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**Abstract**

Epstein-Zin preferences have attracted significant attention within the macro-finance literature based on DSGE models as they allow to substantially increase risk aversion, and consequently generate non-trivial risk premia, without compromising the ability of standard models to achieve satisfactory macroeconomic data coherence. Such appealing features certainly hold for structural modelling frameworks where monetary policy is set according to Taylor-type rules or seeks to minimize an *ad hoc* loss function under commitment. However, Epstein-Zin preferences may have significant quantitative implications for both asset pricing and macroeconomic allocation under a welfare-based monetary policy conduct. Against this background, the paper focuses on the impact of such preferences on the Ramsey approach to monetary policy within a medium-scale model based on Smets and Wouters (2007) including a wide range of nominal and real frictions that have proven to be relevant for quantitative business cycle analysis. After setting an empirical benchmark that generates a mean value of 100 bp for the ten-year term premium, we show that Epstein-Zin preferences significantly affect the macroeconomic outcome when optimal policy is considered. The level and the dynamic pattern of risk premia are also markedly altered. We show that the effect of Epstein-Zin preferences is extremely sensitive to the presence of real rigidities in the form of quasi-kinked demands. We also analyse how this effect can be linked to a combined effect of capital accumulation and wage rigidities.

***Keywords:*** Optimal monetary policy, macroeconometric equivalence, non time-separable preferences, term premium.

***JEL classification:*** E44, E52, E61, G12

**Non-Technical Summary**

The paper examines the implications of Epstein-Zin preferences for both asset pricing – with a special focus on term premium– and macroeconomic dynamics when monetary policy maximizes the social welfare under commitment. The quantitative effects of Epstein-Zin preferences are explored within a medium-scale model which embeds a wide range of nominal and real frictions and has proven to be relevant for quantitative business cycle analysis.

The original contributions of our paper cover several dimensions. First, we set an empirical benchmark where we show how a fully specified model like Smets and Wouters (2007) can be augmented with Epstein-Zin preferences so as to generate a ten-year term premium of 100 bp on average. Compared with Rudebusch and Swanson (2009), we consider a much richer baseline macro-model since we include several structural shocks, endogenous capital accumulation with adjustment costs on investment and variable capital utilisation, wage rigidities and real frictions in the form of quasi-kinked demands. In doing so, we follow up the approach of De Graeve et al. (2009a) who claim that medium-scale DSGE models can describe bond yield dynamics in a satisfactory manner. We also analyse how our calibration of the Epstein-Zin parameter crucially depends on the intertemporal elasticity of substitution and the deterministic discount factor associated with the use of detrended variables. This helps us explain the wide dispersion of values found in the empirical literature.

Second, our paper is linked to the literature analysing the optimal monetary policy within DSGE models. Specifically we focus on the link between Epstein-Zin preferences and optimal policy. To our knowledge only Levin et al. (2008) attempt to assess the influence of Epstein-Zin preferences on optimal macroeconomic allocations. They consider a small stylised New Keynesian model with one shock and they explicitly show that Epstein-Zin preferences enter the first-order approximation of the Ramsey policymaker's first order conditions. In our medium-scale model, we rather rely on numerical simulations that allow us to generalize their approach to a much larger and commonly used model. Again, compared with Levin et al. (2008), we add many specifications that have proven to be empirically relevant. In line with the conclusions of Levin et al. (2008), we provide numerical evidence that in general Epstein-Zin preferences strongly affect the tradeoffs faced by the optimizing policymaker. Our analysis allows to highlight two features. First, the effect of Epstein-Zin preferences is strongly affected by the presence of quasi-kinked demands. Our paper is therefore related to the analysis of *strategic com-*

*plementarities* – quasi-kinked demands or firm-specific factors – and their implications for monetary policy and welfare as conducted by Levin et al. (2007, 2008). Second, we investigate the role of capital accumulation and nominal rigidities. We show that, in a world with Dixit Stiglitz aggregators, with price rigidities only, the deviation from price stability is amplified by Epstein-Zin preferences, be it with capital or not. Adding wage rigidities tend to reduce this amplification.

Finally, unlike both the traditional macro-finance approach and the literature on optimal policy, we analyse the behaviour of the term premium under optimal monetary policy. We find that the effect of Epstein-Zin preferences on the level and the dynamics of the term premium is much stronger under optimal policy and is qualitatively different from the Taylor rule case: the term premium is a non-linear function of the Epstein-Zin parameter and we show how it is substantially shifted up in presence of quasi-kinked demands.

# 1 Introduction

The paper examines the implications of Epstein-Zin preferences for both asset pricing and macroeconomic dynamics when monetary policy maximizes the social welfare under commitment. The quantitative effects of Epstein-Zin preferences are explored within a medium-scale model which embeds a wide range of nominal and real frictions and has proven to be relevant for quantitative business cycle analysis.

Macro-finance literature aims at studying the interactions between the macroeconomy and the pricing of financial claims (yield curve, equity). A structural approach consists in using Dynamic Stochastic General Equilibrium (DSGE) models as a representation of the macroeconomy and then deriving model-consistent non arbitrage constraints so as to price financial assets (see Rudebusch and Swanson, 2008, 2009, De Graeve, Emiris and Wouters, 2009, Ravenna and Seppala, 2006, Doh, 2009, Amisano and Tristani, 2010, to name but a few[1]). Over the last decade considerable progress has been made regarding the specification and the empirical validation of structural macroeconomic models. For example models like Christiano, Eichenbaum and Evans (2005) or Smets and Wouters (2007) have become workhorses for the macroeconomic analysis due to their successful empirical properties. However, if the ability of such models to match a satisfactory level of macroeconomic data coherence is now commonly admitted, their weak performance on the financial side clearly calls for further improvement (see Rudebusch and Swanson, 2008 for example and Tovar, 2008 for a survey). In a nutshell, current generation of structural macroeconomic models generate risk premia that are too small and too little volatile compared with non-structural measures[2]. As emphasized by Cochrane (2007) and Rudebusch and Swanson (2008), asset pricing and macroeconomic behaviours are inextricably linked so that unsatisfactory implications of standard DSGE frameworks for the financial side may reveal crucial misspecifications.

In order to cope with the need to obtain both macroeconomic and financial coherence, recent papers by Andreasen (2009), Rudebusch and Swanson (2009), Guvenen (2009), Amisano and Tristani (2010) – among others – have made use of non time-separable preferences, namely Epstein Zin preferences, in the structural specification of agents'

[1] We can divide the macro-finance literature into three categories. First, numerous papers deal with a purely statistical approach which involves a non-structural representation of the macroeconomy associated with an ad-hoc pricing-kernel (see Ang et al., 2006 for example). Second, some models are built on a structural modelling of the economy but keep the specification of the pricing-kernel ad-hoc (see Hördhal et al. 2007). Third, as reported in the main text, several papers analyse the implications of fully structural models consisting of a structural macro part and its consistent pricing-kernel. In this paper we are interested in the latter approach.

[2] See Rudebusch, Sack and Swanson (2007) for a survey.

utility, within New Keynesian frameworks. As explicitly shown by Swanson (2010), Epstein-Zin preferences provide an additional degree of freedom that allows to increase risk aversion and hence generate substantial time-varying risk premia. In this respect, Rudebusch and Swanson (2009) augment a baseline New Keynesian model with such preferences and are then able to match both macroeconomic and financial moments, including a 106 basis points term premium average for the U.S. ten-year nominal bond. Likewise, Andreasen (2009) provides a joint quadratic estimation of a DSGE macro-model including Epstein-Zin preferences along with a consistent derivation of the yield curve. Epstein-Zin preferences help him generate a mean value of 69 basis points for the term premium. In both papers, the same conclusion holds: Epstein-Zin preferences are a key component to make the link between the macroeconomy and the financial side, as they improve the fit with financial data without significantly deteriorating the already well fitted macroeconomic dynamic. Such appealing features certainly hold for structural modelling frameworks where monetary policy is set according to Taylor-type rule or seeks to minimize an *ad hoc* loss function under commitment. However, Epstein-Zin preferences may have significant quantitative implications for both asset pricing and macroeconomic allocation under a welfare-based monetary policy conduct. As explored by Levin et al (2008), models relying on Epstein-Zin preferences may yield some *microeconomic dissonance*: to a first order approximation, the underlying microeconomic difference in the agents' preferences may affect the macroeconomic allocations under Ramsey policy. Therefore, if Epstein-Zin preferences are to become broadly used within DSGE models – for example, recent papers like Binsbergen et al. (2008), Caldara et al. (2009) provide a methodology to solve and estimate DSGE with non-standard preferences – an analysis of the non-linear properties through their impact on the welfare and the optimal policy is a relevant research avenue.

Against this background, the our paper aims to shed light on the influence of Epstein-Zin preferences on asset pricing and optimal monetary policy within a calibrated medium-scale DSGE model based on Smets and Wouters (2007) and De Graeve et al. (2009). Our study is therefore related to diverse areas of economic research to which we make several contributions. First, our paper contributes to the empirical literature dedicated to the analysis of the implications of macroeconomic models on asset pricing, and especially on the evaluation of the term premium. Our methodology regarding the analysis of the term premium is close to that of Rudebusch, Sack and Swanson (2007), Rudebusch and Swanson (2008, 2009), De Graeve et al. (2009b) and to the second approach presented in Rudebusch (2010). In order to get non-zero term premia, we use sufficiently

high order approximation to a model which juxtaposes the macroeconomic dynamic and a consistent derivation of the yield curve[3]. We contribute to this literature by using a much richer baseline model than that of Rudebusch and Swanson (2009). We show how a fully specified model like Smets and Wouters (2007) can also be augmented with Epstein-Zin preferences so as to generate a ten-year term premium of 100 bp on average. In doing so, we follow up the approach of De Graeve et al. (2009a) who claim that medium-scale DSGE models can describe bond yield dynamics in a satisfactory manner. We also analyse how our calibration of the Epstein-Zin parameter (930) crucially depends on the intertemporal elasticity of substitution and the deterministic discount factor associated with detrended variables. This helps us explain the wide dispersion of values found in the empirical literature. Second, our paper is linked to the literature analysing the optimal monetary policy within DSGE models. Specifically we focus on the link between Epstein-Zin preferences and optimal policy. To our knowledge only Levin et al. (2008) attempt to assess the influence of Epstein-Zin preferences on optimal macroeconomic allocations. They consider a small stylised New Keynesian model with one shock and they explicitly show that Epstein-Zin preferences enter the first-order approximation of the Ramsey policymaker's first order conditions. However, even in a small model, this derivation by hand of the first order approximation of the Ramsey conditions is somewhat cumbersome and provides no further insights into the way Epstein-Zin preferences operate. Consequently, we do not aim to obtain such explicit linearization. We rather rely on numerical simulations that allow us to generalize their approach to a much larger and commonly used model[4]. To our knowledge, our study is the first attempt to derive the optimal policy in such a medium-scale model including Epstein-Zin preferences. Compared with Levin et al. (2008), we add many specifications that have proven to be empirically relevant: endogenous capital accumulation with adjustment costs and investment, several structural shocks, wage rigidities and real frictions. This improvement of the model constitutes an important contribution of our paper since no analytical characterisation of the optimal policy with Epstein-Zin preferences has been so far established. In line with the conclusions of Levin et al. (2008), we

[3] Unlike Jerman (1998), Wu (2006), De Graeve et al. (2009a), Doh (2009), Andreasen (2009) or Amisano and Tristani (2010), this approach does not rely on additional assumptions such as stochastic volatility or the joint-lognormality of pricing kernels and bond prices.

[4] Like Schmitt-Grohe and Uribe (2005), Levin et al. (2005), Adjemian et al (2007, 2008) and Faia (2008, 2009), among others, we follow the Ramsey approach to optimal monetary policy within a medium-scale model. We derive the exact non-linear solution that maximizes the aggregate welfare subject to the Epstein-Zin constraints as well as the equations characterizing the competitive equilibrium. We then derive a Taylor expansion of the optimal policy around the deterministic steady state. Given the degree of sophistication of our model, the policymaker's objective cannot be treated analytically and we consequently rely on numerical simulations.

provide numerical evidence that in general Epstein-Zin preferences have a substantial impact on the first order dynamic of the macroeconomic outcome under optimal monetary policy. In other words, Epstein-Zin preferences strongly affect the tradeoffs faced by the optimizing policymaker. Our analysis allows to highlight two features. First, the effect of Epstein-Zin preferences is strongly affected by the presence of quasi-kinked demands. Our paper is therefore related to the analysis of *strategic complementarities* – quasi-kinked demands or firm-specific factors – and their implications for monetary policy and welfare as conducted by Levin et al. (2007, 2008). Considering relatively small New Keynesian models, they prove that the specific form of strategic complementarities has crucial consequences on the optimal policy. Here we focus on the quasi-kinked demands assumed by Smets and Wouters (2007). As a first attempt to study strategic complementarities in an Epstein-Zin world, we show that, in our empirical benchmark, the effect of Epstein-Zin preferences on the transmission of the optimal monetary policy is extremely sensitive to this form of real rigidities. This underscores that the implications of strategic complementarities for the design of monetary policy are even more crucial in an Epstein-Zin world. Second, we investigate the role of capital accumulation and nominal rigiditie. We show that, in a world with Dixit Stiglitz aggregators, with price rigidities only, the optimal reaction is amplified by Epstein-Zin preferences, be it with capital or not. Adding wage rigidities tend to reduce this amplification. Finally, unlike both the traditional macro-finance approach and the literature on optimal policy, another novelty of our paper consists in analysing of the behaviour of the term premium under optimal monetary policy. We find that the effect of Epstein-Zin preferences on the level and the dynamics of the term premium is much stronger under optimal policy and is qualitatively different from the Taylor rule case: the term premium is a non-linear function of the Epstein-Zin parameter and we show how it is substantially increased in presence of quasi-kinked demands.

The remainder of the paper is organised as follows. In section 2, we describe the assumptions our model with Epstein-Zin preferences is built upon. In section 4 we analyse their implications for the macroeconomic outcome as well as the implied term premium. In section 5, we discuss how the impact is affected by assumptions pertaining to real rigidities, capital and nominal rigidities. Section 6 concludes.

# 2 The DSGE model

Here we summarize the baseline model considered all along the paper and we pose the Ramsey optimization problem with Epstein-Zin preferences. We point out the impact of such preferences on the equilibrium conditions as well as the optimal policy. Our model is a slightly modified version of Smets and Wouters (2007) and De Graeve et al. (2009a) in which we consider Epstein-Zin preferences. The Smets and Wouters (2007) model is here considered as a benchmark and its microfoundations are therefore not discussed. We assume staggered nominal wage and price contracts à la Calvo (1983) with partial indexation, adjustment costs on investment and capacity utilization, internal habit persistence. We also include real rigidities by using a Kimball (1995) agregator for both goods and labour markets, which results in quasi-kinked demand functions (see Levin et al. 2007).

## 2.1 Summary ot the theoretical model

### 2.1.1 Households behaviour

The economy is populated by a continuum of heterogenous infinitely-lived households. Each household is characterized by the quality of its labour services, $h \in [0,1]$. At time $t$, the instantaneous utility function of a generic household $h$ is:

$$\mathcal{U}_t(h) = \varepsilon_t^b \frac{(C_{t+j}(h) - \eta C_{t+j-1}(h))^{1-\sigma_c}}{1-\sigma_c} \exp\left(\left(\varepsilon_t^l\right)^{-1} \widetilde{L} \frac{(\sigma_c - 1)}{(1+\sigma_l)} {L_{t+j}^h}^{1+\sigma_l}\right) \tag{1}$$

Household $h$ obtains utility from consumption of an aggregate index $C_t(h)$, relative to an *internal* habit depending on its past consumption, while receiving disutility from the supply of their homogenous labor $L_t^h$. $\tilde{L}$ is a positive scale parameter. Utility also incorporates a consumption preference shock $\varepsilon_t^b$ as well as a labour supply shock $\varepsilon_t^l$.

Following Epstein and Zin (1989) and adopting the formulation of Rudebusch and Swanson (2009) and Andreasen (2009), we introduce Epstein-Zin preferences by assuming that the welfare follows the dynamic:

$$\mathcal{W}_t(h) = \mathcal{U}_t(h) + \beta \mathbb{E}_t \left[\mathcal{W}_{t+1}(h)^{1-\alpha_{EZ}}\right]^{\frac{1}{1-\alpha_{EZ}}} \tag{2}$$

The parameter $\beta$ is the deterministic discount factor and $\alpha_{EZ}$ denotes the Epstein-Zin parameter. As shown by Swanson (2010), when $\mathcal{W}_t(h)$ is positive, the higher $\alpha_{EZ}$, the more risk-adverse the agent, and conversely when $\mathcal{W}_t(h)$ is negative.

Each household $h$ maximizes its welfare $\mathcal{W}_0(h)$ under the Epstein-Zin constraint (2) and the following budget constraint:

$$\frac{B_t(h)}{P_t R_t} + C_t(h) + I_t(h) = \frac{B_{t-1}(h)}{P_t} + \frac{(1-\tau_{w,t})\, W_t^h L_t^h + A_t(h) + T_t(h)}{P_t} + r_t^k u_t(h) K_{t-1}(h) - \Psi\left(u_t(h)\right) K_{t-1}(h) + \Pi_t(h) \tag{3}$$

where $P_t$ is an aggregate price index, $R_t = 1 + i_t$ is the one period ahead nominal interest factor, $B_t(h)$ is a nominal bond, $I_t(h)$ is the investment level $W_t^h$ is the nominal wage, $T_t(h)$ and $\tau_{w,t}$ are government transfers and time-varying labor tax, and

$$r_t^k u_t(h) K_{t-1}(h) - \Psi\left(u_t(h)\right) K_{t-1}(h) \tag{4}$$

represents the return on the real capital stock minus the cost associated with variations in the degree of capital utilization. The income from renting out capital services depends on the level of capital augmented for its utilization rate. The cost (or benefit) $\Psi$ is an increasing function of capacity utilization and is zero at steady state, $\Psi(u^\star) = 0$. $\Pi_t(h)$ are the dividend emanating from monopolistically competitive intermediate firms. Finally $A_t(h)$ is a stream of income coming from state contingent securities and equating marginal utility of consumption across households $h \in [0, 1]$.

In choosing the capital stock, investment and the capacity utilization rate the households take into account the following capital accumulation equation:

$$K_t = (1-\delta) K_{t-1} + \varepsilon_t^I \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{5}$$

where $\delta \in (0, 1)$ is the depreciation rate, $S$ is a non negative adjustment cost function such that $S(\gamma) = 0$ and $\varepsilon_t^I$ is an efficiency shock on the technology of capital accumulation.

In equilibrium, households choices in terms of consumption, hours, bond holdings, investment and capacity utilization are identical (see Smets and Wouters, 2007, Adjemian et al., 2008). Therefore, the welfare is also identical accross households, and the first order conditions are reported in Appendix A.1, dropping the $h$ index.

The functional forms used for the adjustment costs on capacity utilization and investment are given by $\Psi(X) = \frac{r^{k\star}}{\varphi}\left(\exp\left[\varphi\left(X-1\right)\right] - 1\right)$ and $S(x) = \phi/2\ \left(x - \gamma\right)^2$.

### 2.1.2 Labour supply and wage setting

Intermediate goods producers make use of a labour input $L_t^D$ produced by a segment of labour packers. Those labour packers operate in a competitive environment and

aggregate a continuum of differentiated labour services $L_t(i)$, $i \in [0,1]$ using a Kimball (1995) technology. The Kimball aggregator is defined by

$$\int_0^1 H\left(\frac{L_t(i)}{L_t^D};\theta_w,\psi_w\right)\mathrm{d}i = 1 \tag{6}$$

where as in Dotsey and King (2005), we consider the following functional form:

$$\begin{aligned} H\left(\frac{L_t(i)}{L_t^D}\right) &= \frac{\theta_w}{(\theta_w(1+\psi_w)-1)}\left[(1+\psi_w)\frac{L_t(i)}{L_t^D}-\psi_w\right]^{\frac{\theta_w(1+\psi_w)-1}{\theta_w(1+\psi_w)}} \\ &\quad -\left[\frac{\theta_w}{(\theta_w(1+\psi_w)-1)}-1\right] \end{aligned} \tag{7}$$

This function, where the parameter $\psi_w$ determines the curvature of the demand curve, reduces to the standard Dixit-Stiglitz aggregator under the restriction $\psi_w = 0$.

The differentiated labour services are produced by a continuum of unions which transform the homogeneous household labor supply. Each union is a monopoly supplier of a differentiated labour service and sets its wage on a staggered basis, paying households the nominal wage rate $W_t^h$. Every period, any union faces a constant probability $1-\alpha_w$ of optimally adjusting its nominal wage, say $W_t^*(i)$, which will be the same for all suppliers of differentiated labor services. We denote thereafter $w_t$ the aggregate real wage that intermediate producers pay for the labor input provided by the labor packers and $w_t^*$ the real wage claimed by re-optimizing unions.

When they cannot re-optimize, wages are indexed on past inflation and steady state inflation according to the following indexation rule:

$$W_t(i) = \gamma\left[\pi_{t-1}\right]^{\xi_w}\left[\pi^\star\right]^{1-\xi_w} W_{t-1}(i) \tag{8}$$

with $\pi_t = \frac{P_t}{P_{t-1}}$ the gross rate of inflation. Taking into account that they might not be able to choose their nominal wage optimally in a near future, $W_t^*(i)$ is chosen to maximize their intertemporal profit under the labor demand from labor packers. Unions are subject to a time-varying tax rate $\tau_{w,t}$ which is affected by a shock defined by $1-\tau_{w,t} = (1-\tau_w^\star)\,\varepsilon_t^w$. The corresponding first order conditions are reported in Appendix A.2.

### 2.1.3 Producers behaviour

Final producers are perfectly competitive firms producing an aggregate final good $Y_t$ that may be used for consumption and investment. This production is obtained using a

continuum of differentiated intermediate goods $Y_t(z)$, $z \in [0,1]$ with the Kimball (1995) technology. Here again, the Kimball aggregator is defined by

$$\int_0^1 G\left(\frac{Y_t(z)}{Y_t};\theta_p,\psi\right)\mathrm{d}z = 1 \tag{9}$$

with

$$\begin{aligned} G\left(\frac{Y_t(z)}{Y_t}\right) &= \frac{\theta_p}{(\theta_p(1+\psi)-1)}\left[(1+\psi)\frac{Y_t(z)}{Y_t}-\psi\right]^{\frac{\theta_p(1+\psi)-1}{\theta_p(1+\psi)}} \\ &\quad -\left[\frac{\theta_p}{(\theta_p(1+\psi)-1)}-1\right]. \end{aligned} \tag{10}$$

The representative final good producer maximizes profits $P_tY_t - \int_0^1 P_t(z)Y_t(z)\mathrm{d}z$ subject to the production function, taking as given the final good price $P_t$ and the prices of all intermediate goods.

In the intermediate goods sector, firms $z \in [0,1]$ are monopolistic competitors and produce differentiated products by using a common Cobb-Douglas technology:

$$Y_t(z) = \varepsilon_t^a \left(u_tK_{t-1}(z)\right)^\alpha \left[\gamma^t L^D(z)\right]^{1-\alpha} - \gamma^t\Omega \tag{11}$$

where $\varepsilon_t^a$ is an exogenous productivity shock, $\Omega > 0$ is a fixed cost and $\gamma$ is the trend technological growth rate. A firm $z$ hires its capital, $\widetilde{K}_t(z) = u_tK_{t-1}(z)$, and labor, $L_t^D(z)$, on a competitive market by minimizing its production cost. Due to our assumptions on the labor market and the rental rate of capital, the real marginal cost is identical across producers. We introduce a time varying tax on firm's revenue is affected by a shock defined by $1-\tau_{p,t} = \left(1-\tau_p^\star\right)\varepsilon_t^p$.

In each period, a firm $z$ faces a constant (across time and firms) probability $1-\alpha_p$ of being able to re-optimize its nominal price, say $P_t^*(z)$. If a firm cannot re-optimize its price, the nominal price evolves according to the rule $P_t(z) = \pi_{t-1}^{\xi_p}\left[\pi^\star\right]^{(1-\xi_p)}P_{t-1}(z)$, *ie* the nominal price is indexed on past inflation and steady state inflation. In our model, all firms that can re-optimize their price at time $t$ choose the same level, denoted $p_t^*$ in real terms. The corresponding first order conditions are reported in Appendix A.3.

### 2.1.4 Government

Public expenditures $G^\star$ are subject to random shocks $\varepsilon_t^g$. The government finances public spending with labour tax, product tax and lump-sum transfers:

$$P_tG^\star\gamma^t\varepsilon_t^g - \tau_{w,t}W_tL_t - \tau_{p,t}P_tY_t - P_tT_t = 0 \tag{12}$$

### 2.1.5 Market clearing conditions

Market clearing condition on goods market is given by:

$$Y_t = C_t + I_t + G^{\star}\varepsilon_t^g + \Psi\left(u_t\right) K_{t-1} \tag{13}$$

$$\Delta_{pk,t} Y_t = \varepsilon_t^a \left(u_t K_{t-1}\right)^{\alpha} \left(\gamma^t L_t^D\right)^{1-\alpha} - \gamma^t \Omega \tag{14}$$

with $\Delta_{pk,t}$ is a price dispersion index whose dynamics is presented in the appendix A.3.

Equilibrium in the labour market implies that

$$\Delta_{wk,t} L_t^D = L_t \tag{15}$$

with $L_t^D = \int_0^1 L_t^D(z)dz$ and $L_t = \int_0^1 L_t^h dh$. The dynamics of the wage dispersion index $\Delta_{wk,t}$ is also described in the appendix A.2.

### 2.1.6 Competitive equilibrium conditions

Rudebusch and Swanson (2009) provide details of the derivation of the equilibrium conditions with Epstein-Zin preferences. The same methodology is employed here and the first order conditions are reported in Appendix. Epstein-Zin preferences introduce a convexity term in the recursive equation of the household 's welfare. This in turn modifies the pricing-kernel $M_{t+1}$ which can be written in the form:

$$M_{t+1} = \left[\left(\frac{\mathcal{W}_{t+1}}{\mathbb{E}_t\left[{\mathcal{W}_{t+1}}^{1-\alpha_{EZ}}\right]^{\frac{1}{1-\alpha_{EZ}}}}\right)^{-\alpha_{EZ}}\right] \times \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \tag{16}$$

where $\lambda_t$ is the marginal utility of consumption[5]. The standard case – hereafter referred to as *expected utility case* – corresponds to $\alpha_{EZ} = 0$.

Regarding the macroeconomic block the pricing-kernel is used to determine the optimal price and wage as well as the capital and investment dynamic. Up to a first order approximation the equilibrium conditions are strictly equivalent to the expected utility case. Therefore, as a theoretical matter, a log-linearized macroeconomic model with a monetary authority acting according to a standard Taylor rule yields the same results, whether Epstein-Zin preferences are used or not. This is what Levin et al. (2008) call *macroeconometric equivalence.*

[5] The marginal utility of consumption is also affected by the Epstein-Zin preferences due to the presence of internal habits (see equation 20 in Appendix).

### 2.1.7 Taylor rule

In what follows, the case of a central bank following a Taylor rule shall constitute a benchmark for our analysis. In that case, the nominal interest rate is adjusted in response to deviations of inflation and output from their respective target levels:

$$\frac{R_t}{R^*} = \left(\frac{R_t}{R^*}\right)^{\rho} \left[\left(\frac{\pi_t}{\pi^*}\right)^{r_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{r_y}\right]^{1-\rho} \left(\frac{Y_t/Y_{t-1}}{Y_t^*/Y_{t-1}^*}\right)^{r_{\Delta_y}}$$

where $R^*$ is the steady state nominal (gross) rate and $Y_t^*$ the natural output, ie, the flexible price output. The parameter $\rho$ reflects the degree of interest rate smoothing.

### 2.1.8 Ramsey policy

As detailed by Levin and al. (2008) we define the Ramsey policy as the monetary policy under commitment which maximizes the household's aggregate welfare $\mathcal{W}_0$, subject to the competitive equilibrium conditions and the Epstein-Zin constraint (2), given the exogenous stochastic processes $\varepsilon_t^a$, $\varepsilon_t^b$, $\varepsilon_t^l$, $\varepsilon_t^I$, $\varepsilon_t^g$, $\varepsilon_t^w$, $\varepsilon_t^p$ values of the state variables dated $t < 0$, and values of the Lagrange multipliers associated with the constraints dated $t < 0$.

The Ramsey programme involves a Lagrangian multiplier $\mu_t$ associated with the Epstein-Zin constraint on the welfare (2) whose dynamic follows:

$$\mu_t = \left(\frac{\mathcal{W}_t}{\mathbb{E}_{t-1}\left[\mathcal{W}_t{}^{1-\alpha_{EZ}}\right]^{\frac{1}{1-\alpha_{EZ}}}}\right)^{-\alpha_{EZ}} \left(\mu_{t-1} + \alpha_{EZ}\mathcal{T}_t\right)$$

where $\mathcal{T}_t$ is a term whose conditional expectation at time $t$ is zero. As explicitly shown by Levin et al. (2008) in a smaller model, the first-order approximation of this equation still involves the Epstein-Zin parameter $\alpha_{EZ}$. Therefore, *ex ante*, Epstein-Zin preferences enter the first-order approximation of the optimal policy and may alter the optimal macroeconomic allocation, reflecting the so-called *microeconomic dissonance*.

## 2.2 Calibration

*Structural parameters of the macro-model*

The parameter values that we use for our economic structure – apart from the Epstein-Zin parameter – are reported in table 1 and are relatively standard in the literature.

As in Smets and Wouters (2007), we set $r_\beta = 100(1/\beta - 1)$ to 0.16. The model is detrended with a derministic trend ($\gamma$) set to 0.43. The inverse of the households '

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $r_\beta$ | 0.16 | $\psi$ | 10 | $\mu_w$ | 1.5 | $\rho$ | 0.81 | $\rho_L$ | 0.98 |
| $\gamma$ | 0.43 | $\mu_p$ | 1.24 | $\xi_w$ | 0.59 | $r_\pi$ | 2.03 | $\rho_P$ | 0.98 |
| $\sigma_c$ | 1.37 | $\pi^*$ | 0.81 | $\alpha$ | 0.19 | $r_y$ | 0.08 | $\rho_G$ | 0.97 |
| $\sigma_l$ | 1.98 | $\xi_p$ | 0.22 | $\delta$ | 0.025 | $r_{\Delta y}$ | 0.22 | $\rho_I$ | 0.71 |
| $h$ | 0.71 | $\alpha_w$ | 0.70 | $\phi$ | 5.48 | $\rho_A$ | 0.95 | | |
| $\alpha_p$ | 0.83 | $\psi_w$ | 10 | $\varphi$ | 1.17 | $\rho_B$ | 0.18 | | |

Table 1: Baseline calibration

intertemporal elasticity of substitution ($\sigma_c$) is set to 1.37 and the inverse of the Frish elasticity ($\sigma_l$) to 1.92. The habit formation is captured by the parameter $h$ set to 0.71.

In line with Galì, Gertler and Lopez-Salido (2005) and Levin et al. (2008), the slope of the linearized Phillips curve for prices is set to 0.01. In this respect, the Calvo frequency of price adjustment ($\alpha_p$) is set to 0.83, the associated Kimball parameter ($\psi$) to 10, the price mark-up ($\mu_p$) to 1.24 and the indexation coefficient ($\xi_p$) to 0.22. The steady state inflation ($\pi^\star$) is assumed to be 0.81[6]. The Calvo frequency of wage adjustment ($\alpha_w$) is set to 0.70 and the associated Kimball parameter ($\psi_w$) to 10. With a wage mark-up ($\mu_w$) set to 1.5 and a indexation coefficient ($\xi_w$) to 0.59., this implies a slope of the corresponding linearized Phillips curve for wages close to 0.01 as well.

We set the Cobb-Douglas parameter ($\alpha$) to 0.19 as estimated by Smets and Wouters (2007) and the depreciation rate of capital ($\delta$) to 0.025. The adjustment cost parameter for investment ($\phi$) is set to 5.48 and the capacity utilization elasticity ($\varphi$) is set to 1.17. In the steady state government spending are assume to compose 18% of the output.

Regarding the Taylor rule, the persistence parameter ($\rho$) is set to 0.81, the long run coefficient on inflation and output, ($r_\pi$) and ($r_y$), are respectively set to 2.03 and 0.08. The coefficient for the short run reaction to output gap ($r_{\Delta y}$) is set to 0.22.

*Exogenous shocks*

We restrict the analysis to several structural shocks, assuming that the stochastic processes for the exogenous disturbances follow AR(1) dynamics:

$$\log \varepsilon_t^i = \rho_i \log \varepsilon_{t-1}^i + \epsilon_t^i$$

where $\epsilon_t^i$ are i.i.d. shocks following a $\mathcal{N}(0, \sigma_i)$ and $i$ refers to $a$, $b$, $l$, $p$, $g$ and $i$. We respectively set $\rho_A$, $\rho_B$, $\rho_L$, $\rho_P$, $\rho_G$ and $\rho_I$ to 0.95, 0.18, 0.98 0.98, 0.97 and 0.71. The standard deviations of the structural shocks are adjusted to reproduce the relative

[6] Here we wish to draw attention to the fact that the dynamic of the optimal policy is not affected by the steady-state value of inflation, due to the complete indexation which holds in the steady state.

variance decomposition of Smets and Wouters (2007)[7].

### 2.3 The term premium

The term premium on a nominal bond yield with a certain maturity is defined as the difference between the actual yield and the corresponding yield which would prevail in a risk-neutral world (see Rudebusch, Sack and Swanson, 2007, Rudebusch and Swanson, 2008-2009). To compute the term premium in our model we follow the approach of Rudebusch and Swanson (2008, 2009) and De Graeve et al. (2009) by assuming the existence of an infinite-lived consol paying off geometric coupons. We calibrate the coupon so that the duration is 10 years and we use the corresponding yield as an approximation for the 10 year zero coupon yield. At time $t$, in our DSGE model, the term premium measures the compensation required by the agents who consider as risky the fact that future shocks may lead the short rate to deviate from the expected path. The term premium is therefore intrinsically linked with agents' risk aversion.

Swanson (2010) provides clarifications regarding the link between the term premium implied by DSGE models and risk aversion. He proves that any premium can be written, to second order approximation around the non-stochastic steady state, as:

$$tp_t = A(\alpha_{EZ}) \times cov_t \left(dA_{t+1}, dp_{t+1}\right) + B \times cov_t \left(d\Psi_{t+1}, dp_{t+1}\right) \tag{17}$$

where $A(\alpha_{EZ})$ is the Arrow-Pratt coefficient reflecting the risk aversion[8], $dp_{t+1}$ is the first order dynamic of the asset price – here the 10 year bond price –, $dA_{t+1}$ denotes the change in households ' wealth and $d\Psi_{t+1}$ the change in current and future wages and interest rates[9]. The coefficient $A(\alpha_{EZ})$ is a linear function of $\alpha_{EZ}$ whose coefficients depend on the steady state. In particular, equation (17) shows that the stochastic steady state of the term premium depends only on the first order approximation of the macroeconomic dynamics.

[7] The impulse response functions for macroeconomic variables we shall present use the first order approximation of the model and are therefore proportional to the size of the shocks. Likewise, there exists a similar proportional relationship with the unconditional mean of the term premium as well as its third order dynamic.

[8] Here the term risk aversion refers to the concept properly defined by Swanson (2010) taking account of both consumption and labour margins.

[9] This is the formula (39) using the equation (A15) in Swanson (2010).

## 3 What calibration for the Epstein-Zin parameter?

A few studies have so far attempted to estimate or calibrate the Epstein-Zin parameter in DSGE models and there seems to be no consensus on a "reasonable" interval the values of $\alpha_{EZ}$ would have to lie in. On the one hand, using grid search, Rudebusch and Swanson (2009) estimate values of $\alpha_{EZ}$ lying between 75 and 88 in a small DSGE model with three shocks. These values are of similar magnitude to those found by Binsbergen et al. (2008)[10] and Campanale et al. (2009). Amisano and Tristani (2010) find a much smaller value – about 8 with our specification– but their empirical approach is slightly different as they rely on stochastic volatility. On the other hand, using Quasi-Maximum Likelihood techniques in a larger model[11], Andreasen (2009) obtains a value of 1981. Therefore, the value of $\alpha_{EZ}$ needed to match a certain level of term premium seems to crucially depend on the sophistication and the size of the underlying macro-model. We shall argue that this is actually not the case.

In the spirit of Rudebusch and Swanson (2009), given the calibration of the macro-model presented in subsection 2.2, we first choose a value of $\alpha_{EZ}$ that generates a ten-year term premium equal to 100 bp on average. The average of the term premium – or stochastic steady state – is calculated using a second order approximation to our macro-model[12]. We obtain:

$$\alpha_{EZ} = 930$$

Figure 5 displays the impulse response functions of the term premium to four selected structural shocks: technology, preferences, labour supply and price markup[13]. With Epstein-Zin preferences the reactions are amplified and more persistent. The term premium reacts negatively to positive technology, labour supply and price markup shocks, and positively to a preference shock, which represents a shock on the pricing kernel. However, if the effect is very marked, the responses of the premium, expressed in basis points, remain small. Accordingly, the volatility of the term premium amounts to 1.6 bp in our baseline calibration, which is well under the volatility targeted by Rudebusch

[10] See their comparison with the literature.

[11] The model estimated by Andreasen (2009) is closer to ours than that of Rudebusch and Swanson (2009), although it does not include wage rigidities and quasi-kinked demands.

[12] Computing DSGE models using perturbation methods has been shown to deliver a very high degree of accuracy (see Caldara et al., 2009). We therefore rely on this method to solve our model to the second – and third order – approximation.

[13] The impulse response functions of the term premium are obtained using a third order approximation to the model.

and Swanson (2009)[14]. These results regarding the small volatility of the term premium implied by DSGE models, even augmented with Epstein-Zin preferences, are in line with the empirical findings of Andreasen (2009).

As our value of $\alpha_{EZ}$ is about ten times as high as the one found by Rudebusch and Swanson (2009)[15], we now examine which frictions and parameters may explain this difference of value between a small model and a medium-scale model like ours or Andreasen (2009). For different submodels and calibrations, we determine which value of $\alpha_{EZ}$ is needed in order to match a 100 bp ten-year term premium. The assumptions we focus on pertain to endogenous capital accumulation, habit formation and wage rigidities which are the main differences with Rudebusch and Swanson (2009)[16]. Here it is worth noting that, in a Taylor rule benchmark, the level of the term premium does not depend on real rigidities in a form of quasi-kinked demands once the *overall* degree of rigidity, i.e., the slope of the Phillips curves, is fixed. Indeed, in that case, the first order dynamics is independent of the degree of real rigidities and so too is the level of the premium (see equation 17)[17]. We also focus on three parameters that have a significant impact on the term premium, and more generally on risk aversion: the deterministic discount factor ($\beta$), the trend ($\gamma$) and the inverse of intertemporal elasticity of subsitution ($\sigma_c$). These parameters directly enter the pricing kernel (16) and are therefore important determinants of the risk premia. We consider two calibrations: the first one – $\beta = 0.99$, $\gamma = 0$ and $\sigma_c = 2$ –corresponds to that of Rudebusch and Swanson (2009), and the second one – $\beta = \beta_{SW} = (1 + \frac{0.16}{100})^{-1_c}$, $\gamma = 0.43$ and $\sigma_c = 1.37$ – to that of Smets and Wouters (2007). The other parameters are left unchanged (see Table 1).

Table 2 reports the results of our simulations. This table calls for several comments. First, the important difference of the value needed for $\alpha_{EZ}$ in a model close to Rudebusch and Swanson (2009) and in a model like ours does *not* come from the addition of specifications like endogenous capital accumulation or wage rigidities. Indeed, given a value of $(\sigma_c, \beta, h, \gamma)$, it is possible to generate a substantial term premium for any submodel with roughly the same value of $\alpha_{EZ}$. Although we observe that capital accumulation tends to increase term premia, it does not fully explain the difference of calibration. In fact, the main change results from modifications of $\beta$, $\gamma$ or $\sigma_c$. An increase in one of

[14] Rudebusch and Swanson (2009) aim at generating a volatility of 54 bp for the term premium. Their calibration generates a volatility of 16.2 bp.

[15] Our value is however well lower than that estimated by Andreasen (2009).

[16] Compared with Rudebusch and Swanson (2009), we also add three shocks (preferences, labour supply and price markup).

[17] Therefore, quasi-kinked demands do not explain the difference of calibration.

| | | $\sigma_c = 1.37$ | | $\sigma_c = 2.$ | |
|---|---|---|---|---|---|
| | $(\beta, \gamma) =$ | (.99,0) | ($\beta_{SW}$,.43) | (.99,0) | ($\beta_{SW}$,.43) |
| no capital | no hab. | 407 | 1130 | 114 | 180 |
| no wage rig. | hab. | 398 | 1110 | 111 | 177 |
| capital | no hab. | 360 | 970 | 112 | 170 |
| no wage rig. | hab. | 343 | 930 | 106 | 163 |
| capital | no hab. | 360 | 960 | 112 | 168 |
| wage rig. | hab. | 348 | 930 | 108 | 165 |

Table 2: Value of the Epstein-Zin parameter needed to match a level of 100 bp for the ten-year term premium

these parameters amplifies the dependence of risk aversion on $\alpha_{EZ}$. As a consequence, the value $\alpha_{EZ} = 88$ used by Rudebusch and Swanson (2009) results from their specific calibration of $\sigma_c$, $\gamma$ and $\beta$. In other words, with smaller calibrations of $\sigma_c$ and the use of detrended variables, like in Smets and Wouters (2007) risk premia are much lower (up to 10 times). Second, the introduction of internal habits tends to slightly decrease the value of $\alpha_{EZ}$ needed in our exercise. This confirms that the introduction of habit formations is not enough to generate substantial risk premia although they do have a small and positive impact (see Rudebusch and Swanson, 2008).

# 4 Optimal policy with Epstein-Zin preferences

In this section we show how Epstein-Zin preferences alter the optimal macroeconomic allocations and the implied term premium by comparing our empirical benchmark ($\alpha_{EZ} = 930$) to the expected utility case ($\alpha_{EZ} = 0$) . In this respect, we focus on four structural shocks: technology, preference, labour supply and price markup.

## 4.1 Optimal macroeconomic allocations

In section 2.1.8 we have shown, like Levin et al. (2008), that the Epstein-Zin parameter directly enters the first-order approximation of the optimal policy, regardless of the set of frictions or the presence of subsidies. However, this does not guarantee that Epstein-Zin preferences will effectively affect the macroeconomic allocation. For example, it is worth noting that the presence of subsidies aimed at offsetting the distortion in the steady state due to monopolistic competition in the goods and labour markets cancels the first-order Epstein-Zin effects. This result should not be too surprising. In

the case of an efficient steady state, Woodford (2003) obtains a second order approximation to the welfare using only first-order approximations to the structural equations. This in turn implies that the first order approximation to the optimal policy depends only on the first order approximation to the equilibrium conditions. The latter is independent of Epstein-Zin preferences. And so too is the optimal policy. We find that the result obtained by Woodford (2003) is still verified in our model including Epstein-Zin preferences. Overall, up to a first order approximation, Epstein-Zin preferences yield exactly the same allocation when monetary policy follows a standard Taylor rule, irrespectively from the efficiency of the steady state, or under optimal monetary policy and when the steady state is efficient. In what follows, those two cases shall constitute useful benchmarks.

We begin our analysis of optimal policy by considering the impulse response functions (IRFs) of macroeconomic aggregates (see charts 1 to 4). To assess the impact specific to Epstein-Zin preferences, we focus on the optimal allocation with a distorted steady state[18]. We use a first order approximation to the model. As a benchmark, the responses obtained with a standard Taylor rule are also represented. The value of $\alpha_{EZ}$ is set to 930 as in our empiral benchmark. For every shock, the main feature is that the optimal policy is substantially affected by Epstein-Zin preferences. Specifically, regarding the productivity shock, responses of inflation, interest rate, labour and output gap tend to be amplified by Epstein-Zin preferences while responses of output, consumption, investment and real wage are lessened. Apart from inflation, we observe that the initial impact of the shock on variables is not significantly affected by the preferences. On the contrary, the response of inflation is completely shifted downwards and inflation reacts even stronger than in the Taylor rule benchmark. To understand why such shift occurs, it is useful to recall that Epstein-Zin preferences modify the expected-utility household's pricing kernel by introducing a term taking into account the utility surprise – the term between square brackets in (16). Since the same pricing-kernel is used by firms that are able to readjust their prices, it follows that Epstein-Zin preferences involve a discounting of the non-expected losses of utility which affects price determination and aggregate inflation. Firms do not know exactly when they can readjust their price, which implies losses of utility that are taken into account with Epstein-Zin preferences. Although this effect is absent when the model is linearized, it does appear when optimal policy is considered with a distorted steady-state. Indeed, the equations characterizing the optimal price and inflation are exactly those whose second order approximation is needed by Benigno

[18] For an in-depth analysis of the optimal monetary policy in the case of an efficient steady state, see Adjemian et al. (2008) for example.

| | $\pi_t$ | $W_t$ | $R_t$ | $Y_t$ | $C_t$ | $L_t$ | $OG_t$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|
| tech. | 0.0342 | 1.9402 | 0.6448 | 2.6620 | 2.4357 | 0.5308 | 0.1607 | 7.3805 |
| tech. | *0.2438* | *1.4175* | *0.7373* | *2.2772* | *2.1157* | *0.6258* | *0.2956* | *6.3834* |
| pref. | 0.0009 | 0.0448 | 1.5132 | 0.1728 | 0.4713 | 0.1557 | 0.0105 | 1.1021 |
| pref. | *0.0343* | *0.1232* | *1.5124* | *0.1830* | *0.4677* | *0.1472* | *0.0633* | *1.1869* |
| lab. | 0.0054 | 0.0714 | 0.0838 | 0.5437 | 0.5849 | 0.4078 | 0.0229 | 1.1274 |
| lab. | *0.0493* | *0.0724* | *0.1045* | *0.4559* | *0.5112* | *0.3540* | *0.0692* | *0.8994* |
| price | 0.2268 | 0.8256 | 0.8837 | 0.4305 | 0.3771 | 0.2151 | 0.4305 | 1.0646 |
| price | *0.2440* | *1.5826* | *0.8885* | *0.9434* | *0.8069* | *0.5155* | *0.9434* | *2.3894* |

Table 3: Volatility under optimal policy with a distorted steady state. The Epstein-Zin parameter is set to 930 (in italic).

and Woodford (2005) to obtain a quadratic welfare measure. As Epstein-Zin preferences affect the second order approximation to those equations, this results in a modification of the weights in the loss function and hence, of the optimal policy[19]. Among the responses to a positive preference shock, the main effect appears on inflation which, unlike the expected utility case and the Taylor rule benchmark, reacts negatively. The responses of real variables and interest rate remain broadly unchanged. As for the labour supply shock, compared with the expected utility case, the responses of output, consumption, labour and investment are lessened. Regarding inflation, output gap and real wage, the reactions are opposite. Finally, regarding the inefficient price markup shock, the effect of Epstein-Zin preferences on optimal policy exhibits a different feature. The reactions of inflation, output, consumption, labour, output gap and investment are markedly shifted upwards and are much more persistent. Interestingly, apart from inflation and interest rate, the reactions resemble those of the Taylor rule benchmark.

Table 3 reports the contribution of each selected shock to the volatility of the macroeconomic aggregates under optimal policy with a distorted steady state. As previously mentioned, when the steady-state is efficient, the volatility is not affected by Epstein-Zin preferences. On the contrary, in the case of a distorted steady state, depending on the shock, the volatility is strongly affected by Epstein-Zin preferences. In some cases, especially for inflation, the volatility is multiplied by a factor 10: in order to take account of the unexpected losses of utility in an Epstein-Zin world, the monetary policy should optimally generate stronger fluctuations of inflation.

We now examine how Epstein-Zin preferences affect the correlations between the

[19] A similar mechanism is involved for the wages Phillips curve.

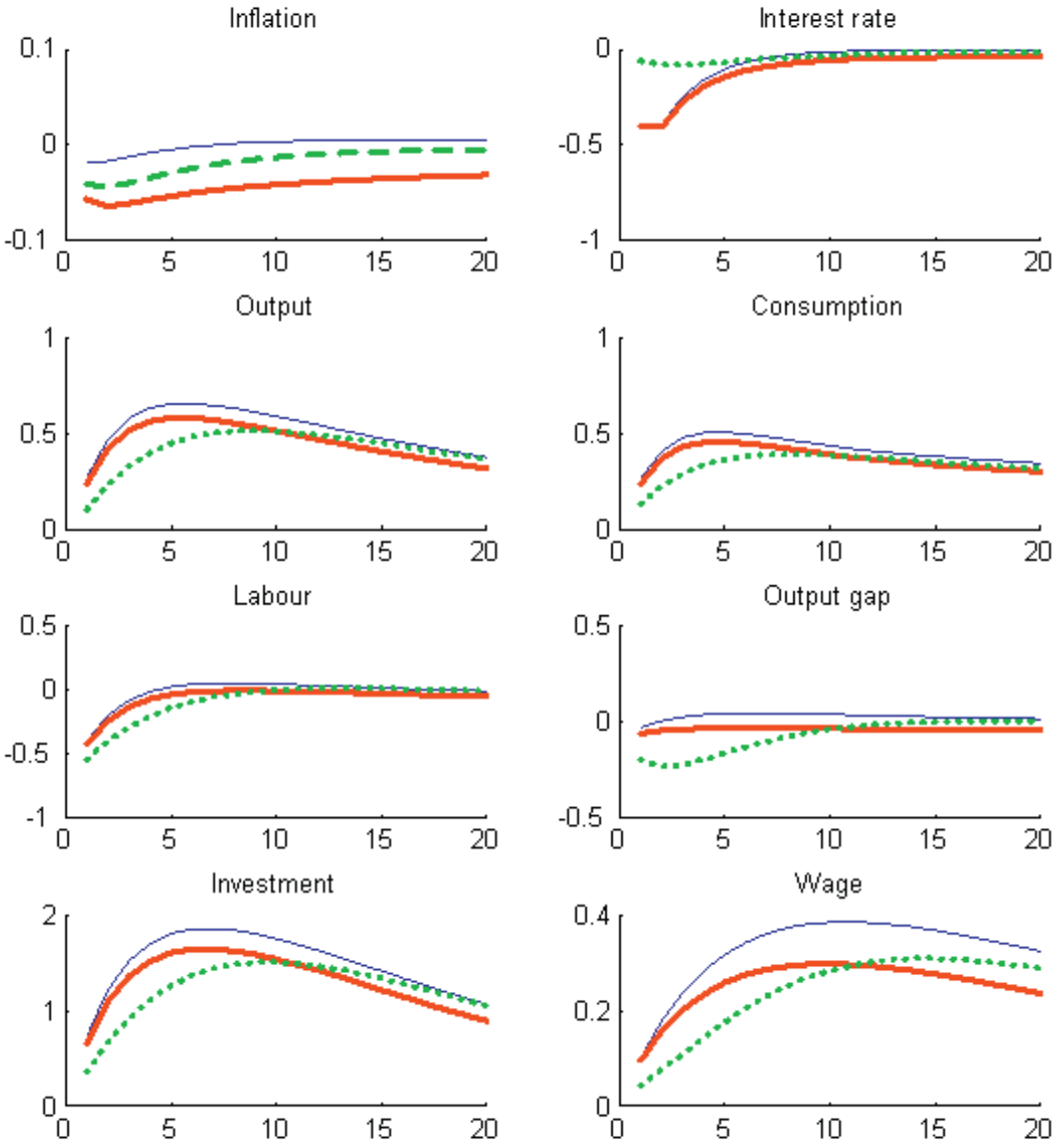


Figure 1: Impulse Responses to a Technology Shock. Taylor (green doted lines), Ramsey expected utility (thin blue lines), Ramsey Epstein-Zin preferences (large red lines). Baseline calibration in the case of a distorted steady state.

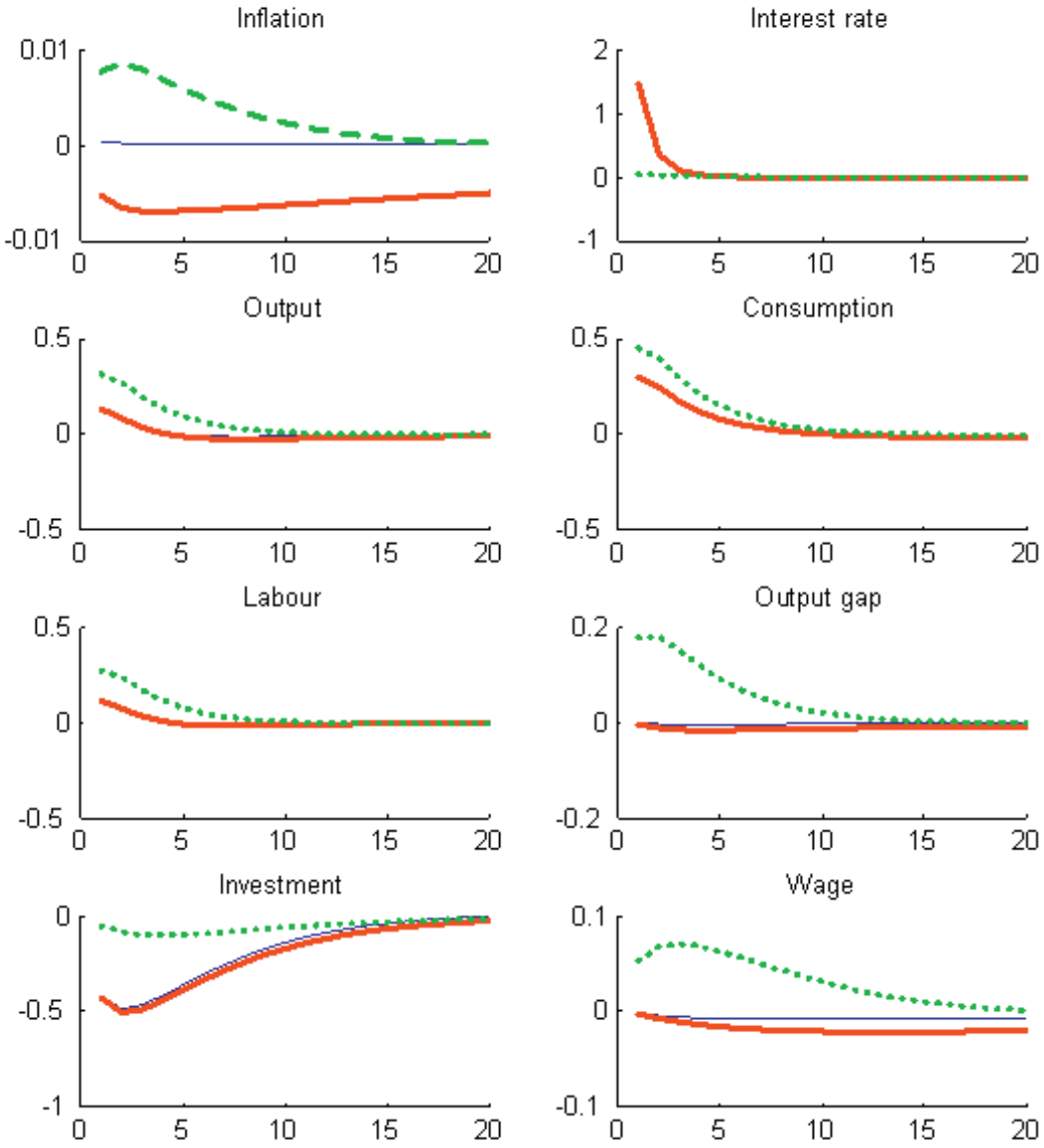


Figure 2: Impulse Responses to a Preference Shock. Taylor (green doted lines), Ramsey expected utility (thin blue lines), Ramsey Epstein-Zin preferences (large red lines). Baseline calibration in the case of a distorted steady state.

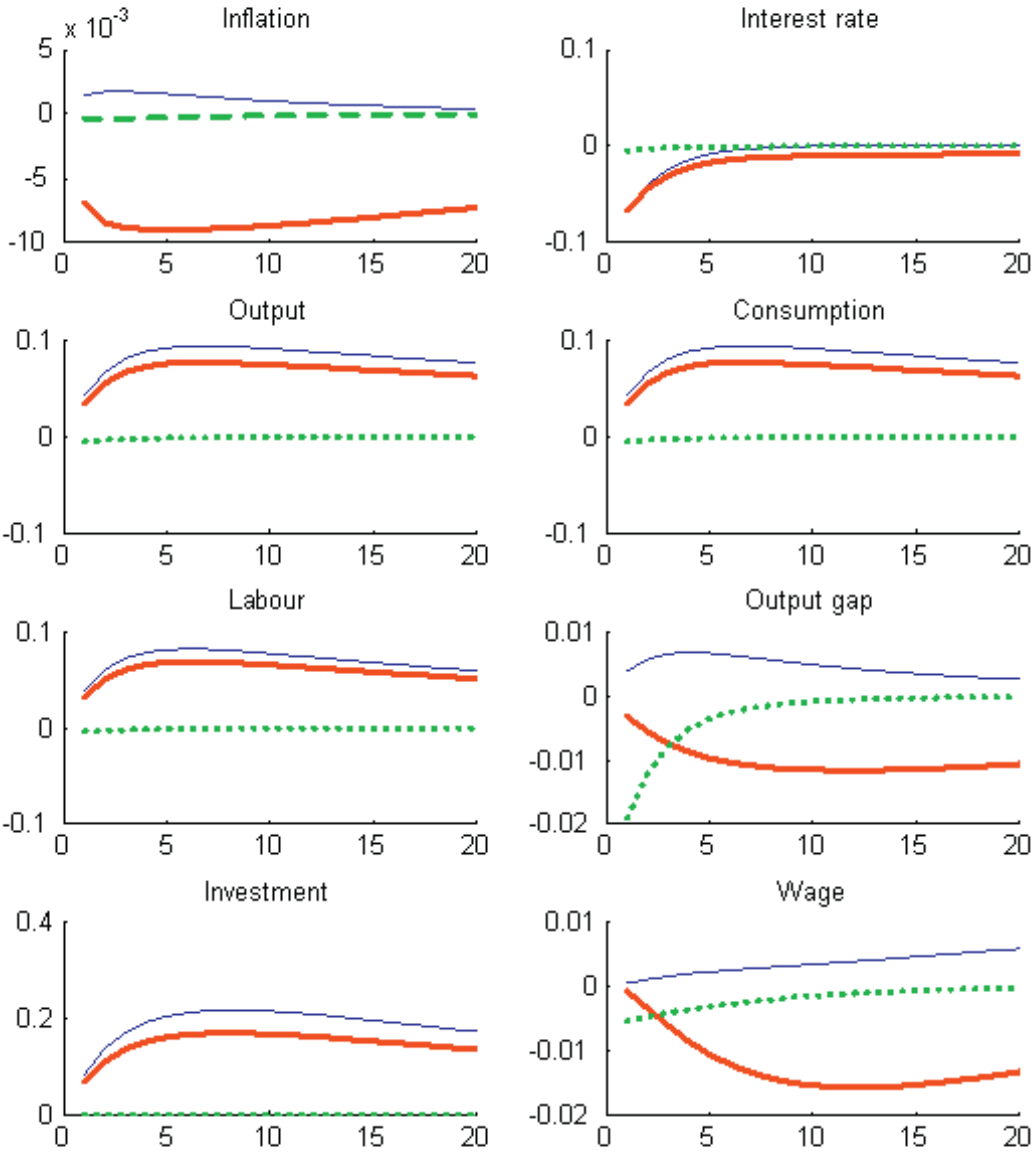


Figure 3: Impulse Responses to a Labour supply Shock. Taylor (green doted lines), Ramsey expected utility (thin blue lines), Ramsey Epstein-Zin preferences (large red lines). Baseline calibration in the case of a distorted steady state.

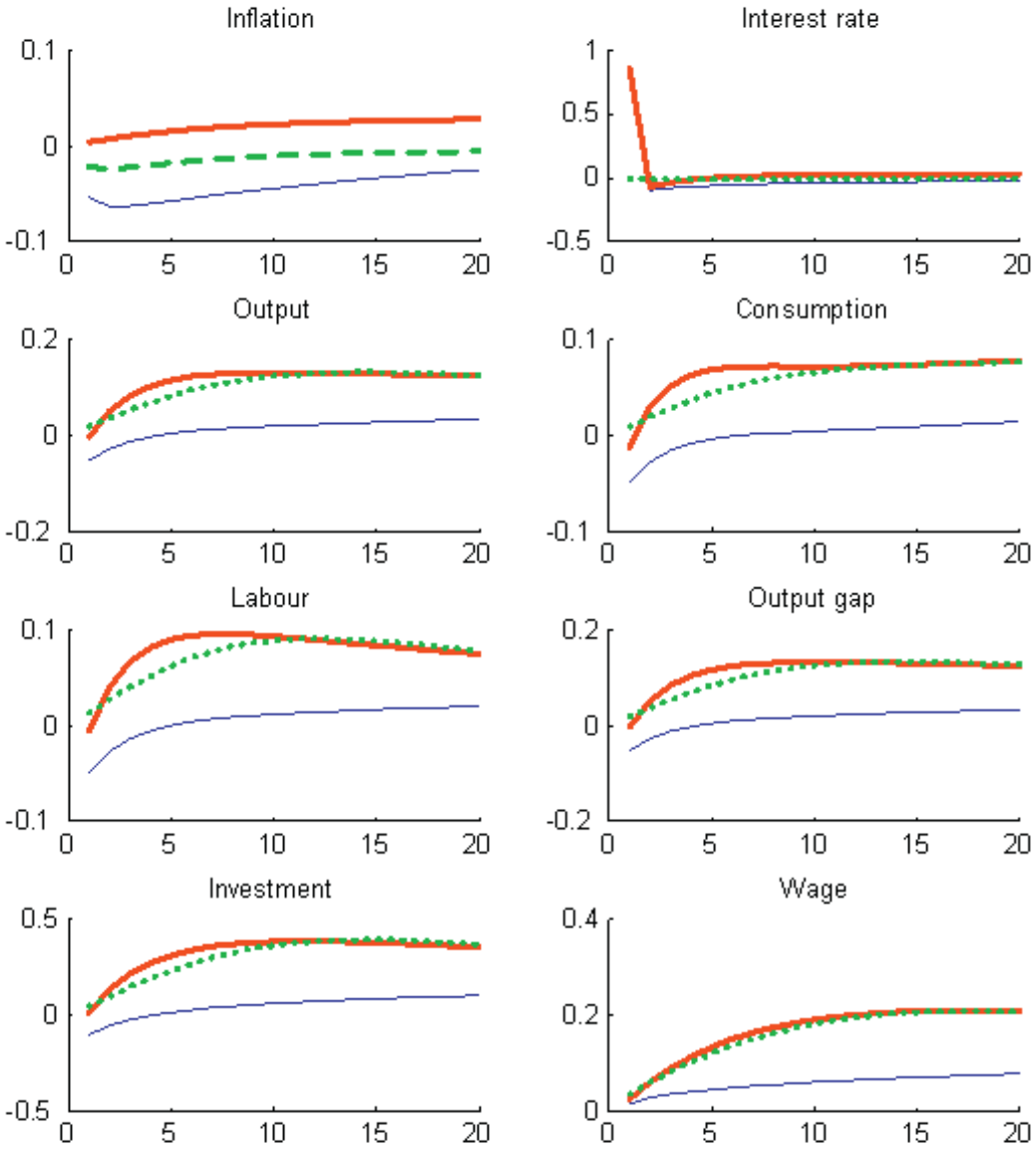


Figure 4: Impulse Responses to a Price Markup Shock. Taylor (green doted lines), Ramsey expected utility (thin blue lines), Ramsey Epstein-Zin preferences (large red lines). Baseline calibration in the case of a distorted steady state.

macroeconomic aggregates. Those correlations play an important role for risk premia, as will be shown in the next section. Once again, when the steady-state is distorted, Epstein-Zin preferences strongly modify the structure of the correlation matrix. Tables 7 to 10 show how the correlation matrices are twisted when $\alpha_{EZ}$ increases. Considering the technology shock (table 7), we distinguish between three types of reactions. First, some correlations are markedly amplified. This is the case of all the correlations involving labour and the correlations between inflation and output gap, and between interest rate and real wage, output, consumption and investment. Second, some others are completly reversed. This is the case of the correlations between inflation and real wage, output, consumption and investment, as well as those between output gap and real wage, interest rate, output, consumption and investment. Finally, the remaining correlations are left broadly unchanged. Roughly similar features – some correlations are amplified, others are reversed or left unchanged– hold for the three other shocks. These features highlight the non-linear effect of Epstein-Zin preferences on optimal allocations.

### 4.2 Application to the term premium

Since Epstein-Zin preferences have been introduced to improve on the general equilibrium pricing of financial assets, we provide here some insights into the term premium implied by our model as a function of the Epstein-Zin parameter. Equation (17) allows to analyse the behaviour of the term premium by distinguishing between two effects. First, Epstein-Zin preferences induce a linear increase in the risk aversion through the Arrow-Pratt coefficient. Thus, *given a first order macroeconomic dynamic*, the term premium linearily increases with the Epstein-Zin sensitivity with a slope depending on the steady state of the economy. In a Taylor-rule-based benchmark or under optimal policy with an efficient steady state, the macroeconomic outcome and hence the covariances in (17) are not impacted by Epstein-Zin preferences (up to a first order approximation). Therefore, in that case, as underlined by Swanson (2010), the level of the term premium is a linear function of $\alpha_{EZ}$. This *quantitative effect* on the risk premium is mechanical. Second, as noted in section 4.1, the optimal policy in general depends on Epstein-Zin preferences. Thus, taking into account such preferences leads to modifications of the macroeconomic allocation. In particular, as the first order dynamic of a bond price is completely determined by the first order expected path of the short rate, the covariances in equation (17) are affected by $\alpha_{EZ}$, which in turn induces a *qualitative effect* on the term premium. The overall effect on risk premia may therefore be non-linear.

| | policy | steady state | $\alpha_{EZ} = 0$ | $\alpha_{EZ} = 88$ | $\alpha_{EZ} = 930$ | $\alpha_{EZ} = 1981$ |
|---|---|---|---|---|---|---|
| Tech. | Taylor | efficient | 2.78 | 10.42 | 83.51 | 174.74 |
| | | distorted | 2.71 | 9.41 | 73.53 | 153.56 |
| | Ramsey | efficient | 12.26 | 26.52 | 164.27 | 331.71 |
| | | distorted | 11.48 | 13.46 | 217.54 | 428.91 |
| Pref. | Taylor | efficient | 0.15 | 0.77 | 6.75 | 14.21 |
| | | distorted | 0.12 | 0.61 | 5.36 | 11.29 |
| | Ramsey | efficient | 15.35 | 20.90 | 74.21 | 140.28 |
| | | distorted | 15.19 | 21.30 | 69.18 | 131.49 |
| Lab. | Taylor | efficient | $5.10^{-4}$ | 0.0013 | 0.0096 | 0.0199 |
| | | distorted | $3.10^{-4}$ | 0.0006 | 0.0036 | 0.0074 |
| | Ramsey | efficient | 0.22 | 0.79 | 6.47 | 13.04 |
| | | distorted | 0.19 | 0.06 | 10.40 | 20.50 |
| Price | Taylor | efficient | −0.0345 | −0.0345 | −0.0345 | −0.0345 |
| | | distorted | −0.01 | 0.32 | 3.44 | 7.35 |
| | Ramsey | efficient | $-3.10^{-3}$ | $-3.10^{-3}$ | $-3.10^{-3}$ | $-3.10^{-3}$ |
| | | distorted | 0.47 | −4.10 | −26.96 | −51.23 |

Table 4: Contribution of selected structural shocks to the term premium (mean, in bp)

We focus on four different calibrations for $\alpha_{EZ}$: 0, 88, 920 and 1981, which respectively correspond to the expected utility case, the value used by Rudebusch and Swanson (2009), our empirical benchmark and the value estimated by Andreasen (2010). Table 4 reports how the average level of term premium implied by our model is affected by Epstein-Zin preferences. As previously, we isolate the contribution of each shock to the unconditional mean of the term premium[20]. We present the results obtained under optimal policy and using a Taylor rule, when the steady state is efficient and distorted. We observe the two effects described above. In the Taylor-rule benchmark and under optimal policy with an efficient steady state, the level of the term premium is a linear function of $\alpha_{EZ}$. For the three efficient shocks – technology, preference and labour supply – an efficient steady state leads to higher risk premia for all values of $\alpha_{EZ}$. Interestingly, regarding the inefficient shock – price markup shock – when the steady state is efficient, the level of the term premium is not affected by Epstein-Zin preferences. We would obtain the same feature with a wage mark up shock. Hence, when the monopolistic distortions are offset by subsidies in the steady state, the contribution of inefficient shocks to the average term premium does not depend on the Epstein-Zin sensitivity. This means that

[20] The mean of the term premium is the sum of unconditional terms of the form $E(\varepsilon_t^i \Omega_{i,j} \varepsilon_t^j)$ where $i$ and $j$ refer to structural shocks. When $i \neq j$, those terms are zero.

the first covariance in (17) is zero when the economy is hit by inefficient markup shock: on average, fluctuations of the markup around an efficient steady state do not yield any significant correlations between the asset price and the wealth and hence no dependance of the risk premium on the risk aversion[21]. Turning to the optimal policy case in the case of a distorted steady state, we observe that the relationship is in general no longer linear, which results from the modification of the correlations of macro-aggregates. Interestingly, the shock on labour supply, whose contribution is almost zero with a Taylor rule, can reach non-negligible levels under optimal policy. In addition, we observe that the contribution of the price markup shock is markedly negative under optimal policy with a distorted steady state. This stems from the fact that, in such context, the central bank has to strongly increase the interest rate while consumption, for high values of $\alpha_{EZ}$, remains relatively close to its path with a Taylor rule (see figure 4 and table 10). Finally, for every shock, we note that the absolute value of the level of the term premium is markedly higher under optimal policy than with a Taylor rule. This result is not surprising. Indeed, it is well known that, within such a medium-scale framework, the short rate is more volatile under optimal policy[22] (see Adjemian et al., 2007, 2008). This higher volatility results in higher risk premia (see table 6).

Finally, we examine the influence of Epstein-Zin preferences on the dynamic of the term premium under optimal policy. To do so, we use a third order approximation to the model – again, relying on the perturbation method –as the second order approximation yields only a constant term premium (see Rudebusch and Swanson, 2008, 2009, De Graeve et al., 2009). Charts 6 displays the impulse response functions obtained under optimal policy without subsidies. Compared with the Taylor rule case, Epstein-Zin preferences considerably affect the responses of the term premium: the initial impact is much stronger for every shocks and the persistence is markedly higher for the technology and labour supply shocks. Again, regarding the price markup shock, we note a qualitatively opposite effect of Epstein-Zin preferences which imply a positive response of the premium. This result regarding inefficient shocks naturally comes from a mechanism similar to the one explained above (strong increase in the interest rate).

[21] However, as underlined by Swanson (2010), this does not necessarily mean that the implied term premium is zero. In our model, the contribution of the price markup shock is slightly negative in this case.

[22] In our empirical benchmark with Taylor rule, the volatility of the short rate is 0.28% whereas it amounts to 2.15% under optimal policy with a distorted steady state and $\alpha_{EZ} = 930$.

# 5 What drives the Epstein-Zin effects in the optimal allocation?

As mentioned in the introduction, one of the main purpose of this paper is to analyse Epstein-Zin preferences within an operational and commonly used, medium-scale DSGE model. In this section we wish to know the extent to which the results presented in the previous section rely on the numerous underlying assumptions. Given the high number of parameters and assumptions, we highlight two effects which appear to be the most relevant for the understanding of the way Epstein-Zin preferences operate: the dependence of optimal policy on the presence of real rigidities via quasi-kinked demands and the combined effect of capital accumulation and wage rigidities.

## 5.1 The role of quasi-kinked demands

Here we examine the influence of real rigidities and Kimball aggregators on optimal monetary policy with or without Epstein-Zin preferences. As emphasized by Levin et al. (2008), once the overall degree of rigidity – i.e. the slope of the Phillips curves for prices and wages inflation – is fixed, the combination of nominal and real rigidities (the choice of $\psi_i$, $\alpha_i$) does not matter for the first order approximation to the equilibrium conditions, i.e., yields the same first order dynamic for price and wage inflation. In addition, as shown in section 3, in a Taylor rule benchmark, the implied term premium does not depend on the presence of such real rigidities. However, we know that the introduction of Kimball aggregators ($\psi_i > 0$) have markedly different implications when optimal policy is considered (see Levin et al., 2008). We contribute to this literature by investigating how the welfare is affected by real rigidities when the agents have Epstein-Zin preferences. In the previous section, we have considered the case of a complete model including Kimball aggregators. We now examine the optimal policy in the case of standard Dixit-Stiglitz aggregators ($\psi_i = 0$), setting the degrees of nominal rigidity so that the overall degree of real rigidity remains unchanged.

First, our numerical simulations suggest that the strong sensitivity of the volatility of optimal policy with respect to Epstein-Zin preferences is mainly due to Kimball aggregators. In a model with standard Dixit-Stiglitz aggregators, Epstein-Zin preferences still have an impact on the macroeconomic outcome, but, for a given combination of parameters, its magnitude is less important as can be seen in table 5: the volatility of the macroeconomic aggregates are not substantially affected by Epstein-Zin preferences (compared with table 3). Second, for the same value of $\alpha_{EZ}$ (here 930), figures 7 to 10

| | $\pi_t$ | $W_t$ | $R_t$ | $Y_t$ | $C_t$ | $L_t$ | $OG_t$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|
| tech. | 0.0545 | 1.7893 | 0.5442 | 2.4280 | 2.2571 | 0.6213 | 0.2203 | 6.7430 |
| *tech.* | *0.0429* | *1.8989* | *0.6374* | *2.5435* | *2.3429* | *0.5691* | *0.1321* | *7.0602* |
| pref. | 0.0009 | 0.0433 | 1.5381 | 0.1760 | 0.4761 | 0.1590 | 0.0027 | 1.0773 |
| *pref.* | *0.0059* | *0.0305* | *1.5134* | *0.1931* | *0.4940* | *0.1769* | *0.0387* | *0.9826* |
| lab. | 0.0012 | 0.0624 | 0.0813 | 0.5279 | 0.5724 | 0.3976 | 0.0074 | 1.0833 |
| *lab.* | *0.0068* | *0.0772* | *0.1116* | *0.5575* | *0.5953* | *0.4184* | *0.0521* | *1.1646* |
| price | 0.0381 | 1.4326 | 0.4189 | 0.8365 | 0.6879 | 0.4625 | 0.8365 | 2.2009 |
| *price* | *0.0458* | *1.3970* | *0.4557* | *0.8103* | *0.6665* | *0.4465* | *0.8103* | *2.1322* |

Table 5: Volatility under optimal policy with a distorted steady state. The Epstein-Zin parameter is set to 930 (in italic). Dixit-Stiglitz aggregators.

compared with figures 1 to 10 show that the *sign* of the Epstein-Zin effect may depend on the degree of real rigidities. For example, regarding the productivity shock, the reaction of inflation is amplified when quasi-kinked demands are assumed whereas it is lessened when Dixit-Stiglitz aggregators are considered. Similar features hold for the other macroeconomic aggregates and the other shocks. Unlike the Kimball case, we also notice that the effect is in general concentrated on the initial periods. This is especially the case for the price markup shock which leads to much less persistence responses than in section 4.1[23].

We can draw an important conclusion from this empirical evidence. For a given value of $\alpha_{EZ}$, the results presented in section 4 are *not* robust to a change in the specification of real rigidities. This underscores the very strong non-linear implications of real rigidities. Although they have identical implications for the first order dynamic of macroeconomic aggregates in both expected utility and Epstein-Zin worlds, they induce markedly different optimal policy for a given value of the Epstein-Zin parameter. We therefore generalize the message of Levin et al. (2008): in an Epstein-Zin world, the microeconomic dissonance of real rigidities have considerable implications on the welfare. As a consequence, such dissonance is also non-neutral with respect to the term premium. In an expected utility world, quasi-kinked demands imply slightly – a few basis points – higher term premia. In an Epstein-Zin world, this difference is sharpened, especially when the steady state is distorted (see table 6).

[23] This feature is also noticed by Levin et al. (2008).

| Steady State | Aggregators | $\alpha_{EZ} = 0$ | $\alpha_{EZ} = 930$ |
|---|---|---|---|
| Efficient | Qs. Kinked | 29 | 282 |
| | Dixit Stiglitz | 27 | 264 |
| Distorted | Qs. Kinked | 34 | 381 |
| | Dixit Stiglitz | 28 | 273 |

Table 6: Influence of quasi-kinked demands on term premium (mean, in bp)

### 5.2 Capital, wage rigidities and the Epstein-Zin mechanics

In this subsection we detail how Epstein-Zin preferences operate under optimal policy within our medium-scale model. In this respect we perform a sensitivity analysis by focusing on capital, price and wage rigitities. Here we consider only Dixit Stiglitz aggregators in order to make our results comparable with those of Levin et al. (2008). Since many papers dealing with optimal policy in a New Keynesian framework do not allow for capital accumulation – Benigno and Woodford (2005), Levin et al. (2008), for example – the first restriction we impose pertains to capital. Starting from the full model, we find that dropping capital accumulation leads to *qualitative* differences in the reactions to shocks. As shown in figures 11 and 12, regarding the technology and labour supply shocks, the sign of the shift due to Epstein-Zin preferences depends on the presence of capital. However, this phenomenon does not happen for preference and price markup shocks.

We now argue that this difference results from the fact that, with Dixit-Stiglitz aggregators, Epstein-Zin preferences amplify the policy tradeoff due to price rigidities while this amplification tends to be weakened by wage rigidities. To see this, we conduct an analysis of the Epstein-Zin impact using submodels. For brevety, we focus on the response of inflation to a positive technology shock, but a similar analysis would also apply to other shocks and variables. We naturally study the case of a distorted steady state. Let us first consider a submodel including only price rigidities[24]. In the expected utility world, the policymaker faces a tradeoff and cannot stabilize inflation and welfare-relevant output gap. Following a positive technology shock, inflation reacts negatively under optimal policy. With Epstein-Zin preferences, this impact is amplified as shown in chart 13 (left side). Now, adding endogenous capital accumulation leads to the opposite reaction. Even in the expected utility world, inflation positively react to the same technology shock. This is fully in line with the results obtained by Faia (2008). In

[24] From the full model, we drop capital accumulation, wages rigidities and subsidies.

this case again, we note that Epstein-Zin preferences tend to amplify the reactions (see chart 13, right side). We note that the degree of price rigidities does not significantly affect the Epstein-Zin impact. Finally, progressively allowing for wage rigidities – and therefore returning to the full model with Dixit-Stiglitz aggregators – we observe i) a downward shift of the response of inflation in both expected utility and Epstein-Zin cases, along with ii) a gradual decrease in the Epstein-Zin effect. This decrease also occurs in absence of capital. This can be seen in chart 14. This shows that price and nominal wage rigidities do not have the same impact on the policymaker's tradeoff. As a conclusion, we have shown that, in presence of Dixit-Stiglitz aggregators, with price rigidities only, the optimal reaction is amplified, be it with capital or not. Adding wage rigidities tend to reduce this amplification[25].

# 6 Conclusion

In this paper, through the use of Epstein-Zin preferences in a Smets and Wouters (2007) framework, we provide an empirical benchmark in which we are able to generate a ten-year term premium of 100 bp on average. We show that the calibration of $\alpha_{EZ}$ needed to match such a level mainly depends on the intertemporal elasticity of substitution and the deterministic discount factor associated with the use of detrended variables. As a theoretical matter, Epstein-Zin preferences as well as real rigidities in the form of quasi-kinked demands do not have any impact on the first order dynamic of the macro-model and the implied term premium, on average, is linear as a function of the Epstein-Zin parameter. Against this background, we perform numerical simulations in order to analyse the optimal monetary policy implied by such a benchmark and how Epstein-Zin preferences distort the optimal allocation. Our results highlight several effects. First, within medium-scale DSGE frameworks, Epstein-Zin preferences do have a significant impact on the first order dynamics of optimal policy – especially inflation, output gap and real wage. Second, our simulations show that this effect strongly depends on the presence of quasi-kinked demands which, in our benchmark, alter the sign of the Epstein-Zin effects and the volatility. Third, our numerical approach allows us to span many submodels relatively easily: expliciting the first order approximation of the Ramsey problem by hand would be impossible in the full model and very cumbersome in small submodels. In addition, in any case, such an approach would not deliver more

[25] As a check of robustness, we have verified that the results presented in section 5.2 are valid for any value of $\alpha_{EZ}$ lying between 0 and 930.

transparency on the way Epstein-Zin preferences operate. We are then able to provide an analysis on how sensitive the Epstein-Zin effect is to assumptions pertaining to capital accumulation, price and wage rigidities. Finally, an important novelty of this paper is the investigation of the term premium under optimal policy. We have shown that, in general, Epstein-Zin preferences as well as other forms of microeconomic dissonance such as quasi-kinked demand have a non-linear impact on the term premium. Overall, the message delivered by our analysis is clear: non time-separable preferences are in general *not* neutral with respect to the (first order) macroeconomic dynamics of DSGE models as well as the implied term premium, as could have been thought in considering Taylor rules. To a large extent, Epstein-Zin preferences in conjunction with specific rigidities or underlying microfoundations, affect the optimal macroeconomic allocation and the implied risk premia.

If the aim of this paper is to shed light on the use of Epstein-Zin preferences within a structural framework, our approach calls for further investigations. First, we have focused on one value of the Epstein-Zin parameter which allows us to generate a term premium of 100 bp on average in a attempt at solving the *bond premium puzzle*. It would be interesting to explore how other financial assets – such as real bonds or equity – would be affected by Epstein-Zin preferences and optimal policy. Second, our methodology could be applied to models including other types of real rigidities (firm specific capital, for example) and more generally to richer models including financial frictions or several countries.

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# A Equilibrium conditions

We define

$$X_{\mathcal{W}t} = \left( \frac{\mathcal{W}_t}{\mathbb{E}_{t-1}\left[\mathcal{W}_t^{1-\alpha_{EZ}}\right]^{\frac{1}{1-\alpha_{EZ}}}} \right)^{-\alpha_{EZ}} \tag{18}$$

## A.1 Households' programme

The first order conditions of the households ' programme can be written in the form:

$$\lambda_t = R_t \beta \mathbb{E}_t \left[ X_{\mathcal{W}t+1} \lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \tag{19}$$

with

$$\begin{aligned} \lambda_t \quad = \quad & \varepsilon_t^b (C_t - hC_{t-1})^{\sigma_c} \exp\left( \left(\varepsilon_t^l\right)^{-1} \widetilde{L} \frac{(\sigma_c - 1)}{(1+\sigma_l)} {L_t}^{1+\sigma_l} \right) \\ & -\beta h \mathbb{E}_t \left[ X_{\mathcal{W}t+1} \varepsilon_{t+1}^b (C_{t+1} - hC_t)^{\sigma_c} \exp\left( \left(\varepsilon_{t+1}^l\right)^{-1} \widetilde{L} \frac{(\sigma_c - 1)}{(1+\sigma_l)} {L_{t+1}}^{1+\sigma_l} \right) \right] \end{aligned} \tag{20}$$

$$Q_t = \varepsilon_t^Q \beta \mathbb{E} \left[ \frac{\lambda_{t+1}}{\lambda_t} X_{\mathcal{W}t+1} \left( Q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - \Psi\left(u_{t+1}\right) \right) \right] \tag{21}$$

$$\begin{aligned} & Q_t \left[ 1 - S\left( \frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S'\left( \frac{I_t}{I_{t-1}} \right) \right] \varepsilon_t^I \\ + \quad & \beta \mathbb{E}_t \left[ Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} X_{\mathcal{W}t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 S'\left( \frac{I_{t+1}}{I_t} \right) \varepsilon_{t+1}^I \right] = 1 \end{aligned} \tag{22}$$

$$r_t^k = \Psi'\left(u_t\right) \tag{23}$$

## A.2 Unions' programme

In the following, given that the steady state model features a balanced growth path, all variables are appropriately deflated to be stationary in the stochastic equilibrium. The first order condition of the union's program for the re-optimized wage $w_t^*$ can be written recursively as follows:

$$w_t^* = \frac{\theta_w(1+\psi_w)}{\left(\theta_w(1+\psi_w)-1\right)} \frac{\mathcal{H}_{1,t}}{\mathcal{H}_{2,t}} + \frac{\psi_w}{(\theta_w - 1)} \left(w_t^*\right)^{1+\theta_w(1+\psi_w)} \frac{\mathcal{H}_{3,t}}{\mathcal{H}_{2,t}} \tag{24}$$

with

$$\mathcal{H}_{1,t} = \varepsilon_t^B \widetilde{L} L_t^{1+\sigma_l} w_t^{\theta_w(1+\psi_w)} \left(C_t - \eta C_{t-1}/\gamma\right)^{(1-\sigma_c)} \exp^{\left((\varepsilon_t^l)^{-1}\widetilde{L}\frac{(\sigma_c-1)}{(1+\sigma_l)}L_t^{(1+\sigma_l)}\right)} \Delta_{w\lambda,t}^{\theta_w(1+\psi_w)/(1-\theta_w(1+\psi_w))}/\lambda_t$$
$$+ \beta\gamma^{(1-\sigma_c)}\alpha_w \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} X_{\mathcal{W}t+1} \left(\frac{\pi_{t+1}}{\pi_t^{\xi_w}\left[\pi^\star\right]^{(1-\xi_w)}}\right)^{\theta_w(1+\psi_w)} \mathcal{H}_{1,t+1}\right] \quad (25)$$

$$\mathcal{H}_{2,t} = (1-\tau_{w,t}) L_t w_t^{\theta_w(1+\psi_w)} \Delta_{w\lambda,t}^{\theta_w(1+\psi_w)/(1-\theta_w(1+\psi_w))}$$
$$+ \beta\gamma^{(1-\sigma_c)}\alpha_w \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} X_{\mathcal{W}t+1} \left(\frac{\pi_{t+1}}{\pi_t^{\xi_w}\left[\pi^\star\right]^{(1-\xi_w)}}\right)^{\theta_w(1+\psi_w)-1} \mathcal{H}_{2,t+1}\right] \quad (26)$$

$$\mathcal{H}_{3,t} = (1-\tau_{w,t}) L_t + \beta\gamma^{(1-\sigma_c)}\alpha_w \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} X_{\mathcal{W}t+1} \left(\frac{\pi_t^{\xi_w}\left[\pi^\star\right]^{(1-\xi_w)}}{\pi_{t+1}}\right) \mathcal{H}_{3,t+1}\right] \quad (27)$$

The aggregate wage dynamics could also be expressed as

$$\begin{aligned}(w_t)^{1-\theta_w(1+\psi_w)} \Delta_{w\lambda,t} &= (1-\alpha_w)\left(w_t^*\right)^{1-\theta_w(1+\psi_w)} \\ &+\alpha_w \left(\frac{\pi_t}{\pi_{t-1}^{\xi_w}[\pi^\star]^{1-\xi_w}}\right)^{\theta_w(1+\psi_w)-1} (w_{t-1})^{1-\theta_w(1+\psi_w)} \Delta_{w\lambda,t}\end{aligned} \quad (28)$$

The previous equations include a dispersion index $\Delta_{w\lambda,t}$ which is related to the re-optimizing wage and the aggregate wage through the following conditions

$$1 = \frac{1}{1+\psi_w}\Delta_{w\lambda,t}^{1/(1-\theta_w(1+\psi_w))} + \frac{\psi_w}{1+\psi_w}\Delta_{wl,t} \quad (29)$$

$$\Delta_{wl,t} = (1-\alpha_w)\left(\frac{w_t^*}{w_t}\right) + \alpha_w \left(\frac{w_t}{w_{t-1}}\frac{\pi_t}{\pi_{t-1}^{\xi_w}[\pi^\star]^{1-\xi_w}}\right)^{-1} \Delta_{wl,t-1} \quad (30)$$

The market clearing condition linking total labor demand of intermediate firms and total labor supply of households includes a wage dispersion index given by

$$\Delta_{wk,t} = \frac{1}{1+\psi_w}\Delta_{w,t} \cdot \Delta_{w\lambda,t}^{\theta_w(1+\psi_w)/(1-\theta_w(1+\psi_w))} + \frac{\psi_w}{1+\psi_w} \quad (31)$$

with

$$\Delta_{w,t} = (1-\alpha_w)\left(\frac{w_t^*}{w_t}\right)^{-\theta_w(1+\psi_w)} + \alpha_w \left(\frac{w_t}{w_{t-1}}\frac{\pi_t}{\pi_{t-1}^{\xi_w}[\pi^\star]^{1-\xi_w}}\right)^{\theta_w(1+\psi_w)} \Delta_{w,t-1} \quad (32)$$

### A.3 Intermediate firms' programme

The first order condition of the intermediate firms profit maximization leads to

$$p_t^* = \frac{\theta_p(1+\psi)}{(\theta_p(1+\psi)-1)}\frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}} + \frac{\psi}{(\theta_p-1)}\left(p_t^*\right)^{1+\theta_p(1+\psi)}\frac{\mathcal{Z}_{3,t}}{\mathcal{Z}_{2,t}} \quad (33)$$

with

$$\mathcal{Z}_{1,t} = mc_t Y_t \Delta_{p\lambda,t}^{\theta_p(1+\psi)/(1-\theta_p(1+\psi))} + \beta\gamma^{(1-\sigma_c)}\alpha_p \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} X_{\mathcal{W}t+1} \left( \frac{\pi_{t+1}}{\pi_t^{\xi_p} [\pi^\star]^{(1-\xi_p)}} \right)^{\theta_p(1+\psi)} \mathcal{Z}_{1,t+1} \right] \quad (34)$$

$$\mathcal{Z}_{2,t} = (1-\tau_{p,t}) Y_t \Delta_{p\lambda,t}^{\theta_p(1+\psi)/(1-\theta_p(1+\psi))} + \beta\gamma^{(1-\sigma_c)}\alpha_p \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} X_{\mathcal{W}t+1} \left( \frac{\pi_{t+1}}{\pi_t^{\xi_p} [\pi^\star]^{(1-\xi_p)}} \right)^{\theta_p(1+\psi)-1} \mathcal{Z}_{2,t+1} \right] \quad (35)$$

$$\mathcal{Z}_{3,t} = (1-\tau_{p,t}) Y_t + \beta\gamma^{(1-\sigma_c)}\alpha_p \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} X_{\mathcal{W}t+1} \left( \frac{\pi_t^{\xi_p} [\pi^\star]^{(1-\xi_p)}}{\pi_{t+1}} \right) \mathcal{Z}_{3,t+1} \right] \quad (36)$$

Aggregate price dynamics can then be written as

$$\Delta_{p\lambda,t} = (1-\alpha_p) (p_t^*)^{1-\theta_p(1+\psi)} + \alpha_p \left( \frac{\pi_t}{\pi_{t-1}^{\xi_p} [\pi^\star]^{1-\xi_p}} \right)^{\theta_p(1+\psi)-1} \Delta_{p\lambda,t-1} \quad (37)$$

The dispersion index $\Delta_{p\lambda,t}$ is given by

$$1 = \frac{1}{1+\psi} \Delta_{p\lambda,t}^{1/(1-\theta_p(1+\psi))} + \frac{\psi}{1+\psi} \Delta_{pl,t} \quad (38)$$

$$\Delta_{pl,t} = (1-\alpha_p) (p_t^*) + \alpha_p \left( \frac{\pi_t}{\pi_{t-1}^{\xi_p} [\pi^\star]^{1-\xi_p}} \right)^{-1} \Delta_{pl,t-1} \quad (39)$$

The market clearing conditions in the goods market also involves a price dispersion index given by

$$\Delta_{pk,t} = \frac{1}{1+\psi} \Delta_{p,t} \cdot \Delta_{p\lambda,t}^{\theta_p(1+\psi)/(1-\theta_p(1+\psi))} + \frac{\psi}{1+\psi} \quad (40)$$

with

$$\Delta_{p,t} = (1-\alpha_p) (p_t^*)^{-\theta_p(1+\psi)} + \alpha_p \left( \frac{\pi_t}{\pi_{t-1}^{\xi_p} [\pi^\star]^{1-\xi_p}} \right)^{\theta_p(1+\psi)} \Delta_{p,t-1} \quad (41)$$

# B Derivation of the optimal policy

The derivation of the optimal policy involves the differentiation of the constraints (20), (21), (22), (25), (26), (27), (34), (35) and (36) with respect to $\mathcal{W}_t$. In the Lagrangian, all these constraints can be written in the form:

$$E_{t-1} [X_{\mathcal{W}t} d_t]$$

Denoting $Wkp_t = E_{t-1}\left[X_{\mathcal{W}tt}^{1-\alpha_{EZ}}\right]^{\frac{1}{1-\alpha_{EZ}}}$, it can be easily seen that

$$\frac{\partial}{\partial \mathcal{W}_t} Wkp_t = X_{\mathcal{W}t}$$

and

$$\frac{\partial}{\partial \mathcal{W}_t} E_{t-1}\left[X_{\mathcal{W}t} d_t\right] = \alpha_{EZ} X_{\mathcal{W}t}\left(Wkp_{t-1}^{-1} E_{t-1}\left[X_{\mathcal{W}t} d_t\right] - \mathcal{W}_t^{-1} d_t\right)$$

This allows to derive the non-linear optimal policy.

| | $\pi_t$ | $W_t$ | $R_t$ | $Y_t$ | $C_t$ | $L_t$ | $OG_t$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|
| $\pi_t$ | 1 | 0.2570 | 0.8168 | 0.0777 | 0.0838 | 0.6345 | 0.2460 | 0.1012 |
| | *1* | *-0.9400* | *0.7976* | *-0.9450* | *-0.9840* | *0.7218* | *0.9157* | *-0.8943* |
| $W_t$ | | 1 | -0.2971 | 0.9579 | 0.9802 | -0.1554 | 0.8350 | 0.9247 |
| | | *1* | *-0.5881* | *0.9684* | *0.9786* | *-0.5080* | *-0.8719* | *0.9418* |
| $R_t$ | | | 1 | -0.4507 | -0.4307 | 0.7332 | -0.2057 | -0.4299 |
| | | | *1* | *-0.6868* | *-0.6846* | *0.8562* | *0.6176* | *-0.6532* |
| $Y_t$ | | | | 1 | 0.9618 | -0.1261 | 0.9165 | 0.9892 |
| | | | | *1* | *0.9599* | *-0.4940* | *-0.7833* | *0.9896* |
| $C_t$ | | | | | 1 | -0.2710 | 0.8050 | 0.9129 |
| | | | | | *1* | *-0.6180* | *-0.9208* | *0.9116* |
| $L_t$ | | | | | | 1 | 0.266 | -0.0433 |
| | | | | | | *1* | *0.7479* | *-0.4147* |
| $OG_t$ | | | | | | | 1 | 0.9445 |
| | | | | | | | *1* | *-0.6968* |
| $I_t$ | | | | | | | | 1 |
| | | | | | | | | *1* |

Table 7: Technology shock. Correlations under optimal policy with a distorted steady state. The Epstein-Zin parameter is set to 930 (in italic)

| | $\pi_t$ | $W_t$ | $R_t$ | $Y_t$ | $C_t$ | $L_t$ | $OG_t$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|
| $\pi_t$ | 1 | -0.6096 | 0.4088 | 0.2986 | 0.5513 | 0.4859 | -0.4870 | -0.7500 |
| | *1* | *0.9598* | *-0.1972* | *0.2281* | *-0.2028* | *-0.1762* | *0.9899* | *0.6638* |
| $W_t$ | | 1 | -0.0828 | 0.0663 | -0.1174 | -0.2501 | 0.3178 | 0.4300 |
| | | *1* | *-0.0275* | *0.4145* | *0.0654* | *0.0078* | *0.9491* | *0.4290* |
| $R_t$ | | | 1 | 0.9017 | 0.7895 | 0.9064 | -0.0065 | -0.5202 |
| | | | *1* | *0.8198* | *0.7840* | *0.9162* | *-0.0988* | *-0.4872* |
| $Y_t$ | | | | 1 | 0.8845 | 0.9490 | -0.0992 | -0.5578 |
| | | | | *1* | *0.8137* | *0.9130* | *0.3094* | *-0.2992* |
| $C_t$ | | | | | 1 | 0.8859 | -0.5378 | -0.8719 |
| | | | | | *1* | *0.8574* | *-0.1660* | *-0.7918* |
| $L_t$ | | | | | | 1 | -0.1721 | -0.6625 |
| | | | | | | *1* | *-0.0798* | *-0.5104* |
| $OG_t$ | | | | | | | 1 | 0.8267 |
| | | | | | | | *1* | *0.6739* |
| $I_t$ | | | | | | | | 1 |
| | | | | | | | | *1* |

Table 8: Preferences shock. Correlations under optimal policy with a distorted steady state. The Epstein-Zin parameter is set to 930 (in italic)

| | $\pi_t$ | $W_t$ | $R_t$ | $Y_t$ | $C_t$ | $L_t$ | $OG_t$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|
| $\pi_t$ | 1 | -0.0503 | -0.5365 | 0.5119 | 0.3607 | 0.6251 | 0.8987 | 0.6010 |
| | *1* | *0.8641* | *0.7513* | *-0.9863* | *-0.9491* | *-0.9971* | *0.9798* | *-0.9953* |
| $W_t$ | | 1 | -0.1480 | 0.8107 | 0.9043 | 0.7183 | 0.3720 | 0.7213 |
| | | *1* | *0.5135* | *-0.8049* | *-0.7132* | *-0.8605* | *0.8402* | *-0.8751* |
| $R_t$ | | | 1 | -0.2982 | -0.2739 | -0.3253 | -0.4515 | -0.2841 |
| | | | *1* | *-0.6894* | *-0.6476* | *-0.7137* | *0.6365* | *-0.6899* |
| $Y_t$ | | | | 1 | 0.9829 | 0.9895 | 0.8319 | 0.9893 |
| | | | | *1* | *0.9864* | *0.9912* | *-0.9930* | *0.9905* |
| $C_t$ | | | | | 1 | 0.9464 | 0.7247 | 0.9466 |
| | | | | | *1* | *0.9560* | *-0.9788* | *0.9554* |
| $L_t$ | | | | | | 1 | 0.9004 | 0.9980 |
| | | | | | | *1* | *-0.9840* | *0.9988* |
| $OG_t$ | | | | | | | 1 | 0.8851 |
| | | | | | | | *1* | *-0.9886* |
| $I_t$ | | | | | | | | 1 |
| | | | | | | | | *1* |

Table 9: Labour supply shock. Correlations under optimal policy with a distorted steady state. The Epstein-Zin parameter is set to 930 (in italic)

| | $\pi_t$ | $W_t$ | $R_t$ | $Y_t$ | $C_t$ | $L_t$ | $OG_t$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|
| $\pi_t$ | 1 | -0.0792 | 0.0236 | 0.1198 | 0.3045 | 0.0991 | 0.1198 | 0.0270 |
| | *1* | *0.9896* | *0.2800* | *0.9737* | *0.9946* | *0.8942* | *0.9737* | *0.9356* |
| $W_t$ | | 1 | 0.0051 | 0.9737 | 0.9103 | 0.9392 | 0.9737 | 0.9797 |
| | | *1* | *0.2766* | *0.9943* | *0.9768* | *0.9463* | *0.9943* | *0.9762* |
| $R_t$ | | | 1 | -0.0836 | -0.0481 | -0.1909 | -0.0836 | -0.0804 |
| | | | *1* | *0.2485* | *0.2461* | *0.2149* | *0.2485* | *0.2478* |
| $Y_t$ | | | | 1 | 0.9648 | 0.9799 | 1.0000 | 0.9898 |
| | | | | *1* | *0.9643* | *0.9725* | *1.0000* | *0.9895* |
| $C_t$ | | | | | 1 | 0.9107 | 0.9648 | 0.9183 |
| | | | | | *1* | *0.8790* | *0.9643* | *0.9165* |
| $L_t$ | | | | | | 1 | 0.9799 | 0.9872 |
| | | | | | | *1* | *0.9725* | *0.9923* |
| $OG_t$ | | | | | | | 1 | 0.9872 |
| | | | | | | | *1* | *0.9895* |
| $I_t$ | | | | | | | | 1 |
| | | | | | | | | *1* |

Table 10: Price Markup shock. Correlations under optimal policy with a distorted steady state. The Epstein-Zin parameter is set to 930 (in italic)

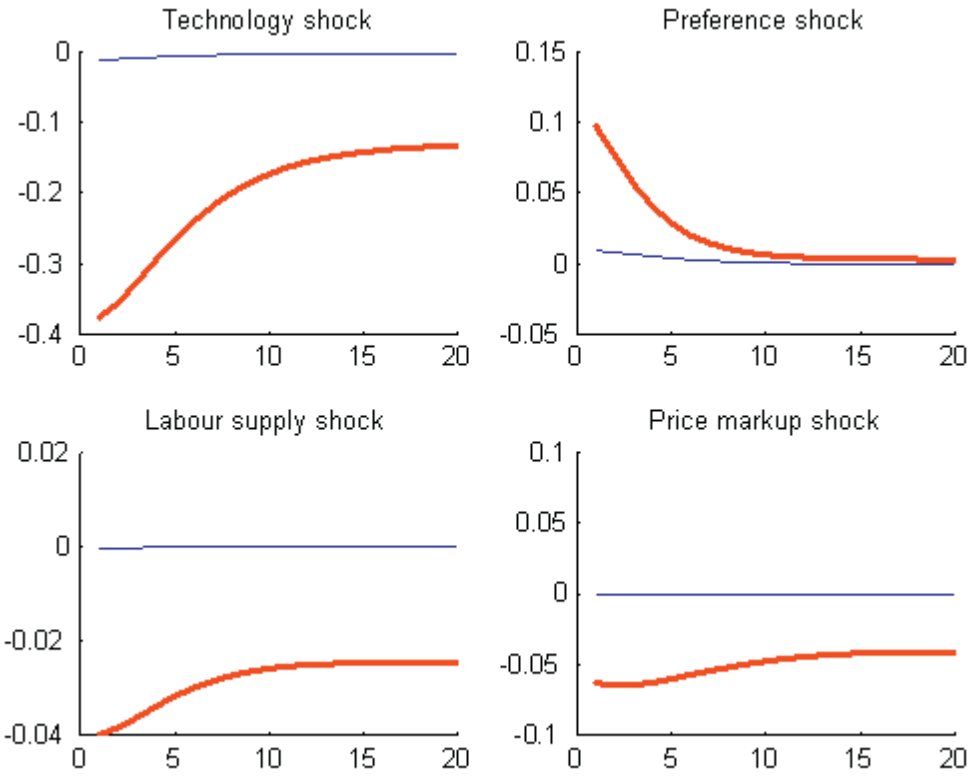


Figure 5: Impulse response functions of the term premium, distorted steady state. Taylor rule expected utility (thin blue lines), Taylor rule Epstein-Zin preferences, $\alpha_{EZ} = 930$ (large red lines).

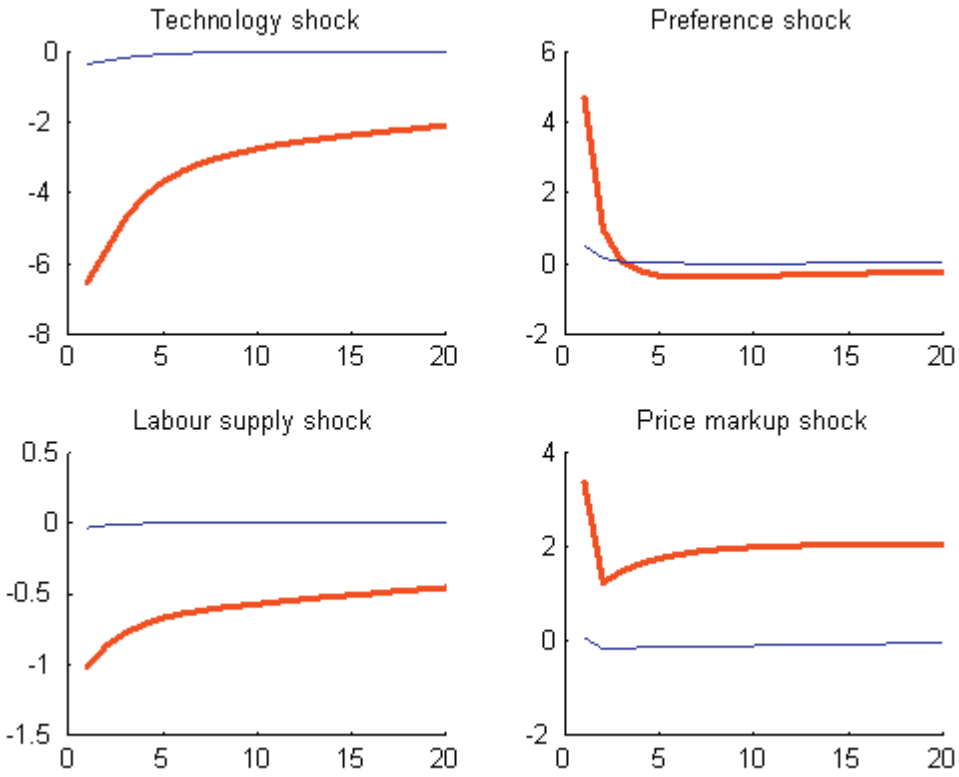


Figure 6: Impulse response functions of the term premium, distorted steady state. Ramsey expected utility (thin blue lines), Ramsey Epstein-Zin preferences $\alpha_{EZ} = 930$ (large red lines).

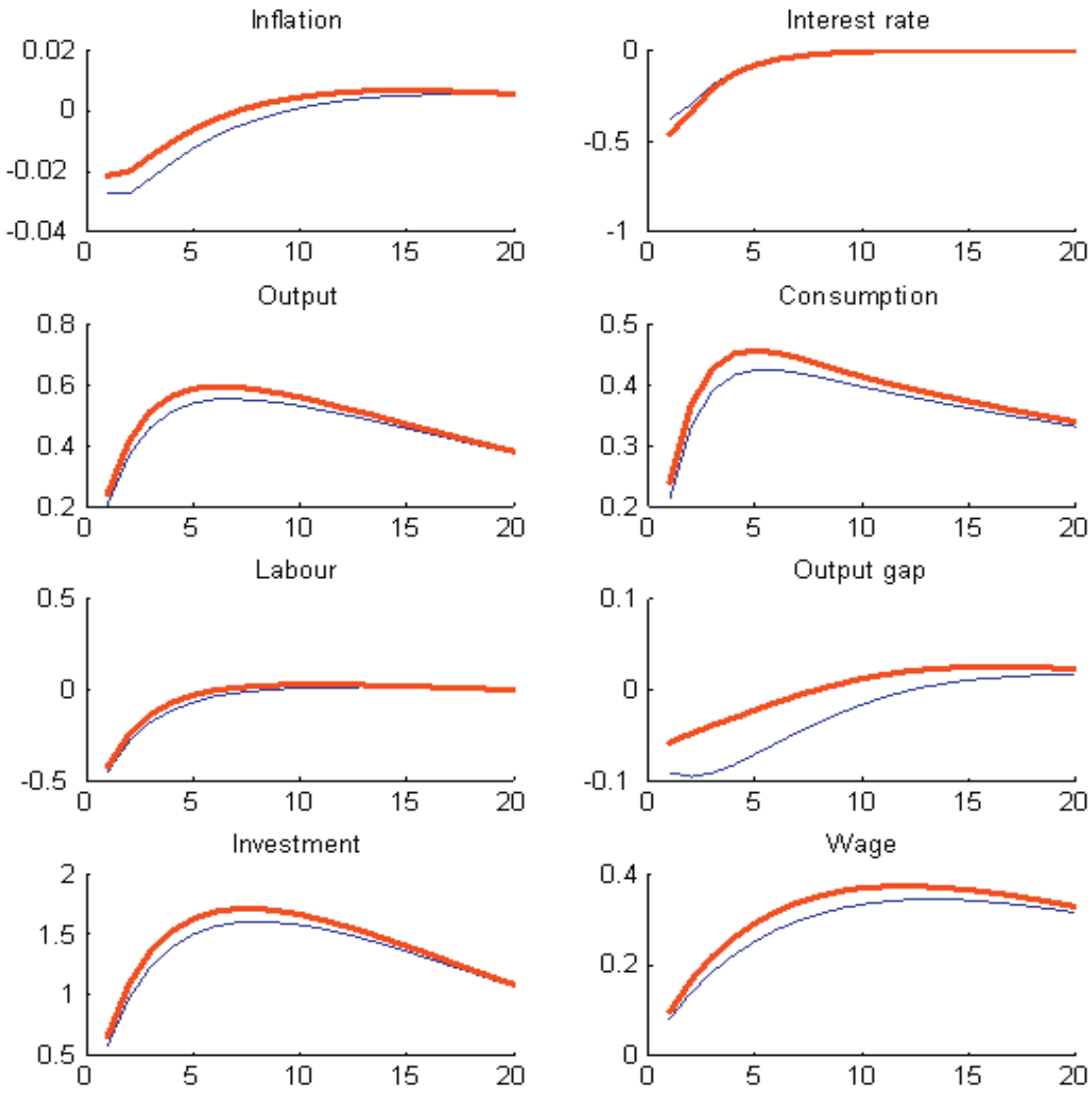


Figure 7: Technology shock, Dixit-Stiglitz aggregators, distorted steady state. Ramsey expected utility (thin blue lines), Ramsey Epstein-Zin preferences (large red lines).

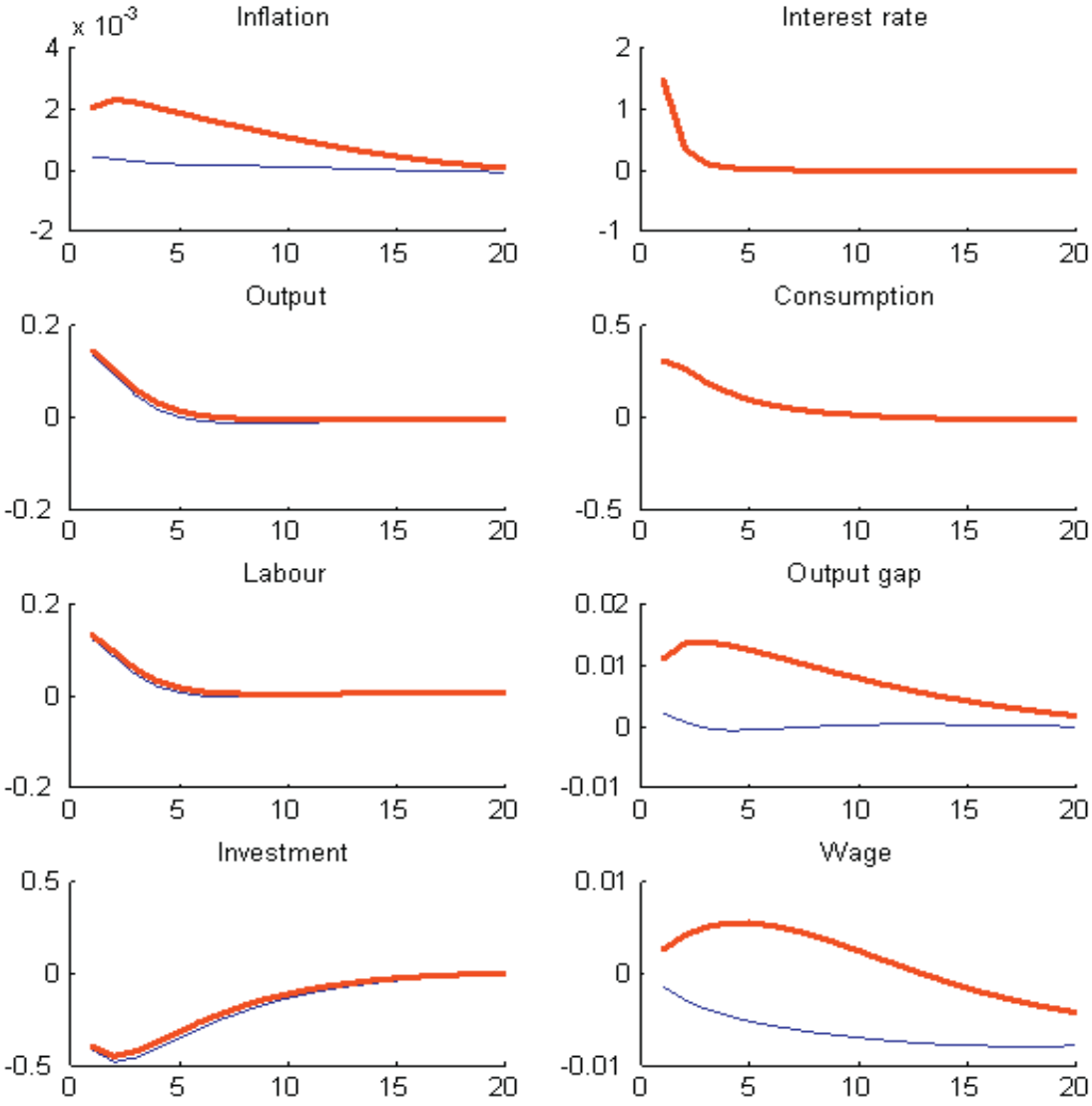


Figure 8: Preference shock, Dixit-Stiglitz aggregators, distorted steady state. Ramsey expected utility (thin blue lines), Ramsey Epstein-Zin preferences (large red lines).

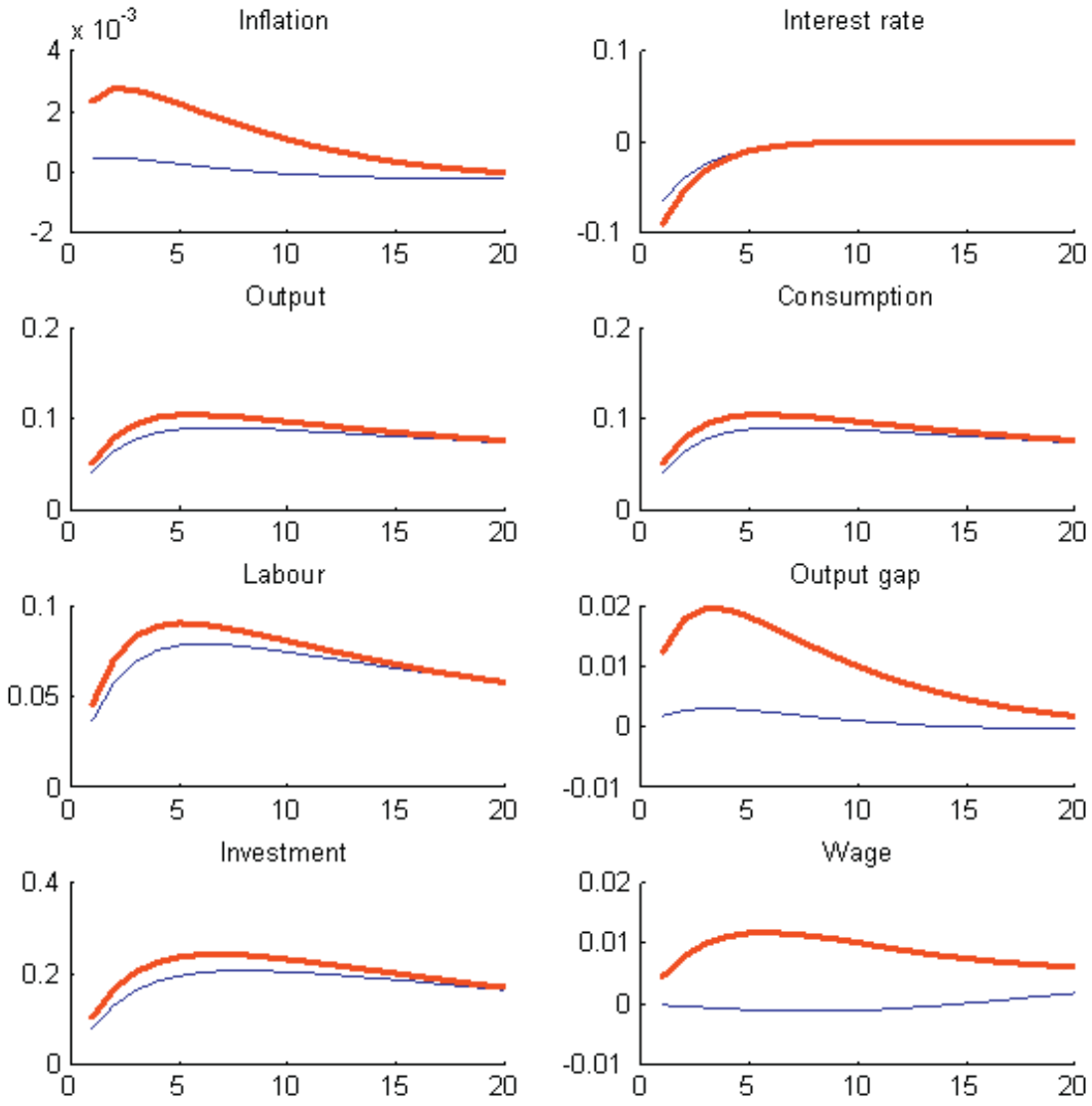


Figure 9: Labour supply shock, Dixit-Stiglitz aggregators, distorted steady state. Ramsey expected utility (thin blue lines), Ramsey Epstein-Zin preferences (large red lines).

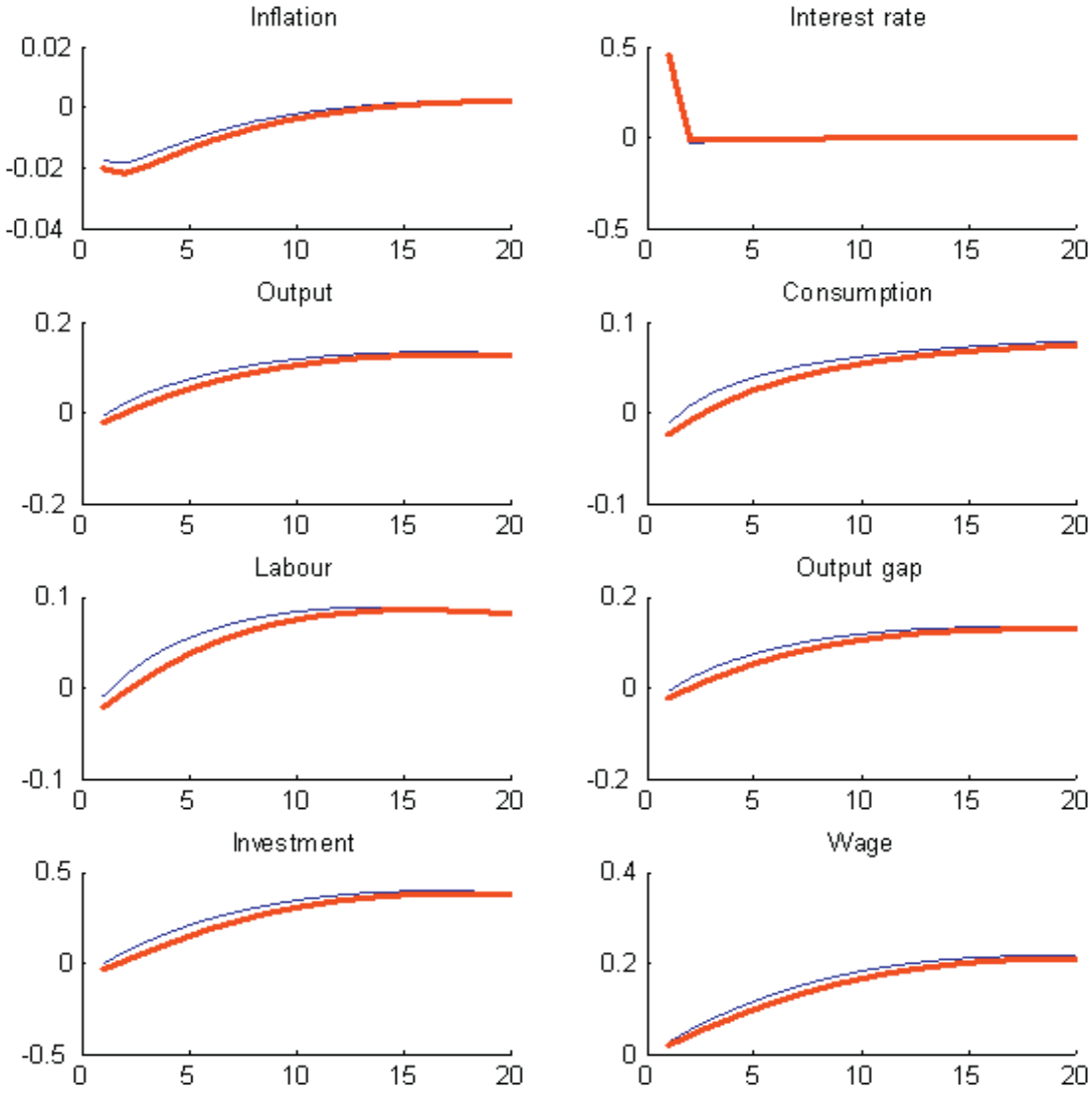


Figure 10: Price markup shock, Dixit-Stiglitz aggregators, distorted steady state. Ramsey expected utility (thin blue lines), Ramsey Epstein-Zin preferences (large red lines).

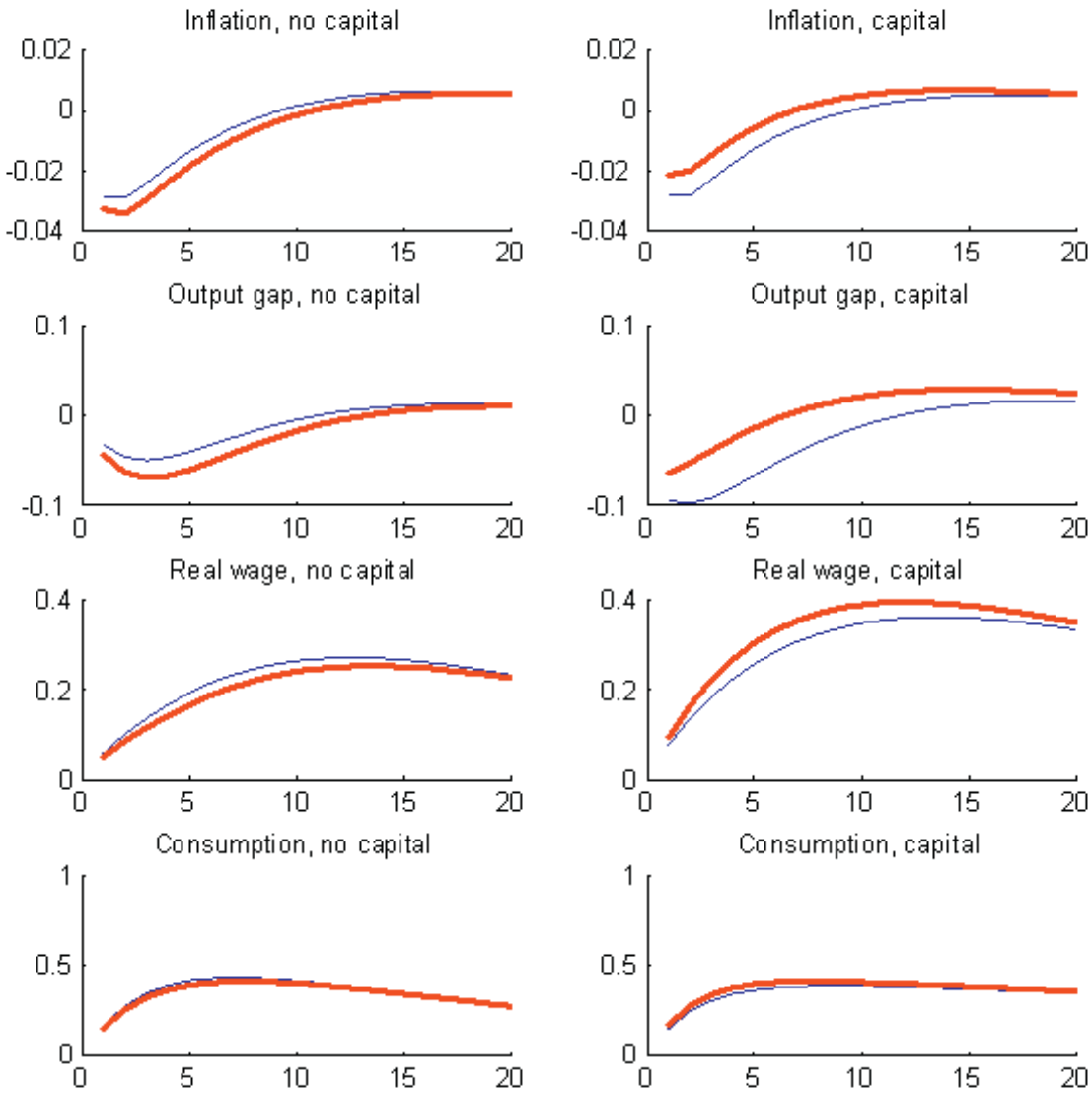


Figure 11: Optimal policy in models with capital and without capital. Response to a technology shock. Ramsey expected utility (blue thin lines), Ramsey Epstein-Zin preferences (red large lines).

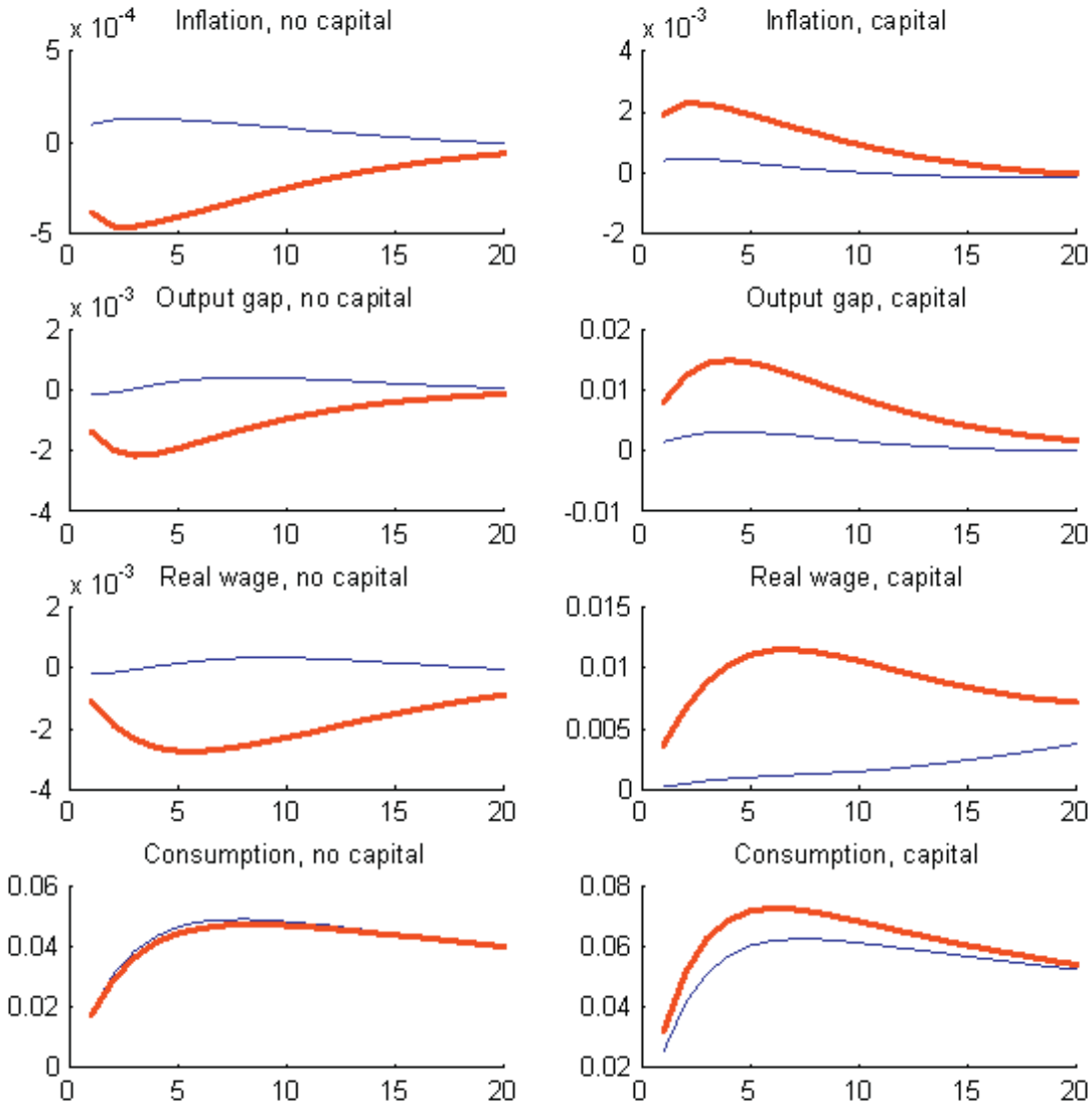


Figure 12: Optimal policy in models with capital and without capital. Response to a labour supply shock. Ramsey expected utility (blue thin lines), Ramsey Epstein-Zin preferences (red large lines).

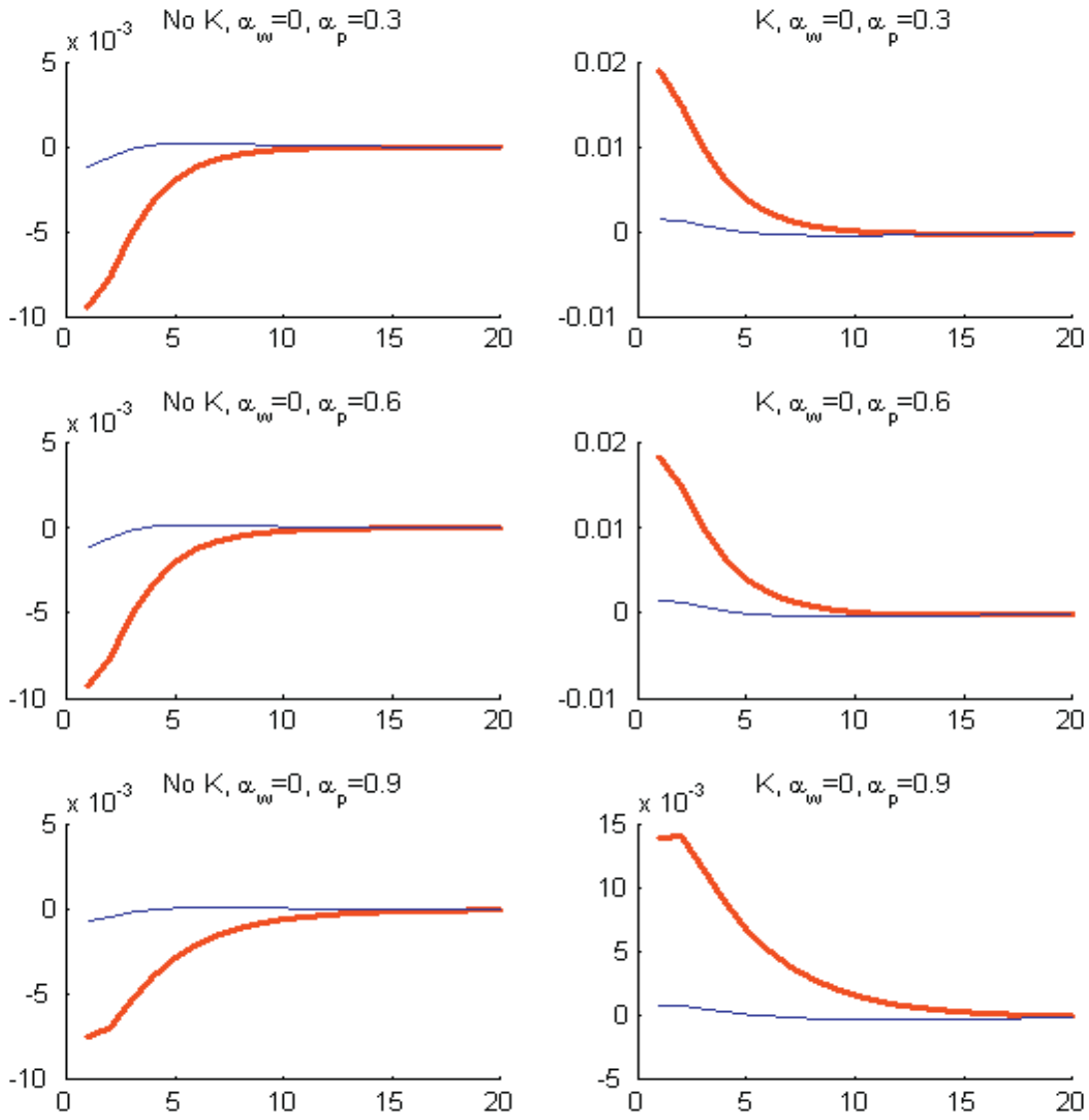


Figure 13: Inflation impulse response to a positive technology shock. Optimal policy with a distorted steady state: $\alpha_{EZ} = 930$ (red bold line) and $\alpha_{EZ} = 0$ (blue thin line). Dixit-Stiglitz aggregators.

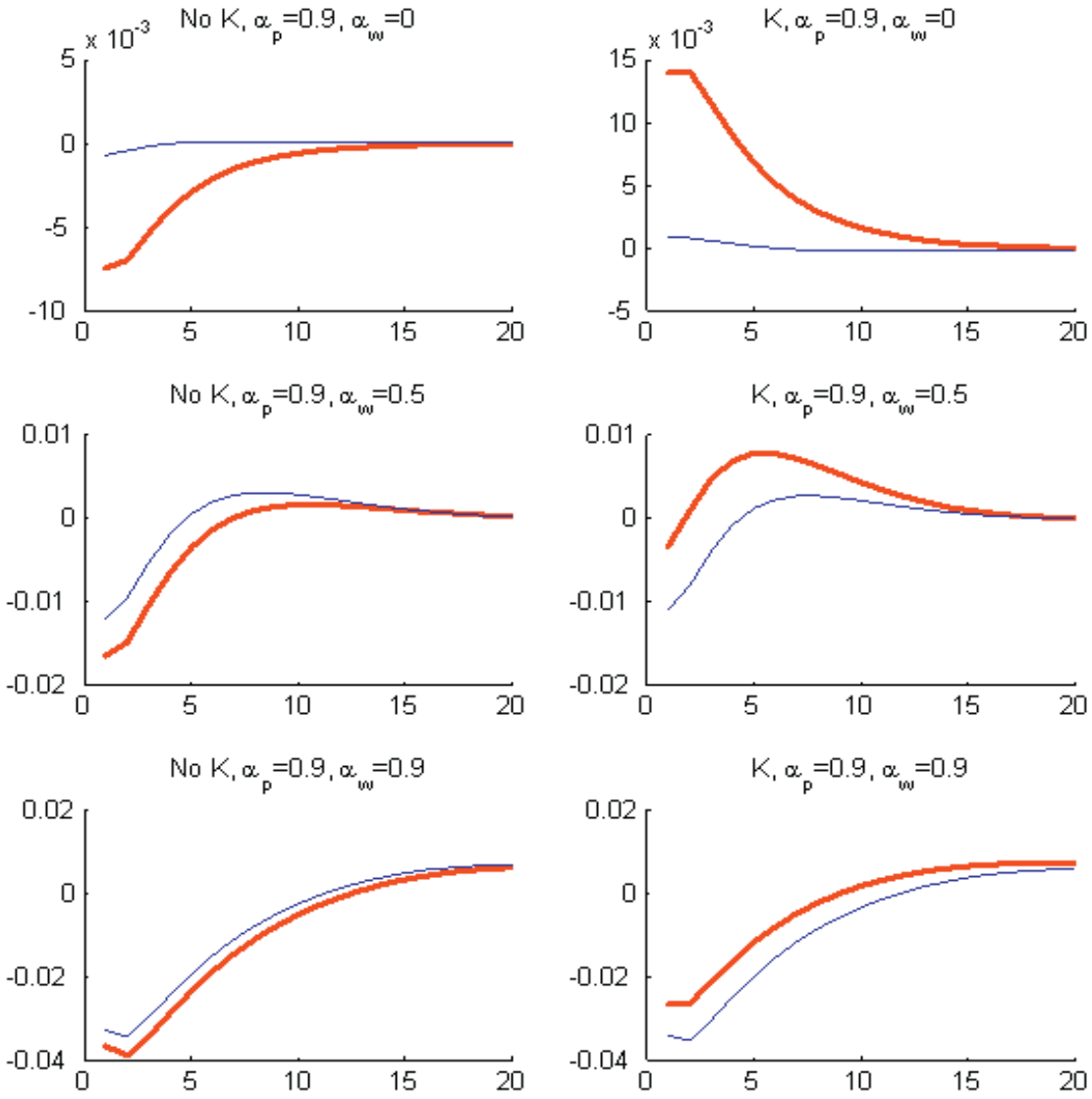


Figure 14: Inflation impulse response to a positive technology shock. Optimal policy with a distorted steady state: $\alpha_{EZ} = 930$ (red bold line) and $\alpha_{EZ} = 0$ (blue thin line). Dixit-Stiglitz aggregators.