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In 2008 all ECB publications feature a motif taken from the €10 banknote.

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Abstract

We analyze the properties of the natural rate of interest in an economy where nominal debt contracts generate a spread between loan rates and the policy interest rate. In our model, monetary policy has real effects in the flexible-price equilibrium, because it affects the credit spread. Relying on a definition suitable for this environment, we demonstrate that: (i) the natural rate is independent of monetary policy and (ii) it delivers price stability, if used as the intercept of a monetary policy rule. The second result holds exactly if real balances are remunerated at a constant spread below policy interest rates, approximately otherwise. We also highlight, however, that the natural rate is not robust to model uncertainty. The natural rate reacts differently to aggregate shocks – not only quantitatively but also qualitatively – depending on the underlying model assumptions (e.g. whether or not financial markets are frictionless).

Keywords: Monetary policy; natural rate of interest; credit frictions.

JEL Classification: E40, E50, G10.

Non-technical abstract

In the benchmark new-Keynesian model, Woodford (2003) defines the natural rate of interest as *the equilibrium real rate of return when prices are fully flexible*, and shows that it acts as a summary indicator of inflationary pressures. In that environment, price stability can be achieved at all times by a central bank committed to a Taylor-type rule - which we label *natural rate rule* - where the policy instrument reacts one-to-one to movements in the natural rate of interest.

Because of the overriding emphasis placed on price stability by most central banks, natural rate rules provide an interesting benchmark for policy analysis. However, their ability to achieve price stability is established in the context of models with frictionless financial markets. Yet, the notion of natural rate initially proposed by Wicksell (1936) is more general and recognizes the fact that different interest rates contribute to determine aggregate demand. Loan rates play an important role in this respect, especially in countries where bank credit is the predominant source of financing for firms. How should one think about the natural rate of interest in an economy where many interest rates affect aggregate demand? Does the natural rate of interest continue to be a useful indicator of inflationary pressure?

In this paper, we answer these questions in a model where the interest rate on short-term bank loans to firms differs from the policy interest rate. Some firms need to borrow funds in order to finance their production, but are subject to idiosyncratic productivity shocks, whose realization can only be observed by lenders after exerting a costly monitoring activity. The interest rate on the resulting optimal credit contract incorporates an external finance premium, as in the standard costly state verification setup. However, in order to capture the nominal nature of bank deposits and loans, we depart from the standard setup and assume that credit is denominated in nominal (rather than real) terms. An implication is that in our model monetary policy exerts real effects by changing the spread between the loan rate and the policy rate, as well as through other more standard channels. An increase in the credit spread leads to higher bankruptcy rates, i.e. a larger waste of real resources for the economy. Apart from asymmetric information and nominal debt contracts, our model is characterised by relatively standard features, such as nominal rigidities in the form of Calvo contracts, and transaction frictions arising because real money balances yield liquidity services.

In the benchmark new-Keynesian model, which abstracts from transactions costs and financial frictions, the real interest rate which would prevail under flexible prices can be shown to be a function of the exogenous stochastic processes hitting the economy and - if the model includes capital accumulation - of the current level of the capital stock. Monetary policy exerts no direct influence on the natural rate. In our model, instead, the dichotomy between the real interest rate under flexible prices and monetary policy is lost. The nominal interest rate affects the real side of the economy both through the terms of the financial contract and through changes in the opportunity cost of real balances. This lack of dicothomy implies that the equilibrium real rate of interest under flexible prices can only be computed after specifying the policy rule adopted by the monetary authorities - an undesirable property for a policy indicator. We therefore adopt a different notion of natural equilibrium, which is based on two additional assumptions beyond that of flexible prices: i) debt is denominated in real terms; ii) the central bank is able to maintain the opportunity cost of money constant over time by remunerating money holdings.

Our analysis leads to two main results. First, the natural rate of interest based on the notion of natural equilibrium described above, retains its desirable properties also in the presence of credit frictions and nominal debt. If used in a natural rate rule, it continues to ensure that price stability is maintained at all times, provided that real balances are remunerated at a constant spread below the nominal interest rate. If this spread is not constant over time, the natural rate rule cannot ensure exact price stability, because of the presence of transactions frictions. Nevertheless, in a numerical calibration of the model we show that deviations from price stability are very small. These findings suggest that the natural rate of interest is a useful policy indicator also in models with credit frictions and nominal debt.

Our second main result, however, is that the natural rate is model dependent. Our impulse responses show that the natural rate reacts differently to aggregate shocks – not only quantitatively but also qualitatively – depending on the underlying model assuptions (e.g. whether or not financial markets are frictionless). This hinders its role in the conduct of monetary policy for a central bank that is uncertain about the true model of the economy. The reason behind this result lies in the different transmission of shocks in the model with and without credit frictions. In the model with credit frictions, the dynamic behavior of firms' net worth following a positive technology shock is initially constrained by the fact that entrepreneurial capital is fixed. The increased demand for capital, however, leads to an increase in the price of capital and in the return on internal funds, and thus, over time, to an increase in entrepreneurial capital and net worth. To reap the benefits of the higher return on internal funds and invest more, entrepreneurs reduce their consumption sharply initially to slowly increase it thereafter. The corresponding share of output is partly consumed by households, whose consumption increases much more, on impact, than in the case without credit frictions. As a result, household consumption is expected to fall from the peak of the impact response, generating a reduction of the natural rate of interest. In the case without credit frictions, instead, expected consumption growth is positive but decreasing over time, leading to an increase in the natural rate of interest.

There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. (Wicksell, 1936, p. 102)

1 Introduction

The notion of natural rate of interest initially proposed by Wicksell (1936) has recently attracted renewed attention. In the context of the benchmark new-Keynesian model, Woodford (2003) defines the natural rate of interest as *the equilibrium real rate of return when prices are fully flexible* (op.cit. p.248), and shows that it acts as a summary indicator of inflationary pressures in the economy. Price stability can be achieved at all times by a central bank committed to a Taylor-type rule – which we label *natural rate rule* – where the policy instrument reacts one-to-one to movements in the natural rate of interest.

Because of the overriding emphasis placed on price stability by most central banks, natural rate rules provide an interesting benchmark for policy analysis. However, their ability to achieve price stability is established in the context of models with frictionless financial markets. Yet, the notion of natural rate initially proposed by Wicksell is more general and recognizes the fact that different interest rates contribute to determine aggregate demand. Loan rates play an important role in this respect, especially in countries where bank credit is the predominant source of financing for firms. How should one think about the natural rate of interest in an economy where many interest rates affect aggregate demand? Does the natural rate of interest continue to be a useful indicator of inflationary pressure?

In this paper, we address these questions in a model where the interest rate on short-term bank loans to firms is different from the policy interest rate. Some firms need to borrow funds in order to finance their production, but are subject to idiosyncratic productivity shocks, whose realization can only be observed by lenders after exerting a costly monitoring activity. The interest rate on the resulting optimal credit contract incorporates an external finance premium, as in the costly state verification setup of Carlstrom and Fuerst (1997, 1998) or Bernanke, Gertler and Gilchrist (1999). However, in order to capture the nominal nature of bank deposits and loans, we depart from these models and assume that credit is denominated in nominal (rather than real) terms. An implication is that in our model monetary policy exerts real effects by changing the cost of external finance and the credit spread (as well as through other more standard channels). An increase in the credit spread leads to higher bankruptcy rates, i.e. a larger waste of real resources for the economy.

Apart from asymmetric information and nominal debt contracts, our model is characterised by relatively standard features, whose implications for the natural rate have already been studied by Woodford (2003). More specifically, we allow for nominal rigidities, in the form of Calvo contracts, and transaction frictions, which arise because real money balances yield liquidity services according to the timing convention adopted by Lucas and Stokey (1987).

Our analysis leads to two main results. First, the natural rate of interest, whose definition needs to be tailored to our model environment, retains its desirable properties also in the presence of credit frictions and nominal debt. If used in a natural rate rule, it continues to ensure that price stability is maintained at all times, provided that real balances are remunerated at a constant spread below the nominal interest rate. If this spread is not constant over time, the natural rate rule cannot ensure exact price stability, because of the presence of transactions frictions. Nevertheless, in a numerical calibration of the model we show that deviations from price stability are very small. This set of results suggests that the natural rate of interest is a useful policy indicator also in models with credit frictions and nominal debt.

Our second main result, however, is that the natural rate is model dependent. Our impulse responses show that the natural rate reacts differently to aggregate shocks – not only quantitatively but also qualitatively – depending on the underlying model assuptions (e.g. whether or not financial markets are frictionless). This hinders its role in the conduct of monetary policy for a central bank that is uncertain about the true model of the economy.

Our paper relates to a recent literature that stresses the role of financial market frictions in explaining business cycle volatility. Christiano et al. (2006) and Quejo (2004) estimate DSGE models of the US and the euro area with different sources of frictions and conclude that financial market imperfections are relevant for both areas. Other contributions have estimated models with financial market frictions and confirmed their relevance, particularly in the form of information asymmetries (see e.g. Levin et al. (2004)).

The paper is also related to the evidence on the so-called cost channel, i.e. the fact that firms' marginal costs depend on the nominal interest rate (see e.g. Barth and Ramey (2001)). The implications of the cost channel for optimal monetary policy have been analyzed by Ravenna and Walsh (2006), while Christiano, Eichenbaum and Evans (2005) incorporate the cost channel in a macro model estimated on aggregate US data. In those models, however, there are no agency costs and firms do not pay a premium over the policy rate when borrowing funds. Because of the presence of nominal debt contracts for firms in the investment sector, our model nests the real credit frictions set-up of Carlstrom and Fuerst (2001) or Bernanke, Gertler and Gilchrist (1999) and the cost channel effects emphasized by Ravenna and Walsh (2006) and Christiano, Eichenbaum and Evans (2005) in the transmission of monetary policy.

The paper is organized as follows. In section 2, we outline the model and present the conditions characterizing the equilibrium. In section 3, we define the natural rate starting from the observation that the traditional definition, derived in the flexible price equilibrium of the model, is not useful in our economy. The reason is that it is not independent of monetary policy and would therefore change endogenously if the central bank tried to use it as the intercept of a natural rate rule. We propose an alternative definition, which is independent of monetary policy, and derive the conditions under which it delivers price stability, if used as the intercept of a Taylor rule. In section 4, we present a numerical analysis. In section 4.1, we illustrate the analytical results derived in section 3, namely that an interest rate rule that uses our notion of natural rate as intercept is able to completely stabilize inflation. We also discuss the sensitivity of the natural rate to the underlying model assumptions by comparing the impulse response following a technology shock in our benchmark model and in a model with frictionless financial markets. In section 4.2, we show how the performance of a natural rate rule differs in the case when the spread between the own return on money and the nominal interest rate is constant and when it changes over time. In section 4.3, we analyse the real effects of monetary policy generated by the mere presence of credit frictions and nominal debt contracts, and abstracting from both nominal rigidities and transactions frictions. In section 5, we carry out a robustness analysis. We argue that the approach to modelling financial frictions posposed in the literature and adopted in this paper has two main shortcomings. First, it cannot replicate the countercyclical behaviour of the default rate and of the premium on external finance observed in the data. Second, the assumption of risk neutrality of the entrepreneurs, which is needed to ensure optimality of the financial contract, implies that they have an infinite elasticity of intertemporal substitution of consumption. In a more realistic version of the model, in which entrepreneurs are risk neutral but have finite elasticity of substitution (equal to that of households), our two main results remain valid. Moreover, the difference in the behaviour of the natural rate in the frictionless model and in the model with financial frictions is more protracted over time. In section 5, we provide some concluding remarks.

2 The model

The economy is inhabited by an infinitely-lived representative household, a continuum of infinitely-lived entrepreneurs, a zero-profit financial intermediary, a monetary authority and a fiscal authority.

The household owns a continuum of monopolistically competitive firms producing differentiated intermediate goods. Entrepreneurs manage firms in a competitive investment sector, which produces capital goods. The financial intermediary collects deposits from the household and lends to the entrepreneurs.

Households are risk-averse and have preferences defined over a final consumption good and leisure. At the beginning of the period, they decide how to split their wealth into the available nominal assets. They also decide how much to consume and to invest, but they do not have access to a technology to produce capital goods. Hence, they purchase capital from firms endowed with such technology, which operate in the investment sector. Entrepreneurs also make consumption and investment decisions, but are risk-neutral.

The production sector is composed of a representative firm that produces the final consumption good by aggregating the continuum of intermediate goods. Because of product differentiation, intermediate goods firms have market power and are therefore price makers. In their price-setting activity, however, they are not free to change their price at will, because prices are subject to Calvo contracts.

The investment sector is made of a continuum of competitive firms endowed with a technology, subject to idiosyncratic shocks, which transforms final consumption goods into capital goods. The internal funds of the firms are not sufficient to finance the desired amount of investment, so entrepreneurs need to raise external finance. Lending occurs through the financial intermediary, which is able to diversify the idiosyncratic risk by providing funds to all firms. The loan takes the form of risky debt, which is the optimal contractual arrangement between lenders and borrowers in the presence of asymmetric information and costly state verification. After lending has occurred, the idiosyncratic shock is realized and the firm decides whether or not to repay the debt. In case of bankruptcy, the firm is monitored by the bank and it looses its entire capital output. Firms that have not defaulted sell the newly produced capital to households and rent their own capital stock to intermediate goods firms.

2.1 Households

At the beginning of period t, the financial market opens. First, the interest on nominal financial assets acquired at time $t-1$ is paid. The households, holding an amount W_t of nominal wealth, choose to allocate it among existing nominal assets, namely money M_t, a portfolio of nominal state-contingent bonds A_{t+1} each paying a unit of currency in a particular state in period $t+1$, and one-period deposits denominated in units of currency D_t.

As in Woodford (2003), we enable the government to remunerate end-of-period private holdings of money at the rate i_t^m. We also assume that the government can provide households with a subsidy that compensates for the effect of expected inflation on their wealth. Under such subsidy, households that decide to deposit an amount D_t of currency at the financial intermediaries receive back $\Omega_t \left(1 + i_t^d\right) D_t$ at the end of the period, where i_t^d is the net interest rate paid by the financial intermediary on deposits and $\Omega_t - 1$ is the subsidy paid by the government for each unit of currency returned to households by the financial intermediary.[1] The reason why a non-zero interest rate is required on intra-period deposits is that, at the end of the period, currency cannot be spent to finance purchases and the financial market is closed. Households are therefore forced to keep the redeemed deposit as cash until the financial market reopens in the following period. From a portfolio-choice viewpoint, deposits would be strictly dominated by the 1-period state-contingent bonds, if they did not yield an interest rate.

In the second part of the period, the goods market opens. Households' money balances are increased by the nominal amount of their revenues and decreased by the value of their expenses. Taxes are also paid or transfers received. The amount of nominal balances brought into period $t+1$ is equal to

$$M_t + P_t \left(w_t h_t + \rho_t k_{h,t}\right) - P_t \left(I_{h,t} + c_t\right) + Z_t - T_t, \tag{1}$$

where c_t is the amount of final consumption good purchased, P_t is its price, h_t is hours worked, $k_{h,t}$ is the capital stock held by the household, w_t is the real wage, ρ_t is the return on capital,

[1] As we will discuss in section 3.1, we do not need, nor argue that such a subsidy is realistic. Nor our results on the stabilization properties of a Wicksellian monetary policy rely on the existence of such subsidy. We use it as a conceptual device to provide an appropriate definition of the natural rate of interest.

Z_t are nominal profits transferred from intermediate goods producers to households, $I_{h,t}$ is the amount of consumption good purchased by the household for investment purposes, and T_t are lump-sum nominal taxes collected by the government.

New capital can be purchased from firms in the investment sector at the end of the period, in exchange of consumption goods. The accumulation of households' capital follows the law of motion

$$I_{h,t} = q_t \left[k_{h,t+1} - (1-\delta) k_{h,t}\right],$$

where q_t is the price of capital in units of the consumption good.

The household's problem is to maximize preferences, defined as

$$E_o \left\{ \sum_0^\infty \beta^t \left[u(c_t, m_t; \xi_t) - v(h_t; \psi_t)\right] \right\}, \tag{2}$$

subject to the budget constraints

$$M_t + D_t + E_t \left[Q_{t,t+1} A_{t+1}\right] \leq W_t, \tag{3}$$

for all $t \geq 0$, where

$$W_{t+1} = A_{t+1} + \Omega_t \left(1 + i_t^d\right) D_t + (1 + i_t^m) \left[M_t + P_t (\rho_t k_{h,t} + w_t h_t) + Z_t - T_t - P_t (I_{h,t} + c_t)\right]. \tag{4}$$

Here $u_c > 0$, $u_m \geq 0$, $u_{cc} < 0, u_{mm} < 0$, $v_h > 0$, $v_{hh} > 0$, $m_t \equiv M_t/P_t$ denotes real balances, ξ_t is a preference shock, and ψ_t is a labor supply shock.

The absence of arbitrage opportunities requires the existence of a discount factor $Q_{t,t+1}$ such that the price of any portfolio of financial assets with random value A_{t+1} in the following period is given by $E_t \left[Q_{t,t+1} A_{t+1}\right]$. The riskless nominal interest rate corresponds to the solution of the equation

$$\frac{1}{1 + i_t} = E_t \left[Q_{t,t+1}\right]. \tag{5}$$

Define $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\Delta_{m,t} \equiv \frac{i_t - i_t^m}{1 + i_t}$. Optimality requires that, at each t, (5) is satisfied, either $D_t = 0$ or

$$1 + i_t^d = \frac{1 + i_t}{\Omega_t}, \tag{6}$$

and the following equations hold

$$\frac{v_h(h_t; \psi_t)}{u_c(c_t, m_t; \xi_t)} = w_t \tag{7}$$

$$(1 + i_t)^{-1} = \beta E_t \left\{ \frac{u_c(c_{t+1}, m_{t+1}; \xi_{t+1}) + u_m(c_{t+1}, m_{t+1}; \xi_{t+1})}{u_c(c_t, m_t; \xi_t) + u_m(c_t, m_t; \xi_t)} \frac{1}{\pi_{t+1}} \right\} \tag{8}$$

$$\frac{u_m(c_t, m_t; \xi_t)}{u_c(c_t, m_t; \xi_t)} = \frac{\Delta_{m,t}}{1-\Delta_{m,t}} \tag{9}$$

$$u_c(c_t, m_t; \xi_t) q_t = \beta E_t \left\{ u_c(c_{t+1}, m_{t+1}; \xi_{t+1}) \left[q_{t+1}(1-\delta) + \rho_{t+1} \right] \right\}. \tag{10}$$

Notice that condition (9) can be written as $m_t = L(c_t, \Delta_{m,t}; \xi_t)$. We can then define the following functions

$$U_c(c_t, \Delta_{m,t}; \xi_t) \equiv u_c(c_t, L(c_t, \Delta_{m,t}; \xi_t); \xi_t) \tag{11}$$

$$U_m(c_t, \Delta_{m,t}; \xi_t) \equiv u_m(c_t, L(c_t, \Delta_{m,t}; \xi_t); \xi_t). \tag{12}$$

2.2 The production sector

Final consumption goods are produced by a representative, competitive firm using a Dixit-Stiglitz aggregator of differentiated intermediate goods

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{13}$$

where $y_t(j)$ denotes the quantity of the differentiated good j. Profit maximization of firms producing final goods requires that

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} y_t, \tag{14}$$

where $P_t(j)$ is the price of good j. Using condition (14), we can obtain an expression for the consumer price index, $P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$.

Each differentiated intermediate good j is produced by monopolistic competitive firms using the technology

$$y_t(j) = A_t l_t(j)^{\alpha} k_t(j)^{1-\alpha}, \tag{15}$$

where $l_t(j)$ and $k_t(j)$ denote the amount of labor and capital rented on the market by firm j, while A_t is an aggregate exogenous productivity shock.

Because of product differentiation, each intermediate good firm has some market power. Profits are distributed to the households, who are the owners of the firms. We assume that each retailer can change its price with probability $1-\theta$, following Calvo (1983). Let P_t^* denote the price for good j set by a firm that can change the price at time t, and $y_t^*(j)$ the demand faced given this price. Then each intermediate good firm chooses its price to maximize expected discounted profits, given by

$$E_t \sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \left[P_t^* y_{t+k}(j) - P_{t+k} \overline{\chi}_{t+k}(j) \right],$$

where $\overline{Q}_{t,t+k} = \beta \frac{U_c(c_{t+k},\Delta_{m,t+1};\xi_{t+k})}{U_c(c_t,\Delta_{m,t};\xi_t)} \frac{P_t}{P_{t+1}}$ and $\overline{\chi}_{t+k}(j)$ denote total real costs of production. It follows that $\overline{\chi}_t(j)$ is given by

$$\overline{\chi}_t(j) = \min_{k(j),l(j)} \left[w_t l_t(j) + \rho_t k_t(j) \right]$$

subject to (15). Denote χ_t as the Lagrange multiplier associated to constraint (15), or the real cost of producing one unit of output. The latter cost is not firm-specific, as all firms face the same real wage and price of capital. Optimality implies that

$$\begin{aligned} w_t &= \chi_t \alpha \frac{y_t(j)}{l_t(j)} \\ \rho_t &= \chi_t (1-\alpha) \frac{y_t(j)}{k_t(j)}. \end{aligned}$$

The first-order conditions of the firm's profit maximization problem imply that

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \left\{ \sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{-\varepsilon}}{P_{t+k}^{-\varepsilon}} \chi_{t+k} y_{t+k} \right\}}{E_t \left\{ \sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}} y_{t+k} \right\}}.$$

Now define

$$\begin{aligned} \Theta_{1,t} &\equiv E_t \left\{ \sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{-\varepsilon}}{P_{t+k}^{-\varepsilon}} \chi_{t+k} y_{t+k} \right\}, \\ \Theta_{2,t} &\equiv E_t \left\{ \sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}} y_{t+k} \right\}. \end{aligned}$$

Using the expression for the aggregate price index, $P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$, and substituting out $\frac{P_t^*}{P_t}$, we obtain the following conditions

$$1 = \theta \pi_t^{\varepsilon - 1} + (1-\theta) \left(\frac{\varepsilon}{\varepsilon - 1} \frac{\Theta_{1,t}}{\Theta_{2,t}} \right)^{1-\varepsilon} \quad (16)$$

$$\Theta_{1,t} = \chi_t y_t + \theta E_t \left[\overline{Q}_{t,t+1} \pi_{t+1}^{\varepsilon} \Theta_{1,t+1} \right] \quad (17)$$

$$\Theta_{2,t} = y_t + \theta E_t \left[\overline{Q}_{t,t+1} \pi_{t+1}^{\varepsilon - 1} \Theta_{2,t+1} \right]. \quad (18)$$

2.3 The investment sector

The investment sector and the process of financial intermediation are modelled along the line of Carlstrom and Fuerst (1997), the main difference being the nominal denomination of debt. This difference has two implications. The first is that expected inflation will have an impact on the terms of the contract. The second implication is that that the cost of bank loans will include a premium over the nominal interest rate paid on households' deposits. In contrast, intra-period real deposits are not remunerated in the economy studied by Carlstrom and Fuerst (1997).

The investment sector is composed of an infinite number of competitive firms, each endowed with a stochastic technology that transforms I units of the final consumption good into ωI units of capital. The random variable ω is i.i.d. across time and across entrepreneurs, with distribution Φ, density ϕ and mean unity. The shock ω is private information, but its realization can be observed by the financial intermediary at the cost of μI units of capital.

The amount of internal funds available to firm i is given by its net worth,

$$n_{i,t} = \left[q_t\left(1-\delta\right)+\rho_t\right] z_{i,t}, \tag{19}$$

where $z_{i,t}$ is the stock of capital owned by firm i at the beginning of period t. The firm's net worth (i.e. the value of the accumulated capital stock plus the return from renting the capital stock to intermediate goods firms) is not sufficient to produce the desired amount of investment goods. Hence, the firm needs to raise external finance. However, households are not willing to lend directly to a firm which could default in the face of an idiosyncratic productivity shock. Lending occurs through the financial intermediary, which is able to ensure a safe return by providing funds to the continuum of firms facing idiosyncratic shocks.

2.3.1 The financial contract

Loans are stipulated in units of currency after all aggregate shocks have occurred, and repaid at the end of the same period. The amount of funds lent to firm i is $P_t\left(I_{i,t}-n_{i,t}\right)$. The capital output is then sold directly to households at the price $P_t q_t$ and used to repay the agreed upon amount (unless default occurs). Define

$$f\left(\overline{\omega}\right) \equiv \int_{\overline{\omega}}^{\infty} \omega\Phi\left(d\omega\right) - \overline{\omega}\left[1-\Phi\left(\overline{\omega}\right)\right] \tag{20}$$

$$g(\overline{\omega}) \equiv \int_{0}^{\overline{\omega}} \omega\Phi\left(d\omega\right) - \mu\Phi\left(\overline{\omega}\right) + \overline{\omega}\left[1-\Phi\left(\overline{\omega}\right)\right] \tag{21}$$

as the fraction of the expected net capital output accruing respectively to an entrepreneur and to a lender, after stipulating a contract that sets the fixed repayment at $P_t q_t \overline{\omega}_{it} I_{i,t}$ units of currency. In case of default, a constant fraction μ of the production input is destroyed in monitoring. At the individual firm level, total output is split between the entrepreneur, the lender, and monitoring costs,

$$f(\overline{\omega}) + g(\overline{\omega}) = 1 - \mu\Phi(\overline{\omega}).$$

Hence, on average, $\mu\Phi(\overline{\omega})$ of the produced capital is lost in monitoring.

Since the contract is intra-period, we can drop the price level from the formulation of the problem. The optimal contract is the pair $(I_{i,t}, \overline{\omega}_{i,t})$ that solves the following CSV problem:

$$\max \; q_t f(\overline{\omega}_{i,t}) I_{i,t}$$

subject to

$$q_t g(\overline{\omega}_{i,t}) I_{i,t} \geq \frac{(1+i_t)}{\Omega_t} (I_{i,t} - n_{i,t}) \tag{22}$$

$$f(\overline{\omega}_{i,t}) + g(\overline{\omega}_{i,t}) + \mu\Phi(\overline{\omega}_{i,t}) \leq 1 \tag{23}$$

$$q_t f(\overline{\omega}_{i,t}) I_{i,t} \geq n_{i,t}, \; n_{i,t} \geq 0. \tag{24}$$

The optimal contract maximizes the entrepreneur's expected return subject to the financial intermediary's expected return exceeding the repayment requested by the household, (22), the feasibility condition, (23), and the entrepreneur being willing to sign the contract, (24). Notice that the intermediary needs to pay back to the household a gross return equal to the safe interest on deposits, $1 + i_t^d$. From condition (6), it follows that the financial intermediary's expected return on each unit of loans cannot be lower than $\frac{(1+i_t)}{\Omega_t}$.

The informational structure corresponds to a standard costly state verification (CSV) problem (see e.g. Gale and Hellwig (1985)). The first-order conditions of the problem are

$$q_t = \frac{(1+i_t)/\Omega_t}{1 - \mu\Phi(\overline{\omega}_{i,t}) + \frac{\mu\overline{\omega}_{i,t} f(\overline{\omega}_{i,t})\phi(\overline{\omega}_{i,t})}{f'(\overline{\omega}_{i,t})}}, \tag{25}$$

$$I_{i,t} = \left\{ \frac{(1+i_t)/\Omega_t}{(1+i_t)/\Omega_t - q_t g(\overline{\omega}_{i,t})} \right\} n_{i,t}. \tag{26}$$

Notice from equation (25) that the terms of the contract depend on the state of the economy only through the price of capital q_t and the return $(1+i_t)/\Omega_t$. Hence, they are the same for all firms, i.e. $\overline{\omega}_{i,t} = \overline{\omega}_t$, for all i. From equation (26), it is clear that the contract only differs among firms because of their initial wealth. The larger is $n_{i,t}$, the larger is the investment $I_{i,t}$ that can be financed. Condition (25) can thus be rewritten as

$$q_t = \frac{(1+i_t)/\Omega_t}{1 - \mu\Phi(\overline{\omega}_t) + \frac{\mu\overline{\omega}_t f(\overline{\omega}_t)\phi(\overline{\omega}_t)}{f'(\overline{\omega}_t)}}. \tag{27}$$

The net nominal interest rate implicit in the contract with firm i, $i_{i,t}^l$, is given by

$$1 + i_{i,t}^l = \frac{q_t \overline{\omega}_t I_{i,t}}{I_{i,t} - n_{i,t}}. \tag{28}$$

Given linearity in net worth, firm-specific variables can be easily aggregated. Equation (26) leads to the aggregate condition

$$I_t = \left\{ \frac{(1 + i_t) / \Omega_t}{(1 + i_t) / \Omega_t - q_t g(\overline{\omega}_t)} \right\} n_t. \tag{29}$$

It is clear from equation (27) that, ceteris paribus, an increase in the nominal interest rate tends to push up the real price of capital, as firms try to pass on to customers their higher financing costs. The higher price of capital reduces the demand for capital goods, and leads to a reduction of investment and output.

2.3.2 Entrepreneurial consumption and investment decisions

Entrepreneurs are infinitely lived, risk-neutral and more impatient than households. They discount the future at a rate $\beta\gamma$, where $0 < \gamma < 1$, and their utility is linear in entrepreneurial consumption. The higher degree of impatience relative to households induces entrepreneurs to consume a sufficiently high share of their net worth, so that they do not accumulate capital up to the point where there is no need for external finance and agency costs become irrelevant.

The problem of the entrepreneur is to maximize the expected value of the discounted stream of future utilities,

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t e_{i,t}, \quad 0 < \gamma < 1, \tag{30}$$

subject to the budget constraint

$$e_{i,t} + q_t z_{i,t+1} = \begin{cases} 0 & \text{if } \omega_{it} \leq \overline{\omega}_t \\ q_t(\omega_{it} - \overline{\omega}_t) I_{i,t} & \text{if } \omega_{it} \geq \overline{\omega}_t \end{cases}, \tag{31}$$

Here $e_{i,t}$ denotes entrepreneurial consumption of the final good and $z_{i,t+1}$ is investment in physical capital to be used in period $t+1$.

Assuming an interior solution, optimality requires that

$$q_t = \beta\gamma E_t \left\{ \frac{q_{t+1} f(\overline{\omega}_{t+1}) \left[q_{t+1}(1-\delta) + \rho_{t+1} \right] (1 + i_{t+1}) / \Omega_{t+1}}{(1 + i_{t+1}) / \Omega_{t+1} - q_{t+1} g(\overline{\omega}_{t+1})} \right\}. \tag{32}$$

2.4 A Wicksellian monetary policy regime

The central bank follows a monetary policy rule described by

$$i_t = v_t \vartheta \left(\frac{\pi_t}{\pi_t^*} \right), \tag{33}$$

where $\vartheta(\cdot)$ is a non-negative-valued, non-decreasing function, π_t^* is the target path for the inflation rate, and v_t captures either a random disturbance to the policy rule or a systematic response of monetary policy to exogenous shocks.

Monetary policy also needs to specify an additional rule for either i_t^m or M_t^s. It is convenient to express this rule in terms of $\Delta_{m,t}$. We consider a general rule of the form

$$\Delta_{m,t} = \Gamma(i_t), \tag{34}$$

where $\Gamma(\cdot)$ is a non-negative-valued function.

2.5 Fiscal policy

The fiscal authorities decide on the subsidy Ω_t. They also use lump-sum taxes T_t to balance the government's budget at each period. This latter is given by

$$M_{t+1}^s - M_t^s + (\Omega_t - 1)\left(1 + i_t^d\right) D_t = T_t. \tag{35}$$

2.6 Market clearing

Market clearing for money, bonds, capital, investment, labor, loans and goods requires that

$$M_t = M_t^s \tag{36}$$

$$B_t = 0 \tag{37}$$

$$k_t = k_{h,t} + z_t \tag{38}$$

$$k_{t+1} = (1-\delta)k_t + I_t\left[1 - \mu\Phi(\overline{\omega}_t)\right] \tag{39}$$

$$h_t = l_t \tag{40}$$

$$D_t = P_t(I_t - n_t) \tag{41}$$

$$\left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} = c_t + e_t + I_t. \tag{42}$$

2.7 Equilibrium

An equilibrium is a solution to the system of equations (7)-(10), (13)-(18), (27),(29), (32)-(42), together with definitions (11), (12), (20) and (21). A log-linearized reduced-form system of equilibrium conditions is reported in Appendix A.

3 The natural rate of interest

In the sticky price model considered by Woodford (2003), the natural rate of interest, defined as the equilibrium real rate of return when prices are fully flexible, acts as a summary statistic of the underlying economic conditions and provides a useful tool in the conduct of monetary policy. A monetary policy rule such that the policy instrument reacts proportionally to movements in the natural rate is able to ensure the achievement of price stability.

In the simplest version of the sticky price model, which abstracts from transactions costs and financial frictions and where labor is the only input in production, the natural rate of interest can be expressed solely as a function of the exogenous stochastic processes. In a version of the model that also includes capital, the natural rate depends on the exogenous stochastic processes and on the current level of the capital stock. In none of these cases, monetary policy exerts a direct influence on the natural rate. In fact, it is always possible to rewrite the flexible price version of these models in two separate blocks of equilibrium conditions. The first can be used to solve for the real variables (and for the equilibrium real rate of interest). The second consists of a Fisher relation, linking the real interest rate to the nominal rate and to expected inflation, and a monetary policy rule. This latter block provides a solution for the path of inflation and the nominal interest rate.

In our model, the dichotomy between the real interest rate which would prevail under flexible prices and monetary policy is lost. Even under flexible prices, the nominal interest rate affects the real side of the economy both through the terms of the financial contract and through changes in the opportunity cost of real balances. The first effect arises because of the nominal denomination of debt contracts, which implies that the credit spread $\left(1+i_t^l\right)/\left(1+i_t\right)$ affects real allocations even if prices are fully flexible. The second effect is due to the transactions role of money, which is captured by the timing of households' decisions as adopted in Lucas and Stokey (1987).

The lack of dicothomy implies that the equilibrium real rate of interest under flexible prices can only be computed after specifying the policy rule adopted by the monetary authorities, i.e. the nominal interest rate. This is an undesirable property for a policy indicator. We therefore adopt a different notion of natural equilibrium, which is based on two additional assumptions beyond that of flexible prices.

The first additional assumption is that, in the ideal conditions prevailing in the natural equilibrium, debt is denominated in real terms (in spite of it being denominated in nominal terms in the actual economy). This assumption is analogous to the standard one, in which the natural rate is defined for an ideal flexible-price economy (when prices are assumed to be sticky in the actual economy). When debt is denominated in real terms, the spread which affects credit conditions is real, thus independent of monetary policy.

The second additional assumption is of a different nature. Following Woodford (2003), we note that there are circumstances under which transactions frictions play no role on the real allocation of resources, in spite of Lucas and Stokey's timing and of non-separability of real balances. This circumstances occur when real money balances are remunerated at a constant spread below the policy instrument. In our model, we obtain a similar result by remunerating money holdings – more precisely by setting a constant spread $\Delta_{m,t} = \Delta_m$, for all t. Relying on this property, we construct the natural rate under the assumption that real balances are remunerated at a constant spread in the natural equilibrium. Our definition of natural rate coincides with the real rate of return arising in an equilibirum in which the central bank is able to maintain the opportunity cost of money constant over time and all nominal frictions are absent, i.e.: i) prices have always been fully flexible and are expected to remain so in the indefinite future; ii) external finance takes the form of real debt. We will then analyse the properties of this definition in two cases: the one in which money holdings are remunerated *also* in the real economy, and the one in which they are not.

In the next section, we demonstrate that our notion of natural rate is independent of monetary policy. We also show that a natural rate rule based on this notion ensures that price stability is maintained at all times, if real money balances are remunerated at a constant spread below the policy instrument. If the spread varies over time, e.g. if money is not remunerated, the natural rate rule is not able to fully stabilize inflation. Nevertheless, our numerical results in section 5 suggest that deviations from price stability are likely to remain small.

3.1 A modified definition of natural rate

To characterise the natural equilibrium, in which variables will be denoted by a subscript n, we set the spread $\Delta_{m,t}$ to a constant, we set to zero the probability that firms producing intermediate goods are unable to reoptimise their prices ($\theta = 0$), and we assume that the government is able to compensate households for the presence of expected inflation by providing a subsidy equal to the ratio between the nominal return on a one-period risk-free asset and the real rate of interest, i.e.

$$\Omega_{n,t} = \frac{1 + i_{n,t}}{1 + r_{n,t}}. \tag{43}$$

Notice that under this subsidy, the effective return paid by the financial intermediary to the household, $\frac{1+i_{n,t}}{\Omega_{n,t}}$, corresponds to the real interest rate.

The non-linear system of equilibrium conditions describing this natural equilibrium is reported in Appendix B. Here, we find it convenient to use the log-linearized system around a steady state with zero inflation, which can be written in matrix form as

$$\begin{bmatrix} E_t \widehat{Z}_{n,t+1} \\ \widehat{X}_{n,t+1} \end{bmatrix} = \underset{(10x10)}{\Upsilon} \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix} + \underset{(10x3)}{\Sigma} s_t \tag{44}$$

$$E_t s_{t+1} = \underset{(3x3)}{\Phi_s} s_t + \varepsilon_t, \tag{45}$$

where

$$\begin{aligned} \widehat{Z}_{n,t} &\equiv \begin{bmatrix} \widehat{c}_{n,t} & \widehat{e}_{n,t} & \widehat{h}_{n,t} & \widehat{I}_{n,t} & \widehat{q}_{n,t} & \widehat{\rho}_{n,t} & \widehat{\widetilde{\omega}}_{n,t} & \widehat{r}_{n,t} \end{bmatrix}', \\ \widehat{X}_{n,t} &\equiv \begin{bmatrix} \widehat{k}_t & \widehat{z}_t \end{bmatrix}', \\ s_t &\equiv \begin{bmatrix} A_t & \xi_t & \psi_t \end{bmatrix}', \end{aligned}$$

ε_t is a vector of iid random processes, Υ and Σ are coefficient matrices, and variables with a hat denote percentage deviations of a variable from its steady state level.

Using standard methods (see Appendix C), the system can be solved to yield

$$\widehat{X}_{n,t+1} = \widehat{X}^s_{n,t+1} + \Psi_{xx} \widehat{X}_{n,t}, \tag{46}$$

$$E_t \widehat{Z}_{n,t+1} = E_t \Psi_{zs'} \left(L^{-1}\right) s_t - \left(V_1'\right)^{-1} V_2' \Psi_{xx} \widehat{X}_{n,t}, \tag{47}$$

where $\widehat{X}^s_{n,t+1}$, Ψ_{xx} and $\Psi_{zs'}$ are defined in appendix C. Here $\widehat{X}^s_{n,t+1}$ denotes the component of the evolution of the endogenous state variables that depends only on exogenous real disturbances, while Ψ_{xx} and $\Psi_{zs'}$ are coefficient matrices.

Moreover, the equilibrium condition for the natural rate of interest $\widehat{r}_t^n$ can be written as

$$\widehat{r}_{n,t} = \widehat{r}^s_{n,t} + \Psi_{rx}\widehat{X}_{n,t}, \tag{48}$$

where $\widehat{r}^s_{n,t}$ denotes the component of the natural rate that exclusively depends on exogenous real disturbances and Ψ_{rx} is a coefficient matrix (both defined in Appendix C). Equation (48) solves for the natural rate of interest as a function of the exogenous stochastic processes, given the actual values of the endogenous state variables, $\widehat{X}_{n,t} \equiv \widehat{X}_t$. The natural rate is independent of any nominal variable, and thus of the monetary policy rule. Our definition of the natural rate is therefore apt to be used as a policy indicator.

3.2 Price stability under a Wicksellian monetary policy regime

We now demonstrate that, if the spread between the policy instrument and the return on money is kept constant, a policy such that the short-term real interest rate tracks our definition of the natural rate is able to ensure price stability. If money cannot be remunerated or if the spread varies over time, however, price stability cannot be achieved. We wish to emphasise that, in both cases, the subsidy Ω_t is absent from the actual economy (where $\Omega_t = 1$).

As a first step, it is useful to write the log-linearized system of equilibrium conditions in terms of gaps between the observed variables and those arising in an equilibrium with flexible prices and subsidized nominal debt.

Define the vector of policy instruments as $\widehat{\delta}_t \equiv \begin{bmatrix} \widehat{i}_t & \widehat{\Omega}_t & \widehat{\Delta}_{m,t} \end{bmatrix}'$ and denote with a tilde the gap of each variable relative to its natural level. Notice that $\widetilde{\chi}_t = \widehat{\chi}_t$, $\widetilde{\pi}_t = \widehat{\pi}_t$, $\widetilde{\Omega}_t = -\widehat{\Omega}_{n,t}$ and $\widetilde{\Delta}_{m,t} = \widehat{\Delta}_{m,t}$. The system can thus be written as

$$\begin{aligned}
\begin{bmatrix} E_t\widetilde{Z}_{1,t+1} \\ \widetilde{X}_{t+1} \end{bmatrix} &= \underset{(11x11)}{\Upsilon'} \begin{bmatrix} \widetilde{Z}_{1,t} \\ \widetilde{X}_t \end{bmatrix} + \underset{(11x3)}{\Xi'}\,\widetilde{\delta}_t + \underset{(11x1)}{\Psi'}\,\widetilde{r}_t & (49)\\
\widetilde{r}_t &= \widehat{i}_t - \kappa_0' E_t\widetilde{Z}_{1,t+1} - \widehat{r}_{n,t} & (50)\\
\widehat{i}_t &= \upsilon_t + \vartheta_\pi \kappa_1' \widetilde{Z}_{1,t} & (51)
\end{aligned}$$

where for convenience we have listed separately the condition for the gap between the real rate of interest and its natural level. Here

$$\begin{aligned}
\widetilde{Z}_{1,t} &\equiv \begin{bmatrix} \widehat{c}_t - \widehat{c}_{n,t} & \widehat{e}_t - \widehat{e}_{n,t} & \widehat{h}_t - \widehat{h}_{n,t} & \widehat{I}_t - \widehat{I}_{n,t} & \widehat{q}_t - \widehat{q}_{n,t} & \widehat{\rho}_t - \widehat{\rho}_{n,t} & \widehat{\overline{\omega}}_t - \widehat{\overline{\omega}}_{n,t} & \widehat{\pi}_t & \widehat{\chi}_t \end{bmatrix}' \\
\widetilde{X}_t &\equiv \begin{bmatrix} \widehat{k}_t - \widehat{k}_{n,t} & \widehat{z}_t - \widehat{z}_{n,t} \end{bmatrix}', \; \widetilde{\delta}_t = \begin{bmatrix} 0 & -E_t\widehat{\pi}_{t+1} & \widehat{\Delta}_{m,t} \end{bmatrix}'.
\end{aligned}$$

and $\acute{\kappa}_0$ in equation (50) and $\acute{\kappa}_1$ in equation (51) denote vectors with all zeros except for the coefficients on expected and current inflation, respectively. Since our definition of a natural equilibrium is based on the current value of the predetermined variables, $\widetilde{X}_t = 0$.

We now investigate whether an interest rate rule that uses the natural rate as an intercept, i.e. a rule such that $v_t = \widehat{r}_{n,t}$ is consistent with price stability.

Case I: $\widehat{\Delta}_{m,t} = 0$.

Complete stabilization of inflation implies that $\widehat{\pi}_t = 0$, for all t, so that $\widetilde{\delta}_t = 0$ and

$$\begin{aligned} \begin{bmatrix} E_t \widetilde{Z}_{1,t+1} \\ \widetilde{X}_{t+1} \end{bmatrix} &= \overline{\Upsilon}' \begin{bmatrix} \widetilde{Z}_{1,t} \\ 0 \end{bmatrix} + \overline{\Psi}' \widetilde{r}_t \\ \widetilde{r}_t &= \widehat{i}_t - \widehat{r}_{n,t} \\ \widehat{i}_t &= \widehat{v}_t \end{aligned}$$

A necessary condition for the gaps to be closed at all times, i.e. for $E_t \widetilde{Z}_{1,t+1} = \widetilde{Z}_{1,t} = 0$ for all t, is

$$\widehat{v}_t = \widehat{r}_{n,t}. \tag{52}$$

Notice that a monetary policy rule satisfying condition (52) is consistent with an equilibrium with zero inflation at all times, but it might be consistent also with many other equilibria. To ensure that the zero-inflation equilibrium is the only one decentralized by a natural rate rule, we need to impose restrictions on the policy parameters that achieve determinacy of rational-expectations equilibrium. To do so, substitute condition (52) into equations (49)-(51). Using the definition of the vector of policy instruments $\widetilde{\delta}_t$, we can rewrite the system as

$$\begin{aligned} \begin{bmatrix} E_t \widetilde{Z}_{1,t+1} \\ \widetilde{X}_{t+1} \end{bmatrix} &= \underset{(11x11)}{\Upsilon'} \begin{bmatrix} \widetilde{Z}_{1,t} \\ 0 \end{bmatrix} + \underset{(11x3)}{\Xi'} \begin{bmatrix} 0 \\ E_t \widetilde{Z}_{1,t+1} \\ 0 \end{bmatrix} + \underset{(11x1)}{\Psi'} \widetilde{r}_t, \\ \widetilde{r}_t &= \vartheta_\pi \acute{\kappa}_1 \widetilde{Z}_{1,t} - \acute{\kappa}_0 E_t \widetilde{Z}_{1,t+1}, \end{aligned}$$

or as

$$\begin{bmatrix} E_t \widetilde{Z}_{2,t+1} \\ \widetilde{X}_{t+1} \end{bmatrix} = \underset{(12x12)}{F'} \begin{bmatrix} \widetilde{Z}_{2,t} \\ 0 \end{bmatrix},$$

where now $\widetilde{Z}_{2,t} \equiv \left[\widetilde{Z}_{1,t}, \widetilde{r}_t \right]$ and F is a new, suitably defined matrix of coefficients. Since the system includes two pre-determined variables, equilibrium determinacy under a natural rate rule is ensured if and only if the matrix F' has exactly 10 eigenvalues outside the unit circle (i.e. with modulus greater than one in absolute value).

Case II: $\widehat{\Delta}_{m,t} = \frac{\Gamma\prime}{\Gamma}\widehat{i}_t$.

Complete stabilization of inflation implies that $\widehat{\pi}_t = 0$, for all t, and

$$\begin{aligned}
\begin{bmatrix} E_t\widetilde{Z}_{1,t+1} \\ \widetilde{X}_{t+1} \end{bmatrix} &= \overline{\Upsilon}' \begin{bmatrix} \widetilde{Z}_{1,t} \\ 0 \end{bmatrix} + \Xi' \begin{bmatrix} 0 \\ 0 \\ \frac{\Gamma\prime}{\Gamma}\widehat{v}_t \end{bmatrix} + \overline{\Psi}'\widetilde{r}_t \\
\widetilde{r}_t &= \widehat{i}_t - \widehat{r}_{n,t} \\
\widehat{i}_t &= \widehat{v}_t
\end{aligned}$$

It follows that a policy where condition (52) holds leads to the following path of the endogenous variables

$$E_t\widetilde{Z}_{1,t+1} = \underset{(9x9)}{\Upsilon'_{11}}\widetilde{Z}_{1,t} + \underset{(9x1)}{\Xi'_{13}}\frac{\Gamma\prime}{\Gamma}\widehat{r}_{n,t}.$$

Hence, a natural rate rule is not consistent with complete stabilization of prices, since it does not implement an equilibrium where $E_t\widetilde{Z}_{t+1} = \widetilde{Z}_t = 0$, for all t. In the next section, we study whether deviations from price stability can be large in a calibrated version of the model.

4 Numerical analysis

In this section, we illustrate the properties of the natural rate of interest through a numerical analysis.

The specific form of the utility function is described in Appendix D. With the exception of the risk-aversion parameter, which we calibrate at 3 (instead of 1) for reasons which will be clarified below, the other structural parameter values are set following Carlstrom and Fuerst (1997). More specifically, monitoring costs are set at 25% of the firm's output (i.e. $\mu = 0.25$) and we calibrate the standard deviations of idiosyncratic shocks (σ_ω) and the average entrepreneurs' time preference so that approximately 1% of firms go bankrupt each quarter and the annualized spread between loan rates and the policy interest rate is approximately 2%. In our model, this procedure generates somewhat different values from those in Carlstrom and Fuerst (1997). The reason is that in our model, differently from the Carlstrom and Fuerst setup, the premium is charged on a positive interest rate. For given values of σ_ω and the entrepreneurs' time preference, the higher overall cost of credit in our model would imply that more firms go bankrupt and, in turn, that a higher premium is charged on average. Thus, our model generates the observed default rate and risk premium with a lower variance of idiosyncratic shocks and more impatient entrepreneurs.[2]

As to monopolistic competition and retail pricing, we assume $\varepsilon = 7$, leading to a steady-state mark-up of 17%, and a probability of not being able to re-optimise prices $\theta = 0.66$, implying that prices are changed on average every 3 quarters. Finally, the money demand function $L(c_t, \Delta_{m,t}; \xi_t)$ is calibrated so as to roughly match, in steady state, the real balances to output ratio observed in US data. In order to highlight the role of transactions frictions, we will analyse below the case in which money is not remunerated at all.[3] At the end of 2006, the ratio of currency in circulation to GDP was equal to 6% in the US, while the M1-to-GDP ratio was equal to 10%. We calibrate the model so that, in steady state, non-remunerated money balances are equal to 10% of yearly output (40% of quarterly output).[4]

Concerning shocks, the technology process is specified as in typical RBC calibrations, namely as an autoregressive process with autocorrelation coefficient $\rho_a = 0.95$ and a 1% standard deviation (i.e. $\sigma_a = 0.01$). All the other shock processes are assumed to have an autocorrelation coefficient equal to 0.9; their standard deviations are also set to 1%.

4.1 Price stability and natural rate dynamics

Figure 1 shows the impulse responses to a 1-standard deviation technology shock under a natural rate rule for our benchmark model with credit frictions and constant Δ_m, and for the corresponding model without credit frictions (namely the limiting case of our model in which $\mu = 0$).

In spite of the presence of nominal debt, our results concerning real variables are broadly in line with those in figure 2 of Carlstrom and Fuerst (1997).[5] The notable difference in the impulse responses of the model with agency costs, compared to the model without credit frictions, is the hump-shaped response of investment and output (as well as hours, not shown

[2] More specifically, we need $\gamma = 0.88$ and $\sigma_\omega = 0.065$ to generate a bankrupcy rate of 0.9% per quarter, and an annualised spread $i^l - i = 2.1\%$.

[3] This assumption potentially overestimates the role of transactions frictions. Nonetheless, the numerical results reported later on in this section suggest a limited role of money balances for the dynamics.

[4] This implies $\iota = .9958$ in the utility function specified in the appendix.

[5] Carlstrom and Fuerst (1997) consider an economy with flexible prices. Our model replicates the flexible prices case under a natural rate rule.

in the figure). This is mainly due to the dynamic behavior of net worth, which is initially constrained by the fact that entrepreneurial capital is fixed. The increased demand for capital, however, leads to an increase in the price of capital and in the return on internal funds, thus, over time, an increase in entrepreneurial capital and net worth. Net worth keeps increasing as long as the price of capital is above the baseline, i.e. for two periods. Thereafter, the impulse responses of output and investment mimic those of the model without credit frictions.

To reap the benefits of the higher return on internal funds and invest more, entrepreneurs also reduce their consumption sharply (around 20% on impact). The corresponding share of output is partly consumed by households, whose consumption increases much more, on impact, than in the case without credit frictions. This effect only lasts one period, because in the second period after the shock entrepreneurs also start consuming more than in steady state. Thereafter, households' consumption follows the same pattern as in the model without credit frictions (albeit at a slightly lower level because of the output share consumed by entrepreneurs).

Figure 1 illustrates the appealing property (shown analytically in section 3.2) that a policy rule which tracks the natural interest rate is consistent with the maintenance of price stability at all times. In the model with agency costs, the technology shock would tend to create larger initial deflationary pressures, compared to the case without credit frictions. The impact fall in the policy interest rate provides the stimulus to aggregate demand which is necessary to ensure that no pressures ensue on marginal costs and final prices.

Nonetheless, the figure also shows a sensitivity of the natural rate to the underlying model assumptions. The dynamic path of the natural rate (which coincides with the nominal interest rate, since Δ_m is constant and inflation is zero at all times) is determined by the response of household consumption. In the case without credit frictions, expected consumption growth is positive but decreasing over time, which leads to a protracted increase in the natural rate. On the contrary, in the model with agency costs, household consumption is expected to fall from the peak of the impact response. Thus, the natural rate falls on impact. The dynamics of the natural rate are roughly consistent with those of the model without credit frictions as of the third period after the shock.

4.2 Money remuneration

Figure 2 illustrates, for the model with agency costs, the role of transactions costs by comparing the performance of a natural rate rule in the cases of constant Δ_m and $i_m = 0$. The difference arises because of the behavior of real balances. In the constant Δ_m case, real balances remain roughly constant (they rise on impact by just 0.3 percentage points and then follow a pattern that closely mimics that of household consumption). When $i_m = 0$, however, the demand for real balances is affected by the decrease in their opportunity costs, the nominal interest rate. The figure shows that this component is important in our calibration: real balances increase by over 12 percentage points – i.e. 40 times more than in the constant Δ_m case – before falling persistently below steady state in the second period after the shock. The higher demand for real balances is satisfied through an increase in money supply and a decrease in the price of final goods, which induces a bit of deflation in the economy. This result confirms that the natural rate rule is unable to stabilise inflation perfectly when real balances are not remunerated. Nevertheless, price stability is still achieved in approximate terms. A shock causing more than a 1% increase in output reduces inflation temporarily (1 period) by just 1 basis point.

4.3 Nominal debt contracts and the real effects of monetary policy

Figure 3 shows that the combination of credit frictions and nominal debt can be a significant source of monetary policy non-neutrality. For the case where Δ_m is constant and thus prices are fully flexible, the figure compares the impulse responses to a monetary policy shock under a simple Taylor rule (with constant intercept and an inflation response coefficient equal to 1.5) in the model with credit frictions and in the corresponding model without credit frictions.

In the economy without credit frictions, a change in the policy interest rate produces no real effects: a reduction in the nominal interest rate leads to a fall in inflation so as to leave the real rate constant. In the economy with credit frictions and nominal debt, however, the reduction in the policy rate stimulates investment through a reduction of the price of capital, thus mitigating the deflationary effect of the shock arising under flexible prices. The net effect is a small increase in the real interest rate, and a reduction in households' consumption. Entrepreneurial consumption also falls to allow the initial increase in investment for given net worth. In spite of a small reduction in both household and entrepreneurial consumption, the overall effect is an increase in output.

The economic expansion triggers the slow process of accumulation, and then decumulation, of net worth. As a result, the response of all variables to the shock is very persistent over time.

5 Robustness

In our model, the formulation of the costly state verification problem arising in the investment sector closely follows the one proposed by Carlstrom and Fuerst (1997), the main difference being the nominal denomination of debt. One appealing feature of this setup is that the optimal contract between lenders and borrowers generates a loan rate that differs from the risk-free rate because of a time-varying premium on external finance. Thus, the dynamics of the risk-free rate and of the loan rate need not coincide. Another attractive feature is that the model generates a humped-shaped response of investment to a technology shock, without the need to assume ad-hoc adjustment costs in investment. Nonetheless, the assumption of risk-neutrality of the entrepreneurs, which is needed for the optimality of the debt contract, is responsible for some undesirable properties of the model.

First, the model predicts that the default rate of firms increases in periods of expansions, while the opposite is observed in the data (see e.g. Levin et al. (2004)). The reason is that a favourable technology shock increases the demand for investment while firms' net worth remains initially constant. Under these circumstances, the optimal solution to the costly state verification problem generates an increase in the loan rate, thus raising the occurrence of default on impact. An implication of this property is that the response of output and other aggregate variables is dampened, rather than amplified, by the presence of financial frictions.

A second undesirable property of the model is that households and entrepreneurs are radically different agents. Not only households are risk-averse while entrepreneurs are risk neutral but these two type of agents also differ in their willingness to substitute consumption intertemporally. Hoseholds have a finite intertemporal elasticity of substitution while the same elasticity is infinite for entrepreneurs. As a result, the model generates very sharp fluctuations in entrepreneurial consumption (and consequently in household consumption) in reaction to aggregate shocks.

In this section, we use a variant of the financial accelerator model that we propose in a companion paper (De Fiore and Tristani (2007)). We continue to assume that entrepreneurs are risk neutral, but we allow for a finite elasticity of intertemporal substitution of consumption across different points in time. Since risk aversion and intertemporal substitution are inextricably linked under standard expected utility preferences, we rely on the class of preferences proposed by Epstein and Zin (1989) and Weil (1990). Under these preferences, the optimality of the contract is preserved, while it is possible to impose the same intertemporal elasticity of substitution for households and entrepreneurs.

More specifically, entrepreneurial utility can be written in terms of the aggregator function

$$V_{i,t} = \left[(1-\beta\gamma)\, e_{i,t}^{1-\zeta} + \beta\gamma \left(\mathrm{E}_t V_{i,t+1} \right)^{1-\zeta} \right]^{\frac{1}{1-\zeta}},$$

where $\zeta \geq 0$ is the inverse of the elasticity of intertemporal substitution, which we set to be identical to that of households. The Bellman equation for the maximisation problem of entrepreneurs, subject to the budget constraint (31), can be written as (see e.g. Weil, 1990)

$$\widetilde{V}\left(z_{i,t}\right) = \max_{e_{i,t}} \left[(1-\beta\gamma)\, e_{i,t}^{1-\zeta} + \beta\gamma \left(\mathrm{E}_t \widetilde{V}\left(z_{i,t+1}\right) \right)^{1-\zeta} \right]^{\frac{1}{1-\zeta}}$$

The first-order condition is given by

$$q_t = \beta\gamma \left(\mathrm{E}_t \widetilde{V}\left(z_{i,t+1}\right) \right)^{-\zeta} \mathrm{E}_t \left\{ \frac{\left[(1-\beta\gamma)\, e_{i,t+1}^{1-\zeta} + \beta\gamma \left(\mathrm{E}_{t+1} \widetilde{V}\left(z_{i,t+2}\right) \right)^{1-\zeta} \right]^{\frac{\zeta}{1-\zeta}}}{\left(\frac{e_{i,t+1}}{e_{i,t}} \right)^{\zeta}} q_{t+1} f\left(\overline{\omega}_{t+1}\right) \frac{I_{i,t+1}}{z_{i,t+1}} \right\}$$

where $I_{i,t+1}$ is given by equations (26) and (19).

Contrary to the case analysed in section 2.3.2, aggregation over all entrepreneurs is not straightforward in the nonlinear model. We therefore only study the equilibrium in a first-order approximation of the model around the non-stochastic steady state.[6]

One implication of the finite level of the intertemporal elasticity of substitution of consumption is that, following an aggregate productivity shock, entrepreneurs are not willing to cut their consumption by 20% in order to increase their investment in capital. Since they know

[6] It is straightforward to verify that, in the non-stochastic steady-state, entrepreneurial consumption and utility drop out of the first-order condition for the optimisation problem of the entrepreneurs. The system characterising the non-stochastic steady state is therefore identical to the one we obtain under linear entrepreneurial utility. To characterise the first-order dynamics, we first linearise the first-order conditions for each entrepreneur, and then aggregate over all entrepreneurs.

that they will eventually increase their consumption in the future, entrepreneurs increase it immediately by around 2%. Figure 4 shows that, as a consequence, household consumption also fluctuates less sharply relative to the $\zeta = 0$ case. At the same time, entrepreneurs' desire to smooth their consumption over time implies that the convergence of all variables to the path they follow in the absence of credit frictions is much slower.

The default rate rises on impact following a favourable aggregate productivity shock but falls below baseline after 5 quarters. We use $\zeta = 3$ in the calibration, because it is the minimum value of this parameter for which the unconditional correlation of the default rate and output becomes negative, as in the data. The reason is related to entrepreneurs' unwillingness to cut their consumption sharply after the shock. This results in a more gradual accumulation of net worth and investment, as shown in Figure 4. The peak of net worth is lower than in figure 1 and is reached after 7 quarters instead of 2. The gradual accumulation of net worth together with the reduced demand for investment implies that the spread between the loan and the policy rate increases more on impact, but falls below baseline on the 5-th quarter after the shock. This fall brings about a more persistent response of investment and output, compared to the case without credit frictions.

Figure 4 shows that the results obtained in section 3 are robust to the introduction of this class of preferences. On the one hand, an economy where the central bank commits to a natural rate rule can achieve complete stabilization of prices. On the other hand, the natural rate features qualitatively different behaviour in the models with and without credit frictions. In the former economy it falls on impact, while it rises in the latter. Moreover, the difference between the two cases is more persistent. The behaviour of some variables under the two models, e.g. household's consumption, does not converge even after 3 years.

Figure 5 shows that the dynamic response of the natural rate of interest depends on the underlying model assumptions under various aggregate shocks. In the cases of technology and labour supply shocks, the impact response have the opposite sign in the models with and without credit frictions. For shocks to consumption preferences, the sign of the impact responses is unchanged, but the size is significantly larger in the model with credit frictions. In all cases, the differences are very persistent: they only become smaller after 3 years.

More specifically, a labour supply shock, i.e. an increase in the disutility of labour, ultimately brings about a reduction in output regardless of the presence of credit frictions. In the case with frictions, however, entrepreneurs' net worth is reduced very slowly, so the fall in investment and output is smaller on impact, but more persistent. The more protracted fall in output is reflected in a deeper fall in households' consumption, which drops on impact and then increases towards the steady state. Expectations of future increases in consumption are what triggers the increase in the natural rate.

A preference shock amounts to a persistent increase in households' consumption demand, which is partly satisfied through a substitution of consumption for investment. As in the case of a labour supply shock, the fall in investment is smaller on impact, but more persistent, in the model with credit frictions. This has a dampening effect on future output which, in turn, causes a slightly smaller increase in households' consumption, compared to the case without credit frictions. At the same time, however, the shock has a direct positive impact on the marginal utility of consumption, which increases in spite of the increase in consumption level. This is what determined the increase in the natural rate, which is slightly larger in the case with credit frictions because of the smaller increase in households' consumption.

6 Conclusions

We have presented a sticky price model where monetary policy exerts real effects under flexible prices for two reasons. First, money balances yield liquidity services. Second, firms need to raise external finance in order to produce and this latter takes the form of nominal debt.

We have argued that in our economy the notion of natural rate of interest generally adopted in the literature, namely the equilibrium real rate of return arising under flexible prices, is not a useful indicator for the conduct of monetary policy because it is not independent of the nominal interest rate. Our proposed notion, based on the idea of switching off all nominal (but not real) frictions, is both independent of the monetary policy instrument and useful in the achievement of price stability. More specifically, if the spread between the own return on money and the nominal interest rate is constant over time, our definition of natural interest rate delivers price stability if used as the intercept of an interest rate rule. If the spread cannot be kept constant, also our notion of natural rate is not independent of monetary policy. Nevertheless, under reasonable calibrations of the model, we find that deviations from price stability would be very small if the central bank were to abstract from transaction frictions in the definition of natural rate and track this 'approximate' notion instead.

We have also shown that the natural rate reacts differently to aggregate shocks - not only quantitatively but also qualitatively – depending on the underlying model assuptions (e.g. whether or not financial market are frictionless). Therefore, it might be difficult for a central bank that is uncertain about the true model of the economy to identify its movements and to use it as regular indicator for the conduct of monetary policy.

Appendix

A. The reduced-form system of equilibrium conditions

Define $p_t(j) \equiv \frac{P_t(j)}{P_t}$ and variables with a tilde as the corresponding variables in a log-linear approximation around a steady state with zero inflation. The log-linearized system of equilibrium conditions can be written as a block of intra-temporal conditions

$$
\begin{aligned}
(1+\overline{v})\,\widehat{h}_t &= -\sigma^{-1}\widehat{c}_t - \sigma_{\Delta}^{-1}\widehat{\Delta}_{m,t} + \widehat{y}_t + \widehat{\chi}_t + \overline{h}_c\xi_t - \overline{h}_h\psi_t \\
\widehat{\rho}_t &= \widehat{y}_t + \widehat{\chi}_t - \widehat{k}_t \\
\widehat{y}_t &= \frac{c}{y}\widehat{c}_t + \frac{e}{y}\widehat{e}_t + \frac{I}{y}\widehat{I}_t \\
\widehat{y}_t &= \widehat{A}_t + \alpha\widehat{h}_t + (1-\alpha)\widehat{k}_t \\
\epsilon_\omega\widehat{\overline{\omega}}_t &= \widehat{i}_t - \widehat{q}_t - \widehat{\Omega}_t \\
\widehat{I}_t &= \frac{qg'\overline{\omega}n}{I}\frac{(1+i)}{\Omega}\widehat{\overline{\omega}}_t - \left(\frac{I}{n}-1\right)\left(\widehat{i}_t - \widehat{\Omega}_t\right) + \widehat{z}_t + \frac{q(1-\delta)}{[q(1-\delta)+\rho]}\widehat{q}_t + \frac{\rho}{[q(1-\delta)+\rho]}\widehat{\rho}_t.
\end{aligned}
$$

a block of law of motions for the state variables

$$
\begin{aligned}
\widehat{k}_{t+1} &= (1-\delta)\widehat{k}_t + \delta\widehat{I}_t - \mu\phi\overline{\omega}\frac{I}{k}\widehat{\overline{\omega}}_t \\
\widehat{z}_{t+1} &= \frac{(e+qz)}{qz}\left(\widehat{I}_t + \frac{f'\overline{\omega}}{f}\widehat{\overline{\omega}}_t\right) - \frac{e}{qz}\widehat{e}_t - \frac{q(z-fI)}{qz}\widehat{q}_t
\end{aligned}
$$

a block of intertemporal conditions

$$
\begin{aligned}
\widehat{i}_t &= \eta_c^{-1}\left(E_t\widehat{c}_{t+1} - \widehat{c}_t\right) + \eta_\Delta^{-1}\left(E_t\widehat{\Delta}_{m,t+1} - \widehat{\Delta}_{m,t}\right) - \overline{h}_\xi\left(E_t\xi_{t+1} - \xi_t\right) + E_t\widehat{\pi}_{t+1}, \\
&\quad \sigma_c^{-1}\left(E_t\widehat{c}_{t+1} - \widehat{c}_t\right) + \sigma_\Delta^{-1}\left(E_t\widehat{\Delta}_{m,t+1} - \widehat{\Delta}_{m,t}\right) - \overline{h}_c\left(E_t\xi_{t+1} - \xi_t\right) \\
&= \frac{q(1-\delta)}{[q(1-\delta)+\rho]}E_t\widehat{q}_{t+1} - \widehat{q}_t + \frac{\rho}{[q(1-\delta)+\rho]}E_t\widehat{\rho}_{t+1}, \\
\widehat{q}_t &= \widehat{\gamma}_t + \left(\frac{g'\overline{\omega}I\Omega}{n(1+i)} + \frac{f'}{f}\right)E_t\widehat{\overline{\omega}}_{t+1} + \left(1-\frac{I}{n}\right)E_t\left(\widehat{i}_{t+1} - \widehat{\Omega}_{t+1}\right) \\
&\quad + \left[1 + \frac{q(1-\delta)}{[q(1-\delta)+\rho]} + \frac{qgI\Omega}{n(1+i)}\right]E_t\widehat{q}_{t+1} + \frac{\rho}{[q(1-\delta)+\rho]}E_t\widehat{\rho}_{t+1}, \\
\widehat{\pi}_t &= \lambda\widehat{\chi}_t + \beta E_t\widehat{\pi}_{t+1},
\end{aligned}
$$

and a block of policy functions

$$\begin{aligned}
\widehat{i}_t &= \upsilon_t + \vartheta_\pi \pi_t \\
\widehat{\Delta}_{m,t} &= \frac{\Gamma'}{\Gamma}\widehat{i}_t \\
\widehat{\Omega}_t &= E_t \widehat{\pi}_{t+1},
\end{aligned}$$

where the coefficients need to satisfy

$$\begin{aligned}
\sigma_c^{-1} &\equiv -\frac{U_{cc}c}{U_c},\ \sigma_\Delta^{-1} \equiv -\frac{U_{c\Delta}\Delta_m}{U_c},\ \overline{v} \equiv h\frac{v_{hh}}{v_h} \\
\overline{h}_c &\equiv \frac{U_{c\xi}}{U_c},\ \overline{h}_h \equiv \frac{v_{h\psi}}{v_h},\ \overline{h}_\xi \equiv \frac{U_{c\xi}+U_{m\xi}}{U_c+U_m} \\
\eta_c^{-1} &\equiv -\left(\frac{U_{cc}+U_{mc}}{U_c+U_m}\right)c > 0,\ \eta_\Delta^{-1} \equiv -\left(\frac{U_{c\Delta}+U_{m\Delta}}{U_c+U_m}\right)\Delta_m > 0 \\
\epsilon_\omega &\equiv \frac{\mu\overline{\omega}\phi}{\left(1-\mu\Phi+\frac{f\phi\mu\overline{\omega}}{f'}\right)}\left[\overline{\omega}-1+\frac{f}{f'}\left(1+\frac{\phi'\overline{\omega}}{\phi}-\frac{\phi\overline{\omega}}{f'}\right)\right] \\
\lambda &\equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}.
\end{aligned}$$

The system consists of 15 equations and provides a solution for the path of the endogenous variables $\left\{\widehat{c}_t, \widehat{e}_t, \widehat{h}_t, \widehat{I}_t, \widehat{y}_t, \widehat{q}_t, \widehat{\rho}_t, \widehat{\chi}_t, \widehat{\overline{\omega}}_t, \widehat{\pi}_t\right\}$, the policy variables $\left\{\widehat{i}_t, \widehat{\Omega}_t, \widehat{\Delta}_{m,t}\right\}$ and the state variables $\left\{\widehat{k}_{t+1}, \widehat{z}_{t+1}\right\}$, given the law of motion of the exogenous stochastic processes $\{A_t, \xi_t, \psi_t\}$.

B. The natural equilibrium

We define with a subscript n the variables obtained in a natural equilibrium, where prices are flexible, $\Delta_{n,m,t} = \Delta_{n,m}$, and $\Omega_{n,t}$ is described by (43), given the currently observed level of all exogenous and predetermined variables.

Recall that in a natural equilibrium, $\chi_t = \chi = \frac{\varepsilon-1}{\varepsilon}$. We can thus derive the path of the natural rate of interest by solving the following system of equations,

$$\frac{v_h(h_{n,t};\psi_t)}{U_c(c_{n,t};\xi_t)} = \chi\alpha A_t h_{n,t}^{\alpha-1}k_t^{1-\alpha}$$

$$(1+r_{n,t})^{-1} = \beta E_t\left\{\frac{U_c\left(c_{n,t+1};\xi_{t+1}\right)+U_m\left(c_{n,t+1};\xi_{t+1}\right)}{U_c(c_{n,t};\xi_t)+U_m(c_{n,t};\xi_t)}\right\} \tag{53}$$

$$U_c(c_{n,t};\xi_t)\,q_{n,t} = \beta E_t\left\{U_c\left(c_{n,t+1};\xi_{t+1}\right)\left[q_{n,t+1}(1-\delta)+\rho_{n,t+1}\right]\right\}$$

$$\rho_{n,t} = \chi(1-\alpha)A_t h_{n,t}^{\alpha}k_t^{-\alpha}$$

$$A_t h_{n,t}^{\alpha} k_t^{1-\alpha} = c_{n,t} + e_{n,t} + I_{n,t}$$

$$e_{n,t} + q_{n,t} z_{n,t+1} = q_{n,t} f(\overline{\omega}_{n,t}) I_{n,t}$$

$$k_{n,t+1} = (1-\delta) k_{n,t} + I_{n,t} \left[1 - \mu \Phi\left(\overline{\omega}_{n,t}\right)\right]$$

$$q_{n,t} \left[1 - \mu \Phi\left(\overline{\omega}_{n,t}\right) + \frac{\mu \overline{\omega}_{n,t} f\left(\overline{\omega}_{n,t}\right) \phi\left(\overline{\omega}_{n,t}\right)}{f'\left(\overline{\omega}_{n,t}\right)}\right] = 1 + r_{n,t}$$

$$I_{n,t} \left[(1 + r_{n,t}) - q_{n,t} g\left(\overline{\omega}_{n,t}\right)\right] = \left[q_{n,t}(1-\delta) + \rho_{n,t}\right] z_t \left(1 + r_{n,t}\right)$$

$$q_{n,t} = \beta \gamma_t E_t \left\{ q_{n,t+1} f\left(\overline{\omega}_{n,t+1}\right) \left[\frac{\left[q_{n,t+1}(1-\delta) + \rho_{n,t+1}\right](1 + r_{n,t})}{(1 + r_{n,t}) - q_{n,t+1} g\left(\overline{\omega}_{n,t+1}\right)}\right] \right\}$$

The system has 10 equations and can be solved for the variables $\left\{c_{n,t}, e_{n,t}, h_{n,t}, I_{n,t}, q_{n,t}, \rho_{n,t}, \overline{\omega}_{n,t}, r_{n,t}\right\}$ and $\left\{k_{n,t+1}, z_{n,t+1}\right\}$, for given law of motion of the stochastic processes $\left\{A_t, \xi_t, \psi_t\right\}$ and initial value of the predetermined variables $\left\{k_t, z_t\right\}$.

C. Solution of the natural equilibrium: the log-linearized system

The log-linearized system characterizing the natural equilibrium can be written as in equation (44) and (45), where

$$\underset{(10x10)}{\Upsilon} \equiv \begin{bmatrix} \underset{(8x8)}{\Upsilon_{11}} & \underset{(8x2)}{\Upsilon_{12}} \\ \underset{(2x8)}{\Upsilon_{21}} & \underset{(2x2)}{\Upsilon_{22}} \end{bmatrix}, \underset{(10x3)}{\Sigma} \equiv \begin{bmatrix} \underset{(8x3)}{\Sigma_1} \\ \underset{(2x3)}{\Sigma_2} \end{bmatrix},$$

are coefficient matrices. Let

$$V' \Upsilon = \Lambda V',$$

where all the eigenvalues of Λ are outside the unit circle. The system (44) can be rewritten as

$$V' \begin{bmatrix} E_t \widehat{Z}_{n,t+1} \\ \widehat{X}_{n,t+1} \end{bmatrix} = \Lambda V' \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix} + V' \Sigma s_t.$$

Define now

$$\widehat{\varphi}_{n,t} \equiv V' \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix} = \begin{bmatrix} V_1' & V_2' \end{bmatrix} \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix}.$$

It follows that

$$\begin{aligned} \widehat{\varphi}_{n,t} &= \Lambda^{-1} \left\{ E_t \widehat{\varphi}_{n,t+1} - V' \Sigma s_t \right\} \\ &= E_t \left[\Psi_{\varphi s}\left(L^{-1}\right) s_t \right] \end{aligned}$$

where $\Psi_{\varphi s}\left(L^{-1}\right) \equiv -\Lambda^{-1}\left(\frac{1}{1-\Lambda^{-1}L^{-1}}\right) V'\Sigma$, and

$$\begin{aligned}
\widehat{Z}_{n,t} &= \left(V_1'\right)^{-1}\left(\widehat{\varphi}_{n,t} - V_2'\widehat{X}_{n,t}\right) \\
&= \left(V_1'\right)^{-1}\left[E_t\left[\Psi_{\varphi s}\left(L^{-1}\right)s_t\right] - V_2'\widehat{X}_{n,t}\right]. \qquad (54)
\end{aligned}$$

Moreover,

$$\begin{aligned}
\widehat{X}_{n,t+1} &= \Upsilon_{21}\widehat{Z}_{n,t} + \Upsilon_{22}\widehat{X}_{n,t} + \Sigma_2 s_t \qquad (55)\\
&= \widehat{X}^s_{n,t+1} + \Psi_{xx}\widehat{X}_{n,t}, \qquad (56)
\end{aligned}$$

where $\widehat{X}^s_{n,t+1} \equiv \left\{\Upsilon_{21}\left(V_1'\right)^{-1}E_t\Psi_{\varphi s}\left(L^{-1}\right) + \Sigma_2\right\}s_t$ is the component of the evolution of the endogenous state variables that depend only on exogenous real disturbances, and $\Psi_{xx} \equiv \Upsilon_{22} - \Upsilon_{21}\left(V_1'\right)^{-1}V_2'$. Also, $E_t\widehat{\varphi}_{n,t+1} = E_t\left[\Psi_{\varphi s}\left(L^{-1}\right)\Phi_s s_t\right]$. It follows that

$$\begin{aligned}
E_t\widehat{Z}_{n,t+1} &= \left(V_1'\right)^{-1}E_t\widehat{\varphi}_{n,t+1} - \left(V_1'\right)^{-1}V_2'\widehat{X}_{n,t+1} \\
&= E_t\Psi_{zs'}\left(L^{-1}\right)s_t - \left(V_1'\right)^{-1}V_2'\Psi_{xx}\widehat{X}_{n,t}, \qquad (57)
\end{aligned}$$

where $E_t\Psi_{zs'}\left(L^{-1}\right) = \left(V_1'\right)^{-1}\left[E_t\Psi_{\varphi s}\left(L^{-1}\right)\Phi_s - V_2'\left\{\Upsilon_{21}\left(V_1'\right)^{-1}E_t\left[\Psi_{\varphi s}\left(L^{-1}\right)\right] + \Sigma_2\right\}\right]$. The complete solution to the system of equilibrium conditions is thus given by (46)-(47) in the main text.

Notice that the equilibrium condition for the equilibrium real interest rate $\widehat{r}^n_t$, equation (53), can be written as

$$\widehat{r}_{n,t} = g_0'\widehat{Z}_{n,t} + g_1'E_t\widehat{Z}_{n,t+1} + g_2's_t.$$

Substituting the solution for $\widehat{Z}_{n,t}$ and $E_t\widehat{Z}_{n,t+1}$, we obtain

$$\widehat{r}_{n,t} = \widehat{r}^s_{n,t} + \Psi_{rx}\widehat{X}_{n,t}.$$

where $\widehat{r}^s_{n,t} \equiv \left[g_0'E_t\Psi_{\varphi s}\left(L^{-1}\right) + g_1'E_t\Psi_{zs'}\left(L^{-1}\right) + g_2'\right]s_t$ denotes the component of the natural rate that exclusively depends on exogenous real disturbances and $\Psi_{rx} \equiv -g_0'\left(V_1'\right)^{-1}V_2' - g_1'\left(V_1'\right)^{-1}V_2'\Psi_{xx}$.

D. Calibration

For the numerical calibration, we use the following functional form for households' utility

$$\begin{aligned}
u\left(c_t, m_t; \xi_t\right) &= \xi_{1t}\frac{\left(C^{\iota}m^{1-\iota}\right)^{1-\zeta}}{1-\zeta} \\
v\left(h_t; \xi_t\right) &= \psi\xi_{2t}h^{\varphi}
\end{aligned}$$

Shocks ξ_{1t} and ξ_{2t}, the two elements of vector ξ_t, are labelled "preference shock" and "labour supply shock", respectively, in Figure 5.

Under this assumption, we can show that the system allows for two steady state equilibria. In the $\Omega_t = 1$ case, the steady state is characterised, amongst other equations, by

$$A\left(\frac{h}{k}\right)^{\alpha} = q\chi\left[\frac{\frac{1}{\beta}-(1-\delta)}{1-\alpha}\right] \tag{58}$$

$$1-\Phi(\overline{\omega})\mu + \frac{f(\overline{\omega})\phi(\overline{\omega})\mu}{f'(\overline{\omega})} = \frac{1+i}{q} \tag{59}$$

$$1 = \beta\gamma f(\overline{\omega})\frac{q(1-\delta)+(1-\alpha)\frac{A\left(\frac{h}{k}\right)^{\alpha}}{\chi}}{1+i-qg(\overline{\omega})}(1+i) \tag{60}$$

where $1+i = \Pi/\beta$ and Π is the steady state gross inflation rate.

These equations can be used to show that $\overline{\omega}$ and q can be solved recursively. Using the first in the third, we obtain

$$1 = \gamma f(\overline{\omega})\frac{q}{1+i-qg(\overline{\omega})}(1+i)$$

which determines q given $\overline{\omega}$ as

$$\frac{1+i}{q} = g(\overline{\omega}) + f(\overline{\omega})\gamma(1+i)$$

This expression for $(1+i)/q$ can be used in (59) to derive an equation which can be solved for the steady state value of $\overline{\omega}$

$$f(\overline{\omega})\left(\mu\frac{\phi(\overline{\omega})}{f'(\overline{\omega})} + 1 - \gamma(1+i)\right) = 0$$

where we also used $g(\overline{\omega}) = 1-\mu\Phi(\overline{\omega}) - f(\overline{\omega})$. Note that this equation has two solutions. The first would be obtained in a model without financial frictions, where no output accrues to entrepreneurs and $f(\overline{\omega}) = 0$. The second solution, which we focus on, is such that

$$\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})} = \frac{1-\gamma(1+i)}{\mu}$$

Since the expression on the left hand side of this equation is the hazard rate, it is always positive. Hence, the second solution only exists if γ is sufficiently small, namely $\gamma < 1/(1+i)$.

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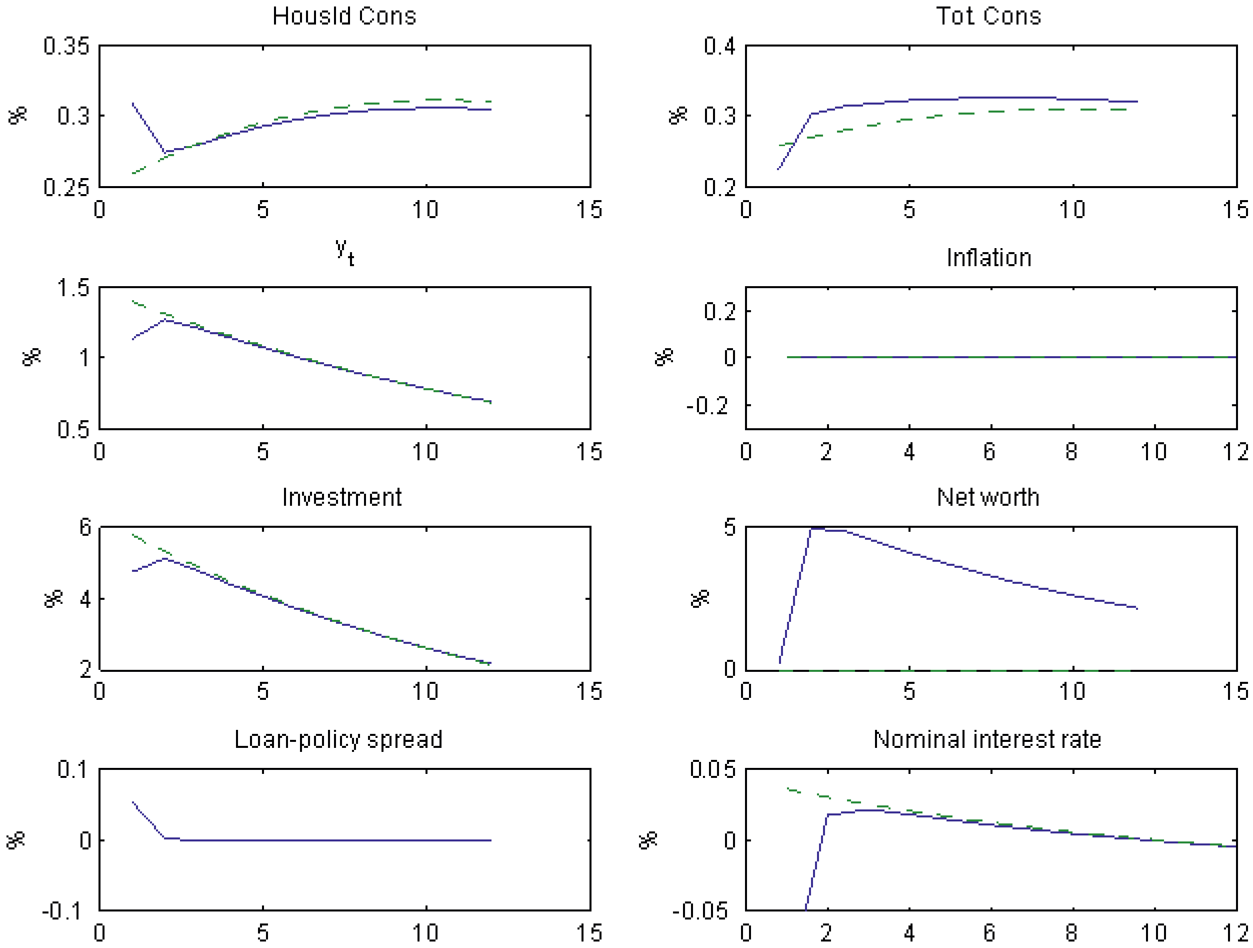

Figure 1: Impulse responses to a technology shock under the natural rate rule (constant Δ_m).

Solid line: model with credit frictions; dashed line: model without credit frictions.

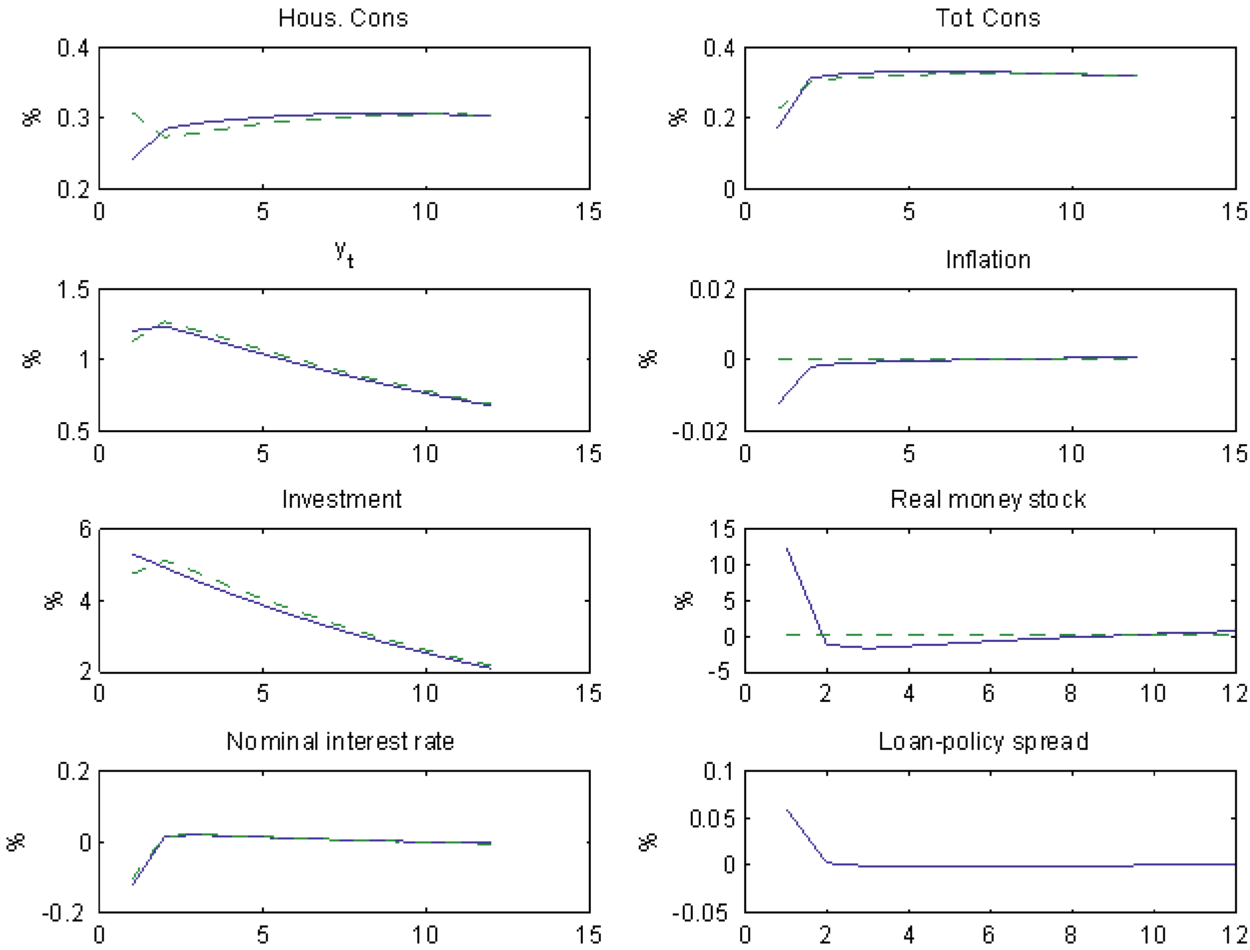

Figure 2: Impulse responses to a technology shock in the model with credit frictions. Solid line: constant Δ_m case; dashed line: $i_m = 0$ case.

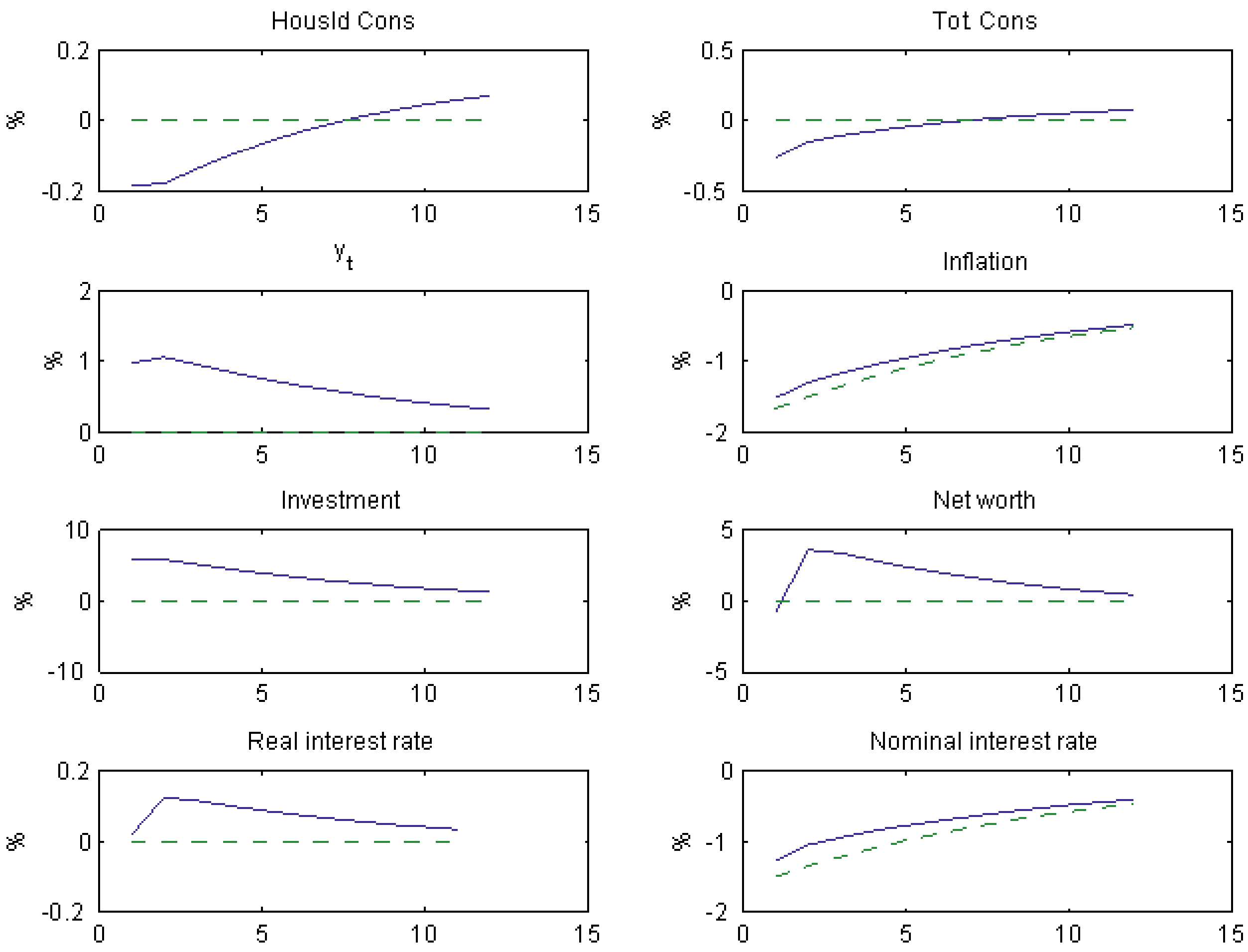

Figure 3: Impulse responses to a monetary policy shock under flexible prices and a Taylor rule. Solid line: model with credit frictions; dashed line: model without credit frictions.

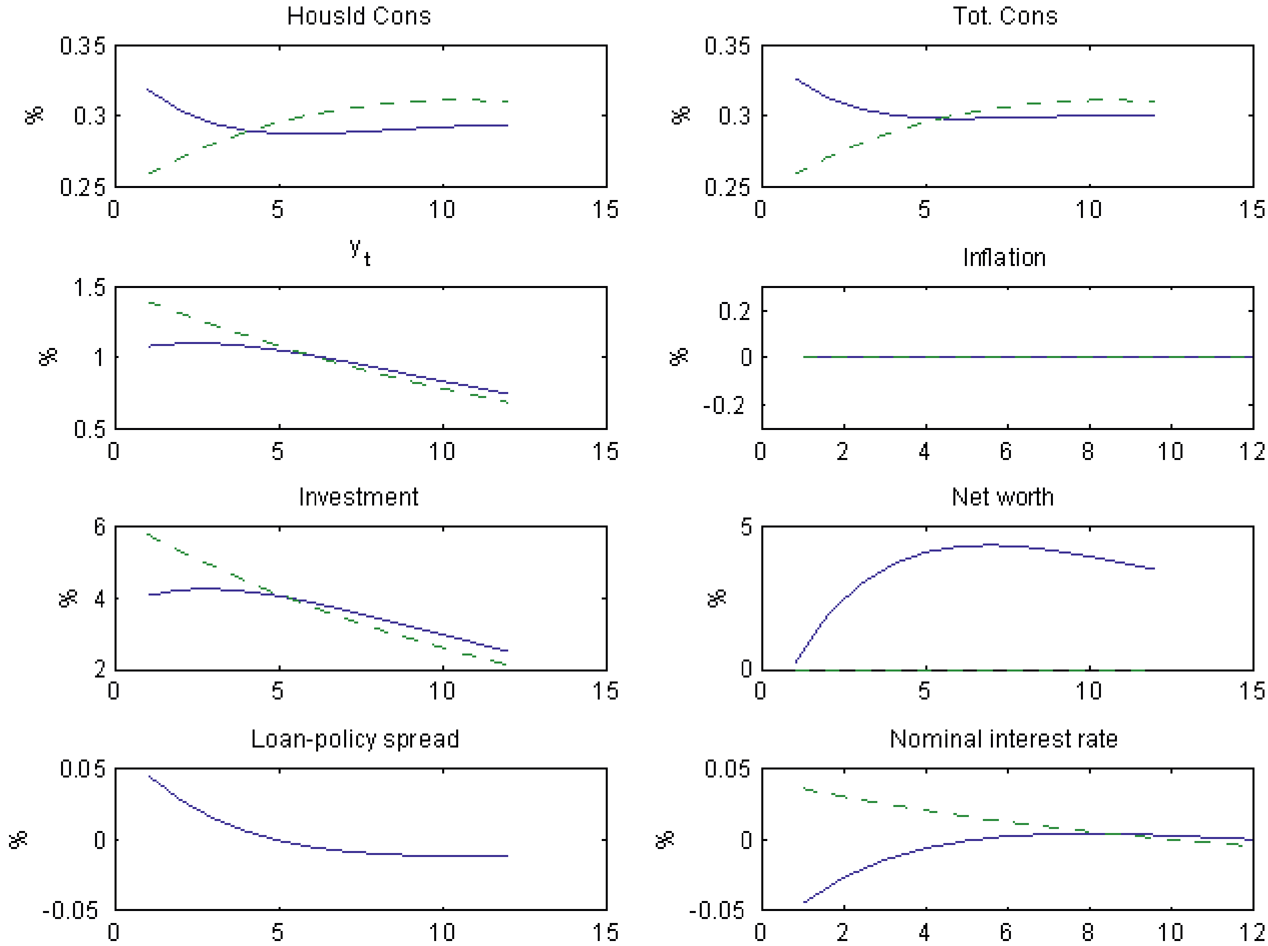

Figure 4: Impulse responses to a technology shock with non-expected utility preferences ($\zeta = 3$). Solid line: model with credit frictions; dashed line: model without credit frictions.

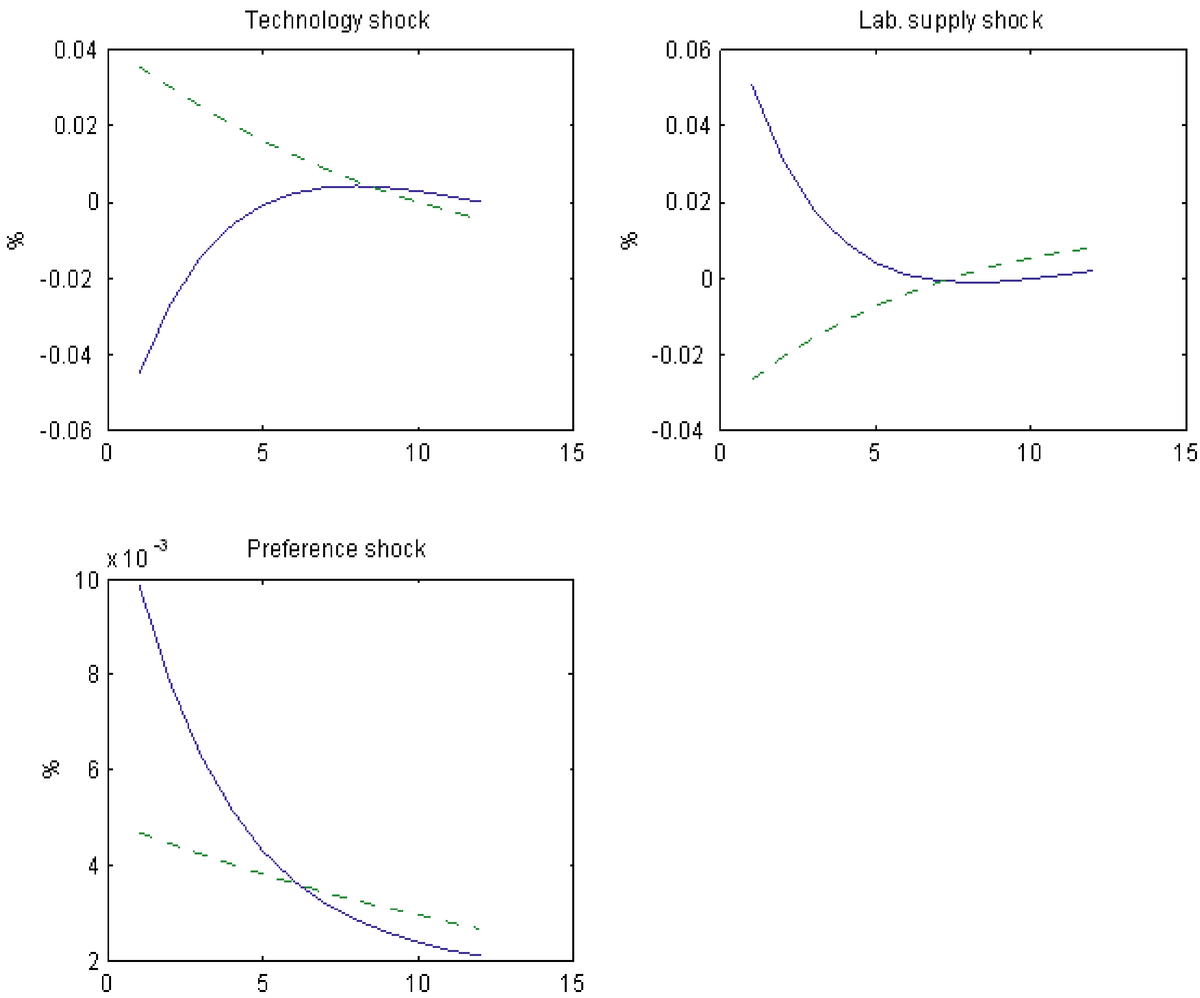

Figure 5: Impulse responses of the natural rate to various shocks. Solid line: model with credit frictions; dashed line: model without credit frictions.

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