EUROPEAN CENTRAL BANK

EUROSYSTEM

Working Paper Series

Angela Abbate, Dominik Thaler

Optimal monetary policy with the risk-taking channel

No 2772 / February 2023

Abstract

Empirical research suggests that lower interest rates induce banks to take higher risks. We assess analytically what this risk-taking channel implies for optimal monetary policy in a tractable New Keynesian model. We show that this channel creates a motive for the planner to stabilize the real rate. This objective conflicts with the standard inflation stabilization objective. Optimal policy thus tolerates more inflation volatility. An inertial Taylor-type reaction function becomes optimal. We then quantify the significance of the risk-taking channel for monetary policy in an estimated medium-scale extension of the model. Ignoring the channel when designing policy entails non-negligible welfare costs (0.7% lifetime consumption equivalent).

Keywords: Risk-taking channel, Optimal monetary policy, Inertial policy rate

JEL classification: E44, E52

Non-technical summary

A broad empirical literature documents that lower interest rates induce banks to make riskier investments, a mechanism known as the risk-taking channel of monetary policy. Furthermore, evidence suggests that this additional risk taking may be economically inefficient. This raises the question of whether the central bank should take the risk-taking channel into account when setting the policy rate, and how.

To address this question, we add a simple model of bank risk taking to the workhorse model of monetary macroeconomics, the New Keynesian model. Financial frictions distort banks' incentives, leading banks to choose excessively risky investments. When real rates drop, these distortions become more important and risk taking increases. On an aggregate level, this implies that investments become less efficient and aggregate productivity drops when real interest rates fall.

We analyse optimal monetary policy in this context. Ideally, the central bank would like to raise the level of the policy rate systematically, in order to incentivise safer and more efficient investment choices. However, this is not possible: expectations would adjust, leading to higher inflation and an unchanged real interest rate. But this doesn't mean that monetary policy can't do anything about risk taking at all. What the central bank can do, is to keep the real interest rates stable. In the model, this reduces fluctuations in risk taking and actually raises average productivity. Thus, stabilisation of the real rate emerges as an additional policy objective.

How can the central bank achieve this new objective? The optimal interest rate policy with a risk-taking channel calls for (i) less strong responses to inflation and for (ii) adjusting the policy rate less abruptly but more persistently to changes in economic conditions. As a result, the central bank delivers a more stable real interest rate relative to a standard macro model. This however comes at the cost of more fluctuations in inflation. Thus, the new objective of real interest rate stabilisation conflicts with the traditional central bank objective of inflation and stabilisation.

These theoretical insights are developed in a very stylised model. To quantify their relevance, we extend the model to a quantitatively plausible medium-scale model, estimated on US data. We find that the risk-taking channel is economically significant for optimal monetary policy. Accounting for the risk-taking channel when designing optimal policy yields a markedly different optimal policy rule and delivers significantly different macroeconomic outcomes: Inflation is about 50% more volatile, but the real rate about 50% less volatile. We quantify the loss of social welfare associated to ignoring the risk-taking channel to be equivalent to a loss of between 0.5% and 1% of aggregate household consumption.

1 Introduction

The risk-taking channel of monetary policy – the mechanism by which lower interest rates encourage banks to take on additional risk – is a well-established empirical regularity. Studies have shown that this channel was active both before and after the 2008 financial crisis.[1] Despite policymakers' awareness of the risk-taking channel, its normative implications for monetary policy remain to be determined.[2] Should central banks consider their influence on bank risk taking when setting their policy rates, and if so, how?

We explore these questions in two steps. First, we embed a tractable model of bank risk taking into the textbook New Keynesian model (NKM), and analytically characterize optimal monetary policy under a linear-quadratic approximation. Second, we embed the same model of bank risk taking into a larger New Keynesian DSGE model estimated on US data, and use it to explore the quantitative importance of the risk-taking channel for optimal monetary policy. We show analytically that the risk-taking channel provides an incentive for the central bank to minimize the volatility of the real interest rate, conflicting with the standard New Keynesian policy prescription to minimize inflation volatility. Hence, the risk-taking channel introduces a new trade-off for the central bank. We find this new trade-off to be quantitatively significant in the large model: Ignoring the risk-taking channel when designing optimal monetary policy entails welfare costs of approximately 0.7% of lifetime consumption equivalent.

To derive the analytical conclusions, we set up a simple NKM with financial intermediation and a bank risk-taking channel. Firms must borrow in advance to finance production, as in Ravenna and Walsh (2006). Firms' technologies are risky and differ in their risk-return characteristics. Banks provide the necessary external financing to firms by funding themselves through equity and deposits, and they choose the riskiness of the firm in which they invest. As in Dell'Ariccia et al. (2014), frictions in the banking system – limited liability, the unobservability of risk taking, and an equity premium – cause a risk shifting problem. Banks choose to lend to excessively risky firms, in the sense that a reduction in risk would increase the expected social return on their investment. The level of the real interest rate influences the degree of risk taking: Lower real interest rates induce banks to choose even riskier

[1]E.g. Maddaloni and Peydro (2011), Buch et al. (2014), Ioannidou et al. (2014), Jimenez et al. (2014), Heider et al. (2019), Bubeck et al. (2020).

[2]E.g. the risk-taking channel was acknowledged in the ECB's 2021 strategy review (ECB 2021).

investments, thus giving rise to the risk-taking channel of monetary policy.

How can monetary policy reduce the effects of bank risk taking on welfare? The average efficiency of banks' investments depends not only on the *level* of the real interest rate, which cannot be influenced by the central bank in the long run, but also on its *volatility*. In particular, by reducing the volatility of the real interest rate, the central bank can increase the average efficiency of investment. The risk-taking channel thus constitutes a motive for the central bank to stabilize the real interest rate around its policy-independent average level.

In linearized form, the model boils down to a modification of the textbook three-equation NKM, which allows us to characterize optimal monetary policy analytically. We derive four key results. If the risk-taking channel is active, (i) welfare depends not only on output gap and inflation volatility, as in the standard NKM, but also on the volatility of the real interest rate; (ii) it is optimal for the central bank to respond less to inflation fluctuations; (iii) optimal policy implies less real interest rate volatility, but greater inflation volatility; (iv) the implicit instrument rule that implements Ramsey-optimal policy features inertia in the policy rate. The risk-taking channel thus provides a novel explanation for interest rate inertia, which is observed empirically and routinely built into models.

To quantify the importance of the risk-taking channel for optimal monetary policy, we use the medium-scale DSGE model of Abbate and Thaler (2019). The latter embeds the same banking-sector model described above in an otherwise standard NKM à la Smets and Wouters (2007). It is estimated on US data, and thus provides a better laboratory for quantitative analysis. We numerically determine optimal simple policy rules, and derive four results that quantify the four theoretical results above. When the risk-taking channel is active, (i) the central bank accepts approximately 50% more inflation volatility; (ii) the optimal Taylor rule features a significantly lower response to inflation and (iii) an autoregressive coefficient of approximately 1. Importantly, (iv) the welfare gains of considering the risk-taking channel when designing optimal monetary policy are significant, and amount to around 0.7% of lifetime consumption equivalent. This contrasts findings in the literature that other types of financial frictions do not affect optimal monetary policy significantly (e.g. Bernanke and Gertler, 2001 or De Fiore and Tristani, 2013).

Our model builds on an extensive empirical literature on the risk-taking channel, which finds that low interest rates increase the riskiness of banks' new investments.[3]

[3]See, for instance, Maddaloni and Peydro (2011), Buch et al. (2014), Ioannidou et al. (2014),

Buch et al. (2014) and Ioannidou et al. (2014) moreover find that banks do not offset higher risk with a sufficiently large increase in the risk premium. Risk taking is thus inefficient, as in our model.

The main contribution of this paper is twofold. First, we add *normative* conclusions for monetary policy to a so far largely *positive* theoretical literature on the risk-taking channel.[4] The latter models different versions of the risk-taking channel, with risk originating either on the liabilities side of banks' balance sheets (via the leverage choice, as in Gertler et al., 2012, de Groot, 2014, Angeloni and Faia, 2013 and Angeloni et al., 2015) or on the asset side (as in Christensen et al. 2011, Collard et al. 2017, Abbate and Thaler, 2019 and Afanasyeva and Guentner, 2020). An important exception is Martinez-Miera and Repullo (2019), who provide a simple two-period macro model in which banks' asset risk taking is modeled and affected by monetary policy in a very similar way to our paper. Since risk taking is excessive from a social point of view, the authors argue that this mechanism constitutes a motive for the central bank to increase the real interest rate through monetary policy. Because we consider monetary policy in the long run under rational expectations, our message contrasts with theirs. We argue that monetary policy cannot systematically raise the real interest rate due to monetary neutrality, but can still affect risk taking by influencing the volatility of the real interest rate.[5]

The second contribution is to the normative literature on interest rate inertia. While well documented empirically and a standard feature in many monetary models, it is theoretically not straightforward that an inertial Taylor rule is optimal for the social planner. By showing that the risk-taking channel adds an interest rate variation term to the welfare function and leads to interest rate inertia under Ramsey policy, we provide a novel theory that can explain inertial interest rate policy. In doing so, we complement other explanations for why interest rate volatility matters for welfare, such as the zero lower bound or transaction frictions (Woodford, 2003).[6]

This normative paper and in particular its last section build on our companion pa-

Jimenez et al. (2014), Bubeck et al. (2020) and Heider et al. (2019)

[4]The risk-taking channel has also important normative implications for regulatory policy, from which we abstract for simplicity. We view this choice as a shortcut to model the realistic fact that regulation might not be completely effective at muting the risk-taking channel in practice.

[5]We conjecture that our intuition might also apply to Martinez-Miera and Repullo (2019) in the long run. Since the welfare function is concave in the real rate, a mean preserving increase in the volatility of the real rate is detrimental to average welfare.

[6]Note that, while in our case it is the *real* interest rate and not the *nominal* one that appears in the welfare function, the effects are similar.

per Abbate and Thaler (2019). The latter is a purely positive paper, which finds that adding the risk-taking channel to an otherwise standard medium-scale NKM improves the fit on US macroeconomic time series, generates a path of risk taking that matches survey evidence on the risk of new loans and produces procyclical bank leverage, as documented by Adrian and Shin (2014). By contrast, the focus of the present paper is purely normative. Here we set out our theory of the risk-taking channel in a much simpler, tractable model, which allows us to derive policy conclusions analytically. Only in a last step, we quantify the importance of the analytical conclusions using the estimated medium-scale model of Abbate and Thaler (2019). As the two papers have been developed jointly, they reference each other.

The paper proceeds as follows: In section 2 we set up a simple NKM with the risk-taking channel, which we then use in section 3 to explore optimal policy analytically. In section 4, we quantify the importance of the risk-taking channel for optimal policy numerically based on the medium scale model. Section 5 concludes.

2 A simple New Keynesian model of the bank risk-taking channel

In this section, we set up a simple NKM with financial intermediation and a bank risk-taking channel. We build on Ravenna and Walsh's (2006) model of the cost channel, where input good producers need to borrow in advance to finance production. This is a tractable way to introduce intermediation into the textbook three-equation NKM. We then add the risk-taking channel modeled as in Abbate and Thaler (2019), who build on Dell'Ariccia et al. (2014): the production technology of input good producers is risky and banks choose the riskiness of the producer they lend to. The economy is populated by eight types of agents: Households, input, intermediate and final good producers, equity and deposit funds, private banks, and a central bank. We discuss these agents in turn.

2.1 Households

Households choose consumption C_t and working hours N_t in order to maximize:

$$U_t = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] .$$

Timing is as follows: Households enter period t with nominal money holdings M_{t-1}. They then receive the wage income $W_t N_t$ in cash as well as a lump-sum cash injection X_t from the central bank. They then use this cash to invest into equity and deposit funds D_t and E_t, and to buy the consumption good C_t, which has to be paid in advance. Hence, consumption is subject to the cash-in-advance (CIA) constraint:

$$P_t C_t \leq M_{t-1} + W_t N_t - D_t - E_t + X_t \ . \tag{1}$$

At the end of the period, households work and consume the previously chosen quantities N_t and C_t. The equity and deposit funds return the safe nominal (gross) rates R_t^e and R_t^d. Furthermore, households receives a lump sum profit payment Π_t. Hence, cash holdings M_t at the end of the period are:

$$M_t = M_{t-1} + W_t N_t - D_t - E_t - P_t C_t + R_t^d D_t + R_t^e E_t + \Pi_t + X_t \ . \tag{2}$$

Utility maximization implies that the two safe interest rates are the same, so that we can simply refer to the safe rate as R_t: $R_t^e = R_t^d \equiv R_t$. Utility maximization also yields the usual labor supply and Euler equations and implies that the CIA constraint (1) must hold with equality given positive nominal rates.

2.2 Input good producers

There is a continuum of ex-ante identical input good producers indexed by m, who hire labor N^m to produce the input good z_t^m using a risky production technology. Each input producer has access to a continuum of technologies with different risk-return characteristics indexed by $q^m \in [0, 1]$. Given a certain technology q_t^m, the output of producer m is:

$$Z_t^m = \begin{cases} \left(\omega_1 - \frac{\omega_2}{2} q_t^m\right) N_t^m & \text{with probablity } q_t^m \\ 0 & \text{else} \end{cases}$$

Input producers need to pre-pay the wage bill $W_t N_t^m$ at the beginning of the period, but only produce at the end of the period. They therefore need to borrow from the bank in order to finance the wage bill. They promise to repay the loan after production at the gross nominal loan rate $r_{l,t}$ and let the bank choose the riskiness of their technology q_t. If production is successful, the producer sells the input good at price $P_{in,t}$ and repays the loan. If production is not successful, he defaults.

Input producers choose the scale of production to maximise their profits. Due to price taking and the linearity of the production technology in N_t, profit maximization implies that they pass all their revenues on to the bank and make zero profits:

$$P_{in,t} = r_{l,t} W_t / \left(\omega_1 - \frac{\omega_2}{2} q_t^m \right) \ . \tag{3}$$

Assuming that the production outcomes are independent across producers,[7] and given that in equilibrium all producers use the same technology q_t, the quantity of input goods produced in equilibrium is given by:

$$Z_t = \underbrace{q_t \left(\omega_1 - \frac{\omega_2}{2} q_t \right)}_{\mathsf{f}(q_t)} N_t \ . \tag{4}$$

where $Z_t \equiv \int_{m=0}^{1} Z^m dm$, $N_t \equiv \int_{m=0}^{1} N^m dm$, and where $\mathsf{f}(q_t)$ denotes the expected productivity of input good producers.

2.3 Final and intermediate good producers

Final and intermediate good producers are standard. A representative final good producer aggregates intermediate good varieties Y^i to produce the final consumption good Y according to the CES aggregator:

$$Y_t = \left[\int_0^1 \left(Y^i \right)^{(\theta_t - 1)/\theta_t} di \right]^{\theta_t/(\theta_t - 1)} \ .$$

There is a continuum of intermediate good producers indexed by i. They linearly transform input goods into differentiated intermediate varieties: $Y^i = A_t Z_t^i$ where A_t is total factor productivity. They purchase input goods at price $P_{in,t}$ and receive a proportional subsidy τ_t, financed by lump sum taxes. Hence, their nominal marginal cost is given by $MC_t = P_{in,t}(1 - \tau_t)/A_t$. Since we are not interested in the effect of the cost channel, we set the subsidy such that its effect is muted $\tau_t = 1 - (q_t r_{l,t})^{-1}$.[8]

[7] This simplifying assumption rules out non-diversifiable systemic risk.

[8] The cost channel only serves us to tractably introduce intermediation into the 3-equation NKM. Its implications for optimal policy are discussed in Ravenna and Walsh (2006). The subsidy is not relevant for the approximation of welfare in section 3.1, but it simplifies the Phillips curve and, as a result, the derivation of optimal monetary policy. We do not impose the subsidy in section 4, and have verified numerically that it does not affect the results reported there significantly.

Combining this assumption with (3), marginal costs are given by:

$$MC_t = \frac{W_t}{A_t q_t \left(\omega_1 - \frac{\omega_2}{2} q_t\right)} = \frac{W_t}{A_t \mathsf{f}(q_t)} \ . \tag{5}$$

This expression of marginal costs differs from its counterpart in the basic NKM only in the term $\mathsf{f}(q_t)$. Idiosyncratic risk at firm level implies that one unit of labor is transformed into $\mathsf{f}(q_t)$ units of input goods on aggregate. The higher the productivity of inputs producers, the lower the marginal costs of intermediate firms. We discuss the implication of this difference in section 2.7.

Intermediate goods producers operate under monopolistic competition and Calvo pricing, which leads to the standard dynamics of aggregate prices and price dispersion reported in Appendix A2. Profits are rebated lump sum.

2.4 Deposit and equity funds

Deposit and equity funds sell fund shares D_t and E_t to the household at the beginning of the period.[9] They invest the proceeds into deposits D_t^b and equity E_t^b issued by a continuum of banks indexed by b: $D_t = \int_0^1 D_t^b db$ and $E_t = \int_0^1 E_t^b db$. Each bank promises to pay the gross nominal deposit and equity rates r_d^b and r_e^b at the end of the period. However, the bank may be hit by an i.i.d. default shock, occurring with probability q_t, in which case neither deposits nor equity is repaid.

The deposit fund is a frictionless pass-on vehicle. Diversifying across all banks its nominal return is:

$$R_{d,t} = q_t r_{d,t} \ . \tag{6}$$

The equity fund functions similarly but, as a simple way to introduce an equity premium, we assume the equity fund manager needs to be paid a (real) premium ξ per unit of funds under management to incentivize him to act in the best interest of equity providers. This premium is rebated to the household lump-sum. The nominal return on the equity fund is hence given by the average return on bank equity minus the premium:

$$R_{e,t} = q_t r_{e,t} - \xi \mathbb{E}_t \pi_{t+1} \ . \tag{7}$$

[9] These funds serve only to simplify exposition. Equivalently, we could assume that the household perfectly diversifies its deposits and equity across banks, or that there is perfect risk sharing among a continuum of households, each interacting with one bank.

Since both funds are perfectly diversified, their returns are risk free. Hence, the households' FOCs imply that the returns *on fund shares* are equated ($R_{e,t} = R_{d,t} \equiv R_t$). Nevertheless, the costs of deposit and equity financing *for banks* differ from each other ($r_{d,t} < r_{e,t}$), due to the equity premium ξ. The latter invalidates the Modigliani-Miller irrelevance principle and plays an important role in delivering the risk-taking channel, as we discuss next.[10]

2.5 Banks

Banks finance themselves through deposits and equity, and invest these funds into risky assets. We show in this section that the bank risk choice has implications for the allocative efficiency of the economy, and therefore bears implications for monetary policy. The modeling of the banks follows Abbate and Thaler (2019), who build on Dell'Ariccia et al. (2014), and involves three key assumptions: (i) Unobservability of the bank's risk choice and (ii) limited liability of the bank, which give rise to an agency problem between depositors and equity providers, and (iii) the cost advantage of deposits over equity introduced in the previous subsection.

There is a continuum of ex-ante identical competitive banks (for convenience we omit the bank's index b in this subsection). Banks live for one period. At the beginning of the period, each bank raises deposits D_t and equity E_t from the respective funds, and lends these resources to one particular input good producer at a promised nominal (gross) rate $r_{l,t}$. When lending to an input producer, the bank chooses the risk characteristic q_t of the technology employed by the producer.[11] Depositors cannot observe this risk choice. Hence deposit contracts cannot be made contingent upon the bank's risk choice, and the bank cannot credibly commit to a certain risk choice. At the end of the period, if the input good producer is successful, which happens with probability q_t, the bank receives $r_{l,t}(E_t + D_t)$. It then repays deposits and equity at the promised nominal (gross) rates $r_{d,t}$ and $r_{e,t}$. With probability $1 - q_t$ production fails, and the loan is not repaid. In this case, equity providers and depositors (because of limited liability) receive nothing.

The bank maximizes excess profits, i.e. the expected return of equity providers net

[10] This simple way of modeling the equity premium, which can be reinterpreted as transaction costs of equity or a convenience yield of deposits, is common, e.g. Allen et al. (2011) or Hellmann et al. (2000).

[11] The choice of q_t may be reinterpreted both as picking among borrowers of different risk levels, or as monitoring the borrower so as to make repayment more likely. We abstract from any agency problem between banks and firms.

of the user cost of equity. The objective function in nominal terms is:

$$q_t \left\{ r_{l,t} - r_{d,t} \frac{D_t}{E_t + D_t} - r_{e,t} \frac{E_t}{E_t + D_t} \right\} (E_t + D_t) \ .$$

When choosing the riskiness of its investment q_t, the bank understands the risk return trade-off implied by the input good producer's optimality condition (3). We can hence substitute $r_{l,t}$ in the above expression. Furthermore, define the equity ratio as $k_t \equiv E_t / (E_t + D_t)$ and the total balance sheet size by $o_t \equiv D_t + E_t$ and divide everything by expected inflation to obtain:

$$\mathbb{E}_t \left[\frac{1}{\pi_{t+1}} \left\{ \left(\omega_1 q_t - \frac{\omega_2}{2} q_t^2 \right) \frac{P_{in,t}}{W_t} - q_t r_{d,t} (1 - k_t) - q_t r_{e,t} k_t \right\} o_t \right] \ .$$

To simplify notation, we rewrite the objective function in real variables using the following definitions: $v_t^r = \mathbb{E}_t \left[P_{in,t} / (\pi_{t+1} W_t) \right]$, $r_{d,t}^r = \mathbb{E}_t \left[r_{d,t} / \pi_{t+1} \right]$, $r_{e,t}^r = \mathbb{E}_t \left[r_{e,t} / \pi_{t+1} \right]$, $R_t^r = \mathbb{E}_t \left[R_t / \pi_{t+1} \right]$ The objective function can be reexpressed as:

$$\left\{ \left(\omega_1 q_t - \frac{\omega_2}{2} q_t^2 \right) v_t^r - q_t r_{d,t}^r (1 - k_t) - q_t r_{e,t}^r k_t \right\} o_t \ .$$

Given the agency problem between depositors and equity providers, it is convenient to think about the bank's problem as a two-stage problem. At stage 1, the bank chooses the scale of its balance sheet and the capital structure and depositors price deposits. At stage 2, once the balance sheet structure and the deposit rate have been fixed, the bank chooses the risk level q_t. Crucially, given the unobservability of the risk choice, at stage 2 the deposit rate is taken as given. We now set out the bank's recursive problem, see Appendix A1 for a more detailed discussion.

At **stage 2**, the bank has already raised $E_t + D_t$ funds and now needs to choose the riskiness of its investment q_t. As already mentioned, we assume that the bank cannot write contracts conditional on q_t with the depositors at stage one. Therefore, at the second stage the bank takes the deposit rate as given. Furthermore, since the capital structure is already determined, maximizing excess profits coincides with maximizing the gross return on equity. The second stage problem is thus:

$$\max_{q_t} \underbrace{(\omega_1 q_t - \frac{\omega_2}{2} q_t^2) v_t^r - q_t r_{d,t}^r (1 - k_t)}_{V(q_t | v_t^r, r_{d,t}^r, k_t)} \ .$$

At **stage 1**, the bank chooses the capital structure k_t and the balance sheet size

o_t to maximize expected excess profits, subject to the participation constraints (i.e. the funding supply schedules) for depositors and equity providers. Since agents have rational expectations, everyone correctly infers the level of risk q_t that will be chosen by the bank at the second stage as a function of k_t, $r^r_{d,t}$ and v^r_t. The first stage problem is thus:

$$\max_{k_t, o_t, q_t, r^r_{d,t}, r^r_{e,t}} \; o_t \left\{ v^r_t \left(q_t \omega_1 - \frac{\omega_2}{2} q_t^2 \right) - q_t r^r_{d,t} (1 - k_t) - q_t r^r_{e,t} k_t \right\}$$

$$\text{s.t.} \;\; r^r_{d,t} = \frac{R^r_t}{q_t} \;\; \text{and} \;\; r^r_{e,t} = \frac{R^r_t + \xi}{q_t} \;\; \text{and} \;\; q_t = \text{argmax}_{q_t} V(q_t \mid v^r_t, r^r_{d,t}, k_t)$$

Solving the bank's recursive problem – see Appendix A1 for the algebra – we derive the bank's risk choice q_t as a function of the safe real interest rate in closed form:

$$q_t = \frac{\omega_1 (\xi + R^r_t)}{\omega_2 (2\xi + R^r_t)} \; . \tag{8}$$

Equation (8) describes the representative bank's risk choice when the financial sector is in equilibrium.[12] Equilibrium risk taking has four important properties:

PROPOSITION 1: Let q_t denote the risk choice of the bank in equilibrium for a given expected real rate R^r_t and assume this choice is interior. Recall the definition of the expected productivity of the input producer $\mathsf{f}(q_t) \equiv \left(\omega_1 - \frac{\omega_2}{2} q_t\right) q_t$. Then:

(1) Risk decreases in the real interest rate: $\frac{\partial q_t}{\partial R^r_t} > 0$.

(2) Risk taking is excessive: $q_t < \text{argmax} \; \mathsf{f}(q_t)$.

(3) Expected productivity increases in the real interest rate: $\frac{\partial \mathsf{f}(q_t)}{\partial R^r_t} > 0$.

(4) Expected productivity is concave in the real interest rate: $\frac{\partial^2 \mathsf{f}(q_t)}{\partial \left(R^r_t\right)^2} < 0$.

Part 1 of the proposition states that a decline in the real risk-free rate R^r_t induces banks to invest into riskier projects (q_t falls). This is the risk-taking channel. What is the intuition behind it? A reduction in the real risk-free rate increases the cost advantage of deposits, by making the equity premium a more important component of the cost of equity, in relative terms. Thus, banks have a stronger incentive to rely

[12] The financial sector is said to be in equilibrium when banks and input good producers solve their optimization problem and households optimally allocate their savings in deposits and equity.

on cheaper deposits and lever up. This in turn induces them to take more risk.[13] This first result contrasts some of the literature on bank competition and risk taking, which finds that higher deposit rates strengthen risk taking incentives because margins are eroded (e.g. Hellmann et al. 2000). In our model, the effect on margins is muted. Perfect competition among banks implies that changes in the risk-free rate are passed on to deposits, equity and loan rates.

Part 2 states that the bank's risk choice is excessive (i.e. suboptimally high), in the sense that expected productivity would increase if the bank chose a safer investment. The inefficiency of the risk choice results from both the agency problem between depositors and equity providers and the cost advantage of deposits. In the absence of these frictions, q_t would be chosen to maximize expected productivity $(\omega_1 - \frac{\omega_2}{2} q_t) q_t$ and would thus be given by $q^o = \frac{\omega_1}{\omega_2}$. The frictions drive a wedge between the optimal risk level q^o and the level that is actually chosen q_t:

$$q_t = q^o \frac{\xi + R_t^r}{2\xi + R_t^r} \, .$$

This wedge is smaller than one, i.e. banks choose excessive risk. Furthermore, it increases in R_t^r. Thus, risk taking gets more excessive as the real rate falls, implying a lower expected productivity $\mathsf{f}(q_t)$ as stated in part 3.

Finally, part 4 states that the effect of R_t^r on expected productivity $\mathsf{f}(q_t)$ decreases in R_t^r. That is, $\mathsf{f}(q_t(R_t^r))$ is concave. This result will be crucial for optimal policy.

2.6 Central bank

To close the model, the central bank needs to set the nominal interest rate according to some criterion and adjust the money supply accordingly. We leave this criterion unspecified for now.

2.7 Comparison to the three-equation New Keynesian model

We have embedded the risk-taking channel into the textbook NKM. This addition ends up altering only two equations: The definitions of aggregate output and of marginal costs. We discuss them in turn. The other equations remain unaltered, and we report them in Appendix A2.

[13] For this result, it is crucial that the equity premium is constant in absolute terms. As Abbate and Thaler (2019) argue, this assumption is both common in the theoretical literature as well as empirically plausible.

First, using equation (4) and aggregating across the 3 types of producers, aggregate output is given by:

$$Y_t = \frac{A_t \left(\omega_1 - \frac{\omega_2}{2} q_t\right) q_t}{\Delta_t} N_t = \frac{A_t \mathsf{f}(q_t)}{\Delta_t} N_t \; . \tag{9}$$

Because of the risk-taking channel, aggregate output is not only a function of labor N_t, exogenous total factor productivity A_t and price dispersion Δ_t, as in the textbook NKM, but also of the average productivity of the input production technology $\mathsf{f}(q_t)$. This new term is a function of the real interest rate by equation (8):

$$\mathsf{f}(q_t) = f\left(R_t^r\right) = \frac{\omega_1^2}{\omega_2} \frac{\xi + R_t^r}{2\xi + R_t^r} - \frac{\omega_1^2}{2\omega_2} \left(\frac{\xi + R_t^r}{2\xi + R_t^r}\right)^2 \; . \tag{10}$$

Aggregate productivity is hence made up of two components, an exogenous one A_t, and an endogenous one $\mathsf{f}(q_t)$. From proposition 1 we know that, because of frictions in the banking sector, $\mathsf{f}(q_t)$ is inefficiently low and increases with the real interest rate. This implies a wedge between the actual and the efficient level of output, which increases as the real interest rate falls.

Second, marginal costs (given by equation 5) are also affected by the risk-taking channel, via its effect on aggregate productivity. By the arguments just discussed, marginal costs are excessively high and increase as the real interest rate falls.

After linearization, the model condenses to an IS and a Phillips curve, which together with a policy rule for the nominal interest rate, define the three-equation NKM. While the IS curve is the same as in the textbook model, the risk-taking channel shows up in the Phillips curve via marginal costs (see Appendix B2):

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa\left(\sigma + \varphi\right) \hat{x}_t - \kappa\left(1 + \varphi\right) \mathcal{R}_1 \hat{R}_t^r + u_t \; . \tag{11}$$

Here $\kappa \equiv (1-\omega)(1-\beta\omega)/\omega$, $\hat{x}_t$ is the log of the welfare relevant output gap (with respect to the efficient level of output), $\hat{R}_t^r$ is the expected real rate, both in deviation from the steady state, and u_t is a cost-push shock driven by θ_t. Furthermore, $\mathcal{R}_1$ is a positive coefficient given by $\mathcal{R}_1 = \frac{\partial f(R^r)/\partial R^r}{f(R^r)}$, where $f(R^r)$ denotes the steady state of equation (10).

To summarize, our model boils down to an extension of the textbook NKM, where aggregate productivity has an endogenous component that is a positive, concave function of the real interest rate.[14]

[14] Our model collapses to the standard NKM when either of the financial frictions (non-

3 Optimal monetary policy in the simple model

To understand the impact of the risk-taking channel on optimal policy, we first derive a second-order approximation of the planner's welfare function, and then use it to derive optimal policy.

3.1 The central bank's problem

The planner maximizes household utility. Because our main focus is on stabilization policies, we follow the literature in assuming that time-invariant subsidies are in place such that the steady state is efficient. This eliminates the steady-state markup and the steady-state inefficiency in risk taking. We later relax this assumption, both in an analytical extension and in the numerical analysis. Under the assumption of an undistorted steady state, a second-order approximation to consumer welfare leads to the following social loss function, as we show in Appendix B1:

$$\mathbb{W} = \frac{1}{2}\mathbb{E}_{\mathrm{t}} \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2 + \frac{\kappa}{\theta} \mathcal{R}_2 \left(\hat{R}_t^r \right)^2 \right] \right\} . \tag{12}$$

In this expression, x_t denotes the log output gap, θ is the steady state elasticity of substitution between goods, and $\lambda \equiv \frac{\kappa}{\theta}(\sigma + \varphi)$ denotes the weight of output gap fluctuations. Moreover, $\hat{R}_t^r$ denotes the time t expectation of the real interest rate in deviations from steady state and $\mathcal{R}_2$ is a positive coefficient discussed below.

The loss function is identical to the one in the textbook NKM, with the exception of the last term, which is related to the risk-taking channel. This is our first key result about optimal policy: The risk-taking channel introduces a real interest rate volatility term into the second-order approximation of welfare.[15] To understand why this term appears, recall proposition 1, which states that the expected return of the bank's investment is concave. This implies that a mean preserving spread in the real interest rate reduces the expected return of investment by Jensen's inequality. Volatility in the expected real interest rate thus affects welfare negatively.

The loss function (12) illustrates this intuition: The weight on real rate volatility is:

$$\mathcal{R}_2 = -\frac{f_{RR}}{f(R^r)} > 0 \, ,$$

contractability of q, equity premium) is removed.

[15] If the equity premium ξ had been defined in nominal terms, the nominal rate would instead appear in the welfare function.

where $f(R^r)$ denotes the steady state of equation (10), andf_{RR} the second-order derivative of $f(R^r)$, which we characterized in proposition 1. This weight is positive due to the concavity of expected productivity in the real rate ($f_{RR} < 0$).

Under the assumption of an undistorted steady state, the new term in the linearized Phillips curve drops out (since $f_R = 0$). A linear quadratic approximation to the central banks problem hence is to minimize (12) subject to the standard IS and Phillips curves and the linearized Fisher equation:

$$\min \tfrac{1}{2}\mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2 + \tfrac{\kappa}{\theta}\mathcal{R}_2 \left(\hat{R}_t^r\right)^2 \right] \right\} \tag{13}$$

$$\begin{aligned} s.t. \qquad x_t &= \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(\hat{R}_t - \mathbb{E}_t \pi_{t+1}\right) & (14)\\ \pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa\left(\sigma + \varphi\right) x_t + u_t & (15)\\ \hat{R}_t^r &= \hat{R}_t - \mathbb{E}_t \pi_{t+1} & (16) \end{aligned}$$

The supply-side cost push shock u_t is assumed to follow an AR process with autoregressive coefficient ρ. For simplicity, but without loss of generality, we have also assumed that A_t is constant so that no shock appears in the IS curve.[16]

3.2 Optimal policy under commitment: Optimal simple rule

We now derive optimal monetary policy, starting with the case of a central bank that commits to a forward-looking Taylor rule.[17] As we show in Appendix B3.1, the optimal simple rule is given by:

$$\hat{R}_t = \phi_\pi^s \mathbb{E}_t \pi_{t+1} = \left[1 + \frac{\theta\kappa\sigma(1-\rho)(\sigma+\varphi)}{\rho(1-\beta\rho)\left(\theta\lambda + \kappa(1-\rho)^2 \mathcal{R}_2 \sigma^2\right)}\right] \mathbb{E}_t \pi_{t+1} \ , \tag{17}$$

The Taylor rule coefficient ϕ_π^s is larger than 1. Deriving ϕ_π^s with respect to the risk-taking channel parameter $\mathcal{R}_2$ delivers our second result: The risk-taking channel

[16] In a standard NKM, it is optimal for the central bank to stabilize both inflation and the output gap perfectly in response to 'demand' shocks (*divine coincidence,* Blanchard and Galí, 2007). This policy requires the real rate to follow the natural rate, which implies non-zero real rate volatility. The risk-taking channel introduces a trade-off that breaks the divine coincidence. Our key results regarding inflation and policy rule coefficients (summarized in section 3.4) are robust to adding a demand shock. The only difference is that output gap volatility would increase in $\mathcal{R}_2$, if A_t were the only shock.

[17] This particular rule is useful to build intuition, since it directly relates the (expected) real rate to inflation. Furthermore, such a rule results from optimal policy under discretion.

lowers the optimal response of the nominal interest rate to expected inflation, i.e:

$$\frac{\partial \phi_\pi^s}{\partial \mathcal{R}_2} < 0 \tag{18}$$

Intuitively, the stronger the risk-taking channel, the stronger the motive to stabilize the real interest rate, hence the closer ϕ_π^s to the value of 1 (which would perfectly stabilize the real rate).

We then solve the model under this rule. That is, we find policy functions of the form: $x_t = au_t$, $\pi_t = bu_t$, $\hat{R}_t = cu_t$ and $\hat{R}^r_{t+1} = du_t$. The absolute values of the coefficients (a, b, c, d), reported in Appendix B3.1, determine the standard deviation of the four variables. To understand how these standard deviations change with the risk-taking channel, we compute the rate of change of the four coefficients with respect to the risk-taking channel parameter $\mathcal{R}_2$ and check the sign.

This leads to our third main result: Optimal policy with the risk-taking channel implies a lower volatility of the output gap and of the real interest rate, but a higher volatility of inflation, relative to the model without the risk-taking channel.[18]

What is the intuition behind these two results? The standard trade-off in the NKM with cost-push shocks is that, through the Phillips curve, inflation stabilization comes at the cost of higher output gap volatility. However, through the IS curve, higher output gap volatility also implies higher real interest rate volatility. While this is irrelevant in the standard NKM, it becomes costly once the risk-taking channel is active, since fluctuations in the real rate lead to less efficient risk choices on average. The risk-taking channel thus tilts the trade-off between output gap and inflation stabilization arising from cost-push shocks in favor of the former. In other words, the risk-taking channel increases the central bank's tolerance to deviations of the real rate from the natural rate.

3.3 Optimal policy under commitment: Ramsey policy

Next, we turn to Ramsey-optimal policy. The central bank's problem is to solve (13) by choosing conditional paths for inflation, the output gap and the interest rate. Appendix B3.2 provides the Lagrangian and the first-order conditions of this Ramsey problem. As the presence of lagged multipliers in the first-order conditions highlights, this Ramsey policy complicates the model, since it leads to the intro-

[18]The effect of the risk-taking channel on the volatility of the nominal rate is ambiguous: For low enough values of ρ, the risk-taking channel also implies a lower volatility of the nominal rate.

duction of state variables. For this reason, no analytical solution is available for the rational expectations equilibrium defined by these conditions. However, it is possible to combine the first-order conditions to derive an implicit instrument rule as in Giannoni and Woodford (2003). This rule applies from $t \geq 2$, is optimal from a timeless perspective and reads:

$$\hat{R}_t = \rho_1 \hat{R}_{t-1} + \rho_2 \Delta \hat{R}_{t-1} + \phi_{E\pi} \mathbb{E}_t \pi_{t+1} + \phi_\pi \pi_t + \phi_{\pi_{-1}} \pi_{t-1} + \phi_x \Delta x_t$$

where $\rho_1 = 1,\ \rho_2 = \frac{1}{\beta},\ \phi_{E\pi} = 1,\ \phi_\pi = \frac{\theta\sigma + \theta\varphi}{\mathcal{R}_2 \sigma} - \frac{1}{\beta} - 1,\ \phi_{\pi\,-1} = \frac{1}{\beta},\ \phi_x = \frac{\theta\lambda}{\mathcal{R}_2^2 \kappa\sigma}$.

As for the optimal simple rule, the weight on inflation in this Taylor-type rule decreases in the strength of the risk-taking channel. Furthermore, the rule exhibits a nontrivial degree of persistence: $\rho_1 = 1, \rho_2 > 1$. That is, the history dependence embedded in Ramsey policy makes it optimal for the central bank to move the policy rate in an inertial manner. Intuitively, moving the policy rate less strongly but for an extended period of time allows the planner to control demand as well as by moving the policy rate more strongly but for a shorter period, but the former implies a lower volatility of the real interest rate, which is welfare enhancing.

As Woodford (2003) shows, the Ramsey-optimal interest rate paths do not involve any explicit reference to the lagged interest rate in the simple NKM.[19] The risk-taking channel introduces instead a case for persistent policy responses under Ramsey-optimal policy.[20] This is our fourth analytical result. The risk-taking channel therefore provides an additional explanation for interest rate inertia, which is routinely built into Taylor rules in models, and which is typically observed in practice. It complements other theories such as the zero lower bound or the cost of holding money, which also implies that the interest rate – the *nominal* one, not the *real* one as in our case – appears in the welfare function and which also lead to inertia under optimal policy (Woodford, 2003).

[19]In the standard NKM, 'optimal policy rules [...] are necessarily pure targeting rules [...] where the target criterion itself is independent of the path of the interest rate instrument.' (Woodford, 2003, p. 560). By contrast, the target criterion given by the above instrument rule does depend on past interest rates.

[20]Note that the above rule does not nest a rule for the standard NKM. However, as the weight on real rate stabilization $\frac{\kappa}{\theta}\mathcal{R}_2^2$ goes towards 0, past interest rates become less important in the determination of the current interest rate, relative to deviations of output and inflation.

3.4 Summary

Before moving on, we summarise the four analytical results concerning the implications of the risk-taking channel for optimal monetary policy:

- R1: Real interest rate volatility affects welfare negatively through the risk-taking channel
- R2: The risk-taking channel lowers the optimal response to inflation in a simple Taylor-type policy rule
- R3: Optimal monetary policy with the risk-taking channel calls for lower real interest rate volatility and higher inflation volatility
- R4: The risk-taking channel introduces a motive for inertia in the policy rate

3.5 Robustness: Optimal discretionary policy

In Appendix B3.3 we consider the case of a discretionary policymaker. Analogously to subsection 3.2, we analyze the impact of the the risk-taking channel parameter $\mathcal{R}_2$ on the Taylor rule parameter and the standard deviations of the endogenous variables. Results confirm those derived for the optimal simple rule under commitment: The risk-taking channel lowers the optimal response of the policy rate to inflation (R2), which leads to lower output gap and real interest rate volatility and higher inflation volatility (R3).

3.6 Robustness: Inefficient steady state

We now relax the assumption of an undistorted steady state, and only assume that the steady-state markup is small. That is, we allow the steady-state distortions from the risk-taking channel to be arbitrary. As we show in Appendix B1, a second order approximation of welfare delivers the following loss function:

$$\begin{aligned} \mathbb{W} \;=\; & \mathbb{E}_{\mathrm{t}}\left\{\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2+\lambda\hat{x}_t^2+\frac{\kappa}{\theta}\left((1+\varphi)\,\mathcal{R}_1^2+\mathcal{R}_2\right)\left(\hat{R}_t^r\right)^2+\right.\right. \qquad (19) \\ & \left.\left. 2\frac{\kappa}{\theta}\left(-\Theta\widehat{x}_t-(1-\Theta)\,\mathcal{R}_1\hat{R}_t^r+\mathcal{R}_1\,(\sigma-1)\,\hat{R}_t^r\hat{y}_t^e-\mathcal{R}_1\,(1+\varphi)\,\hat{R}_t^r\hat{x}_t\right)\right]\right\}, \end{aligned}$$

where $\hat{y}_t^e$ denotes the efficient level of output in log deviations from steady state, the term $\Theta = 1/\theta$ is related to the steady-state distortion from imperfect competition, and the coefficient $\mathcal{R}_2 = -\frac{f_{RR} f(R^r) - (f_R)^2}{f(R^r)^2} > 0$ now contains a first-order term as well. The loss function contains the same volatility terms as before, with an even larger weight on real rate volatility, strengthening result R1. Additionally, it now contains a number of first-order and covariance terms. While the first first-order term $-\Theta \widehat{x}_t$ is standard, the second one $-(1-\Theta)\,\mathcal{R}_1 \hat{R}_t^r$ is related to the risk-taking channel: higher levels of the real interest rate lead to less excessive risk taking, which increases welfare. The covariance terms are discussed in Appendix B1.

The central bank's problem is to minimize the loss function (19) subject to the Phillips curve (11), now affected by risk-taking channel, the IS curve (14) and the Fisher equation (16).[21] As before, we first determine the optimal simple Taylor rule. In appendix B3.4, we show that the response to inflation decreases in the strength of the risk-taking channel (i.e. in $\mathcal{R}_1$ and $\mathcal{R}_2$). This confirms result R2.

In a second step, we determine the equilibrium dynamics determined jointly by the IS curve, the Phillips curve, the Fisher equation and the optimal Taylor rule, as we did before. However, the risk-taking channel parameter $\mathcal{R}_1$ now appears also in the Phillips curve. Hence, the risk-taking channel affects volatilities both through its impact on the behavior of the private sector (the Phillips curve) and through its impact on policy (the Taylor rule). Because our focus is on optimal policy, we focus on the latter. That is, we derive the volatilities with respect to the parameters $\mathcal{R}_1$ and $\mathcal{R}_2$ that appear in the optimal rule, keeping the $\mathcal{R}_1$ in the Phillips curve constant. This exercise tells us how the equilibrium in the risk-taking channel economy changes when the central bank adjusts its policy rule *from* a rule that would be optimal in the absence of the risk-taking channel *towards* the rule that is optimal given the risk-taking channel. This is in line with the numerical exercise we conduct in the last section of the paper. Our findings confirm result R3: Inflation (real rate) volatility increases (decreases), if the central bank's policy optimally accounts for the risk-taking channel.

In sum, results R1-R3 are robust to steady state distortions. However, we cannot make any conclusion about result R4, since an instrument rule is not attainable in closed form.

[21]The latter two are not affected by the steady state distortions.

4 The importance of the risk-taking channel in a quantitative New Keynesian model

To quantify the importance of the risk-taking channel, we turn to the quantitative model of Abbate and Thaler (2019).[22] The latter embeds the model of bank risk-taking from section 2 in an otherwise standard medium-scale New-Keynesian model as in Smets and Wouters (2007). This larger model has two advantages: First, it includes a number of additional features that bring it closer to macroeconomic dynamics. Second, it has been estimated on US data and shown to fit the data well. Thus, we can rely on a plausible set of empirically determined parameters.[23]

Since the larger model is essentially a medium scale extension of the simple model, we refer to Abbate and Thaler (2019) for a full description, and limit ourselves to a brief explanation of the four differences. First, it includes capital, financed by banks, so we can give up the assumption that wages need to be prefinanced. The risky *input* good producer hence becomes a risky *capital* good producer. Inefficient risk taking affects aggregate output through the productivity of capital, via the same mechanisms discussed in the simple model. Second, the banking sector in the larger model has two additional features that improve the model's quantitative fit: Partial deposit insurance and a non-zero liquidation value in case of bank default.[24] Importantly, these additions leave proposition 1 and thus the mechanism unchanged. Third, the larger model features additional frictions and a more shocks, as in Smets and Wouters (2007). Fourth, we allow for a distorted steady-state.

4.1 The numerical experiment

We proceed as follows. We set the model parameters to their posterior mean estimates (cf. table 4 in Appendix C). Then, we numerically determine the *optimal simple implementable* monetary policy rule using a second order approximation as in

[22]Previous studies typically find financial frictions to have only *quantitatively insignificant* effects on optimal monetary policy (e.g. Bernanke and Gertler, 2001 or De Fiore and Tristani, 2013).

[23]Abbate and Thaler (2019) estimate the model on US data over 1984Q1 to 2007Q3, using seven standard macro series, plus a measure of the equity ratio in the US banking sector, which allows the identification of the banking sector parameters. Further details on the estimation and on the goodness of fit can be found there.

[24]The deposit insurance scheme, financed through a variable tax on capital, covers the gap between the insurance cap and the liquidation value. The two features have opposing effects. Deposit insurance improves the cost advantage of deposits, worsening the risk-taking problem. The liquidation value increases the optimal risk level, easing the excessiveness of risk taking.

Schmitt-Grohe and Uribe (2007). We focus on a simple policy rule – as opposed to Ramsey optimal policy – both because it is realistically implementable and because its coefficients can easily be related to our analytical results R2 and R4. We do so in two model versions: The full model with the risk-taking channel (henceforth *bank model*), and a model version without this channel, that corresponds to a standard Smets and Wouters economy (henceforth *benchmark model*).[25] Comparing the two resulting optimal rules establishes how the risk-taking channel affects optimal policy. Furthermore, comparing the performance of these two rules in the bank model allows us to asses how different the two rules are in terms of the behavior of macroeconomic variables and welfare, thus informing us how important it is for the policymaker to account for the risk-taking channel.

We look for the policy rule that maximizes welfare (the household's conditional lifetime utility) among the class of simple, implementable interest-rate feedback rules given by:

$$\hat{R}_t = \phi_\pi \hat{\pi}_{t+s} + \phi_y \hat{y}_{t+s} + \rho_r \hat{R}_{t-1} \,, \tag{20}$$

where the index $s \in \{1, 0\}$ allows for forward-looking or contemporaneous rules, and the hat symbol denotes (expected) log deviations from the steady state.[26] We impose that the inertia parameter ρ has to be non-negative.

4.2 Findings

The numerical analysis delivers four results, which mirror the four theoretical results above.[27] The first two results are evident from Table 1, which reports the optimal coefficients for four different specifications of the monetary policy rule: contemporaneous and forward-looking, without inertia and with optimal inertia.

First, the optimal coefficient on inflation deviations is always significantly smaller in the bank model than in the benchmark model. This confirms that our analytical result R2, a lower optimal weight on inflation in the Taylor rule, carries over to the

[25]The benchmark model can easily be obtained from the bank model by fixing the equity ratio and the risk choice at their steady state levels. In doing so we keep all parameters fixed. Reestimating the benchmark model does not affect the results significantly.

[26]Implementability requires uniqueness of the rational expectations equilibrium, while simplicity requires the interest rate to be a function of readily observable variables (see Schmitt-Grohe and Uribe, 2007). Note that leverage and risk taking are dependent on the nominal rate and inflation, both of which already appear in the Taylor rule.

[27]The results are qualitatively robust with respect to the estimation sample and the choice of the priors and calibrated parameters.

Table 1: **Optimal simple rules**: The second (third) column describes the timing (restrictions) of the policy rule. Italics indicate restricted parameters.

rule			*benchmark model*			*bank model*		
	s	*restriction*	ρ_r	$\phi_{\pi_{t+s}}$	$\phi_{y_{t+s}}$	ρ	$\phi_{\pi_{t+s}}$	$\phi_{y_{t+s}}$
I	0	$\rho_r = 0$	*0*	7.18	0.11	*0*	3.08	0.12
II	0		0.00	7.18	0.11	1.06	0.05	0.01
III	1	$\rho_r = 0$	*0*	17.82	0.14	*0*	4.30	0.17
IV	1		0.23	12.75	0.12	1.11	0.07	0.01

medium scale model used here.

Second, if the central bank can optimize over its smoothing parameter, then full interest rate smoothing is optimal in the bank model, but not in the benchmark model (rows II and IV), supporting result R4.[28]

Third, different policy rules imply different behaviours of macroeconomic variables. Table 2 displays how much the mean and volatility of key variables change when the central bank switches from the benchmark-optimal rule to the bank-optimal rule in the bank model. By responding less aggressively to inflation and by smoothing the nominal interest rate, the central bank optimally limits the volatility of the real interest rate (column 4). The lower volatility of R_t^r translates into a higher average return on investment $\mathsf{f}(q_t)$, due to the concavity of this function in R_t^r (column 10).[29] However, this higher average return on investment comes at the cost of higher inflation volatility (column 5), in line with the analytical result R3. The increase in volatility is sizeable (50-70%). Hence, the new trade-off between inflation and real rate stabilization implies a significant deviation from inflation stabilization: The central bank reacts a lot less strongly to deviations of inflation from the target in order to achieve a more stable real rate

Finally, we assess the welfare cost Ω of ignoring the risk-taking channel, i.e. of applying the benchmark-optimal rule in the bank model.[30] The last column of Table 2 shows that these costs are significant for all policy rule specifications, ranging from 0.5% to 0.9% of lifetime consumption equivalent. The risk-taking channel turns out to affect welfare significantly (result R1). Overall, we can conclude that the risk-taking channel is economically significant for optimal monetary policy both in terms of the prescribed policy and the welfare cost of deviating from it.

[28]Values of ρ_r slightly above 1 are not uncommon e.g. Rotemberg and Woodford (1999).

[29]The slight increase in R_t^r accounts only for a marginal fraction of the increase in $f(q_t)$.

[30]This metric ignores the costs associated with the transition.

Table 2: **Differences in moments and welfare costs**: Columns 4-10 indicate the mean and standard deviation changes (in %) of key variables when the central bank switches from the benchmark-optimal to the bank-optimal rule in the bank model. The last column reports the welfare cost (in % of lifetime consumption stream) associated with implementing the benchmark-optimal policy rule in the bank model.

	rule		*standard deviation*			*mean*				
	s	*restriction*	R^r	π	y	R^r	π	y	$f(q)$	Ω
I	0	$\rho_r = 0$	-49.45	48.75	-0.97	0.00	-0.06	0.32	0.05	0.49
II	0		-78.81	58.53	-9.44	0.01	-0.05	0.44	0.06	0.91
III	1	$\rho_r = 0$	-59.25	55.42	-3.29	0.00	-0.07	0.48	0.06	0.73
IV	1		-79.14	66.71	-10.65	0.01	-0.05	0.43	0.06	0.85

5 Conclusions

This paper analyses the implications of the risk-taking channel for optimal monetary policy. To this end, we first embed a model of asset risk taking into the textbook NKM. Then, we characterize optimal policy analytically using a linear quadratic approximation. We find that the risk-taking channel (i) introduces real rate volatility into the otherwise standard objective function of the central bank, (ii) calls for lower real rate volatility and higher inflation volatility, (iii) lowers the optimal response to inflation in a Taylor-type policy rule, (iv) introduces a motive for inertia in the policy rule. Lastly, we extend the model to a medium-scale DSGE model of the type routinely used at central banks to evaluate the quantitative importance of the risk-taking channel for monetary policy. We show that the four conclusions from the simple model carry over and matter significantly.

Our model of the risk-taking channel is analytically tractable and our analysis delivers clear results for optimal monetary policy. To this end, we have abstracted form other important dimensions such as risks on the liabilities side of banks, effects of the zero lower bound or regulation. At the same time however, our qualitative lessons about optimal policy can be of relevance for any theory that relates TFP to the level of the real interest rate, such as some theories of capital misallocation (e.g. Gopinath et al., 2017).

References

Angela Abbate and Dominik Thaler. Monetary policy and the asset risk-taking channel. *Journal of Money, Credit and Banking*, 51(8):2115–2144, December 2019.

Tobias Adrian and Hyun Song Shin. Procyclical Leverage and Value-at-Risk. *Review of Financial Studies*, 27(2):373–403, 2014.

Elena Afanasyeva and Jochen Guentner. Bank market power and the risk channel of monetary policy. *Journal of Monetary Economics*, 111(C):118–134, 2020.

Franklin Allen, Elena Carletti, and Robert Marquez. Credit market competition and capital regulation. *The Review of Financial Studies*, 24(4):983–1018, 2011.

Ignazio Angeloni and Ester Faia. Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics*, 60(3):311–324, April 2013.

Ignazio Angeloni, Ester Faia, and Marco Lo Duca. Monetary policy and risk taking. *Journal of Economic Dynamics and Control*, 52:285–307, March 2015.

Ben S. Bernanke and Mark Gertler. Should Central Banks Respond to Movements in Asset Prices? *American Economic Review*, 91(2):253–257, May 2001.

Oliver Blanchard and Jordi Galí. Real wage rigidities and the new keynesian model. *Journal of Money, Credit and Banking*, 39(s1):35–65, 2007.

Johannes Bubeck, Angela Maddaloni, and Jose-Luis Peydro. Negative Monetary Policy Rates and Systemic Banks' Risk-Taking: Evidence from the Euro Area Securities Register. *Journal of Money, Credit and Banking*, 52(S1):197–231, October 2020.

Claudia M. Buch, Sandra Eickmeier, and Esteban Prieto. In search for yield? Survey-based evidence on bank risk taking. *Journal of Economic Dynamics and Control*, 43(C):12–30, 2014.

Ian Christensen, Cesaire Meh, and Kevin Moran. Bank Leverage Regulation and Macroeconomic Dynamics. Working Papers 11-32, Bank of Canada, 2011.

Richard Clarida, Jordi Galí, and Mark Gertler. The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature*, 37(4):1661–1707, December 1999.

Fabrice Collard, Harris Dellas, Behzad Diba, and Olivier Loisel. Optimal Monetary and Prudential Policies. *American Economic Journal: Macroeconomics*, 9(1): 40–87, January 2017.

Fiorella De Fiore and Oreste Tristani. Optimal Monetary Policy in a Model of the Credit Channel. *Economic Journal*, 123(571):906–931, 09 2013.

Oliver de Groot. The Risk Channel of Monetary Policy. *International Journal of Central Banking*, 10(2):115–160, June 2014.

Giovanni Dell'Ariccia, Luc Laeven, and Robert Marquez. Real interest rates, leverage, and bank risk-taking. *Journal of Economic Theory*, 149(0):65 – 99, 2014.

ECB. The role of financial stability considerations in monetary policy and the interaction with macroprudential policy in the euro area. *Occasional Papers*, 2021.

Jordi Galí. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications Second edition*. Number 10495 in Economics Books. Princeton University Press, 2015.

Mark Gertler, Nobuhiro Kiyotaki, and Albert Queralto. Financial crises, bank risk exposure and government financial policy. *Journal of Monetary Economics*, 59 (S):S17–S34, 2012.

Marc P. Giannoni and Michael Woodford. Optimal Interest-Rate Rules: II. Applications. NBER Working Papers 9420, National Bureau of Economic Research, Inc, January 2003.

Gita Gopinath, Sebnem Kalemli-Oezcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez. Capital Allocation and Productivity in South Europe. *The Quarterly Journal of Economics*, 132(4):1915–1967, 2017.

Florian Heider, Farzad Saidi, and Glenn Schepens. Life below Zero: Bank Lending under Negative Policy Rates. *The Review of Financial Studies*, 32(10):3728–3761, 02 2019.

Thomas F. Hellmann, Kevin C. Murdock, and Joseph E. Stiglitz. Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough? *The American Economic Review*, 90(1):147–165, 2000.

Vasso Ioannidou, Steven Ongena, and Jose-Lui Peydro. Monetary Policy, Risk-Taking and Pricing: Evidence from a Quasi-Natural Experiment. Discussion Paper 2009-04S, European Banking Center, June 2014.

Gabriel Jimenez, Steven Ongena, Jose Luis Peydro, and Jesus Saurina. Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say About the Effects of Monetary Policy on Credit Risk-Taking? *Econometrica*, 82 (2):463–505, 2014.

Angela Maddaloni and Jose-Luis Peydro. Bank risk-taking, securitization, supervision, and low interest rates: Evidence from the euro-area and the u.s. lending standards. *The Review of Financial Studies*, 24(6):2121–2165, 2011. ISSN 08939454, 14657368.

David Martinez-Miera and Rafael Repullo. Monetary policy, macroprudential policy, and financial stability. *Annual Review of Economics*, 11(1):809–832, 2019.

Federico Ravenna and Carl E. Walsh. Optimal monetary policy with the cost channel. *Journal of Monetary Economics*, 53(2):199–216, March 2006.

Julio J. Rotemberg and Michael Woodford. Interest Rate Rules in an Estimated Sticky Price Model. In *Monetary Policy Rules*, NBER Chapters, pages 57–126. National Bureau of Economic Research, Inc, 1999.

Stephanie Schmitt-Grohe and Martin Uribe. Optimal, Simple, and Implementable Monetary and Fiscal Rules. *Journal of Monetary Economics*, 54(6):1702–1725, September 2007.

Frank Smets and Rafael Wouters. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3):586–606, 2007.

Michael Woodford. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, 2001.

Michael Woodford. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, 2003.

Online Appendix

Appendix A: Details on the model

A1: The bank's problem

At the **second stage**, the bank has already raised $E_t + D_t$ funds and now needs to choose the riskiness of its investment q_t. As mentioned in the main text, we assume that the bank cannot write contracts conditional on q_t with the depositors at the first stage. Therefore, at the second stage the bank takes the deposit rate as given and maximizes the gross return on equity. The second stage problem is thus:

$$\max_{q_t \in [0,1]} \left(\omega_1 q_t - \frac{\omega_2}{2} q_t^2 \right) v_t^r - q_t r_{d,t}^r (1 - k_t) \ . \tag{21}$$

Notice the risk shifting incentives: If the probability of repayment q_t is higher, then the expected payment to depositors ($q_t r_{d,t}^r (1 - k_t)$) is higher. This is due to limited liability.[31]

Also notice that – as mentioned in the main text – maximizing the gross return on equity coincides with maximizing equity holders' return net of the cost of equity, which in turn is determined by the equity supply schedule. This is so because the capital structure and the deposit rate are given at the second stage, but equity providers still want to break even. In other words, there is no difference between solving the unconstrained optimization problem (21) or the following constrained optimization problem:

$$\max_{q_t \in [0,1]} \left(\omega_1 q_t - \frac{\omega_2}{2} q_t^2 \right) v_t^r - q_t r_{d,t}^r (1 - k_t) - q_t r_{e,t}^r k_t \ .$$

$$s.t.$$

$$r_{e,t}^r = \frac{R_t^r + \xi}{q_t}$$

The FOC reads:

[31] With unlimited liability the last q_t would drop out of the expression and the bank would simply maximize the expected return on loans.

$$q_t = \frac{\omega_1 v_t^r - r_{d,t}^r (1 - k_t)}{\omega_2 v_t^r} \ . \tag{22}$$

We assume that parameters guarantee an interior solution. By concavity in q_t, the FOC is then sufficient.

At the **first stage**, the bank chooses the capital structure k_t and the balance sheet size o_t to maximize expected excess profits, subject to the participation constraints (i.e. the funding supply schedules) for depositors and equity providers. Since agents have rational expectations, everyone correctly infers the level of risk q_t that will be chosen by the bank at the second stage as a function of k_t, $r_{d,t}^r$ and v_t^r. The first stage problem is thus

$$\max_{k_t, \in[0,1], \left\{o_t, q_t, r_{d,t}^r, r_{e,t}^r\right\} \in \mathbb{R}_+^4} o_t \left\{ v_t^r \left(q_t \omega_1 - \frac{\omega_2}{2} q_t^2 \right) - q_t r_{d,t}^r (1 - k_t) - q_t r_{e,t}^r k_t \right\} \tag{23}$$

$$\text{s.t.} \quad r_{d,t}^r = \frac{R_t^r}{q_t} \quad \text{and} \quad r_{e,t}^r = \frac{R_t^r + \xi}{q_t} \quad \text{and} \quad q_t = \frac{\omega_1 v_t^r - r_{d,t}^r (1 - k_t)}{\omega_2 v_t^r}.$$

Substituting for q_t, $r_{d,t}^r$ and $r_{e,t}^r$ allows us to rewrite the above constrained optimization problem more compactly as an unconstrained optimization problem:[32]

$$\max_{k_t, \in[0,1], o_t \in \mathbb{R}^+} o_t \left\{ v_t^r \left(\hat{q}_t \omega_1 - \frac{\omega_2}{2} \hat{q}_t^2 \right) - R_t^r - \xi k_t \right\} \tag{24}$$

where

$$\hat{q}_t = \frac{\omega_1 + \sqrt{\omega_1^2 - \left(4\omega_2 (1 - k_t) R_t^r\right) / v_t^r}}{2\omega_2} \ . \tag{25}$$

This simplified objective function reflects the fact that not only equity providers but also depositors anticipate perfectly the banks risk choice in stage 2, such that they always break even: It is precisely for this reason that the cost of capital $R_t^r + \xi k_t$ now is independent of the level of safety q_t. It only depends on the real rate, the equity ratio and the equity premium. There is thus no risk shifting at stage 1, though everyone anticipates that risk shifting will happen at stage 2.

This formulation highlights the trade-off that the bank faces when choosing the

[32]Note that when substituting for q_t we have guessed that the larger of two roots is the relevant one. Part 2 of the below proposition will verify this assumption: Given equilibrium prices, for any $k_{t,} \in [0, 1], o_t \in \mathbb{R}_+$ the objective is larger when the larger root is picked.

equity ratio k_t. The cost of choosing a higher equity ratio is immediately obvious: A higher k_t implies higher costs of capital, due to the equity premium ξ. The benefit of choosing a higher equity ratio is somewhat less trivial to see as it operates through $\hat{q}_t$: A higher k_t implies more skin in the game and thus a lower risk choice in stage 2 (higher q_t). The higher q_t leads to (i) an increase in the expected return on the loan $v_t^r \left(\hat{q}_t\omega_1 - \frac{\omega_2}{2}\hat{q}_t^2\right)$ (ii) reductions in the deposit and equity rates r_d and r_e that leave the expected cost of capital unchanged, apart from the equity premium. Thus, through (i) an increase in k_t increases the banks expected excess profits. This is the benefit of choosing a higher k_t.

The FOCs for leverage k_t reads:

$$k_t = 1 - \frac{\xi(R_t^r + \xi)\omega_1^2 v_t^r}{\omega_2 R_t^r \left(R_t^r + 2\xi^2\right)} \ . \tag{26}$$

Finally, since the first stage problem is linear in the balance sheet size o_t, the corresponding first order condition requires banks to make no expected profits in excess of the costs of funds:

$$\underbrace{v_t^r \left(q_t\omega_1 - \frac{\omega_2}{2}q_t^2\right)}_{\text{revenues}} - \underbrace{(k_t\xi + R_t^r)}_{\text{cost of funds}} = 0 \ . \tag{27}$$

Guessing that the solution for the equity ratio k_t is interior, it can be shown that the FOCs are sufficient.

We can combine the last three equations to derive the banks' risk choice q_t, its equity ratio k_t and the relative equilibrium price ratio v_t^r as a function of the safe real interest rate:

$$k_t = \frac{R_t^r}{R_t^r + 2\xi} \tag{28}$$

$$q_t = \frac{\omega_1(\xi + R_t^r)}{\omega_2(2\xi + R_t^r)} \tag{29}$$

$$v_t^r = \frac{2\omega_2 R_t^r(2\xi + R_t^r)}{\omega_1^2(\xi + R_t^r)} \tag{30}$$

Inspecting these equations, we observe that k_t is always interior, which verifies our guess. Inspecting the equation for q_t and combining it with the definition of $\mathsf{f}(q_t)$ then directly delivers proposition 1 in the text.

A2: Full set of recursive equations in the simple model

The following 12 equations (31) - (42) define the equilibrium. Note that only equations (31) - (33) differ from the standard NKM and that the model collapses to the standard NKM if $\mathsf{f}(q_t)$ is a constant.

Marginal costs:

$$MC_t = \frac{W_t}{A_t \left(\omega_1 - \frac{\omega_2}{2} q_t\right) q_t} = \frac{W_t}{A_t \mathsf{f}(q_t)} \tag{31}$$

Output:

$$A_t \mathsf{f}(q_t) N_t = \Delta_t C_t \tag{32}$$

Risk-taking channel:

$$\mathsf{f}(q_t) = f\left(R^r_{t+1}\right) = \frac{\omega_1^2}{\omega_2} \frac{\xi + R^r_t}{2\xi + R^r_t} - \frac{\omega_1^2}{2\omega_2} \left(\frac{\xi + R^r_t}{2\xi + R^r_t} \right)^2 \tag{33}$$

$$R^r_t = \frac{R_t}{\mathbb{E}_t \pi_{t+1}} \tag{34}$$

Household optimization:

$$u_C(C_t, N_t) = \beta \frac{R_t}{\mathbb{E}_t \pi_{t+1}} u_C(C_{t+1}, N_{t+1}) \tag{35}$$

$$-\frac{u_N(C_t, N_t)}{u_C(C_t, N_t)} = w_t \tag{36}$$

Price setting:

$$\pi^\star_t = \frac{\epsilon_p}{\epsilon_p - 1} \frac{Z_{1,t}}{Z_{2,t}} \tag{37}$$

$$\Lambda_t = \frac{\beta u_C(C_{t+1}, N_{t+1})}{u_C(C_t, N_t)} \tag{38}$$

$$Z_{1,t} = \Lambda_t mc_t y_t + \beta \lambda_p \mathbb{E}_t \left[(\pi_{t+1})^{\epsilon_p} Z_{1,t+1} \right] \tag{39}$$

$$Z_{2,t} = \Lambda_t y_t + \beta \lambda_p \mathbb{E}_t \left[(\pi_{t+1})^{\epsilon_p - 1} Z_{2,t+1} \right] \tag{40}$$

$$1 = (1-\lambda)\left(\pi_t^\star\right)^{1-\epsilon_p} + \lambda\left(\pi_t\right)^{\epsilon_p - 1} \tag{41}$$

$$\Delta_t = (1-\lambda)\left(\pi_t^\star\right)^{-\epsilon_p} + \lambda\Delta_{t-1}\left(\pi_t\right)^{\epsilon_p} \tag{42}$$

The following equations are recursive:

$$d_t + e_t = m_{t-1}/\pi_t + w_t N_t - C_t + x_t \tag{43}$$

$$m_t = m_{t-1}/\pi_t + w_t N_t - d_t - e_t - C_t + R_t\left(d_t + e_t\right) + \Pi_t + x_t + \xi e_t - T_t \tag{44}$$

$$\Pi_t = C_t - w_t z_t r_{l,t} \tag{45}$$

$$q_t = \frac{\omega_1}{\omega_2}\frac{\xi + R_t^r}{2\xi + R_t^r} \tag{46}$$

$$k_t = \frac{R_t^r}{R_t^r + 2\xi} \tag{47}$$

$$d_t + e_t = o_t \tag{48}$$

$$o_t = w_t N_t \tag{49}$$

$$\frac{e_t}{e_t + d_t} = k_t \tag{50}$$

Appendix B: Deriving optimal policy

B1: Deriving the welfare function

Our goal is to derive a second-order approximation to the utility of the household when the economy is close to the steady state, around a zero-inflation steady state and in the case of a small steady state distortion. We follow Galí (2015) and Ravenna and Walsh (2006). The procedure involves 8 steps. We preliminary describe the notation that will be used throughout the derivations:

Table 3: Notation

X_t	variable in level
X	steady state level
x_t	variable in log: $\ln(X_t)$
$\hat{x}_t$	log deviation from steady state: $x_t - x = \ln(X_t) - \ln(X) = \ln(X_t/X)$
$\hat{X}_t$	absolute deviation from steady state
R_t^r	gross real interest rate

Step 1: Take a second-order Taylor expansion of the utility function in time *t* around the steady state C,N:

$$U(C_t,N_t) \simeq U+U_C(C_t-C)+U_N(N_t-N)+\frac{1}{2}U_{CC}(C_t-C)^2+\frac{1}{2}U_{NN}(N_t-N)^2+t.i.p.$$

where t.i.p. stands for terms independent of policy, $U = U(C,N)$ denotes the utility function evaluated at the steady state and $U_x = U_x(C,N)$,$U_{xx} = U_{xx}(C,N)$ respectively denote the first and second order derivative of the utility function with respect to variable x, evaluated at the steady state. Multiply and divide by steady-state consumption or employment, where appropriate:

$$U(C_t,N_t) \simeq U+U_C\frac{(C_t-C)}{C}C+U_N\frac{(N_t-N)}{N}N+\frac{1}{2}U_{CC}\frac{(C_t-C)^2}{C^2}C^2+\frac{1}{2}U_{NN}\frac{(N_t-N)^2}{N^2}N^2+t.i.p.$$

Step 2 Exploit the aggregate resource constraint: $Y_t = C_t$

$$U(C_t,N_t) \simeq U+U_C\left[\frac{(Y_t-Y)}{Y}Y+\frac{1}{2}\frac{U_{CC}}{U_c}\frac{(Y_t-Y)^2}{Y^2}Y^2\right]+U_N\left[\frac{(N_t-N)}{N}N+\frac{1}{2}\frac{U_{NN}}{U_N}\frac{(N_t-N)^2}{N^2}N^2\right]+t.i.p$$

Note that, given our utility function, $\frac{U_{CC}}{U_C}C = -\sigma$ and $\frac{U_{NN}}{U_N}N = \varphi$. Use that $\frac{X_t-X}{X} \simeq \widehat{x}_t+\frac{1}{2}\widehat{x}_t^2$ and drop terms of order higher than 2, i.e. use that $\left(\widehat{x}_t+\frac{1}{2}\widehat{x}_t^2\right)^2 = \widehat{x}_t^2+\widehat{x}_t^3+\frac{1}{4}\widehat{x}_t^4 \simeq \widehat{x}_t^2$:

$$U(C_t,N_t) \simeq U + U_C\left(\left(\widehat{y}_t + \frac{1}{2}\widehat{y}_t^2\right)Y - \frac{1}{2}\sigma\widehat{y}_t^2 Y\right) + U_N\left(\left(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2\right)N + \frac{1}{2}\varphi\widehat{n}_t^2 N\right) + t.i.p.$$

Rearranging:

$$U(C_t,N_t) \simeq U + U_C Y\left(\widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2\right) + U_N N\left(\widehat{n}_t + \frac{1+\varphi}{2}\widehat{n}_t^2\right) + t.i.p.$$

Step 3: Express aggregate employment as a function of output. From our model, we can express aggregate employment as:

$$N_t = \frac{\Delta_t Y_t}{A_t f\left(R_t^r\right)}$$

where Δ_t expresses the resource loss due to the price dispersion term, and where $f\left(R_t^r\right)$ is the dispersion term related to the risk-taking channel, which depends on the expected real interest rate R_t^r. We take logs of both sides:

$$\ln(N_t) = \ln(\Delta_t) + \ln(Y_t) - \ln(A_t) - \ln f(R_t^r)$$

We derive the second-order Taylor expansion of $\ln f(R_t^r)$:

$$\ln f(R_t^r) \approx \ln f(R^r) + \frac{f_R}{f(R^r)}\left(R_t^r - R^r\right) + \frac{1}{2}\frac{f_{RR}f(R^r) - (f_R)^2}{f(R^r)^2}\left(R_t^r - R^r\right)^2$$

Define now the two coefficients:

$$\mathcal{R}_1 = \frac{f_R}{f(R^r)} = \frac{2\xi^2}{(R^r+\xi)(R^r+2\xi)(R^r+3\xi)} > 0 \tag{51}$$

$$\mathcal{R}_2 = -\frac{f_{RR}f(R^r) - (f_R)^2}{f(R^r)^2} = \frac{2(3\left(R^r\right)^2\xi^2 + 12R^r\xi^3 + 11\xi^4)}{(R^r+\xi)^2(R^r+2\xi)^2(R^r+3\xi)^2} > 0 \tag{52}$$

Replace the second-order Taylor expansion of $\ln f(R_t^r)$ into the expression for aggregate employment, and subtract from both sides the log of the steady state. Since $\ln(\Delta) = \ln(1) = \delta = 0$ and $\ln(A) = \ln(1) = a = 0$ we have:

$$\hat{n}_t = \delta_t + \hat{y}_t - a_t - \mathcal{R}_1\hat{R}_t^r + \frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2$$

We can plug this into our utility function:

$$U(C_t, N_t) \simeq U + U_C Y \left(\widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2\right) + U_N N \left[\delta_t + \hat{y}_t - a_t - \mathcal{R}_1 \hat{R}_t^r + \frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2\right]$$
$$+ U_N N \frac{1+\varphi}{2}\left[\delta_t + \hat{y}_t - a_t - \mathcal{R}_1 \hat{R}_t^r + \frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2\right]^2 + t.i.p.$$

Now, we use the following Lemma, proven in ch. 3.4 of Galí (2015): $\delta_t = \frac{\theta}{2} var\left\{p_t(i)\right\}$. This is valid in the neighborhood of a symmetric steady state and up to a second order approximation. Using this Lemma, and the fact that terms of order higher than 2 can be dropped out of the approximation, yields:

$$U(C_t, N_t) \simeq U + U_C Y \left(\widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2\right)$$
$$+ U_N N \left(\frac{\theta}{2} var\left\{p_t(i)\right\} + \hat{y}_t - a_t - \mathcal{R}_1 \hat{R}_t^r + \frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2 + \frac{1+\varphi}{2}\left(\hat{y}_t - a_t - \mathcal{R}_1 \hat{R}_t^r\right)^2\right) + t.i.p.$$

Step 4: Divide everything by $U_c C$ so to express the approximation as a percentage of steady state consumption:

$$\frac{U(C_t, N_t) - U}{U_C Y} \simeq \widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2$$
$$+ \frac{U_N N}{U_C Y}\left(\frac{\theta}{2} var\left\{p_t(i)\right\} + \hat{y}_t - a_t - \mathcal{R}_1 \hat{R}_t^r + \frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2 + \frac{1+\varphi}{2}\left(\hat{y}_t - a_t - \mathcal{R}_1 \hat{R}_t^r\right)^2\right) + t.i.p.$$

Combining the household first order condition with respect to labor, and input producer's labor demand condition and the definition of marginal costs, we get that:

$$-\frac{U_N}{U_C} = \frac{W}{P} = \frac{Aq\left(\omega_1 - \omega_2 q\right)}{\Phi}$$

Where we have defined $\Phi = \theta / (\theta - 1)$ as the steady-state markup.[33] We then get that:

$$\frac{U_N N}{U_C Y} = -\frac{Aq\left(\omega_1 - \frac{\omega_2}{2} q\right)}{\Phi} \frac{N}{Aq\left(\omega_1 - \frac{\omega_2}{2} q\right) N} = -\frac{1}{\Phi}$$

[33] In the absence of the subsidy on input goods, there would be an additional term related to the cost channel in the above equation, but the rest of the derivations would be unaffected.

Define Θ such that:

$$1 - \Theta = \frac{1}{\Phi}$$

We can exploit the definition of Θ as well as the assumption of a small steady state distortion (so any interaction with terms of order ≥ 2 can be eliminated) to re-express the utility approximation as:

$$\frac{U(C_t, N_t) - U}{U_C Y} \simeq \widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2 - \frac{\theta}{2} var\left\{p_t(i)\right\} - \hat{y}_t + a_t + \mathcal{R}_1 \hat{R}_t^r - \frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2 - \frac{1+\varphi}{2}\left(\hat{y}_t - a_t - \mathcal{R}_1 \hat{R}_t^r\right)^2 + \Theta\left(\hat{y}_t - a_t - \mathcal{R}_1 \hat{R}_t^r\right) + t.i.p.$$

Note also that a_t is independent of policy and hence can go into the t.i.p. After some rearranging of terms, we get:

$$\frac{U(C_t, N_t) - U}{U_C Y} \simeq \Theta \hat{y}_t + (1-\Theta)\,\mathcal{R}_1 \hat{R}_t^r - \frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2 - \frac{1}{2}\left[\theta var\left\{p_t(i)\right\} - (1-\sigma)\,\widehat{y}_t^2 + (1+\varphi)\left(\hat{y}_t - a_t - \mathcal{R}_1 \hat{R}_t^r\right)^2\right] + t.i.p.$$

Step 5: Collect terms related to output deviations and re-express them as output gap deviations. Open the square bracket and collect terms related to output deviations squared:

$$\frac{U(C_t, N_t) - U}{U_C Y} \simeq \Theta \hat{y}_t + (1-\Theta)\,\mathcal{R}_1 + 1 - \frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2 - \frac{1}{2}\left[\theta var\left\{p_t(i)\right\} + (\sigma+\varphi)\,\hat{y}_t^2 + (1+\varphi)\,a_t^2\right] - \frac{1}{2}\left[(1+\varphi)\left(\mathcal{R}_1 \hat{R}_t^r\right)^2 - 2\,(1+\varphi)\,\hat{y}_t \mathcal{R}_1 \hat{R}_t^r + 2\,(1+\varphi)\,a_t \mathcal{R}_1 \hat{R}_t^r\right] + t.i.p.$$

Re-express productivity as a function of the efficient level of output y_t^e where needed. Recall that y_t^e is independent of policy and can be expressed as:

$$\widehat{y}_t^e = \frac{1+\varphi}{\sigma+\varphi} a_t \tag{53}$$

Denoting the output gap as $x_t = y_t - y_t^e$ we can express the utility approximation as:

$$\frac{U(C_t,N_t)-U}{U_C Y} \simeq \Theta\widehat{x}_t+(1-\Theta)\,\mathcal{R}_1\hat{R}_t^r-\frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2-\frac{1}{2}\left[\theta var\left\{p_t(i)\right\}+(\sigma+\varphi)\,\hat{x}_t^2\right]$$
$$-\frac{1}{2}(1+\varphi)\left(a_t^2+\mathcal{R}_1^2\left(\hat{R}_t^r\right)^2-2\hat{y}_t\mathcal{R}_1\hat{R}_t^r+2a_t\mathcal{R}_1\hat{R}_t^r\right)+t.i.p.$$

Step 6: Express $var\left\{p_t(i)\right\}$ as a function of inflation. From Woodford (2000) and Lemma 2 in Ch. 4 of Galí (2015) we know that: $var\left\{p_t(i)\right\}\approx \omega var\left\{p_{t-1}(i)\right\}+\frac{\omega}{1-\omega}\pi_t^2$, where ω is the Calvo parameter. So we have that:

$$\sum_{t=0}^{\infty}\beta^t var\left\{p_t(i)\right\}=\frac{\omega}{(1-\beta\omega)(1-\omega)}\sum_{t=0}^{\infty}\beta^t\pi_t^2+t.i.p$$

Step 7: Get the approximated present discounted value of the welfare loss function:

$$\mathbb{W}=-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{U(C_t,N_t)-U}{U_cC}=-E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\Theta\widehat{x}_t+(1-\Theta)\,\mathcal{R}_1\hat{R}_t^r-\frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2\right.\right.$$
$$\left.\left.-\frac{1}{2}\left[\theta var\left\{p_t(i)\right\}+(\sigma+\varphi)\,\hat{x}_t^2+(1+\varphi)\left(a_t^2+\mathcal{R}_1^2\left(\hat{R}_t^r\right)^2-2\hat{y}_t\mathcal{R}_1\hat{R}_t^r+2a_t\mathcal{R}_1\hat{R}_t^r\right)\right]\right]\right\}$$

Using step 6 we get:

$$\mathbb{W}=-\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\Theta\widehat{x}_t+(1-\Theta)\,\mathcal{R}_1\hat{R}_t^r-\frac{\mathcal{R}_2}{2}\left(\hat{R}_t^r\right)^2-\frac{1}{2}\frac{\omega\theta}{(1-\omega)(1-\omega\beta)}\pi_t^2-\frac{1}{2}(\sigma+\varphi)\,\hat{x}_t^2\right.\right.$$
$$\left.\left.-\frac{1}{2}(1+\varphi)\left(a_t^2+\mathcal{R}_1^2\left(\hat{R}_t^r\right)^2-2\hat{y}_t\mathcal{R}_1\hat{R}_t+2a_t\mathcal{R}_1\hat{R}_t^r\right)\right]\right\}$$

Collect the terms related to the real interest rate, remember that a_t is independent of policy and re-express $\hat{y}_t$ in terms of the output gap. This yields:

$$\mathbb{W}=-\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\Theta\widehat{x}_t-\frac{1}{2}\frac{\omega\theta}{(1-\omega)(1-\omega\beta)}\pi_t^2-\frac{1}{2}(\sigma+\varphi)\,\hat{x}_t^2+(1-\Theta)\,\mathcal{R}_1\hat{R}_t^r\right.\right.$$

$$-\frac{1}{2}\left(\left(1+\varphi\right)\mathcal{R}_1^2+\mathcal{R}_2\right)\left(\hat{R}_t^r\right)^2-\mathcal{R}_1\left(\sigma-1\right)\hat{R}_t^r\hat{y}_t^e+\mathcal{R}_1\left(1+\varphi\right)\hat{R}_t^r\hat{x}_t\Big]\Big\}$$

The first three terms in this approximation are standard. Welfare loss increases with distortions in the current output gap (from the first best level), and with the volatility of inflation and in the output gap. The remaining terms derive from the inclusion of the risk-taking channel:

- $-\left(1-\Theta\right)\mathcal{R}_1\hat{R}_t^r$: A higher real interest rate, decreases the inefficiency of risk taking, reducing the welfare loss.
- $\left(\left(1+\varphi\right)\mathcal{R}_1^2+\mathcal{R}_2\right)\left(\hat{R}_t^r\right)^2$: The real rate affects the efficiency of the banks' investment choice and through that the productivity of labor (TFP). Volatility in the real interest rate makes TFP more volatile and reduces it on average (due to the concavity of f). These two effects, which are captured by the two coefficients, imply that real rate volatility lowers welfare.
- $\mathcal{R}_1\left(\sigma-1\right)\hat{R}_t^r\hat{y}_t^e$: For a risk aversion parameter σ greater than unity, this term is clearly positive, implying that welfare losses increase in the covariance between the real interest rate gap and the efficient level of output (i.e. productivity). A negative productivity shock (a fall in the efficient level of output) coupled with a fall in the real interest rate would imply an even larger negative productivity shock, amplifying the welfare loss. This is because a lower real rate increases the inefficiency of risk-taking, lowering the marginal productivity of labor.
- $-\mathcal{R}_1\hat{R}_t^r\left(1+\varphi\right)\hat{x}_t$: Welfare losses decrease in the covariance between the output gap the real interest rate gap. The intuition is the same as for the previous covariance term: A positive output gap coupled with an increase in the real interest rate implies an additional increase in the output gap, increasing welfare. This is because a higher real rate decreases the inefficiency of risk-taking, increasing the marginal productivity of labor and through that aggregate output.

Step 8: Assume correction of the steady state distortion through fiscal instruments. If we assume that the steady state is undistorted then $\Theta=0$ and $\mathcal{R}_1=0$ and $\hat{x}_t=x_t$. The second equality follows from the first derivative of $f(R^r)$

being equal to zero, given the optimal steady state risk choice. The approximated present discounted value of the welfare loss simplifies to:

$$\mathbb{W} = -\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \frac{\omega\theta}{(1-\omega)(1-\omega\beta)} \pi_t^2 - \frac{1}{2}(\sigma+\varphi) x_t^2 - \frac{1}{2} \mathcal{R}_2 \left(\hat{R}_t^r \right)^2 \right] \right\}$$

Hence, only the variance of the real interest rate remains as an additional term in the welfare loss function.

Defining $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ and $\lambda = \frac{\kappa}{\theta}(\sigma+\varphi)$ we can equivalently write the loss function as

$$\mathbb{W} = \frac{1}{2} \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2 + \frac{\kappa}{\theta} \mathcal{R}_2 \left(\hat{R}_t^r \right)^2 \right] \right\}$$

B2: The linearized Phillips curve

We can express the Phillips curve as (Galí (2015), ch.3):

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] - \kappa \tilde{\phi}_t \tag{54}$$

with $\tilde{\phi}_t$ being the deviation between the average and the desired markup and $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$. Note that the average price markup is equal to the inverse of real marginal costs, defined in equation (31):

$$\phi_t = p_t - (w_t - a_t - \ln f(R_t^r))$$

where small-case letters denote logs. We can then substitute the household's labor choice ($\varphi n_t + \sigma c_t = w_t - p_t$) and use $y_t = c_t$, yielding:

$$\phi_t = -(\varphi n_t + \sigma y_t) + a_t + \ln f(R_t^r)$$

Substitute for n_t using $n_t = y_t - a_t - \ln f(R_t^r)$

$$\phi_t = -(\sigma+\varphi) y_t + (1+\varphi) a_t + (1+\varphi) \ln f(R_t^r)$$

Under flexible prices, the markup is equal to the desired level ($\Phi_t^n = \theta_t / (\theta_t - 1)$):

$$\phi_t^n = -(\sigma+\varphi) y_t^n + (1+\varphi) a_t + (1+\varphi) \ln f(R_t^{r,n}) \tag{55}$$

Get an expression for $\tilde{\phi}_t$, the deviation between the average and the desired markup:

$$\tilde{\phi}_t = -(\sigma+\varphi)(y_t - y_t^n) + (1+\varphi)(\ln f(R_t^r) - \ln f(R_t^{r,n}))$$

Use the identity $y_t - y_t^n = (y_t - y_t^e) + (y_t^e - y_t^n)$:

$$\tilde{\phi}_t = -(\sigma+\varphi)(y_t - y_t^e) - (\sigma+\varphi)(y_t^e - y_t^n) + (1+\varphi)(\ln f(R_t^r) - \ln f(R_t^{r,n}))$$

Use the definition of the welfare relevant output gap:

$$\tilde{\phi}_t = -(\sigma+\varphi)x_t - (\sigma+\varphi)(y_t^e - y_t^n) + (1+\varphi)(\ln f(R_t^r) - \ln f(R_t^{r,n}))$$

Subtract the steady state, and denote with "hat" deviations from the steady state.

$$\begin{aligned}\tilde{\phi}_t - 0 = &-(\sigma+\varphi)\hat{x}_t - (\sigma+\varphi)(\hat{y}_t^e - \hat{y}_t^n) \\ &+ (1+\varphi)[\ln f(R_t^r) - \ln f(R^r) - \ln f(R_t^{r,n}) + \ln f(R^{r,n})]\end{aligned}$$

A first order Taylor expansion of the bracket in the second line yields $\mathcal{R}_1\hat{R}_t^r - \mathcal{R}_1\hat{R}_t^{r,n}$ (see Appendix B1 step 3) so we can write

$$\tilde{\phi}_t = -(\sigma+\varphi)\hat{x}_t + (1+\varphi)\mathcal{R}_1\hat{R}_t^r - (1+\varphi)\mathcal{R}_1\hat{R}_t^{r,n} - (\sigma+\varphi)(\hat{y}_t^e - \hat{y}_t^n)$$

Using the definitions of $\hat{y}_t^e$ and $\hat{y}_t^n$ (53) and (55) we can rewrite this as:

$$\tilde{\phi}_t = -(\sigma+\varphi)\hat{x}_t + (1+\varphi)\mathcal{R}_1\hat{R}_t^r - \hat{\phi}_t$$

Substitute the above equation into the first version of the Phillips curve (54):

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \kappa(\sigma+\varphi)\hat{x}_t - \kappa(1+\varphi)\mathcal{R}_1\hat{R}_t^r + u_t$$

Where we have defined the cost-push shock as $u_t = \kappa\hat{\phi}_t$, i.e. the term capturing short-run deviations of the desired markup caused by movements in the parameter θ_t.

Under the assumption of an efficient steady state the expression simplifies to the standard Phillips curve, since $\mathcal{R}_1 = 0$.

$$\pi_t = \beta \mathbb{E}_t \left[\pi_{t+1}\right] + \kappa \left(\sigma + \varphi\right) x_t + u_t$$

B3: Optimal monetary policy

Assuming no steady-state distortions, the monetary policy problem is:

$$\min \frac{1}{2} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2 + \frac{\kappa}{\theta} \mathcal{R}_2 \hat{R}_t^{r\,2} \right] \right\} \tag{56}$$

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma} \left(\hat{R}_t - \mathbb{E}_t \pi_{t+1} \right) \qquad \forall t \tag{57}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left(\sigma + \varphi\right) x_t + u_t \qquad \forall t \tag{58}$$

$$\hat{R}_t^r = \hat{R}_t - \mathbb{E}_t \pi_{t+1} \qquad \forall t \tag{59}$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ is the coefficient on marginal costs in the New Keynesian Phillips curve, $\lambda = \frac{\kappa}{\theta} \left(\sigma + \varphi\right)$ denotes the weight of output gap fluctuations relative to inflation fluctuations in the loss function, θ is the elasticity of substitution between goods, and where we have already substituted $\hat{R}_t^r = \hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1}$. The term u_t is a cost-push shock that follows an AR process with autoregessive coefficient ρ.

B3.1: Optimal simple rule

To determine the Taylor-type optimal simple rule, we follow Clarida et al. (1999) and find an optimal simple rule that depends on only the exogenous state. We analyze the associated equilibrium and then solve for the equivalent Taylor-type rule. We present the results in the main text in the reverse order.

Consider a rule for the target output gap, contingent on the fundamental shock u_t

$$x_t = -\gamma u_t, \ \forall t \tag{60}$$

This particular rule is motivated by the fact that the optimal policy under discretion leads to a rule of such shape. Furthermore it is analytically convenient.

Combine this equation with the Phillips curve (58) and the IS curve (57), and get expressions for π_t and $\hat{R}_t^r$ that also depend on the fundamental shock:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} - \kappa (\sigma + \varphi) \gamma u_t + u_t$$

$$\hat{R}_t^r = \gamma \sigma (1 - \rho) u_t + \mathbb{E}_t \pi_{t+1} \tag{61}$$

Manipulate the Phillips curve to get:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + [1 - \kappa (\sigma + \varphi) \gamma] u_t$$

$$\pi_t = \mathbb{E}_t \sum_{i=0}^{\infty} \left\{ \beta^i \rho^i [1 - \kappa (\sigma + \varphi) \gamma] u_t \right\}$$

$$\pi_t = \frac{1 - \kappa (\sigma + \varphi) \gamma}{1 - \beta \rho} u_t \tag{62}$$

Using this we can rewrite the IS curve

$$\hat{R}_t^r = \gamma \sigma (1 - \rho) u_t + \frac{1 - \kappa (\sigma + \varphi) \gamma}{1 - \beta \rho} \rho u_t \tag{63}$$

The central bank chooses γ in order to minimize the loss function $U_t = \pi_t^2 + \lambda x_t^2 + \frac{\kappa}{\theta} \mathcal{R}_2 \left(\hat{R}_t^r \right)^2$ subject to the contemporaneous Phillips and IS curve (62) and (63) and the policy rule (60). Plugging in, the optimal policy problem can be re-expressed as choosing the value of γ that maximizes the following objective function:

$$\left(\frac{1 - \kappa (\sigma + \varphi) \gamma}{1 - \beta \rho} \right)^2 u_t^2 + \lambda \gamma^2 u_t^2 + \frac{\kappa}{\theta} \mathcal{R}_2 (\gamma \sigma (1 - \rho))^2 u_t^2$$

The FOC yields the following solution for γ:

$$\gamma = \frac{(\sigma + \varphi) \kappa}{(1 - \rho\beta)^2 \left(\frac{\kappa^2 (\sigma+\varphi)^2}{(1-\beta\rho)^2} + \lambda + \frac{\kappa (1-\rho)^2 \mathcal{R}_2 \sigma^2}{\theta} \right)} \tag{64}$$

This solution can be substituted in equations (60), (63) and (62) to get the policy functions for the output gap, inflation and the nominal and real interest rate.

$$x_t = a u_t = -\gamma u_t \tag{65}$$

$$\pi_t = bu_t = \frac{\left(1 - \frac{\theta\kappa^2(\sigma+\varphi)^2}{\theta\left(\lambda(1-\beta\rho)+\kappa^2(\sigma+\varphi)^2\right)+\kappa(1-\rho)^2\mathcal{R}_2\sigma^2(1-\beta\rho)^2}\right)}{1-\beta\rho} u_t \tag{66}$$

$$R_t = cu_t = -\frac{\left(\theta(\lambda\rho(1-\beta\rho) - \kappa(1-\rho)\sigma(\sigma+\varphi)) - \kappa(1-\rho)^2\rho\mathcal{R}_2\sigma^2(1-\beta\rho)\right)}{\theta\left(\lambda(1-\beta\rho)^2 + \kappa^2(\sigma+\varphi)^2\right) + \kappa(1-\rho)^2\mathcal{R}_2\sigma^2(1-\beta\rho)^2} u_t \tag{67}$$

$$R_t^r = du_t = \frac{\theta\kappa(1-\rho)\sigma(\sigma+\varphi)}{\theta\left(\lambda(1-\beta\rho)^2 + \kappa^2(\sigma+\varphi)^2\right) + \kappa(1-\rho)^2\mathcal{R}_2\sigma^2(1-\beta\rho)^2} u_t \tag{68}$$

The parameters multiplying the shock also determine the standard deviation of the variables of interest up to a constant, which is the standard deviation of the exogenous shock. We can compute the rate of change of these standard deviations with respect to the risk-taking channel parameter $\mathcal{R}_2$ to understand how the risk-taking channel affects these standard deviations:

$$\sigma_{x,\mathcal{R}_2} = -\frac{\kappa(1-\rho)^2\sigma^2}{\theta\left(\frac{\kappa^2(\sigma+\varphi)^2}{(1-\beta\rho)^2} + \lambda + \frac{\kappa(1-\rho)^2\mathcal{R}_2\sigma^2}{\theta}\right)} < 0 \tag{69}$$

$$\sigma_{\pi,\mathcal{R}_2} = \frac{\theta\kappa^3(1-\rho)^2\sigma^2(\sigma+\varphi)^2}{\left(\theta\lambda + \kappa(1-\rho)^2\mathcal{R}_2\sigma^2\right)\left(\theta\left(\lambda(1-\beta\rho)^2 + \kappa^2(\sigma+\varphi)^2\right) + \kappa(1-\rho)^2\mathcal{R}_2\sigma^2(1-\beta\rho)^2\right)} > 0 \tag{70}$$

$$\sigma_{R,\mathcal{R}_2} = -\frac{\theta\kappa^2(1-\rho)^2\sigma^2(1-\beta\rho)(\sigma+\varphi)\left(\sigma\left(-\rho(\beta+\kappa) + \beta\rho^2 + (1-\rho)\right) - \kappa\rho\varphi\right)}{\left(\theta(\lambda\rho(1-\beta\rho) + \kappa(1-\rho)\sigma(\sigma+\varphi)) + \kappa(1-\rho)^2\rho\mathcal{R}_2\sigma^2(1-\beta\rho)\right)\left(\theta\left(\lambda(1-\beta\rho)^2 + \kappa^2(\sigma+\varphi)^2\right) + \kappa(1-\rho)^2\mathcal{R}_2\sigma^2(1-\beta\rho)^2\right)} \tag{71}$$

$$\sigma_{R^r,\mathcal{R}_2} = -\frac{\kappa(1-\rho)^2\sigma^2(1-\beta\rho)^2}{\theta\left(\lambda(1-\beta\rho)^2 + \kappa^2(\sigma+\varphi)^2\right) + \kappa(1-\rho)^2\mathcal{R}_2\sigma^2(1-\beta\rho)^2} < 0 \tag{72}$$

$\sigma_{x,\mathcal{R}_2}$ and $\sigma_{R^r,\mathcal{R}_2}$ are evidently negative while $\sigma_{\pi,\mathcal{R}_2}$ is evidently positive, as in the case without commitment. The sign of $\sigma_{R,\mathcal{R}_2}$ is ambiguous as before. However, for low enough values of ρ the derivative of the standard deviation with respect to the risk-taking channel parameter is negative. We can see this by setting ρ to zero, which yields $\frac{\partial\sigma_r/\partial\mathcal{R}_2}{\sigma_r} = -\frac{\kappa\sigma^2}{\theta(\kappa^2(\sigma+\varphi)^2+\lambda)+\kappa\mathcal{R}_2\sigma^2} < 0$. Overall, we can conclude that the inclusion of the risk-taking channel implies a lower output gap and real interest rate volatility and a higher inflation volatility under optimal policy.

As in Clarida et al. (1999), we can reexpress the policy rule for the nominal interest rate as a function of expected future inflation. The parameter multiplying expected inflation is the optimal Taylor rule parameter ϕ_π^c, describing how the central bank should react to expected inflation:

$$\hat{R}_t = \phi_\pi^s \mathbb{E}_t \pi_{t+1} = \left[1 + \frac{\theta\kappa\sigma(1-\rho)(\sigma+\varphi)}{\rho(1-\beta\rho)\left(\theta\lambda + \kappa(1-\rho)^2\mathcal{R}_2\sigma^2\right)}\right] \mathbb{E}_t \pi_{t+1} \tag{73}$$

We can see that the presence of the risk-taking channel lowers the optimal response of the nominal interest rate to expected inflation, i.e.:

$$\frac{\partial \phi_\pi^s}{\partial \mathcal{R}_2} = -\frac{\theta\kappa(\rho-1)^3\sigma^3(\sigma(\kappa+\rho-1)+\kappa\varphi)}{(\rho(\beta+\kappa)-1)\left(\theta\lambda+\kappa(\rho-1)^2\mathcal{R}_2\sigma^2\right)^2} - \frac{\theta\kappa^2(1-\rho)^3\sigma^3(\sigma+\varphi)}{\rho(1-\beta\rho)\left(\theta\lambda+\kappa(1-\rho)^2\sigma^2\right)^2} < 0 \tag{74}$$

B3.2: Ramsey-optimal policy

Under full commitment, the central bank's problem is to maximizes (56) , by choosing conditional paths for inflation, the output gap and the interest rate. The associated Lagrangian is given by

$$\begin{aligned}\mathcal{L}_0 = -\frac{1}{2}\mathbb{E}_0 \Bigg\{ \sum_{t=0}^{\infty} \beta^t \bigg[\pi_t^2 + \lambda x_t^2 + \frac{\kappa}{\theta}\mathcal{R}_2\left(\hat{R}_t^r\right)^2\bigg] \\ + \chi_t \left[x_t - x_{t+1} + \frac{1}{\sigma}\left(\hat{R}_t - \pi_{t+1}\right)\right] \\ + \psi_t \left[\pi_t - \beta\pi_{t+1} - \kappa(\sigma+\varphi)x_t - u_t\right] \\ + \varsigma_t\left(-\hat{R}_t^r + \hat{R}_t - \pi_{t+1}\right)\Bigg\}\end{aligned} \tag{75}$$

The multipliers associated to the Phillips curve, the IS curve and the definition of the real rate are respectively ψ_t, χ_t and ς_t. The FOCs wrt. π_t, x_t, $\hat{R}_t$, $\hat{R}_t^r$, ς_{tt} are:

$$\begin{aligned}\pi_t + \psi_t - \psi_{t-1} - \frac{1}{\sigma\beta}\chi_{t-1} - \frac{1}{\beta}\varsigma_{t-1} = 0 \\ \lambda x_t - \kappa(\sigma+\varphi)\psi_t + \chi_t - \frac{1}{\beta}\chi_{t-1} = 0 \\ \frac{1}{\sigma}\chi_t + \varsigma_t = 0 \\ \frac{\kappa}{\theta}\mathcal{R}_2\hat{R}_t^r - \varsigma_t = 0 \\ x_t - x_{t+1} + \frac{1}{\sigma}\left(\hat{R}_t - \pi_{t+1}\right) = 0\end{aligned}$$

$$\pi_t - \beta\pi_{t+1} - \kappa(\sigma+\varphi)x_t - u_t = 0$$
$$-\hat{R}_t^r + \hat{R}_t - \pi_{t+1} = 0$$

with $\chi_{-1} = \psi_{-1} = 0$. We can eliminate $\hat{R}_t^r$ and ς_{tt} to simplify the system somewhat

$$\pi_t + \psi_t - \psi_{t-1} - \frac{1}{\sigma\beta}\chi_{t-1} - \frac{\kappa}{\beta\theta}\mathcal{R}_2\left(\hat{R}_{t-1} - \pi_t\right) = 0$$
$$\lambda x_t - \kappa(\sigma+\varphi)\psi_t + \chi_t - \frac{1}{\beta}\chi_{t-1} = 0$$
$$\frac{1}{\sigma}\chi_t + \frac{\kappa}{\theta}\mathcal{R}_2\left(\hat{R}_t - \pi_{t+1}\right) = 0$$
$$x_t - x_{t+1} + \frac{1}{\sigma}\left(\hat{R}_t - \pi_{t+1}\right) = 0$$
$$\pi_t - \beta\pi_{t+1} - \kappa(\sigma+\varphi)x_t - u_t = 0$$

Unfortunately, no simple analytical solution is available for the rational expectations equilibrium defined by these conditions.
However, is possible to combine the first three equations to derive the following implicit instrument rule as in Giannoni and Woodford (2003). This rule applies from $t \geq 2$ and is optimal from a timeless perspective:

$$\hat{R}_t = \rho_1\hat{R}_{t-1} + \rho_2\Delta\hat{R}_{t-1} + \phi_{E\pi}\mathbb{E}_t\pi_{t+1} + \phi_\pi\pi_t + \phi_{\pi_{-1}}\pi_{t.-1} + \phi_x\Delta x_t$$

where

$$\begin{aligned}
\rho_1 &= 1 \\
\rho_2 &= \frac{1}{\beta} \\
\phi_{E\pi} &= 1 \\
\phi_\pi &= \frac{\theta\sigma + \theta\varphi}{\mathcal{R}_2\sigma} - \frac{1}{\beta} - 1 \\
\phi_{\pi-1} &= \frac{1}{\beta} \\
\phi_x &= \frac{\theta\lambda}{\mathcal{R}_2\kappa\sigma}
\end{aligned}$$

As in Giannoni and Woodford (2003), this Taylor-type rule exhibits a nontrivial degree of persistence: $\rho_1 = 1, \rho_2 \gg 0$. By construction, the optimal Taylor rule under no commitment or under optimal simple rules does not feature any persistence. As Woodford (2001) shows, in the standard NKM – i.e. in this model without the risk-taking channel – the fully optimal interest rate paths do not involve any explicit reference to the path of interest rates either.[34] Thus, under fully optimal policy the risk-taking channel requires persistent policy responses. This is a result of the fact that the interest rate appears in the welfare function.

The risk-taking channel thus provides an additional explanation for interest rate inertia, which is routinely built into Taylor rules in models, and which is typically observed in practice. It augments other theories such as the zero lower bound or the cost of holding money, which regularly motivate researchers to include the interest rate – the nominal one, not the real one as in our case – in the welfare function and which also lead to inertia under optimal policy.

B3.3: Optimal discretionary policy

We now assume that the central bank cannot credibly commit itself to any future action and cannot therefore influence expectations on future variables. The central bank problem simplifies to one of sequential optimisation, i.e. the central bank chooses output and inflation in order to minimise the period losses $U_t = \pi_t^2 + \lambda x_t^2 + \frac{\kappa}{\theta}\mathcal{R}_2\left(\hat{R}_t - \mathbb{E}_t\pi_{t+1}\right)^2$ subject to the contemporaneous IS and Phillips curve, whose multipliers are denoted by χ_t and ψ_t . Under optimal discretion, the first-order conditions for the central bank problem are:

$$\frac{\partial U_t}{\partial \pi_t} = -\pi_t + \psi_t = 0$$

$$\frac{\partial U_t}{\partial x_t} = -\lambda x_t + \chi_t - \kappa\left(\sigma + \varphi\right)\psi_t = 0$$

$$\frac{\partial U_t}{\partial \hat{R}_t} = -\frac{\kappa}{\theta}\mathcal{R}_2\left(\hat{R}_t - \mathbb{E}_t\pi_{t+1}\right) + \frac{1}{\sigma}\chi_t = 0$$

The above conditions imply the following equilibrium relationship between inflation, the output gap and the real interest rate, under optimal discretionary monetary policy:

[34] Note that the above rule does not nest a rule for the standard NKM. However, as the weight on real rate stabilization $\frac{\kappa}{\theta}\mathcal{R}_2^2$ goes towards 0, past interest rates become less important in the determination of current interest rate, relative to deviations of output and inflation.

$$\pi_t = -\frac{\lambda}{\kappa(\sigma+\varphi)} x_t + \frac{\sigma\frac{\kappa}{\theta}\mathcal{R}_2}{\kappa(\sigma+\varphi)} \left(\hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1}\right) \tag{76}$$

Next, we derive the policy functions for the key variables of interest. We find them using the method of undetermined coefficients. Since there are no endogenous states, the policy functions must be linear functions in the cost-push shock u_t. We assume therefore the following policy functions: $x_t = au_t$, $\pi_t = bu_t$ $\hat{R}_t = cu_t$ and $\hat{R}^r_t = du_t$. Since the cost-push shock is AR(1), we also know that $Ex_{t+1} = a\rho u_t$ and $E\pi_{t+1} = b\rho u_t$. Combining these functions with the Phillips curve (58), the IS curve (57), the definition of the real rate and the central bank's optimality condition (76), we can derive the following coefficients:

$$a = -\frac{\theta\kappa(\sigma+\varphi)}{\theta\left(-\beta\lambda\rho + \kappa^2(\sigma+\varphi)^2 + \lambda\right) + \kappa(\rho-1)\mathcal{R}_2\sigma^2(\beta\rho-1)} \tag{77}$$

$$b = \frac{\theta\lambda - \kappa(\rho-1)\mathcal{R}_2\sigma^2}{\theta\left(-\beta\lambda\rho + \kappa^2(\sigma+\varphi)^2 + \lambda\right) + \kappa(\rho-1)\mathcal{R}_2\sigma^2(\beta\rho-1)} \tag{78}$$

$$c = \frac{\theta(\lambda\rho - \kappa(\rho-1)\sigma(\sigma+\varphi)) - \kappa(\rho-1)\rho\mathcal{R}_2\sigma^2}{\theta\left(-\beta\lambda\rho + \kappa^2(\sigma+\varphi)^2 + \lambda\right) + \kappa(\rho-1)\mathcal{R}_2\sigma^2(\beta\rho-1)} \tag{79}$$

$$d = -\frac{\theta\kappa(\rho-1)\sigma(\sigma+\varphi)}{\theta\left(-\beta\lambda\rho + \kappa^2(\sigma+\varphi)^2 + \lambda\right) + \kappa(\rho-1)\mathcal{R}_2\sigma^2(\beta\rho-1)} \tag{80}$$

The absolute values of these coefficients also determine the standard deviation of the output gap, inflation and the nominal and real interest rates, up to a scaling factor which is the standard deviation of the cost-push shock. We are interested in establishing how these standard deviations change with the risk-taking channel. To do so, we derive the rate of change of the four coefficients with respect to the risk-taking channel parameter $\mathcal{R}_2$ – e.g. $\sigma_{x,\mathcal{R}_2} = \frac{\partial a}{\partial \mathcal{R}_2} a^{-1}$ – and check the sign:

$$\sigma_{x,\mathcal{R}_2} = -\frac{\kappa(1-\rho)\sigma^2(1-\beta\rho)}{\theta\left(\lambda(1-\beta\rho) + \kappa^2(\sigma+\varphi)^2\right) + \kappa(1-\rho)\mathcal{R}_2\sigma^2(1-\beta\rho)} < 0 \tag{81}$$

$$\sigma_{\pi,\mathcal{R}_2} = \frac{\theta\kappa^3(1-\rho)\sigma^2(\sigma+\varphi)^2}{\left(\theta\lambda + \kappa(1-\rho)\mathcal{R}_2\sigma^2\right)\left(\theta\left(\lambda(1-\beta\rho) + \kappa^2(\sigma+\varphi)^2\right) + \kappa(1-\rho)\mathcal{R}_2\sigma^2(1-\beta\rho)\right)} > 0 \tag{82}$$

$$\sigma_{R,\mathcal{R}_2} = \frac{\theta\kappa^2(1-\rho)\sigma^2(\sigma+\varphi)(\sigma(\rho(\beta(1-\rho)+\kappa+1)-1)+\kappa\rho\varphi)}{\left(\theta(\lambda\rho + \kappa(1-\rho)\sigma(\sigma+\varphi)) + \kappa(1-\rho)\rho\mathcal{R}_2\sigma^2\right)\left(\theta\left(\lambda(1-\beta\rho) + \kappa^2(\sigma+\varphi)^2\right) + \kappa(1-\rho)\mathcal{R}_2\sigma^2(1-\beta\rho)\right)} \tag{83}$$

$$\sigma_{R^r,\mathcal{R}_2} = -\frac{\kappa(1-\rho)\sigma^2(1-\beta\rho)}{\theta\left(\lambda(1-\beta\rho)+\kappa^2(\sigma+\varphi)^2\right)+\kappa(1-\rho)\mathcal{R}_2\sigma^2(1-\beta\rho)} < 0 \tag{84}$$

Given that $\rho < 1$ and $\beta < 1$, it is straightforward to see that all terms in the numerators and denominators of (81), (82) and (84) are positive. Thus $\sigma_{x,\mathcal{R}_2}$and $\sigma_{R^r,\mathcal{R}_2}$ are negative, implying that the standard deviation of the output gap under optimal policy is lower when the risk-taking channel is present. By contrast, $\sigma_{\pi,\mathcal{R}_2}$ is negative, indicating that the standard deviation of inflation increases with the risk-taking channel.

The sign of $\sigma_{R,\mathcal{R}_2}$ in equation (83) is ambiguous. While the denominator is clearly positive, the sign of the numerator depends on the value of the autoregressive parameter ρ. For low enough values of this parameter, the derivative of the standard deviation with respect to the risk-taking channel parameter is negative. We can see this, by considering the case of an i.i.d. shock (ρ=0). In this case, equations (81) to (84) become:

$$\sigma_{x,\mathcal{R}_2} = \sigma_{R,\mathcal{R}_2} = \sigma_{R^r,\mathcal{R}_2} = -\frac{\kappa\sigma^2}{\theta\left(\kappa^2(\sigma+\varphi)^2+\lambda\right)+\kappa\mathcal{R}_2\sigma^2} < 0$$

$$\sigma_{\pi,\mathcal{R}_2} = \frac{\theta\kappa^3(\sigma+\varphi)^2}{(\theta\lambda+\kappa\mathcal{R}_2\sigma^2)\left(\theta\left(\kappa^2(\sigma+\varphi)^2+\lambda\right)+\kappa\mathcal{R}_2\sigma^2\right)} > 0$$

To derive a policy rule that implements the above equilibrium, we re-express the nominal interest rate as a function of expected future inflation (see Clarida et al. (1999)). The parameter multiplying expected inflation can be interpreted as an optimal Taylor rule parameter ϕ_π^d, describing how the central bank should react to expected inflation under optimal discretionary policy:

$$\hat{R}_t = \phi_\pi^d \mathbb{E}_t\pi_{t+1t} = \frac{\theta(\lambda\rho+\kappa(1-\rho)(\sigma+\varphi)\varphi)+\kappa(1-\rho)\rho\mathcal{R}_2\sigma^2}{\rho\theta\lambda+\kappa(1-\rho)\rho\mathcal{R}_2\sigma^2}\mathbb{E}_t\pi_{t+1} \tag{85}$$

We can see that the presence of the risk-taking channel lowers the optimal response of the nominal interest rate to inflation, i.e.:

$$\frac{\partial\phi_\pi^d}{\partial\mathcal{R}_2} = -\frac{\theta\kappa^2(1-\rho)^2\sigma^3(\sigma+\varphi)}{\rho\left(\theta\lambda+\kappa(1-\rho)\mathcal{R}_2\sigma^2\right)^2} < 0 \tag{86}$$

Note that it suffices to take a derivative since the sign of equation (85) is clearly negative.

B3.4 Optimal simple rules with a distorted steady state

Assuming a small steady-state markup, the monetary policy problem is:

$$\min \mathbb{E}_{\mathrm{t}} \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\pi_t^2 + \frac{1}{2}\lambda \hat{x}_t^2 + \frac{1}{2}\frac{\kappa}{\theta} \left((1+\varphi)\,\mathcal{R}_1^2 + \mathcal{R}_2 \right) \left(\hat{R}_t^r \right)^2 \right.$$
$$\left. -\frac{\kappa}{\theta}\,(1-\Theta)\,\mathcal{R}_1 \hat{R}_t^r - \mathcal{R}_1 \frac{\kappa}{\theta}\,(1+\varphi)\,\hat{R}_t^r \hat{x}_t - \frac{\kappa}{\theta}\Theta \widehat{x}_t \right] \tag{87}$$

$$x_t = \mathbb{E}_{\mathrm{t}} x_{t+1} - \frac{1}{\sigma}\left(\hat{R}_t - \mathbb{E}_t \pi_{t+1} \right) \qquad \forall t \tag{88}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa\,(\sigma+\varphi)\,\hat{x}_t - \kappa\,(1+\varphi)\,\mathcal{R}_1 \hat{R}_t^r + u_t \qquad \forall t \tag{89}$$

$$\hat{R}_t^r = \hat{R}_t - \mathbb{E}_t \pi_{t+1} \qquad \forall t \tag{90}$$

Note that we have dropped $\hat{y}_t^e$ since the efficient level of output is constant in the absence of productivity shocks.

We proceed in two steps: First, we determine the optimal simple rule as in appendix B3.1 to analyze how the risk-taking channel affects the optimal behaviour of the central bank. Second, we determine how the risk-taking channel affects the volatility of macro variables via its impact on policy.

Optimal simple rule

We begin by finding the optimal simple rule. As before, we restrict ourselves to a one-instrument Taylor rule with zero steady-state inflation. This latter assumption is without loss of generality. To see this, recall that zero steady-state inflation is optimal in the three equations New Keynesian model (see Gali 2008). Since the real rate is exogenous to policy, it must also be optimal in the model with the risk-taking channel.

Proceeding similarly as in appendix B3.1, we derive an optimal simple rule in the form of a forward-looking Taylor rule. It is given by:

$$\hat{R}_t = \phi_\pi^{dss} \mathbb{E}_t \pi_{t+1} \tag{91}$$

where

$$\phi_\pi^{dss} = 1 + \frac{\theta\kappa(1-\rho)\sigma\left[\sigma\left((1-\rho)\mathcal{R}_1(\varphi+1)+1\right)+\varphi\right]}{\kappa(1-\rho)\sigma\rho(1-\beta\rho)\left((1-\rho)(\mathcal{R}_1)^2\sigma(\varphi+1)+2\mathcal{R}_1(\varphi+1)+(1-\rho)\mathcal{R}_2\sigma\right)+\rho(1-\beta\rho)\theta\lambda}$$

The risk-taking channel now manifests itself in two parameters $\mathcal{R}_1$ and $\mathcal{R}_2$. The derivatives of the Taylor rule coefficient ϕ_π^{dss} wrt. $\mathcal{R}_1$ and $\mathcal{R}_2$ are given by:

$$\frac{\partial\phi_\pi^{dss}}{\partial\mathcal{R}_1} = -\frac{\theta(1-\rho)^2\sigma^2(\varphi+1)\left(\sigma\left((1-\rho)\left((1-\rho)(\mathcal{R}_1)^2\sigma(\varphi+1)+2\mathcal{R}_1(\sigma+\varphi)-(1-\rho)\mathcal{R}_2\sigma\right)+1\right)+\varphi\right)}{\rho(1-\beta\rho)\left((1-\rho)^2\sigma^2\left((\mathcal{R}_1)^2(\varphi+1)+\mathcal{R}_2\right)+\sigma(1+2(1-\rho)\mathcal{R}_1(\varphi+1))+\varphi\right)^2} < 0$$

$$\frac{\partial\phi_\pi^{dss}}{\partial\mathcal{R}_2} = -\frac{\theta\kappa^2(1-\rho)^3\sigma^3(\sigma((1-\rho)\mathcal{R}_1(\varphi+1)+1)+\varphi)}{\rho(1-\beta\rho)\left(\theta\lambda+\kappa(1-\rho)\sigma\left((1-\rho)(\mathcal{R}_1)^2\sigma(\varphi+1)+2\mathcal{R}_1(\varphi+1)+(1-\rho)\mathcal{R}_2\sigma\right)\right)^2} < 0$$

The latter is clearly negative. The former is negative if the term in the numerator is positive, i.e. if:[35]

$$\left(\sigma\left((1-\rho)\left((1-\rho)\mathcal{R}_1^2\sigma(\varphi+1)+2\mathcal{R}_1(\sigma+\varphi)-(1-\rho)\mathcal{R}_2\sigma\right)+1\right)+\varphi\right) > 0 \tag{92}$$

From (51) and 52 it is evident that $\mathcal{R}_1^2 > \mathcal{R}_2$ since:

$$\mathcal{R}_1^2 \quad = \quad \frac{f_R^2}{f(R^r)^2} > \mathcal{R}_2 = \frac{f_R^2}{f(R^r)^2} - \frac{f_{RR}f(R^r)}{f(R^r)^2} > 0$$

Hence, (92) is indeed positive. An increase in the severity of both the first and the second-order effects of the risk-taking channel $\mathcal{R}_1$ and $\mathcal{R}_2$ unambiguously reduces the Taylor rule coefficient, just as in appendix B3.1.

[35] This condition has a mathematical interpretation: The parameters $\mathcal{R}_1, \mathcal{R}_2$ in the objective function (87) reduce the Taylor rule coefficient ϕ_π^{dss}, while the parameter $\mathcal{R}_1$ in the Phillips curve (89) increases this coefficient. This condition is satisfied if the effect through the Phillips curve is smaller than the countervailing effect through the objective function.

Dynamics under the optimal simple rule

Next, we analyze dynamics under this optimal simple rule. To do so we solve the following system, using the method of undetermined coefficients:

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(\hat{R}_t - \mathbb{E}_t \pi_{t+1}\right) \qquad \forall t \tag{93}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa (\sigma + \varphi) \hat{x}_t - \kappa (1 + \varphi) \check{\mathcal{R}}_1 \hat{R}_t^r + u_t \tag{94}$$

$$\hat{R}_t^r = \hat{R}_t - \mathbb{E}_t \pi_{t+1} \qquad \forall t \tag{95}$$

$$\hat{R}_t = \phi_\pi^{dss}(\bar{\mathcal{R}}_1, \bar{\mathcal{R}}_2) \mathbb{E}_t \pi_{t+1} \tag{96}$$

Note that in writing down the above system we have distinguished between (i) $\mathcal{R}_1$ in the Phillips curve (94), which we relabelled $\check{\mathcal{R}}_1$, and (ii) $\mathcal{R}_1$ and $\mathcal{R}_2$ in the central bank's policy function (96), which we relabelled $\bar{\mathcal{R}}_1$ and $\bar{\mathcal{R}}_2$. This allows us to distinguish the effect that the risk-taking channel has (i) through the change in the dynamics of the private sector (ii) through the change in the optimal behavior of the central bank. If the central bank is fully aware of the risk-taking channel naturally $\bar{\mathcal{R}}_x = \check{\mathcal{R}}_x$ for $x = 1, 2$. If the central bank behaves as if the channel didn't exist, then $\bar{\mathcal{R}}_x = 0$ (but still $\check{\mathcal{R}}_1 \neq 0$) .

We want to assess what different policy rules imply for the behaviour of the economy. To do so, we derive equilibrium outcomes wrt. $\bar{\mathcal{R}}_1$ and $\bar{\mathcal{R}}_2$. This tells us how the equilibrium in the risk-taking channel economy characterized by (93)-(95) changes when the central bank adjusts its policy rule *from* a rule that the central banks finds optimal when it underestimates the strength of the risk-taking channel *towards* the rule that is optimal given the actual strength of the risk-taking channel. This is in line with the numerical exercise we conduct in the last section of the paper.

The exercises yields the following policy functions: $x_t = au_t$, $\pi_t = bu_t$ $\hat{R}_t = cu_t$ and $\hat{R}_t^r = du_t$ where

$$a = \frac{\theta\kappa\left[\sigma\left((1-\rho)\bar{\mathcal{R}}_1(\varphi+1)+1\right)+\varphi\right]}{e} \tag{97}$$

$$b = \frac{(1-\beta\rho)\left(\theta\lambda + \kappa(1-\rho)\sigma\left((1-\rho)\left(\bar{\mathcal{R}}_1\right)^2\sigma(\varphi+1) + 2\bar{\mathcal{R}}_1(\varphi+1) + (1-\rho)\bar{\mathcal{R}}_2\sigma\right)\right)}{e} \tag{98}$$

$$\begin{aligned} c \;=\; & \frac{\theta\left(\lambda\left(\rho-\beta\rho^2\right) + \kappa(\rho-1)\sigma(\sigma((\rho-1)\bar{\mathcal{R}}_1(\varphi+1)-1)-\varphi)\right)}{e} \\ & - \frac{\kappa(\rho-1)\rho\sigma(\beta\rho-1)\left((\rho-1)\left(\bar{\mathcal{R}}_1\right)^2\sigma(\varphi+1) - 2\bar{\mathcal{R}}_1(\varphi+1) + (\rho-1)\bar{\mathcal{R}}_2\sigma\right)}{e} \end{aligned} \tag{99}$$

$$d = a \tag{100}$$

where

$$\begin{aligned} e \;=\; & \kappa(1-\rho)\sigma(1-\beta\rho)^2\left((1-\rho)\left(\bar{\mathcal{R}}_1\right)^2\sigma(\varphi+1) + (1-\rho)\bar{\mathcal{R}}_2\sigma + 2\bar{\mathcal{R}}_1(\varphi+1)\right) \\ & + \theta\left\{\lambda(1-\beta\rho)^2 + \kappa^2\left[\sigma\left((1-\rho)\bar{\mathcal{R}}_1(\varphi+1)+1\right)+\varphi\right]\left[\sigma\left((1-\rho)\check{\mathcal{R}}_1(\varphi+1)+1\right)+\varphi\right]\right\} > 0 \end{aligned}$$

Recall from section B3.1 that the coefficients a to d determine the standard deviations of the respective variables. Analogously to what we did in sections B3.1 and B3.3, we compute the rate of change of these standard deviations with respect to the policy related risk-taking channel parameters $\bar{\mathcal{R}}_1$ and $\bar{\mathcal{R}}_2$. Consider first the rates of change of standard deviations wrt. $\bar{\mathcal{R}}_2$.

$$\sigma_{x,\bar{\mathcal{R}}_2} = \sigma_{R^r,\bar{\mathcal{R}}_2} = -\frac{\kappa(1-\rho)^2\sigma^2(1-\beta\rho)^2}{e} < 0 \tag{101}$$

$$\sigma_{\pi,\bar{\mathcal{R}}_2} = \frac{\theta\kappa^3(1-\rho)^2\sigma^2(\sigma((1-\rho)\bar{\mathcal{R}}_1(\varphi+1)+1)+\varphi)(\sigma((1-\rho)\check{\mathcal{R}}_1(\varphi+1)+1)+\varphi)}{\left(\theta\lambda + \kappa(1-\rho)\sigma\left((1-\rho)\left(\bar{\mathcal{R}}_1\right)^2\sigma(\varphi+1) + 2\bar{\mathcal{R}}_1(\varphi+1) + (1-\rho)\bar{\mathcal{R}}_2\sigma\right)\right)e} > 0 \tag{102}$$

We find that $\sigma_{x,\bar{\mathcal{R}}_2}$ and $\sigma_{R^r,\bar{\mathcal{R}}_2}$ are negative while $\sigma_{\pi,\bar{\mathcal{R}}_2}$ is positive, just as in section B3.1.

Turn next to the rates of change wrt. $\bar{\mathcal{R}}_1$, which are given by

$$\sigma_{x,\bar{\mathcal{R}}_1} = \sigma_{R^r,\bar{\mathcal{R}}_1} = -\frac{(1-\rho)\sigma(\varphi+1)(1-\beta\rho)^2\left(\sigma\left((1-\rho)\left((1-\rho)\left(\bar{\mathcal{R}}_1\right)^2\sigma(\varphi+1) + 2\bar{\mathcal{R}}_1(\sigma+\varphi) - (1-\rho)\bar{\mathcal{R}}_2\sigma\right)+1\right)+\varphi\right)}{\left(\sigma((1-\rho)\bar{\mathcal{R}}_1(\varphi+1)+1)+\varphi\right)g} < 0 \tag{103}$$

$$\sigma_{\pi,\bar{\mathcal{R}}_1} = \frac{\theta\kappa(1-\rho)\sigma(\varphi+1)\left[\sigma\left((1-\rho)\mathcal{R}_1(\varphi+1)+1\right)+\varphi\right]\left(\sigma\left((1-\rho)\left((1-\rho)\left(\bar{\mathcal{R}}_1\right)^2\sigma(\varphi+1)+2\bar{\mathcal{R}}_1(\sigma+\varphi)-(1-\rho)\bar{\mathcal{R}}_2\sigma\right)+1\right)+\varphi\right)}{\left((1-\rho)^2\sigma^2\left(\left(\bar{\mathcal{R}}_1\right)^2(\varphi+1)+\bar{\mathcal{R}}_2\right)+\sigma(1+2(1-\rho)\bar{\mathcal{R}}_1(\varphi+1))+\varphi\right)g} > 0 \tag{104}$$

where

$$\begin{aligned} g &= (1-\beta\rho)^2\left((1-\rho)\sigma^2\left(\bar{\mathcal{R}}_1^2(\varphi+1)+\bar{\mathcal{R}}_2\right)+\sigma(1+2(1-\rho)\bar{\mathcal{R}}_1(\varphi+1))+\varphi\right) \\ &\quad +\theta\kappa\left(\sigma((1-\rho)\bar{\mathcal{R}}_1(\varphi+1)+1)+\varphi\right)\left(\sigma((1-\rho)\check{\mathcal{R}}_1(\varphi+1)+1)+\varphi\right) > 0 \end{aligned}$$

In (103) and (104) the denominators are clearly positive and the numerator is positive if the aforementioned term (92) is positive, which we have estabilished before. Thus, $\sigma_{x,\bar{\mathcal{R}}_1}$ and $\sigma_{R^r,\bar{\mathcal{R}}_1}$ are negative while $\sigma_{\pi,\bar{\mathcal{R}}_1}$ is positive. That is, the signs of the rates of change wrt. to the first order term line up with those of the second order term.

In sum, when the central bank accounts for the risk-taking channel in optimizing its policy function, it responds less strongly to inflation, which leads to more inflation and less real rate and output gap volatility. This coincides with the case of an undistorted steady state.

Table 4: **Appendix C: Coefficients for the numerical optimisation**: Priors and posterior values of Bayesian estimation. For details see Abbate and Thaler (2019).

	parameter	prior shape	prior mean	prior std	post. mean	90% HPD interval	
			structural parameters				
μ_y	trend growth	norm	0.4	0.1	0.4270	0.3913	0.4618
μ_l	labor normalization	norm	0	2	-0.0885	-1.6666	1.4739
α	output share	norm	0.3	0.05	0.2004	0.1619	0.2396
$100\frac{1-\beta}{\beta}$	real rate in %	norm	0.25	0.1	0.4252	0.3000	0.5497
$\bar{\varepsilon}^P$	price markup	norm	1.25	0.12	1.5059	1.3584	1.6516
$\bar{\pi}$	inflation in %	gamma	0.62	0.1	0.6285	0.4902	0.7640
ϕ_π	TR weight on inflation	norm	1.5	0.25	1.8712	1.5489	2.1951
ϕ_y	TR weight on output	norm	0.12	0.05	0.0194	-0.0359	0.0742
ρ	TR persistence	beta	0.75	0.1	0.8412	0.8062	0.8775
κ	investment adj. costs	norm	4	1.5	7.4517	5.5854	9.2988
ι	habits	norm	0.7	0.1	0.7776	0.7055	0.8518
σ_c	risk aversion	gamma	1.5	0.375	1.7366	1.2684	2.1780
σ_l	disutility from labor	gamma	2	0.75	2.0390	0.9705	3.0888
λ_p	price calvo parameter	beta	0.5	0.1	0.6201	0.5404	0.6996
λ_w	wage calvo parameter	beta	0.5	0.1	0.8478	0.8099	0.8860
γ_p	price indexation	beta	0.5	0.15	0.1528	0.0529	0.2471
γ_w	wage indexation	beta	0.5	0.15	0.4469	0.2104	0.6858
ξ	equity premium	norm	0.015	0.01	0.0226	0.0077	0.0355
θ	liquidation value	norm	0.5	0.1	0.7416	0.6447	0.8407
$\bar{k}$	equity ratio	norm	0.12	0.05	0.1231	0.1207	0.1253
			structural shock processes				
σ_A	stdev TFP	unif	0	10	0.3667	0.3175	0.4144
σ_B	stdev preference	unif	0	10	3.4561	2.257	4.6229
σ_G	stdev govt. spending	unif	0	10	2.2669	1.9838	2.536
σ_I	stdev investment	unif	0	10	4.7234	3.036	6.3548
σ_P	stdev price markup	unif	0	1	0.1334	0.1086	0.1570
σ_R	stdev monetary	unif	0	1	0.1165	0.1009	0.1319
σ_W	stdev wage markup	unif	0	10	0.4744	0.4098	0.5380
σ_ξ	stdev equity premium	unif	0	10	0.5157	0.2055	0.8752
ρ_A	persistence TFP	beta	0.5	0.2	0.9008	0.8476	0.9565
ρ_B	persistence preference	beta	0.5	0.2	0.1879	0.0340	0.3289
ρ_G	persistence gov. spending	beta	0.5	0.2	0.9770	0.9619	0.9923
ρ_I	persistence investment	beta	0.5	0.2	0.7720	0.6721	0.8743
ρ_P	persistence price markup	beta	0.5	0.2	0.9586	0.9211	0.9963
ρ_R	persistence monetary	beta	0.5	0.2	0.4605	0.3461	0.5752
ρ_W	persistence wage markup	beta	0.5	0.2	0.9009	0.8556	0.9473
ρ_ξ	persistence equity premium	beta	0.5	0.2	0.8161	0.7636	0.8707
$\rho_{G,A}$	correlation gov. spending & TFP	beta	0.5	0.2	0.6545	0.3861	0.9352
m_p	MA component of price markup	beta	0.5	0.2	0.7759	0.6803	0.8745
m_w	MA component of wage markup	beta	0.5	0.2	0.9744	0.9519	0.9967

Acknowledgements

We thank Harris Dellas, Beatriz Gonzalez, Thomas Lejeune, Galo Nuño, Evi Pappa, Federico Ravenna, Tiziano Ropele and participants in a seminar at the Bank of Spain, the 23rd Central Bank Macro Modeling Workshop and the 2020 ESCB research cluster 1 conference for their comments.
The views, opinions, findings, and conclusions or recommendations expressed in this paper are strictly those of the authors. They do not necessarily reflect the views of the Swiss National Bank (SNB) or the European Central Bank (ECB). The SNB and the ECB take no responsibility for any errors or omissions in, or for the correctness of, the information contained in this paper.

Angela Abbate
Swiss National Bank, Zurich, Switzerland; email: Angela.Abbate@snb.ch

Dominik Thaler
European Central Bank, Frankfurt am Main, Germany; email: Dominik.Thaler@eui.eu

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

PDF ISBN 978-92-899-5514-0 ISSN 1725-2806 doi:10.2866/3899 QB-AR-23-009-EN-N