\frac{1}{\varepsilon+1}} \\
mc_t^T &= \left(\frac{w_t^T}{(1-\psi)A_t^T} \right)^{1-\psi} \left(\frac{r_t}{\psi} \right)^{\psi} \\
mc_t^N &= \frac{1}{Z_t} \left(\frac{w_t^N}{(1-\psi)A_t^N} \right)^{1-\psi} \left(\frac{r_t}{\psi} \right)^{\psi} \\
\widetilde{K}_t^T &= \frac{w_t^T \psi}{r_t(1-\psi)} \widetilde{L}_t^T \\
\widetilde{K}_t^N &= \frac{w_t^N \psi}{r_t(1-\psi)} \widetilde{L}_t^N \\
\widetilde{Y}_t^T &= (\widetilde{K}_t^T)^{\psi} (A_t^T \widetilde{L}_t^T)^{1-\psi} \\
\widetilde{Y}_t^N &= (\widetilde{K}_t^N)^{\psi} (A_t^N \widetilde{L}_t^N)^{1-\psi} \\
\theta_T \left(\frac{\Pi_t^T}{\Pi^T} - 1 \right) \frac{\Pi_t^T}{\Pi^T} &= (1-\eta^T) + \eta^T mc_t^T + \theta_T E_t \left[\Lambda_{t,t+1} \left(\frac{\Pi_{t+1}^T}{\Pi^T} - 1 \right) \frac{\Pi_{t+1}^T}{\Pi^T} \frac{\widetilde{Y}_{t+1}^T}{\widetilde{Y}_t^T} L_{t+1}^g \right] \\
\theta_N \left(\frac{\Pi_t^N}{\Pi^N} - 1 \right) \frac{\Pi_t^N}{\Pi^N} &= (1-\eta^N) + \eta^N mc_t^N + \theta_N E_t \left[\Lambda_{t,t+1} \left(\frac{\Pi_{t+1}^N}{\Pi^N} - 1 \right) \frac{\Pi_{t+1}^N}{\Pi^N} \frac{Z_{t+1} \widetilde{Y}_{t+1}^N}{Z_t \widetilde{Y}_t^N} L_{t+1}^g \right] \\
\widetilde{K}_{t-1} &= \widetilde{K}_t^N + \widetilde{K}_t^T \\
\widetilde{C}_t^N &= \widetilde{Y}_t^N - \frac{\theta_N}{2} \left(\frac{\Pi_t^N}{\Pi^N} - 1 \right)^2 \widetilde{Y}_t^N \\
\widetilde{C}_t^T + L_{t+1}^g \widetilde{K}_t &= (1-\delta)\widetilde{K}_{t-1} + \widetilde{Y}_t^T - \frac{\theta_T}{2} \left(\frac{\Pi_t^T}{\Pi^T} - 1 \right)^2 \widetilde{Y}_t^T \\
\Lambda_{t,t+1} &\equiv \beta \frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1-\delta+r_{t+1}}
\end{aligned}
$$

$$
\begin{aligned}
\Pi_t^N &= \frac{Z_t}{Z_{t-1}} \Pi_t^T \\
\overline{\Pi}_t &= \frac{\sum_j N_{t,j}}{\sum_s N_{t-1,j}} \frac{\sum_j N_{t-1,j} \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z_{t-1}}\right)^{1-\alpha_j}}{\sum_j N_{t,j} \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z_t}\right)^{1-\alpha_j}} \\
\Pi_t &= \overline{\Pi}_t \Pi_t^T \\
R_t &= \left(\frac{R_t}{R}\right)^{\phi_R} \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi(1-\phi_R)}
\end{aligned}
$$

Notice that the presence of age-varying sectoral consumption shares generates non-trivial dynamics in inflation (Π_t) as captured by variation in $\overline{\Pi}_t \equiv \frac{\overline{P}_t}{\overline{P}_{t-1}}$ as a consequence of varying age-structure and relative price of N-goods. Therefore, it is not obvious how, for example, a decrease of the same magnitude in both sectoral inflations (Π_t^N, Π_t^T) translates into variations of aggregate inflation (Π_t). Instead, more obvious the case when the sectoral consumption share is constant across the ages:

$$
\alpha_j = \alpha, \quad \forall j
$$

In this case (see footnote 35):

$$
\overline{\Pi}_t \equiv \frac{\overline{P}_t}{\overline{P}_{t-1}} = \left(\frac{Z_t}{Z_{t-1}}\right)^{1-\alpha}
$$

Hence, since $\frac{Z_t}{Z_{t-1}} = \frac{\Pi_t^N}{\Pi_t^T}$ it follows that:

$$
\Pi_t = \left(\frac{\Pi_t^N}{\Pi_t^T}\right)^{1-\alpha} \Pi_t^T = (\Pi_t^T)^{\alpha} (\Pi_t^N)^{1-\alpha}
$$

which is the usual way of aggregating sectoral inflations when sectoral consumption shares are constant.

B.3 Steady states

The ultimate goal is to study a transition from an initial to a final steady state where the (all exogenous) demographic variables permanently set at a different level from the initial one. A steady state here is such that for each variable X_t, $X_t = X$ for all t given the permanent level of demographic variables. Hence, the value X might differ in the two steady states depending on the level exogenously taken by the demographic variables. Both the initial and the final steady state are such that the demographic variables do not change:

$$
\gamma_t^{T,g} = \gamma_t^{N,g} = L_t^g = 1, \quad \forall t
$$

and the central bank hits always target inflation Π^* such that:

$$\Pi_t = \Pi_t^T = \Pi_t^N = \Pi^*, \quad \forall t$$

The level of the demographic variables will be different in the two steady states: γ_t^T, γ_t^N, L_t will all have a different value in the final steady state as compared to the initial one (unless in the data it is found the contrary). As a consequence of this demographic change, for example, the relative price of N-goods, Z_t, might differ in the final steady state. But this will not prevent sectoral inflations to be always at target level. Finally, to fully isolate the effect of demographic change it is further assumed that the productivity parameters are always constant (both in the initial and final steady state): $A_t^T = A^T$, $A_t^N = A^N$ for all t. It is convenient to consider the initial and the final steady state separately.

B.3.1 Initial steady state

Following Cantelmo and Melina (2017), it is assumed that in the initial steady state $\chi = \frac{L^T}{L} = \widetilde{L}^T$, namely that real wages are equalized across sectors ($w^N = w^T = w$). In this way, χ results a variable to be determined in the initial steady state which thus has an analytical solution, here derived. The optimal conditions in the previous section, in steady state read:

$$
\begin{aligned}
r &= \frac{1}{\beta} - (1-\delta) \\
\Pi^T &= \Pi^* \\
\Pi^N &= \Pi^T \\
\Pi &= \Pi^T \\
R &= \frac{\Pi^T}{\beta} \\
mc^T &= \frac{\eta^T - 1}{\eta^T} \\
mc^N &= \frac{\eta^N - 1}{\eta^N} \\
w^T &= (mc^T)^{\frac{1}{1-\psi}} \left(\frac{r}{\psi}\right)^{-\frac{\psi}{1-\psi}} (1-\psi)A^T \\
w^N &= w^T \\
w &= w^T \\
Z &= \frac{mc^T}{mc^N} \left(\frac{A^T}{A^N}\right)^{1-\psi} \\
\widetilde{L}^T &= \chi \\
\widetilde{L}^N &= (1-\chi) \\
\widetilde{K}^T &= \frac{w^T\psi}{r(1-\psi)}\widetilde{L}^T \\
\widetilde{K}^N &= \frac{w^N\psi}{r(1-\psi)}\widetilde{L}^N \\
\widetilde{Y}^T &= (\widetilde{K}^T)^{\psi}(A^T\widetilde{L}^T)^{1-\psi} \\
\widetilde{Y}^N &= (\widetilde{K}^N)^{\psi}(A^N\widetilde{L}^N)^{1-\psi} \\
\widetilde{K} &= \widetilde{K}^N + \widetilde{K}^T \\
\widetilde{C}^T &= \frac{\gamma^T}{L\lambda} \\
\widetilde{C}^N &= \frac{\gamma^N\widetilde{C}^T}{\gamma^T Z}
\end{aligned}
$$

To have an analytical solution of the steady state, the two missing expressions above are the ones for λ and χ that can be retrieved from the clearing conditions in steady state:

$$
\begin{aligned}
\widetilde{C}^N &= \widetilde{Y}^N \\
\widetilde{C}^T + \delta\widetilde{K} &= \widetilde{Y}^T
\end{aligned}
$$

First notice that the capital-labor ratio is easily identified:

$$\widetilde{K} = \widetilde{K}^N + \widetilde{K}^T = \frac{w\psi}{r(1-\psi)}(1-\chi) + \frac{w\psi}{r(1-\psi)}\chi = \frac{w\psi}{r(1-\psi)}$$

Hence:

$$\begin{aligned} \widetilde{K}^T &= \widetilde{K}\chi \\ \widetilde{K}^N &= \widetilde{K}(1-\chi) \\ \widetilde{Y}^T &= \chi\widetilde{K}^\psi (A^T)^{1-\psi} \\ \widetilde{Y}^N &= (1-\chi)\widetilde{K}^\psi (A^N)^{1-\psi} \end{aligned}$$

Then, using the first clearing condition, it results:

$$\frac{\gamma^N \frac{\gamma^T}{L\lambda}}{\gamma^T Z} = (1-\chi)\widetilde{K}^\psi (A^N)^{1-\psi}$$

i.e.

$$\lambda = \frac{\gamma^N}{ZL(1-\chi)\widetilde{K}^\psi (A^N)^{1-\psi}}$$

From the second clearing condition:

$$\frac{\gamma^T}{L\lambda} + \delta\widetilde{K} = \chi\widetilde{K}^\psi (A^T)^{1-\psi}$$

i.e.

$$\lambda = \frac{\gamma^T}{L(\chi\widetilde{K}^\psi (A^T)^{1-\psi} - \delta\widetilde{K})}$$

Equating the two found expressions for λ:

$$\frac{\gamma^N}{ZL(1-\chi)\widetilde{K}^\psi (A^N)^{1-\psi}} = \frac{\gamma^T}{L(\chi\widetilde{K}^\psi (A^T)^{1-\psi} - \delta\widetilde{K})}$$

i.e.

$$\gamma^N(\chi\widetilde{K}^\psi (A^T)^{1-\psi} - \delta\widetilde{K}) = \gamma^T Z(1-\chi)\widetilde{K}^\psi (A^N)^{1-\psi}$$

i.e.

$$\chi\gamma^N\widetilde{K}^\psi (A^T)^{1-\psi} - \gamma^N\delta\widetilde{K} = \gamma^T Z\widetilde{K}^\psi (A^N)^{1-\psi} - \chi\gamma^T Z\widetilde{K}^\psi (A^N)^{1-\psi}$$

i.e.

$$\chi = \frac{\frac{\gamma^N}{\gamma^T}\delta\widetilde{K}^{1-\psi} + Z(A^N)^{1-\psi}}{\frac{\gamma^N}{\gamma^T}(A^T)^{1-\psi} + Z(A^N)^{1-\psi}}$$

$$\lambda = \frac{\gamma^N}{ZL(1-\chi)\widetilde{K}^{\psi}(A^N)^{1-\psi}}$$

These expressions make clear that one does not need values for labor in efficiency units, L, to identify prices and variables expressed in units of labor efficiency in steady state. Indeed, by substituting the found expression for λ into the above expression for $\widetilde{C}^T$ (the only expression where λ and L appear) it results:

$$\widetilde{C}^T = \frac{\gamma^T Z(1-\chi)\widetilde{K}^{\psi}(A^N)^{1-\psi}}{\gamma^N}$$

namely, L cancels out. That is, the exogenous labor supply (in efficiency units) does not matter to determine prices in the initial steady state. An higher labor supply would only lead quantities to adjust to their constant value in units of labor supply determined by relative prices (which in turn depend only on structural parameters such as production function parameters, ψ, A^T, A^N preferences parameters, β, η^T, η^N, depreciation δ).

Apart from L, the other way demographics enter into the steady state is via the ratio of the two parameters γ^N/γ^T which measures how much demand is biased relatively more on N-goods due to age-varying consumption preferences. Suppose an economy has an older age-structure which gives an higher γ^N/γ^T. How will the initial steady state of such an economy be characterized? The relative consumption of N-goods will be higher:

$$\frac{C^N}{C^T} = \frac{\widetilde{C}^N}{\widetilde{C}^T} = \frac{\gamma^N}{\gamma^T Z}$$

as the relative price of N-goods, Z, is only determined by the relative labor productivity parameter, $(A^T/A^N)^(1-\psi)$, and by the relative real marginal costs, mc^T/mc^N, which in turn depends only on the constant price markup (i.e. on the constant demand elasticity of substitution between differentiated intermediate goods in each sector, η^T, η^N). This must be met by an higher relative production of N-goods which in the initial steady state is reached by an increase in the parameter χ. Indeed, it can be seen from the framed expression for χ above that

$$\frac{\partial\chi}{\partial\frac{\gamma^N}{\gamma^T}} < 0 \Longleftrightarrow \delta\widetilde{K}^{1-\psi} < (A^T)^{1-\psi}$$

which is always satisfied.[36] Hence, an older economy will lead to a higher fraction of the labor force to

[36] It is equivalent to $\delta\widetilde{K} < \widetilde{K}^{\psi}(A^T)^{1-\psi}$: depreciated capital needs to be smaller than output of T-goods, $\widetilde{Y}_t^T = \chi\widetilde{K}^{\psi}(A^T)^{1-\psi}$,

be employed in the N-sector as older cohorts tend to consume relatively more N-goods.

Finally, the expression for steady state aggregate output (in T-goods units) is:

$$\begin{aligned}\widetilde{Y} = \widetilde{Y}^T + Z\widetilde{Y}^N &= \chi\widetilde{K}^{\psi}(A^T)^{1-\psi} + Z(1-\chi)L\widetilde{K}^{\psi}(A^N)^{1-\psi} \\ &= \widetilde{K}^{\psi}\left[\chi(A^T)^{1-\psi} + Z(1-\chi)(A^N)^{1-\psi}\right]\end{aligned}$$

B.3.2 Final steady state

The final steady state is the result of a demographic transition after which the demographic variables (the only source of exogenous variation in the model) remain at the same level reached at that point forever. The three demographic variables that are allowed to change in level in the final steady state as compared to the initial one are: L_t, γ_t^T and γ_t^N. It has already been shown in the previous section that L_t is irrelevant for the determination of prices in steady state. Instead, the ratio γ^N/γ^T is relevant. Furthermore, the final steady state will differ from the initial one because wage equalization is not imposed any more, i.e. the parameter χ is the same of the initial steady state.[37] Hence, potential changes in relative consumption induced by the change in γ^N/γ^T will need to be compensated by changes in relative prices. In the final steady state nominal variables are at their initial level: both aggregate and sectoral inflations are at the target level Π^*, the gross nominal interest rate is at Π^*/β. Real variables can have a different value due to permanent demographic change. While variables in the initial steady state are denoted by no subscript,

(as long as one admits that consumption $\widetilde{C}^T$ is positive), as the clearing on T-market requires $\widetilde{C}_t^T + \delta\widetilde{K} = \chi\widetilde{K}^{\psi}(A^T)^{1-\psi}$, i.e. $\delta\widetilde{K} < \chi\widetilde{K}^{\psi}(A^T)^{1-\psi}$. Hence, *a fortiori*, given $0 < \chi < 1$: $\delta\widetilde{K} < \widetilde{K}^{\psi}(A^T)^{1-\psi}$. $\square$

[37] Notice that this is a direct consequence of how imperfect labor mobility is modelled in this context: for given relative wage, the representative household has a fixed preference to work into either the T-sector or the N-sector which depends on the parameters χ and ε.

denote the final steady state with the subscript f. The following system identifies the final steady state:

$$
\begin{aligned}
mc_f^T &= mc^T = \frac{\eta^T - 1}{\eta^T} \\
mc_f^N &= mc^N = \frac{\eta^N - 1}{\eta^N} \\
r_f &= r = \frac{1}{\beta} - (1-\delta) \\
w_f^T &= w^T = w = (mc^T)^{\frac{1}{1-\psi}} (1-\psi) A^T \left(\frac{r}{\psi}\right)^{-\frac{\psi}{1-\psi}} \\
Z_f &= \frac{mc^T}{mc^N} \left(\frac{w_f^N}{w^T} \frac{A^T}{A^N}\right)^{1-\psi} \\
\widetilde{L}_f^T &= \chi \left(\frac{w^T}{w_f}\right)^{\varepsilon} \\
\widetilde{L}_f^N &= (1-\chi) \left(\frac{w_f^N}{w_f}\right)^{\varepsilon} \\
\widetilde{K}_f^T &= \frac{w^T \psi}{r(1-\psi)} \widetilde{L}_f^T \\
\widetilde{K}_f^N &= \frac{w_f^N \psi}{r(1-\psi)} \widetilde{L}_f^N \\
\widetilde{Y}_f^T &= (\widetilde{K}_f^T)^{\psi} (A^T \widetilde{L}_f^T)^{1-\psi} \\
\widetilde{Y}_f^N &= (\widetilde{K}_f^N)^{\psi} (A^N \widetilde{L}_f^N)^{1-\psi} \\
w_f &= \left[\chi (w^T)^{\varepsilon+1} + (1-\chi)(w_f^N)^{\varepsilon+1}\right]^{\frac{1}{\varepsilon+1}} \\
\widetilde{K}_f &= \widetilde{K}_f^T + \widetilde{K}_f^N
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{C}_f^T &= \widetilde{Y}_f^T - \delta \widetilde{K}_f \\
\widetilde{C}_f^N &= \frac{\gamma_f^N}{\gamma_f^T} \frac{\widetilde{C}_f^T}{Z_f}
\end{aligned}
$$

By recursively substituting the above equation into one another, the clearing condition in the N-goods market:

$$
\widetilde{C}_f^N = \widetilde{Y}_f^N
$$

is 1 equation in 1 unknown, say the real wage in the N-sector w_f^N, which can be easily solved numerically.

B.4 Log-linearized model

The goal is to set-up a log-linearized version of the model in order to study the transition dynamics from the initial steady state to the final one. To do so, the approach used here is to log-linearize the model around the initial steady state. This means that the final steady state will be modelled in terms of permanent deviations from the initial one. Have the following notation for each variable X_t whose initial steady state value is X:

$$\widehat{x}_t \equiv \frac{X_t - X}{X} \approx \log\left(\frac{X_t}{X}\right)$$

Consider a perfect foresight equilibrium so that the expectation operator is dropped. The log-linearization of most of the equations of section B.2 is somewhat standard (the algebra is reported in the following subsection B.4.1). Start by considering the exogenous time-varying demographic variables, once log-linearized they are:

$$\widehat{\gamma}_t^T = \frac{\sum_j \alpha_j N_j \widehat{n}_{t,j}}{\sum_j \alpha_j N_j} \tag{B.1}$$

$$\widehat{\gamma}_t^N = \frac{\sum_j (1-\alpha_j) N_j \widehat{n}_{t,j}}{\sum_j (1-\alpha_j) N_j} \tag{B.2}$$

$$\widehat{\gamma}_t^{T,g} = \frac{\sum_j \alpha_j N_j (\widehat{n}_{t,j} - \widehat{n}_{t-1,j})}{\sum_j \alpha_j N_j} \tag{B.3}$$

$$\widehat{l}_t^g = \frac{\sum_j h_j N_j (\widehat{n}_{t,j} - \widehat{n}_{t-1,j})}{\sum_j h_j N_j} \tag{B.4}$$

$$A_t = \frac{\sum_{j=0}^J N_j (\widehat{n}_t - \widehat{n}_{t-1})}{\sum_{j=0}^J N_j} - \frac{\sum_{j=0}^J \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j} N_j (\widehat{n}_{t,j} - \widehat{n}_{t-1,j})}{\sum_{j=0}^J \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j} N_j} \tag{B.5}$$

where

$$\widehat{n}_{t,j} = \frac{N_{t,j}}{N_j} - 1$$

is the percentage deviation of the number of people in cohort j in period t from the initial steady state. Furthermore, this additional time-invariant parameter is necessary:

$$B = \frac{\sum_{j=0}^J \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j} N_j (1-\alpha_j)}{\sum_{j=0}^J \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j} N_j}$$

Given the above exogenous variables, the following system of equations holds for each period t:

$$
\begin{aligned}
\widehat{\widetilde{c}}_t^N &= \widehat{\widetilde{c}}_t^T - \widehat{z}_t + \widehat{\gamma}_t^N - \widehat{\gamma}_t^T && \text{(B.6)}\\
\widehat{\widetilde{c}}_t^T &= \widehat{\widetilde{c}}_{t+1}^T - \frac{r}{1-\delta+r}\widehat{r}_{t+1} - \widehat{\gamma}_{t+1}^{T,g} + \widehat{l}_{t+1}^g && \text{(B.7)}\\
\widehat{\widetilde{c}}_t^T &= \widehat{\widetilde{c}}_{t+1}^T - (\widehat{\mathrm{R}}_t - \widehat{\Pi}_{t+1}^T) - \widehat{\gamma}_{t+1}^{T,g} + \widehat{l}_{t+1}^g && \text{(B.8)}\\
\widehat{\widetilde{l}}_t^T &= \varepsilon\widehat{\omega}_t^T - \varepsilon\widehat{\omega}_t && \text{(B.9)}\\
\widehat{\widetilde{l}}_t^N &= \varepsilon\widehat{\omega}_t^N - \varepsilon\widehat{\omega}_t && \text{(B.10)}\\
\widehat{\omega}_t &= \chi\widehat{\omega}_t^T + (1-\chi)\widehat{\omega}_t^N && \text{(B.11)}\\
\widehat{mc}_t^T &= (1-\psi)\widehat{\omega}_t^T + \psi\widehat{r}_t - (1-\psi)\widehat{a}_t^T && \text{(B.12)}\\
\widehat{mc}_t^N &= (1-\psi)\widehat{\omega}_t^N + \psi\widehat{r}_t - \widehat{z}_t - (1-\psi)\widehat{a}_t^N && \text{(B.13)}\\
\widehat{\widetilde{k}}_t^T &= \widehat{\omega}_t^T - \widehat{r}_t + \widehat{\widetilde{l}}_t^T && \text{(B.14)}\\
\widehat{\widetilde{k}}_t^N &= \widehat{\omega}_t^N - \widehat{r}_t + \widehat{\widetilde{l}}_t^N && \text{(B.15)}\\
\widehat{\widetilde{y}}_t^T &= \psi\widehat{\widetilde{k}}_t^T + (1-\psi)\widehat{\widetilde{l}}_t^T + (1-\psi)\widehat{a}_t^T && \text{(B.16)}\\
\widehat{\widetilde{y}}_t^N &= \psi\widehat{\widetilde{k}}_t^N + (1-\psi)\widehat{\widetilde{l}}_t^N + (1-\psi)\widehat{a}_t^N && \text{(B.17)}\\
\widehat{\widetilde{k}}_{t-1} &= \chi\widehat{\widetilde{k}}_t^T + (1-\chi)\widehat{\widetilde{k}}_t^N && \text{(B.18)}\\
\widehat{\widetilde{c}}_t^N &= \widehat{\widetilde{y}}_t^N && \text{(B.19)}\\
\widehat{\widetilde{y}}_t^T &= \frac{\widetilde{C}^T}{\widetilde{Y}^T}\widehat{\widetilde{c}}_t^T + \frac{\widetilde{K}}{\widetilde{Y}^T}\left[\widehat{l}_{t+1}^g + \widehat{\widetilde{k}}_t - (1-\delta)\widehat{\widetilde{k}}_{t-1}\right] && \text{(B.20)}\\
\widehat{\Pi}_t^N &= \widehat{z}_t - \widehat{z}_{t-1} + \widehat{\Pi}_t^T && \text{(B.21)}\\
\widehat{\Pi}_t &= \widehat{\widehat{\Pi}}_t + \widehat{\Pi}_t^T && \text{(B.22)}\\
\widehat{\widehat{\Pi}}_t &= A_t + B(\widehat{\Pi}_t^N - \widehat{\Pi}_t^T) && \text{(B.23)}\\
\widehat{\mathrm{R}}_t &= \phi_R\widehat{\mathrm{R}}_{t-1} + \phi_\pi\widehat{\Pi}_t && \text{(B.24)}\\
\frac{\theta_T}{\eta^T-1}\widehat{\Pi}_t^T &= \widehat{mc}_t^T + \frac{\theta_T}{\eta^T-1}\beta\widehat{\Pi}_{t+1}^T && \text{(B.25)}\\
\frac{\theta_N}{\eta^N-1}\widehat{\Pi}_t^N &= \widehat{mc}_t^N + \frac{\theta_N}{\eta^N-1}\beta\widehat{\Pi}_{t+1}^N && \text{(B.26)}
\end{aligned}
$$

Additionally, one would need exogenous processes for the sectoral labor productivities. To isolate the effect of demographics, in this paper it will be assumed that $\widehat{a}_t^T = \widehat{a}_t^N = 0, \quad \forall t$. Then, the above system is composed by 21 equations in 21 endogenous variables:

$$
\left\{\widehat{\widetilde{l}}_t^T, \widehat{\widetilde{l}}_t^N, \widehat{\widetilde{k}}_t^T, \widehat{\widetilde{k}}_t^N, \widehat{\widetilde{k}}_t, \widehat{\widetilde{y}}_t, \widehat{\widetilde{y}}_t^T, \widehat{\widetilde{y}}_t^N, \widehat{\widetilde{c}}_t^N, \widehat{\widetilde{c}}_t^T, \widehat{\omega}_t^T, \widehat{\omega}_t^N, \widehat{\omega}_t, \widehat{r}_t, \widehat{mc}_t^T, \widehat{mc}_t^N, \widehat{z}_t, \widehat{\Pi}_t^T, \widehat{\Pi}_t^N, \widehat{\Pi}_t, \widehat{\mathrm{R}}_t, \widehat{\widehat{\Pi}}_t\right\}
$$

that can be studied in a deterministic transition from period 0 (the initial steady state) to infinity (an arbitrary point sufficiently far in the future when the demographic transition is assumed to stop) where the exogenous variation of the demographic variables above ($\{\widehat{\gamma}_t^T, \widehat{\gamma}_t^N, \widehat{\gamma}_t^{T,g}, \widehat{l}_t^g, A_t\}_{t=0}^{\infty}$) is perfectly anticipated by the agents in the model.

B.4.1 Some log-linearization algebra

Take a first-order Taylor expansion of the natural logarithm of the optimal conditions in section B.2 around the initial steady state. First, consider the exogenous time-varying demographic variable:

$$\gamma_t^T = \sum_j \alpha_j N_{t,j}$$

which is linear already. Hence, in percentage deviations from the initial steady state:

$$\frac{\gamma_t^T - \gamma^T}{\gamma^T} = \frac{\alpha_0 N_0}{\sum_j \alpha_j N_j} \frac{(N_{t,0} - N_0)}{N_0} + \frac{\alpha_1 N_1}{\sum_j \alpha_j N_j} \frac{(N_{t,1} - N_1)}{N_1} + \cdots + \frac{\alpha_J N_J}{\sum_j \alpha_j N_j} \frac{(N_{t,J} - N_J)}{N_J}$$

i.e., using the hat-notation introduced above:

$$\widehat{\gamma}_t^T = \frac{\alpha_0 N_0}{\sum_j \alpha_j N_j} \widehat{n}_{t,0} + \frac{\alpha_1 N_1}{\sum_j \alpha_j N_j} \widehat{n}_{t,1} + \cdots + \frac{\alpha_J N_J}{\sum_j \alpha_j N_j} \widehat{n}_{t,J}$$

i.e., more compactly:

$$\widehat{\gamma}_t^T = \frac{\sum_j \alpha_j N_j \widehat{n}_{t,j}}{\sum_j \alpha_j N_j} \tag{B.27}$$

It follows immediately that the growth rate of this variable in percentage deviation is simply:

$$\widehat{\gamma}_t^{T,g} = \widehat{\gamma}_t^T - \widehat{\gamma}_{t-1}^T = \frac{\sum_j \alpha_j N_j (\widehat{n}_{t,j} - \widehat{n}_{t-1,j})}{\sum_j \alpha_j N_j} \tag{B.28}$$

Symmetrically:

$$\widehat{\gamma}_t^N = \frac{\sum_j (1-\alpha_j) N_j \widehat{n}_{t,j}}{\sum_j (1-\alpha_j) N_j} \tag{B.29}$$

The case of the exogenous labor supply in efficiency units, $L_t = \sum_j h_j N_{t,j}$, is equivalent:

$$\widehat{l}_t = \frac{\sum_j h_j N_j \widehat{n}_{t,j}}{\sum_j h_j N_j}$$

so that:

$$\widehat{l}_t^g = \widehat{l}_t - \widehat{l}_{t-1} = \frac{\sum_j h_j N_j (\widehat{n}_{t,j} - \widehat{n}_{t-1,j})}{\sum_j h_j N_j} \tag{B.30}$$

Notice that the total number of people in the economy in each period t can be obtained by adding the two:

$$\varphi_t \equiv \gamma_t^T + \gamma_t^N = \sum_j N_{t,j}\alpha_j + \sum_j N_{t,j}(1-\alpha_j) = \sum_j N_{t,j} \tag{B.31}$$

Hence:

$$\widehat{\varphi}_t = \frac{\sum_j N_j \widehat{n}_{t,j}}{\sum_j N_j} = \frac{\gamma^T}{\gamma^T+\gamma^N}\widehat{\gamma}_t^T + \frac{\gamma^N}{\gamma^T+\gamma^N}\widehat{\gamma}_t^N$$

and its growth rate:

$$\widehat{\varphi}_t^g = \frac{\sum_j N_j(\widehat{n}_{t,j} - \widehat{n}_{t-1,j})}{\sum_j N_j} \tag{B.32}$$

Notice that in the case of age-invariant consumption shares, i.e. $\alpha_j = \alpha$ for all j, $\gamma_t^T = \gamma_t^N = \varphi_t$, so that $\widehat{\gamma}_t^{T,g} = \widehat{\varphi}_t^g$ is the growth rate of the population.

Next, consider:

$$\overline{P}_t \quad = \quad \frac{\sum_j N_{t,j}}{\sum_j N_{t,j}\alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z_t}\right)^{1-\alpha_j}}$$

Taking the natural logarithm:

$$\log \overline{P}_t = \log\sum_j N_{t,j} - \log\sum_j N_{t,j}\alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z_t}\right)^{1-\alpha_j}$$

Hence, the first order Taylor expansion reads:

$$\begin{aligned}
\frac{\overline{P}_t - \overline{P}}{\overline{P}} \quad &= \quad \frac{N_0}{\sum_j N_j}\frac{(N_{t,0}-N_0)}{N_0} + \cdots + \frac{N_J}{\sum_j N_j}\frac{(N_{t,J}-N_J)}{N_J} + \\
&- \quad \frac{N_0\alpha_0^{\alpha_0}\left(\frac{1-\alpha_0}{Z}\right)^{1-\alpha_0}}{\sum_j N_j\alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}}\frac{(N_{t,0}-N_0)}{N_0} - \cdots - \frac{N_J\alpha_J^{\alpha_J}\left(\frac{1-\alpha_J}{Z}\right)^{1-\alpha_J}}{\sum_j N_j\alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}}\frac{(N_{t,J}-N_J)}{N_J} \\
&- \quad \left[\frac{-(1-\alpha_0)Z^{-(1-\alpha_0)-1}N_0\alpha_0^{\alpha_0}(1-\alpha_0)^{1-\alpha_0}}{\sum_j \alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}}(Z_t - Z) + \cdots + \frac{-(1-\alpha_J)Z^{-(1-\alpha_J)-1}N_J\alpha_J^{\alpha_J}(1-\alpha_J)^{1-\alpha_J}}{\sum_j \alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}}(Z_t - Z)\right]
\end{aligned}$$

i.e.

$$\widehat{\overline{p}}_t \quad = \quad \frac{\sum_j N_j\widehat{n}_{t,j}}{\sum_j N_j} - \frac{\sum_j N_j\alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}\widehat{n}_{t,j}}{\sum_j N_j\alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}} + \frac{\sum_j(1-\alpha_j)N_j\alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}}{\sum_j \alpha_j^{\alpha_j}\left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}}\widehat{z}_t$$

It follows:

$$\hat{\hat{\Pi}}_t = \hat{\hat{p}}_t - \hat{\hat{p}}_{t-1} = \underbrace{\frac{\sum_j N_j(\hat{n}_{t,j} - \hat{n}_{t-1,j})}{\sum_j N_j} - \frac{\sum_j N_j \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j} (\hat{n}_{t,j} - \hat{n}_{t-1,j})}{\sum_j N_j \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}}}_{\equiv A_t}$$

$$+ \quad (\hat{z}_t - \hat{z}_{t-1}) \underbrace{\frac{\sum_j (1-\alpha_j) N_j \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}}{\sum_j \alpha_j^{\alpha_j} \left(\frac{1-\alpha_j}{Z}\right)^{1-\alpha_j}}}_{\equiv B} \tag{B.33}$$

Hence, aggregate inflation results:

$$\hat{\Pi}_t = \hat{\hat{\Pi}}_t + \hat{\Pi}_t^T = A_t + (\hat{\Pi}_t^N - \hat{\Pi}_t^T)B + \hat{\Pi}_t^T = A_t + (1-B)\hat{\Pi}_t^T + B\hat{\Pi}_t^N$$

an expression which shows that a deviation of aggregate inflation from its target is not only a convex combination of deviations of sectoral inflations from their target ($(1-B)\hat{\Pi}_t^T + B\hat{\Pi}_t^N$) but depends also on a time-varying intercept (A_t) which captures deviations of the aggregate sectoral consumption shares from the case of age-invariant sectoral consumption shares due to (exogenously) time-varying age structure. To see this, notice that when the sectoral consumption shares are age-invariant, i.e. $\alpha_j = \alpha, \forall j$, then:

$$\begin{aligned} A_t &= \frac{\sum_{j=0}^J N_j(\hat{n}_t - \hat{n}_{t-1})}{\sum_{j=0}^J N_j} - \frac{\alpha^\alpha \left(\frac{1-\alpha}{Z}\right)^{1-\alpha}}{\alpha^\alpha \left(\frac{1-\alpha}{Z}\right)^{1-\alpha}} \frac{\sum_{j=0}^J N_j(\hat{n}_{t,j} - \hat{n}_{t-1,j})}{\sum_{j=0}^J N_j} = 0 \quad \forall t \\ B &= (1-\alpha)\frac{\alpha^\alpha \left(\frac{1-\alpha}{Z}\right)^{1-\alpha} \sum_{j=0}^J N_j}{\alpha^\alpha \left(\frac{1-\alpha}{Z}\right)^{1-\alpha} \sum_{j=0}^J N_j} = (1-\alpha) \end{aligned}$$

so that the first component of aggregate inflation simplifies to:

$$\hat{\hat{\Pi}}_t = (\hat{\Pi}_t^N - \hat{\Pi}_t^T)(1-\alpha)$$

which in turn implies:

$$\hat{\Pi}_t = \hat{\hat{\Pi}}_t + \hat{\Pi}_t^T = (\hat{z}_t - \hat{z}_{t-1})(1-\alpha) + \hat{\Pi}_t^T = (\hat{\Pi}_t^N - \hat{\Pi}_t^T)(1-\alpha) + \hat{\Pi}_t^T = \alpha\hat{\Pi}_t^T + (1-\alpha)\hat{\Pi}_t^N$$

that is, in this case, a deviation of aggregate inflation from its target is simply a convex combination of deviations of sectoral inflations from their target with weights given by the respective age-invariant consumption share.

B.4.2 Steady states of the log-linearized system

Given that all variables are expressed in deviation from the initial steady state, the log-linearized variables will all have zero value at the initial steady state. Instead, variables in the final steady state are expressed in terms of permanent deviations from the initial steady state, i.e. for each log-variable x_t in deviation from the initial steady state $\widehat{x}_t$, the final steady state is such that:

$$\widehat{x}_t = \widehat{x}, \quad \forall t$$

One can find these permanent deviations analytically (these values will be used to check that the transition leads the economy to the right steady state when the model is solved numerically). Denote with f the time of the final steady state (no subscript for the initial steady state). The only source of exogenous variation that makes the final steady state different from the initial one is the assumption that at a certain point in the future the number of people in each cohort remain at the level reached there forever:

$$\widehat{n}_{t,j} = \widehat{n}_{f,j}, \quad \forall t \geq f, j$$

As a consequence, none of the demographic variables grows in the final steady state:

$$\widehat{\gamma}_f^{T,g} = \widehat{l}_f^g = A_t = 0$$

and the only parameters that differ in the final steady state are the sectoral consumption wedges:

$$\widehat{\gamma}_f^T = \frac{\gamma_f^T}{\gamma^T} - 1, \quad \widehat{\gamma}_f^N = \frac{\gamma_f^N}{\gamma^N} - 1$$

The constancy of consumption coupled with $\widehat{\gamma}_f^{T,g} = \widehat{l}_f^g = 0$ implies that in the final steady state the real interest rate is at its initial level:

$$\widehat{r}_f = 0$$

There is no long-run variation in the nominal variables:

$$\widehat{\Pi}_f^T = \widehat{\Pi}_f^N = \widehat{\Pi}_f = \widehat{R}_f = \widehat{\bar{\Pi}}_f = 0$$

which implies, together with the assumption of no technology growth ($\widehat{a}_t^T = \widehat{a}_t^N = 0, \forall t$) that in the final steady state the sectoral real marginal costs as well as the real wage in the T-sector are at their initial level:

$$\widehat{mc}_f^T = \widehat{mc}_f^N = \widehat{\omega}_f^T = 0$$

To have a reduced-form expression for the other variable, one needs to solve, for example, for the real wage of the N-sector, $\widehat{\omega}_f^N$. From the system above, it results:

$$\widehat{\omega}_f = (1 - \chi)\widehat{\omega}_f^N$$

which, plugged into the expressions for sectoral hours worked, gives:

$$
\begin{aligned}
\widehat{\widetilde{l}}_f^T &= -\varepsilon(1-\chi)\widehat{\omega}_f^N \\
\widehat{\widetilde{l}}_f^N &= \varepsilon\chi\widehat{\omega}_f^N
\end{aligned}
$$

Then:

$$
\begin{aligned}
\widehat{\widetilde{k}}_f^T &= \widehat{\widetilde{y}}_f^T = \widehat{\widetilde{l}}_f^T \\
\widehat{\widetilde{k}}_f^N &= (1+\chi\varepsilon)\widehat{\omega}_f^N
\end{aligned}
$$

which imply:

$$
\widehat{\widetilde{k}}_f = (1-\chi)\widehat{\omega}_f^N
$$

Plug the expressions for $\widehat{\widetilde{k}}_f$ and $\widehat{\widetilde{y}}_f^T$ into the clearing condition in the T-goods market to have

$$
\widehat{\widetilde{c}}_f^T = -(1-\chi)\widehat{\omega}_f^N \left[\varepsilon\frac{\widetilde{Y}^T}{\widetilde{C}^T} + \delta\varepsilon\frac{\widetilde{K}}{\widetilde{C}^T}\right]
$$

then, given that the relative price is:

$$
\widehat{z}_f = (1-\psi)\widehat{\omega}_f^N
$$

from the intra-temporal condition:

$$
\widehat{\widetilde{c}}_f^N = \widehat{\gamma}_f^N - \widehat{\gamma}_f^T - \left[(1-\psi) + (1-\chi)\left(\varepsilon\frac{\widetilde{Y}^T}{\widetilde{C}^T} + \delta\frac{\widetilde{K}}{\widetilde{C}^T}\right)\right]\widehat{\omega}_f^N
$$

Hence, finally, from the cleating condition in the N-goods market ($\widehat{\widetilde{c}}_f^N = \widehat{\widetilde{y}}_f^N$) the reduced-form expression for $\widehat{\omega}_f^N$ is:

$$
\boxed{\widehat{\omega}_f^N = \frac{\widehat{\gamma}_f^N - \widehat{\gamma}_f^T}{\left[1+\chi\varepsilon + (1-\chi)\left(\varepsilon\frac{\widetilde{Y}^T}{\widetilde{C}^T} + \delta\frac{\widetilde{K}}{\widetilde{C}^T}\right)\right]}}
$$

So, as the economy in the long-run shifts to a distribution with permanently more elderly, in comparison to the initial steady state, there will be a consumption bias towards N-goods: $\widehat{\gamma}_f^N - \widehat{\gamma}_f^T > 0$. This will induce an increase of the relative wage of the N-sector, $\widehat{\omega}_f^N > 0$, in order to drain some hours worked in the N-sector, necessary to meet the new relative demand of N-goods. How much the relative wage increases in the long run depends on the representative household's preferences to work in either sector, as captured by the parameters χ, ε. Accordingly, the relative price of N-goods will be higher, $\widehat{z}_f > 0$.

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