Bayesian Multivariate Quantile Regression with alternative Time-varying Volatility Specifications

Matteo Iacopini[a] Francesco Ravazzolo[b,c] Luca Rossini[d]

[a]Queen Mary University of London, UK [b]BI Norvegian Business School, Norway
[c]Free University of Bozen-Bolzano, Italy [d]University of Milan, Italy

UNIVERSITÀ DEGLI STUDI DI MILANO

12th European Central Bank Conference on Forecasting Techniques

12-13 June 2023

Motivation

- Current global economic and financial situation caused by the COVID-19 pandemic and the Russia's invasion of Ukraine has renewed the interest of the economic forecasters' and policy institutions in tail risk.
- Increasing interest in understanding, modeling and forecasting the macroeconomic downside tail risk (see Adrian et al. (2019,2022)) and in quantifying the uncertainty around these predictions.
- Typically time series methods model the conditional mean of variable of interest, making them unsuited to capture features such as skewness, fat tails and outliers, that characterize economic and financial time series in turbulent periods.
- Quantile regression (QR) models (see Koenker and Bassett, 1978) have been exploited to study the heterogeneous impact of covariates on different quantile levels of a variable of interest.

Motivation

- Ferrara et al. (2022) introduce mixed-data sampling (MIDAS) to a Bayesian QR model to leverage on the information content of high-frequency financial conditions indicators
- Carriero et al. (2022) propose to nowcast tail risk to GDP growth by using Bayesian QR with mixed frequency and stochastic volatility. Pfarrhofer (2022) introduces time-varying parameters in QR to trace quantiles of inflation.
- Chavleishvilli and Maganelli (2021) propose quantile Vector Autoregressive (QVAR) to capture nonlinear relations among macroeconomics variables and define quantile impulse response function to perform stress tests.
- Iacopini et al. (2022) propose a novel asymmetric continuous probabilistic score for evaluating and comparing density forecast, which is useful when decision-maker has asymmetric preferences in the evaluation of forecasts.
- Iacopini et al. (2023) introduce a novel mixed-frequency QVAR which combines different frequencies in macroeconomic and financial variables to nowcast conditional quantiles of US GDP.

Our Contribution

- We propose two frameworks for modeling time-varying scale, which is a multiplicative component of the variance in multivariate quantile regression models by means of
  - Stochastic Volatility (SV) - parameter-driven specification
  - GARCH - observation-driven specification
- We define the likelihood of a QVAR model with time-varying volatility via the multivariate asymmetric Laplace (MAL) distribution.
- Coupling SV or GARCH effects with mixture of Gaussian representation of asymmetric Laplace distribution results in the standard deviation (no variance) affecting also the conditional mean, but differently from traditional SV- or GARCH-in-mean models, making previous algorithm inefficient.
- We reformulate the models to make possible the joint sampling of whole trajectory of time-varying volatility, independently along cross-sectional dimension.

Take home results

- We compare several univariate and multivariate quantile regression models with constant and alternative time-varying volatility specifications for forecasting different quantiles for several US macroeconomic and financial indicators
- The results show that the proposed methods beat the constant volatility QVAR benchmark for all the variables investigated.
- However no single specification is found to uniformly dominate the other over time, nor across variables or quantiles.
- We introduce model combinations based on quantile score weighting schemes to handle model uncertainty.
- The combination weights show significant variation over time, especially when quantiles corresponding to tails of the distribution are concerned, and at each point in time most of the mass is assigned a single model.
- QVAR time-varying combinations with time-varying weights perform accurately.

Q(VA)R with constant volatility

Multivariate quantile regression model

$$\mathbf{y}_t = B\mathbf{x}_t + \boldsymbol{\epsilon}_t, \qquad \boldsymbol{\epsilon}_t \sim \text{MAL}_n(\mathbf{0}, D\boldsymbol{\theta}_1, D\Theta_2\Psi\Theta_2 D), \tag{1}$$

where

- $\mathbf{y}_t$ and $\mathbf{x}_t$ are the n-dim vector of responses and the k-dim vector of common covariates;
- B is a $(n \times k)$ coefficient matrix;
- $\text{MAL}_n(\mu, \boldsymbol{\xi}, \Sigma)$ denotes the multivariate asymmetric Laplace distribution with location μ,skew parameter $\boldsymbol{\xi}$ and positive definite scale matrix Σ.

The parametrization of eq. (1) is such that $D = \text{diag}(\delta_1, \ldots, \delta_n)$ with $\delta_j > 0$, Ψ is a correlation matrix, and $\Theta_2 = \text{diag}(\boldsymbol{\theta}_2)$, with:

$$\theta_{1,j} = \frac{1 - 2\tau_j}{\tau_j(1-\tau_j)}, \qquad \theta_{2,j} = \sqrt{\frac{2}{\tau_j(1-\tau_j)}}, \qquad j = 1, \ldots, n, \tag{2}$$

where $\tau_j \in (0,1)$ is the (marginal) quantile of the jth series.

Q(VA)R with constant volatility

- Building on the mixture representation of the multivariate asymmetric Laplace distribution, one obtains:

$$\mathbf{y}_t = X_t \boldsymbol{\beta} + D\boldsymbol{\theta}_1 w_t + \sqrt{w_t} D \Theta_2 \Psi^{1/2} \mathbf{z}_t, \qquad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \mathcal{E}xp(1), \qquad (3)$$

  where $\boldsymbol{\beta} = \text{vec}(B) \in \mathrm{R}^{nk}$, $\boldsymbol{\theta}_1 = (\theta_{1,1}, \ldots, \theta_{1,n})$, and $X_t = (I_n \otimes \mathbf{x}_t)$.

- The multivariate QR includes the quantile VAR (QVAR) model as special case for $\mathbf{x}_t = \mathbf{y}_{t-1}$

- We assume homoskedastic variance for the conditional distribution of the response variable $\mathbf{y}_t$, which is highly restrictive when modeling economic and financial time series as they are typically characterized by highly persistent and clustered volatility

Q(VA)R with time-varying volatility

- Let denote $\Sigma = D\Psi D$ a positive definite matrix, where $D = \text{diag}(\Sigma_{11}^{1/2}, \ldots, \Sigma_{nn}^{1/2})$, then we assume heteroskedasticity:

$$\Sigma_t = AH_tA', \tag{4}$$

where H_t is a diagonal matrix with positive elements on the diagonal and A is a lower triangular matrix with 1 on the main diagonal.

- Recalling definition of D, one has $H_t^{1/2} = \text{diag}(\Sigma_{t,11}^{1/2}, \ldots, \Sigma_{t,nn}^{1/2}) = D_t$. Therefore introducing time-varying volatility in the scale matrix, Σ, the multivariate QR model with time-varying volatility is

$$\mathbf{y}_t = X_t\boldsymbol{\beta} + H_t^{1/2}\boldsymbol{\theta}_1 w_t + \sqrt{w_t}\Theta_2 A H_t^{1/2}\mathbf{z}_t, \quad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \mathcal{E}xp(1). \tag{5}$$

Parameter Driven: Q(VA)R-SV

- When dealing with conditional mean multivariate time series models, the inclusion of stochastic volatility leads to strong improvements with respect to constant volatility models.

- The quantile multivariate regression model with stochastic volatility (QR-SV) is defined as

$$
\begin{aligned}
\mathbf{y}_t &= X_t\boldsymbol{\beta} + D\boldsymbol{\theta}_1 w_t + \sqrt{w_t}D\Theta_2\Psi^{1/2}\mathbf{z}_t, \qquad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \mathcal{E}xp(1), \\
\Sigma_t &= AH_tA', \\
H_t &= \text{diag}\left(e^{h_{1,t}}, \ldots, e^{h_{n,t}}\right), \\
h_{j,t} &= \phi_j h_{j,t-1} + \epsilon^h_{j,t}, \qquad \epsilon^h_{j,t} \sim \mathcal{N}(0, \sigma^2_{h,j}),
\end{aligned}
$$

where $\left|\phi_j\right| < 1$ and $h_{j,1} \sim \mathcal{N}(0, \sigma^2_{h,j}/(1-\phi_j^2))$.

- By introducing lags of the response variable into the covariates, we obtain the QVAR-SV model.

Parameter Driven: Q(VA)R-SV

- It follows that introducing stochastic volatility in Σ results in a model that includes the square root of volatility terms, $e^{h_{i,t}/2}$, in the conditional mean equation for $\mathbf{y}_t$:

$$\mathbf{y}_t = X_t\boldsymbol{\beta} + w_t\Theta_1 e^{\mathbf{h}_t/2} + \sqrt{w_t}\Theta_2 A\,\mathrm{diag}(e^{\mathbf{h}_t/2})\mathbf{z}_t, \qquad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \mathcal{E}xp(1), \quad (6)$$

where $\mathbf{h}_t = (h_{1,t}, \ldots, h_{n,t})'$, $e^{\mathbf{h}_t/2} = (e^{h_{1,t}/2}, \ldots, e^{h_{n,t}/2})'$, and $\Theta_1 = \mathrm{diag}(\boldsymbol{\theta}_1)$.

- Conditional on w_t, it resembles a VAR with stochastic volatility in mean (VAR-SVM) model.

$\Longrightarrow$ The main difference is that VAR-SVM model includes vector of log-volatilities $\mathbf{h}_t$, whereas we have vector of square roots of volatilities, $e^{\mathbf{h}_t/2}$.

$\Longrightarrow$ We design a computationally efficient procedure for making inference on the log-volatility processes $\mathbf{h}_j = (h_{j,1}, \ldots, h_{j,T})'$ for each series $j = 1, \ldots, n$.

Computational advantages

- We remove for simplicity $X_t\beta$, and we can rearrange

$$\mathbf{y}_t = w_t \Theta_1 e^{\mathbf{h}_t/2} + \sqrt{w_t}\Theta_2 A \operatorname{diag}(e^{\mathbf{h}_t/2})\mathbf{z}_t, \qquad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \mathcal{E}xp(1)$$
$$\Longrightarrow \mathbf{y}_t = B_t e^{\mathbf{h}_t/2} + A_t \bar{\mathbf{z}}_t, \qquad \bar{\mathbf{z}}_t \sim \mathcal{N}_n(\mathbf{0}, H_t),$$

  where $B_t = w_t\Theta_1 \in \mathrm{R}^{n\times n}$ and $A_t = \sqrt{w_t}\Theta_2 A \in \mathrm{R}^{n\times n}$ are transformation of Θ_1 and Θ_2.

- After some computations, we obtain the likelihood for the vector $\mathbf{h}_j$

$$\widetilde{\mathbf{y}}_t^j = A_t^{-1}\mathbf{y}_t - \sum_{i\neq j} \widetilde{A}_{t,:i} e^{h_{i,t}/2}, \text{ where } \widetilde{A}_{t,:i} \text{ denotes } i\text{-th column of } \widetilde{A}_t = A_t^{-1} B_t$$

- We use an adaptive RW Metropolis-Hastings algorithm (with Gaussian proposal) to draw samples from posterior distribution of $\mathbf{h}_j$, which allows for computationally more efficient sampling of log-volatility

$\Longrightarrow$ We substitute a standard forward loop over time t, with a cycle over series j which can be easily parallelized. Thus replacing a step of $O(T)$ complexity with one of complexity $O(n)$.

Observation Driven: Q(VA)R-GARCH

- The quantile multivariate regression model with GARCH (QR-GARCH) is defined as

$$\begin{aligned}
&\mathbf{y}_t = X_t\boldsymbol{\beta} + D\boldsymbol{\theta}_1 w_t + \sqrt{w_t}D\Theta_2\Psi^{1/2}\mathbf{z}_t, \qquad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \mathcal{E}xp(1),\\
&\Sigma_t = AH_tA', \qquad H_t = \text{diag}\left(\sigma^2_{1,t}, \dots, \sigma^2_{n,t}\right),\\
&\sigma^2_{j,t} = \omega_j + \alpha_j\epsilon^2_{j,t-1} + \gamma_j\sigma^2_{j,t-1} = \omega_j + \alpha_j\left(y_{j,t-1} - X_{t-1}\boldsymbol{\beta} - w_{t-1}\theta_{1,j}\sigma_{j,t-1}\right)^2 + \gamma_j\sigma^2_{j,t-1},
\end{aligned}$$

  where parameters need to ensure stationarity: $\omega_j > 0$, $\alpha_j \geq 0$, $\gamma_j \geq 0$, and $(\alpha_j + \gamma_j) < 1$.

- It follows that introducing GARCH in Σ results in a model that includes the square root of volatility terms, $\sigma_{i,t}$, in the conditional mean equation for $\mathbf{y}_t$:

$$\mathbf{y}_t = X_t\boldsymbol{\beta} + w_t\Theta_1\boldsymbol{\sigma}_t + \sqrt{w_t}\Theta_2 A\,\text{diag}(\boldsymbol{\sigma}_t)\mathbf{z}_t, \text{ where } \boldsymbol{\sigma}_t = (\sigma_{1,t}, \dots, \sigma_{n,t})'$$

- Conditional on w_t, it resembles a VAR with GARCH in mean (VAR-GARCH-M) model.

$\Longrightarrow$ The main difference is that VAR-GARCH-M includes vector of volatilities $\boldsymbol{\sigma}^2_t$, whereas we have the vector of square roots of volatilities, $\boldsymbol{\sigma}_t$.

Bayesian Inference

- For vectorized matrix of coefficients and vector containing non zero elements of j-th row of the A matrix, $\bar{\mathbf{a}}_j$, we assume

$$\beta \sim \mathcal{N}_{nk}(\underline{\mu}_\beta, \underline{\Sigma}_\beta), \qquad \bar{\mathbf{a}}_j \sim \mathcal{N}_{j-1}(\underline{\mu}_{a,j}, \underline{\Sigma}_{a,j}), \qquad j = 2, \dots, n,$$

where hyperparameters are chosen such as to be noninformative.

- For QR-SV, prior for persistence parameter and innovation variance of log-volatility are

$$\left(\frac{1+\phi_j}{2}\right) \sim \mathcal{B}e(\underline{a}_\rho, \underline{b}_\rho), \qquad \sigma^2_{h,j} \sim \mathcal{IG}(\underline{a}_\sigma, \underline{b}_\sigma).$$

- For QR-GARCH, prior for log-transformation of parameters with stationarity condition

$$\log(\omega_j) \sim \mathcal{N}(\underline{\mu}_\omega, \underline{\sigma}^2_\omega)$$

$$\begin{pmatrix} \log(\alpha_j) \\ \log(\gamma_j) \end{pmatrix} \sim \mathcal{N}_2\left(\begin{pmatrix} \underline{\mu}_\alpha \\ \underline{\mu}_\gamma \end{pmatrix}, \begin{pmatrix} \underline{\sigma}^2_\alpha & 0 \\ 0 & \underline{\sigma}^2_\gamma \end{pmatrix}\right) \mathbb{I}(\alpha_j + \gamma_j < 1).$$

Evaluation: Quantile Score (QS)

- To assess the quality of quantile forecasts, we rely on Quantile score (QS, see Giacomini and Komunjer, 2005) as tail metric.

- The QS for model $k = 1, \ldots, K$, where K is total number of models estimated in forecasting exercise, at forecasting horizon $h = 1, \ldots, H$ and quantile τ, is defined as:

$$\mathrm{QS}_{k,\tau,t+h} = (\mathbf{y}_{t+h} - \hat{Q}_{k,\tau,t+h}) \odot (\tau - \mathbb{I}_{\{\mathbf{y}_t \leq \hat{Q}_{k,\tau,t+h}\}}),$$

  where

  - $\odot$ denotes Hadamard product,
  - $\mathbf{y}_{t+h}$ is observed value of vector response to be forecasted,
  - $\hat{Q}_{k,\tau,t+h}$ is forecast of quantile τ under model k,
  - $\mathbb{I}_{\{C\}}$ is vector-valued indicator function, whose jth element has value of 1 if outcome $y_{j,t+h}$ is at or below forecasted quantile $\hat{Q}_{j,k,tau,t+h}$ and 0 otherwise.

- Notice that better performances are associated to lower values of the QS.

Combination

We propose a combination of different models based on QS:

- Forecast combination based on time-varying weights (T-V):

$$Q^{c,tv}_{\tau,t+h} = \sum_{k=1}^{K} w_{k,\tau,t+h} \times QS_{k,\tau,t+h},$$

where weights of model k at horizon h and quantile τ are function of past performance of each model k known when the forecast is made

$$w_{k,\tau,t+h} = \frac{\sum_{t=t_o}^{t_i+t_o-h} QS^{-1}_{k,\tau,t}}{\sum_{j=1}^{K} \sum_{t=t_o}^{t_i+t_o-h} QS^{-1}_{j,\tau,t}},$$ where t_i = in-sample, t_o = out-of-sample length

- Forecast combination based on constant (average) weights (AVG):

$$Q^{c,avg}_{\tau,t+h} = \sum_{k=1}^{K} \bar{w}_{k,\tau} \times QS_{k,\tau,t+h},$$

where we use temporal average of weights, $\bar{w}_{k,\tau} = \frac{1}{t_o} \sum_{t=1}^{t_o} w_{k,\tau,t+h}$.

Data Description

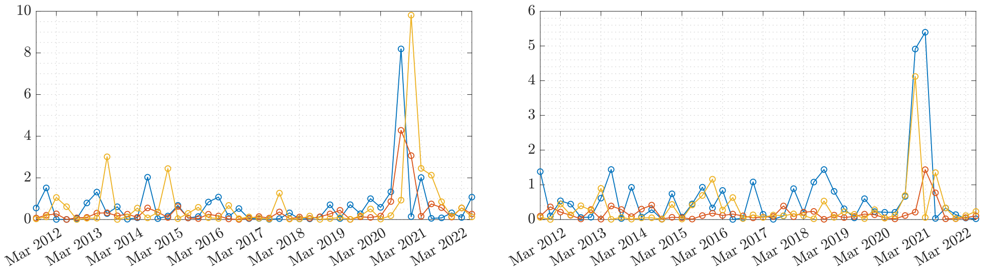
- Eight quarterly macroeconomic variables and one financial variable (NFCI) as in Iacopini et al. (2022):

Description	Fred Mnemonic	Transformation
Average Weekly Hours	AWHMAN	$0.1x_t$
CPI Inflation	CPIAUCSL	$100\Delta \ln(x_t)$
Industrial Production	INDPRO	$100\Delta \ln(x_t)$
S& P 500	S&P500	$100\Delta \ln(x_t)$
Federal Funds Rate	FEDFUNDS	Δx_t
10y Government Treasury yield	GS10	Δx_t
Unemployment Rate	UNRATE	Δx_t
Real Gross Domestic Product	GDPC1	$400\Delta \ln(x_t)$
Chicago Fed National Financial Condition Index	NFCI	Level

- In-sample analysis: 1971Q1-2022Q2.
- Out-of-sample analysis based on both rolling and expanding window of length 160 quarters (alias 40 years): 2011Q1-2022Q2

Quantile Score over-time for QVAR-SV for GDP (left) and NFCI (right)

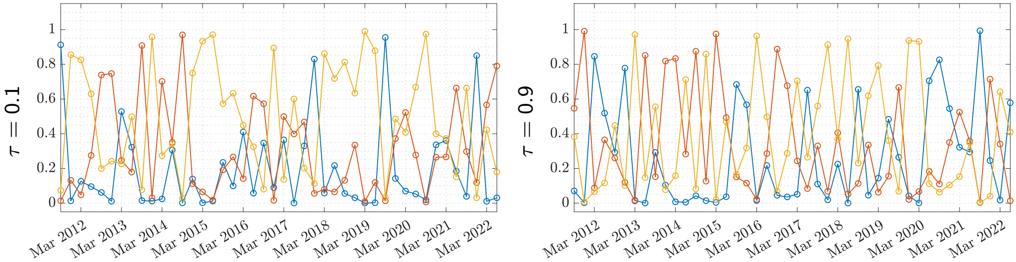
Different quantiles: $\tau = 0.1$ (blue); $\tau = 0.5$ (red) and $\tau = 0.9$ (yellow).



- QS for GDP at 90th pct peaks at 2014:Q4 in correspondence to drop of GDP from 5.0 to 2.5 percent $\Longrightarrow$ driven by upturn in imports + downturn in federal government spending
- Covid-19 pandemic: 10th pct worsened during 2021:Q4, while 90th occured in 2022:Q1 at outbreak of Russian-Ukraine war
- QS for NFCI has sinusoidal trajectory at 10th and 90th. Left tail peaks around 2013, 2016 and 2018 related to US debt-ceiling
- Conditional mean and median miss meaningful changes of macro & financial tail risks

T-V Combination weights for NFCI for $\tau = 0.1$ (left) and $\tau = 0.9$ (right)

Different models: QVAR-SV (red line); QVAR-GARCH (yellow) and QVAR (blue)



- QVAR model has almost zero weight for all percentiles
- Left quantile evidence QVAR-GARCH is best performing except between 2014–15, where QVAR-SV was the best
- Right quantile less persistent across time, where QVAR-GARCH best from 2018–22, while QVAR-SV best from 2014–15.

Table interpretations

- Report QS score for baseline Q(V)AR(1) model with constant volatility and Ratios between computed metric of the current model over baseline Q(V)AR model with expanding window.

$\Longrightarrow$ Entries of less than 1 indicate that given current model yields forecasts more accurate than those provided by baseline.

- Perform Diebold and Mariano (1995) t-test for equality of QS to compare predictions of alternative models with the benchmark (QVAR and QAR) if differences in forecast accuracy are significant

$\Longrightarrow$ $^{*},^{**},^{***}$ mean significance at $10\%, 5\%, 1\%$ levels.

- Perform the Model Confidence Set procedure Hansen et al (2011) to jointly compare the predictive power of all models

$\Longrightarrow$ Bold number indicate models that belong to Superior Set of Models delivered by the MCS at confidence level 10%.

Quantile Score for different variables and percentiles ($\tau = 0.1, 0.9$)

Variable	AWHMAN	CPIAUCSL	INDPRO	S&P500	FEDFUNDS	GS10	UNRATE	NFCI	GDPC1
Quantile: $\tau = 0.1$									
QVAR	1.699	1.128	1.598	1.806	1.855	2.078	1.901	1.497	1.793
QVAR-SV	**0.601*****	**0.568****	**0.627*****	**0.425*****	0.618***	**0.591*****	**0.963*****	**0.643*****	**0.643*****
QVAR-GARCH	**1.091**	**0.476*****	**0.932**	**0.523*****	**0.347*****	**0.570*****	**1.126***	**0.349*****	**0.974***
QVAR Combin (AVG)	1.136**	**0.538*****	**0.731*****	**0.512*****	**0.482*****	**0.574*****	**1.050****	**0.450*****	0.828***
QVAR Combin (T-V)	**0.360*****	1.100	**0.511*****	**0.415*****	**0.480*****	**0.463*****	**0.581*****	**0.591*****	**0.480*****
QAR	1.763	1.869	1.728	1.514	1.277	1.732	1.937	1.604	1.788
QAR-SV	1.721	2.261	1.643	1.933	1.950	1.864	1.979	2.315	1.836
QAR-GARCH	2.054	2.562	2.279	2.370	1.754	2.242	1.953	1.651	2.163
QAR Combin (AVG)	1.792	2.224	1.690	1.673	1.441	2.048	1.937	1.843	1.886
QAR Combin (T-V)	1.705	1.974	1.565	0.997***	0.905***	0.758***	**0.841*****	**0.657*****	**0.405*****
Quantile: $\tau = 0.9$									
QVAR	2.115	1.822	1.804	2.050	1.618	1.853	**1.881**	1.180	1.814
QVAR-SV	0.665***	0.488***	0.553***	0.754***	0.417***	0.526***	**0.616*****	0.344***	0.684***
QVAR-GARCH	**0.725*****	0.396***	0.696***	0.893***	0.564***	0.587***	**1.892**	0.564*	1.022*
QVAR Combin (AVG)	**0.717*****	1.492***	1.207***	0.759***	0.484***	0.515***	**1.364**	0.568**	0.792***
QVAR Combin (T-V)	**0.238*****	**0.187*****	**0.147*****	0.304***	**0.140*****	0.198***	**0.322*****	**0.111*****	**0.315*****
QAR	1.745	1.415	1.666	1.815	1.370	1.518	1.667	1.319	1.734
QAR-SV	1.964	1.911	1.980	1.911	1.502	1.792	1.407**	1.647	1.939
QAR-GARCH	2.205	1.099**	2.012	2.356	1.558	1.842	**1.608**	0.832***	1.751
QVAR Combin (AVG)	1.917	1.218**	1.915	1.906	1.466	1.696	1.519*	1.519	1.810
QVAR Combin (T-V)	**0.203*****	**0.225*****	**0.170*****	**0.115*****	**0.105*****	**0.127*****	**0.318*****	**0.077*****	**0.277*****

Conclusion

- We compare several univariate and multivariate quantile regression models with constant and alternative time-varying volatility specifications for forecasting different quantiles for several US macroeconomic and financial indicators
- The results show that the proposed methods beat the constant volatility QVAR benchmark for all the variables investigated.
- However no single specification is found to uniformly dominate the other over time, nor across variables or quantiles.
- We introduce model combinations based on quantile score weighting schemes to handle model uncertainty.
- QVAR time-varying combinations with time-varying weights perform accurately.

References

THANK YOU FOR THE ATTENTION

The talk was based on

- Iacopini, M., Ravazzolo, F. & Rossini, L. (2023) – Bayesian Multivariate Quantile Regression with alternative Time-varying Volatility Specifications, arXiv:2211.16121

Other project on Asymmetry/ Quantile regression

- Iacopini, M., Ravazzolo, F. & Rossini, L. (2022) – Proper Scoring Rules for Evaluating Density Forecasts with Asymmetric Loss Functions, *Journal of Business and Economic Statistics*, 41(2), 482-496
- Iacopini, M., Poon, A., Rossini, L. and Zhu, D. (2023) – Bayesian Mixed-Frequency Quantile Vector Autoregression: Eliciting tail risks of Monthly US GDP, arXiv:2209.01910
- Iacopini, M., Poon, A., Rossini, L. and Zhu, D. (2023) – Quantile Responsiveness, Work in progress