Exploiting the monthly data flow for structural forecasting

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Motivation

Economic analysis in policy institutions:

- Policy analysis
  - storytelling about the medium-run
  - scenario analysis

  $\rightarrow$ micro-founded models
    - theoretically consistent, interpretable forecasts
    - for (few) key macro variables
    - low frequency forecasts updated infrequently

- Conjunctural analysis
  - assess current conditions
  - exploit real-time data flow
  - reduced-form/judgmental models
    - From very judgmental
    - to very sophisticated nowcasting techniques (Banbura et al. 2013)

This paper

- The separation between structural analysis and now-casting is potentially costly
- New emphasis on state contingent rules in the conduct of monetary policy

⇒ Bridging now-casting and structural modelling ever more crucial.

We bridge variables at different frequencies while maintaining the structural features of the **micro-founded model**

To do so we must derive the monthly dynamics of the model, addressing a classic problem of time aggregation (see, Hansen and Sargent 1991).

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What we do

We exploit the monthly data flow within a DSGE model.

- Show how to obtain a real-valued monthly specification for the DSGE model
  - that maintains the cross-equation restrictions determined by the behavioral assumptions.
  - $\rightarrow$ not mixed frequency in a reduced-form model like Banbura et al (2013), Andreu et al. (2014)

- Bridge the “monthly” DGSE model with a set of timely monthly variables

- Assess the nowcasting performance of the model
  - Observable variables, e.g. Unemployment
  - Underlying unobservable variables, e.g. output gap

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Related literature

- **DSGE models and large information**: Boivin and Giannoni (2006), Giannone, Monti and Reichlin (2009), Schorfheide, Sill and Kryshko (2010)
  - Improve the estimation of the quarterly DSGE structural parameters and states
  - Understand how the model's dynamics propagates to non-modeled variables.
- **"Mixed frequency" DSGE models**: Kim (2011), Christensen *et al.* (2012), Foroni and Marcellino (2012)
  - Improve the estimation of the quarterly DSGE structural parameters
  - by alleviating the temporal aggregation bias
  - by mitigating identification issues

Empirics

- Model we use is (a modification of) the Gali' Smets and Wouters (2011)
  - Smets-Wouters (2007) + theory of unemployment proposed in Galí (2011a,b).
  - $\rightarrow$ explicit introduction of unemployment
  - preference specification à la Jaimovich and Rebelo (nests GHH and KPR preferences)

- 15 high-frequency variables
  - Real: IP, CU, RDPI, PCE, INV, SALES, CONTOT, HSTARTS
  - Prices: PPI, CPI
  - Financial: FFR, AAA, BAA
  - Sentiment: PMI, PHBOS

The ingredients

Structural model (log-linearized)

$$
\begin{aligned}
\text{quarterly variables} \quad y_{t_q} &= \mathcal{M}_\theta \; s_{t_q} \\
\text{states} \quad s_{t_q} &= \mathcal{T}_\theta \; s_{t_q - 1} + B_\theta \; \varepsilon_{t_q}
\end{aligned}
$$

θ: deep parameters. Model is estimated or calibrated at a quarterly frequency.

The conjunctural variables X_{t_m}

But both the auxiliary variables X_{t_m} **and** some of the key variables y_{t_m} might be available at higher frequency (e.g. unemployment)

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The recipe

3 steps:

1. Handle the irregular sampling problem
   What is quarterly data?
   - temporal aggregation of monthly data.
   - periodically sampled.

   a. We align monthly and quarterly data.
      - Transform monthly data so as to correspond to a quarterly quantity when observed at the end of the quarter

   b. We map the quarterly model into its monthly counterpart

2. Estimate the relation between the conjunctural and the model variables

3. Handle the jagged edge problem as in Giannone, Reichlin and Small (JME, 2008)

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From quarterly to monthly model

Quarterly sampling **Monthly sampling**

$$s_{t_q} = \mathcal{T}_\theta \; s_{t_q - 1} + B_\theta \; \varepsilon_{t_q}$$

$$\Downarrow$$

$$s_{t_m} = \mathcal{T}_\theta \; s_{t_m - 3} + B_\theta \; \varepsilon_{t_m} \quad \Rightarrow \quad s_{t_m} = \mathcal{T}_\theta^m \; s_{t_m - 1} + B_\theta^m \; \varepsilon_{t_m}^m$$

where

$$\mathcal{T}_\theta^m = \mathcal{T}_\theta^{\frac{1}{3}}$$

$$vec(B_m B_m') = (I + \mathcal{T}_m \otimes \mathcal{T}_m + \mathcal{T}_m^2 \otimes \mathcal{T}_m^2)^{-1} vec(B_\theta B_\theta').$$

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From quarterly to monthly model

Cube roots of a matrix may be infinite, finite or zero in number.

Paper builds on Higham (2008) to show various results regarding the existence of a real-valued cube root.

- If the $\mathcal{T}_\theta$ is diagonalizable *i.e* i.e $\mathcal{T}_\theta = VDV^{-1}$,
  - then the cube root of $\mathcal{T}_\theta$ can be obtained as

$$\mathcal{T}_\theta^{\frac{1}{3}} = VD^{\frac{1}{3}}V^{-1},$$

  - At most 3^n cube roots
  - we can characterise them and identify the ones that have real coefficients.

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Intuition

Take a diagonal real matrix A, such that $V = I$ and $D = A$

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \Rightarrow A^{\frac{1}{3}} = \begin{bmatrix} a_{11}^{\frac{1}{3}} & 0 & 0 \\ 0 & a_{22}^{\frac{1}{3}} & 0 \\ 0 & 0 & a_{33}^{\frac{1}{3}} \end{bmatrix}$$

Each real number has 3 cube roots, a real one and 2 complex ones $\Rightarrow$ A has $3^3 = 27$ cube roots, but only one of them has real coefficients.

Intuition

Take a general diagonalizable matrix A

$$A^{\frac{1}{3}} = VD^{\frac{1}{3}}V^{-1},$$

D is a diagonal matrix with some real eigenvalues and some complex conjugate couples of eigenvalues (k)

- There are 3^k real cube roots
- Select among these using the likelihood (Deistler et al. 2013)

On the monthly AR

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- Advantages:
  - closed form;
  - No need to re-estimate or re-calibrate the model;
  - Interpretation remains unchanged.
- Caveats:
  - What does this assumption of autoregressive structure for the monthly model imply? What is the implicit timing of the decision making? What are the implied informational assumption?
  - We are currently investigating those issues.

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$$\begin{bmatrix} y_{t_m} \\ X_{t_m} \end{bmatrix} = \begin{bmatrix} \mathcal{M}_\theta^m \\ \Lambda \mathcal{M}_\theta^m \end{bmatrix} s_{t_m} + \begin{bmatrix} u_{t_m} \\ \nu_{t_m} \end{bmatrix}$$

expand the state-space

Deal with missing data and mixed frequencies

$$var(u_{it_m}) = \begin{cases} 0 \text{ if available} \\ \infty \text{ if not available} \end{cases} \qquad var(\nu_{jt_m}) = \begin{cases} R_j \text{ if available} \\ \infty \text{ if not available} \end{cases}$$

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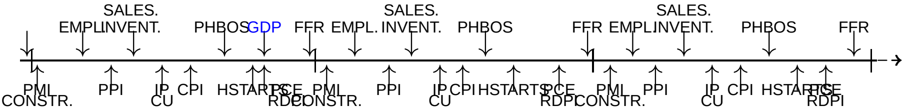
Empirical exercise

the structural model $\rightarrow$ The Gali' Smets and Wouters model

the conjunctural indicators



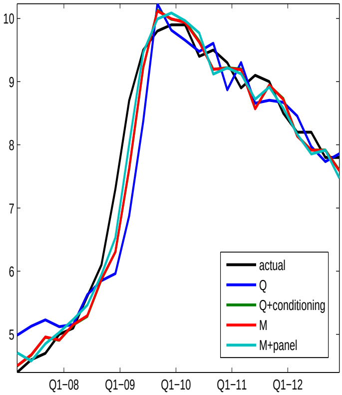
the bridge model $\rightarrow X_{t_m} = f(Y_t) + \nu_t$

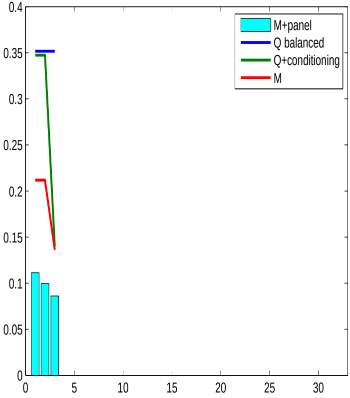
Empirical exercise

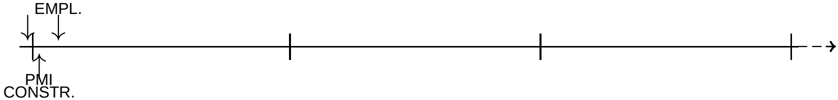
We evaluate the model's out-of-sample performances with a real-time forecasting exercise

- evaluation sample: 2007q1-2012q4
- θ are estimated once at the beginning of the evaluation sample
- Λ is estimated recursively
- use real-time data (e.g. GDP, C,I,GDPDEFL)
- we perform this exercise 32 times, i.e. every time there is a new release
- we produce both point and density forecasts

Nowcasting Unemployment







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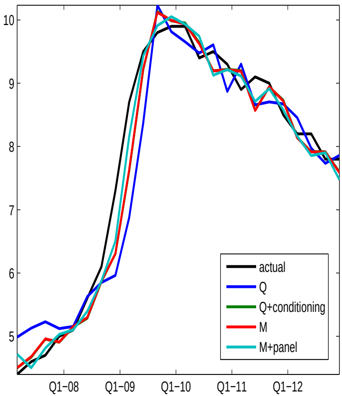

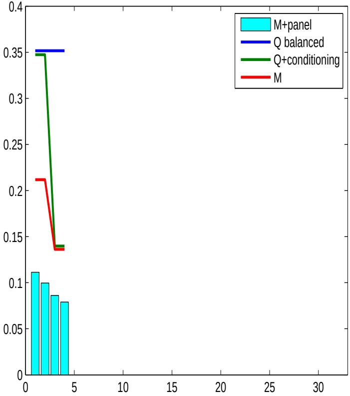

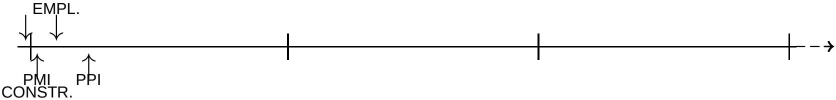

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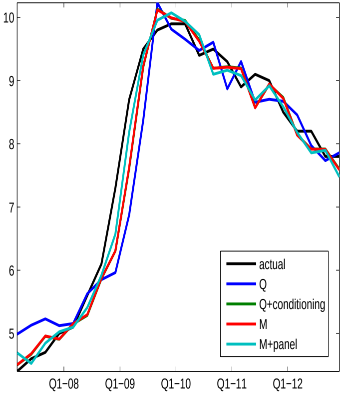

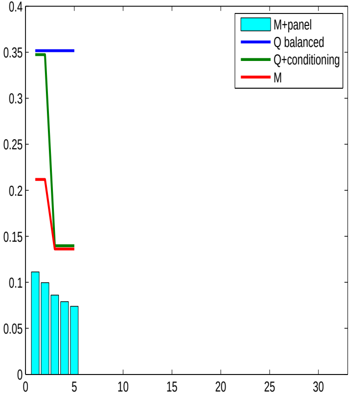

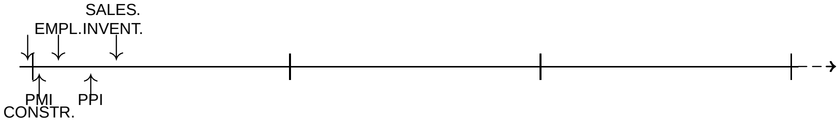

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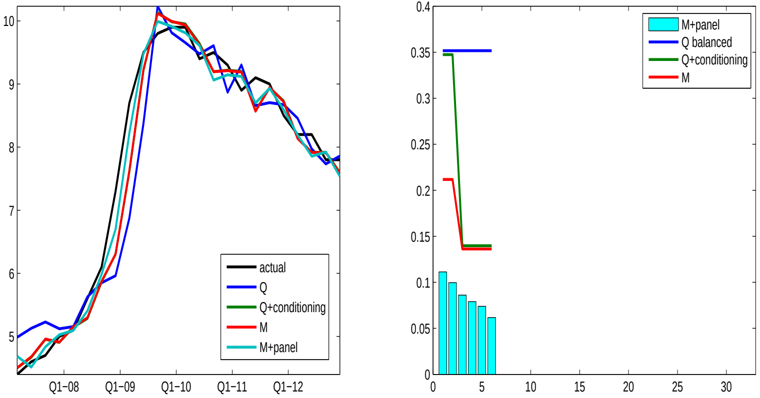

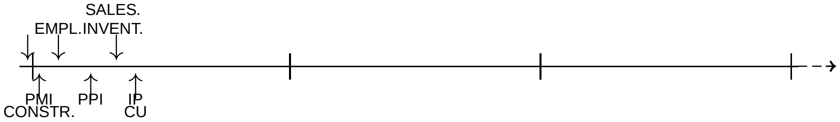

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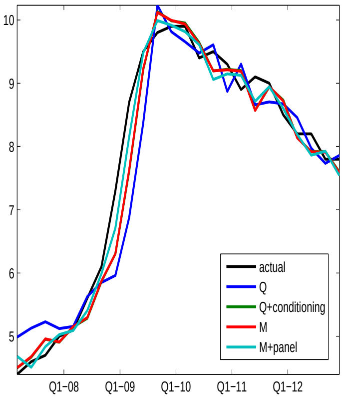

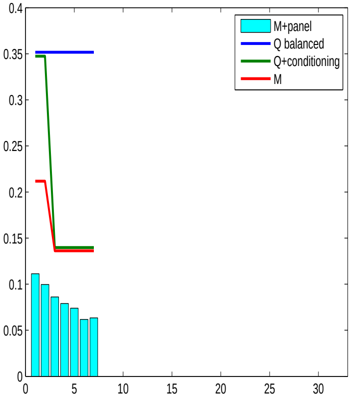

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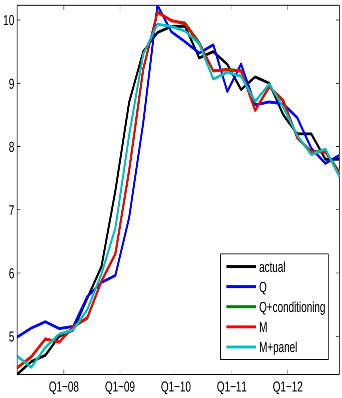

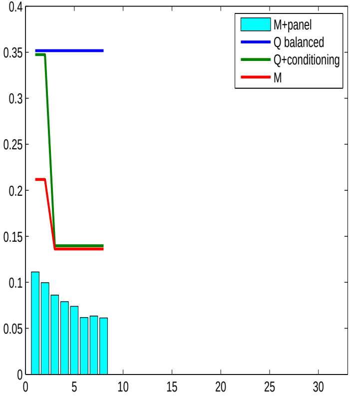

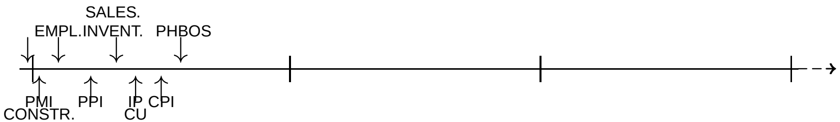

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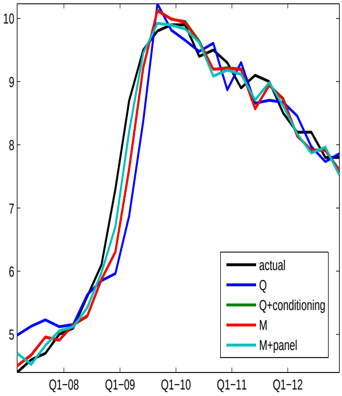

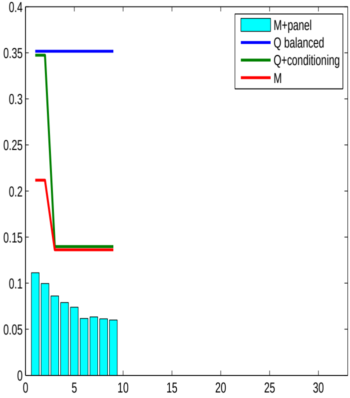

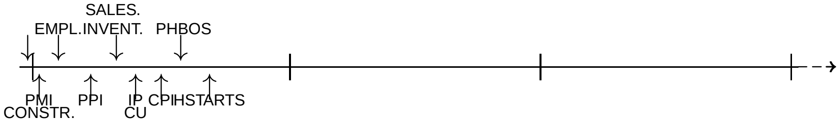

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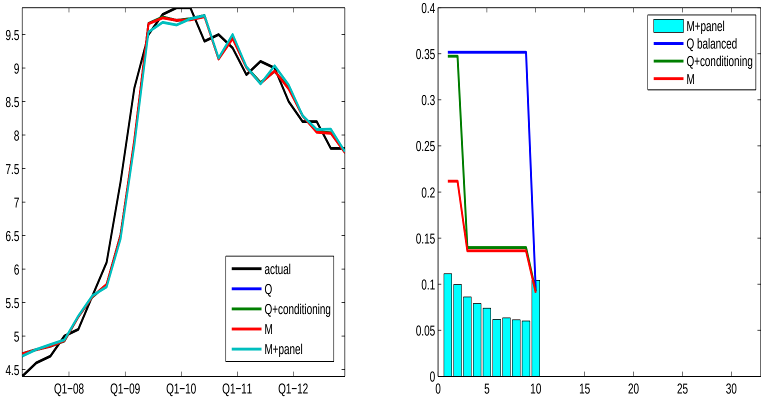

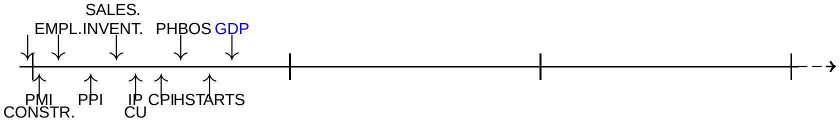

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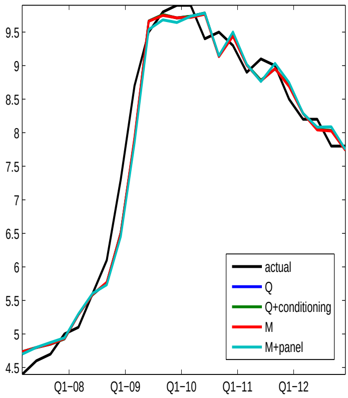

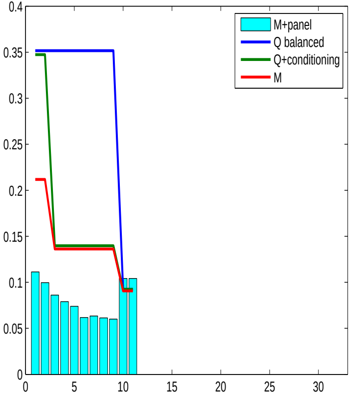

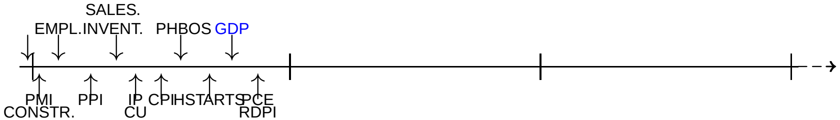

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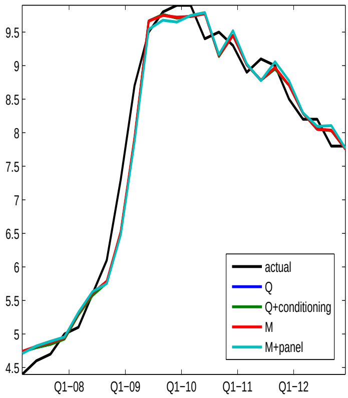

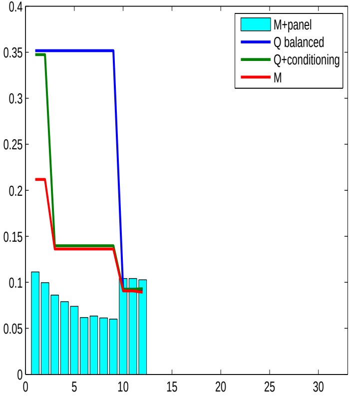

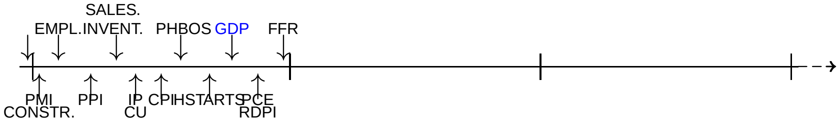

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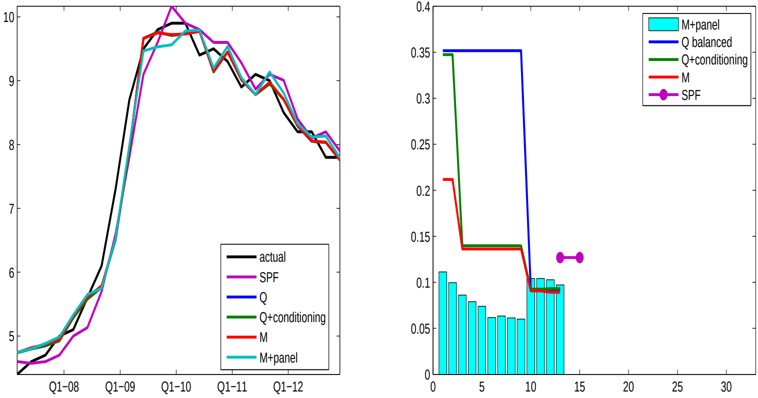

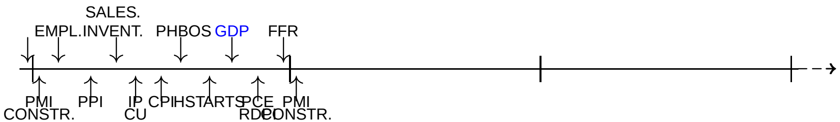

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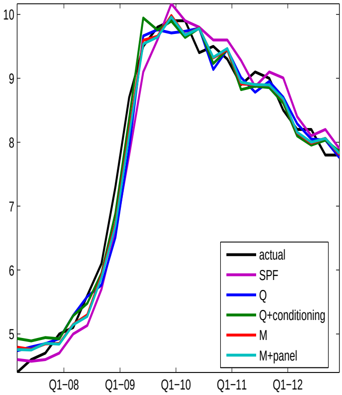

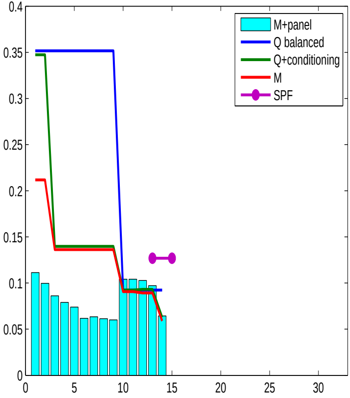

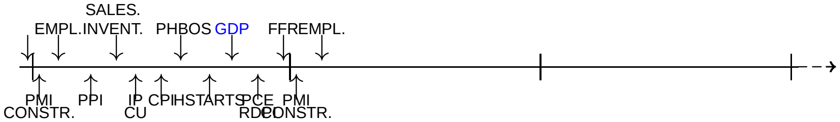

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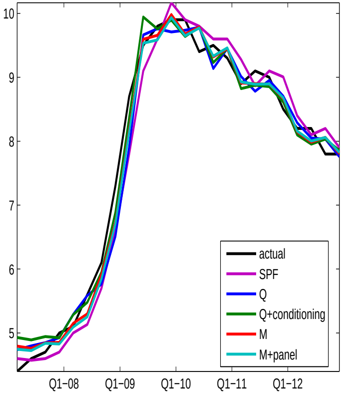

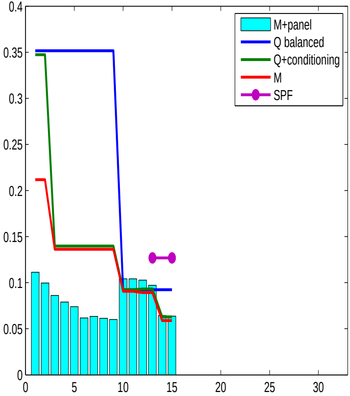

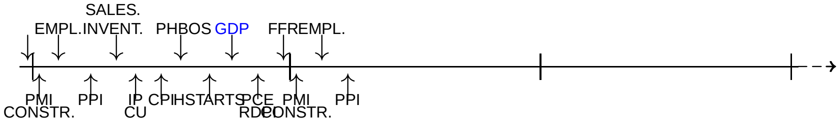

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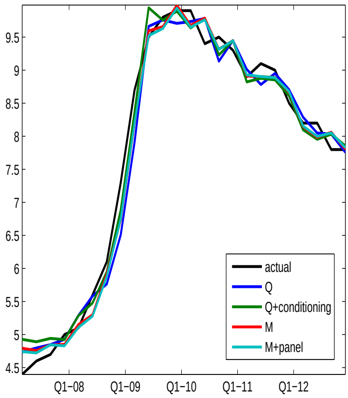

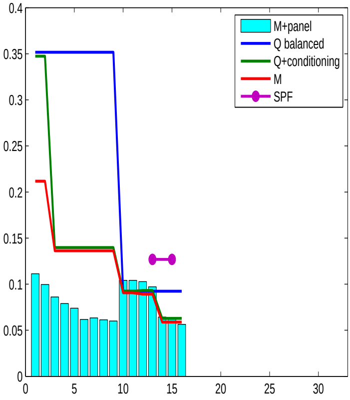

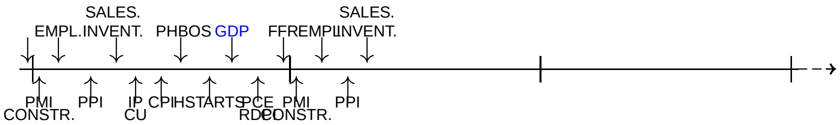

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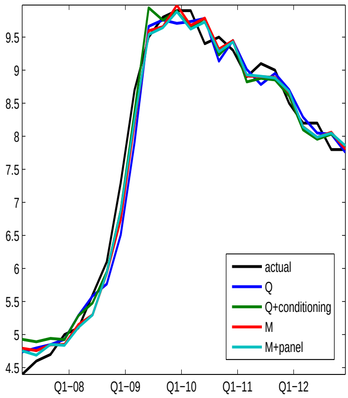

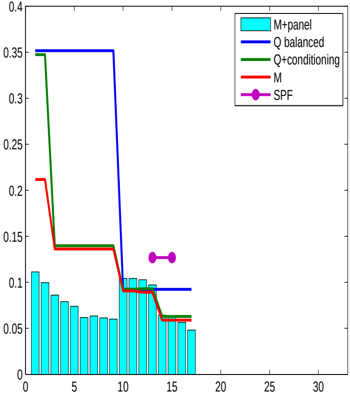

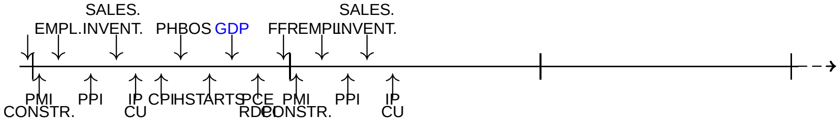

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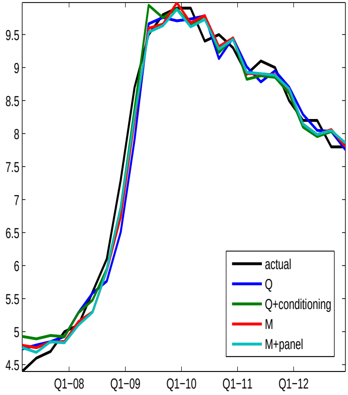
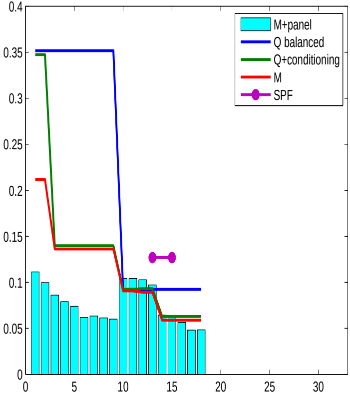
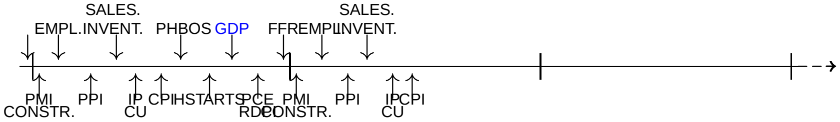
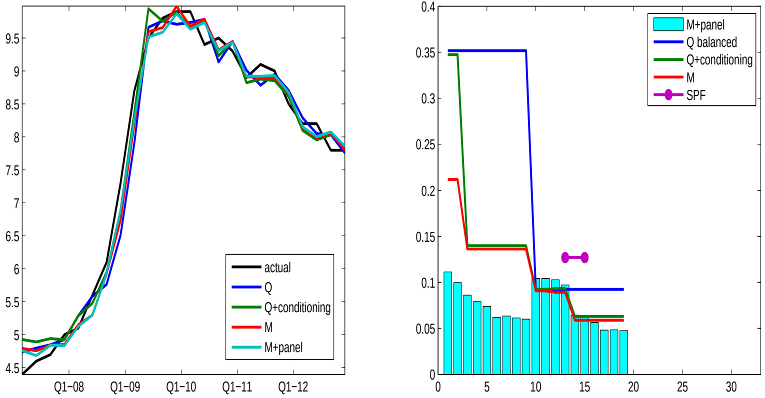
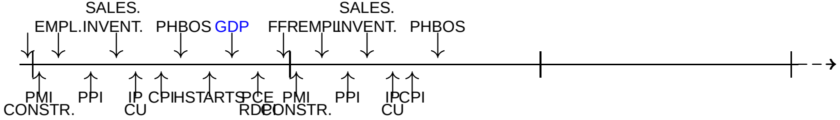






Nowcasting Unemployment





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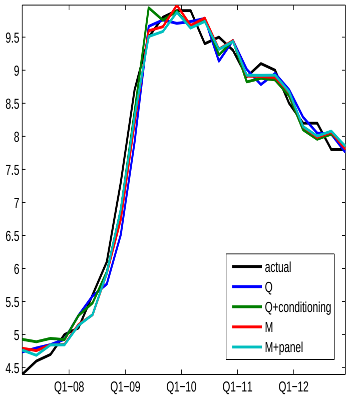

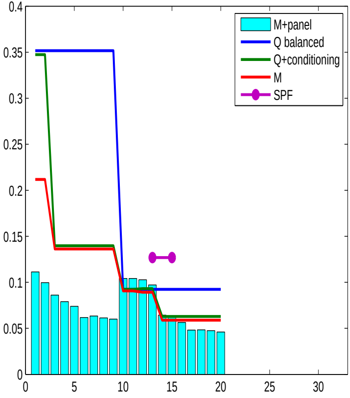

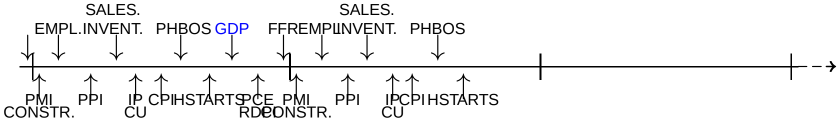

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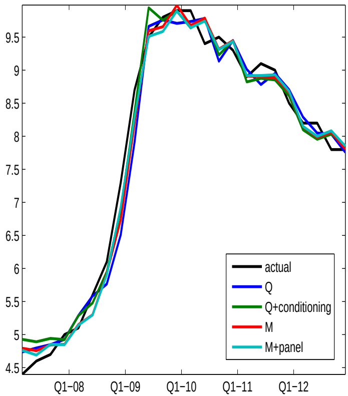

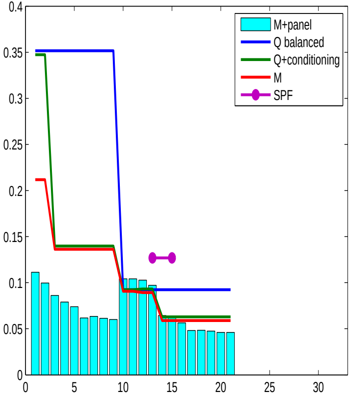

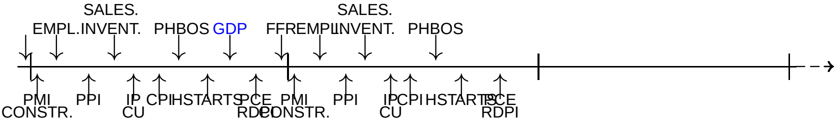

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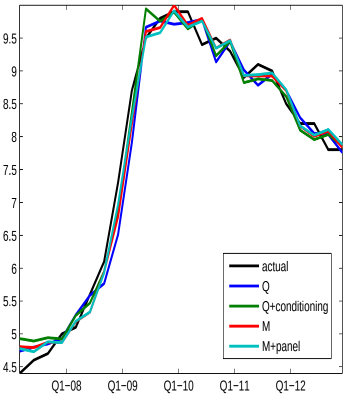

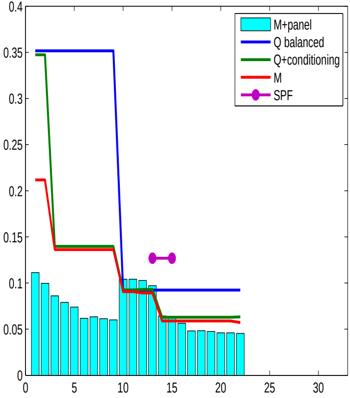

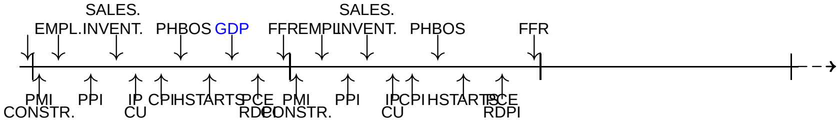

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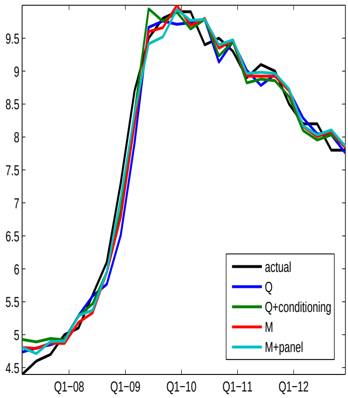

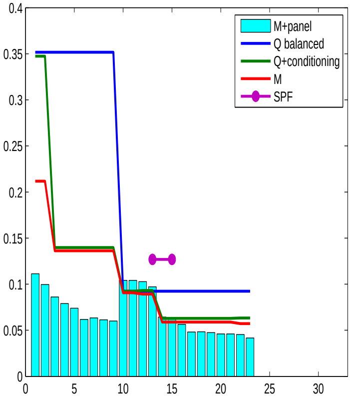

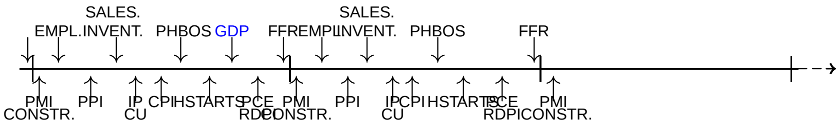

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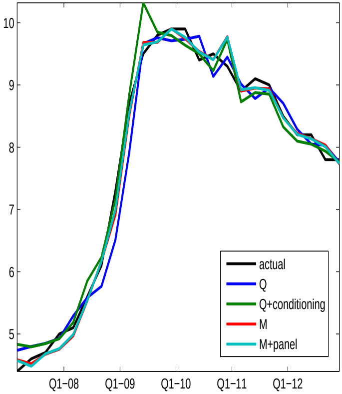

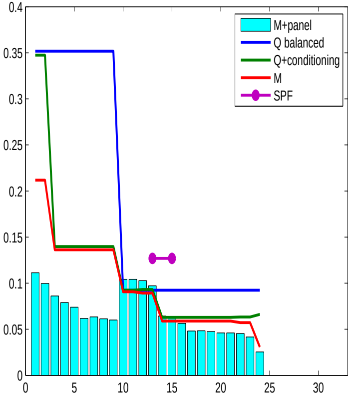

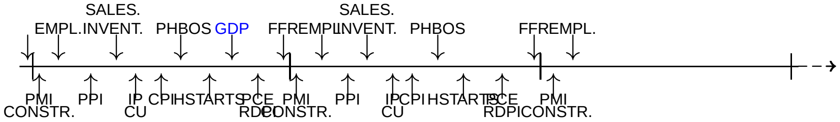

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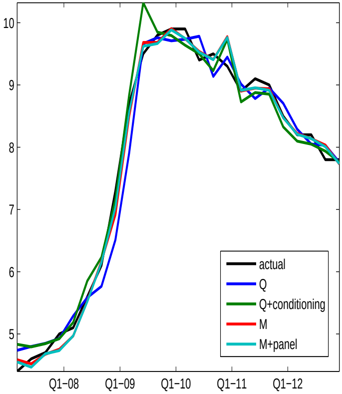

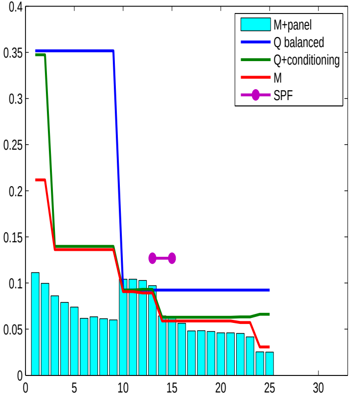

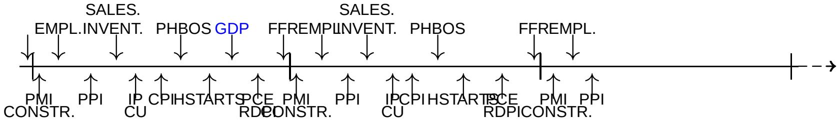

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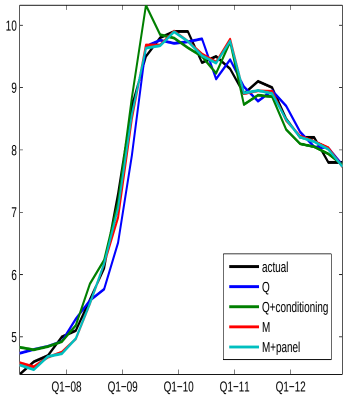

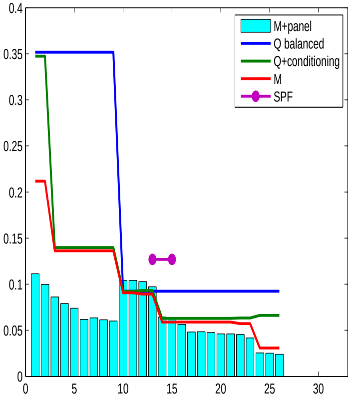

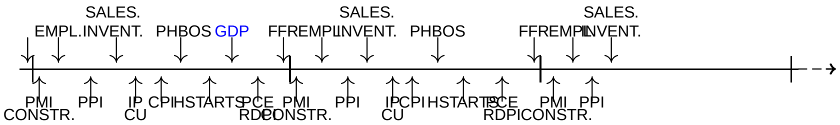

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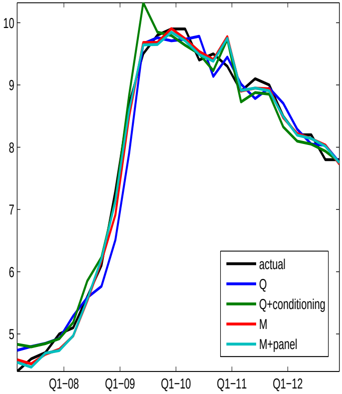

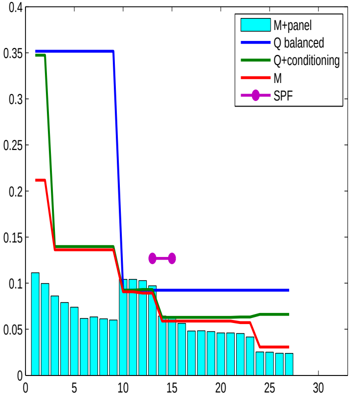

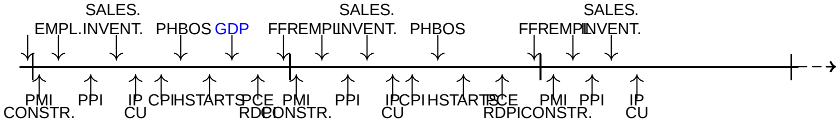

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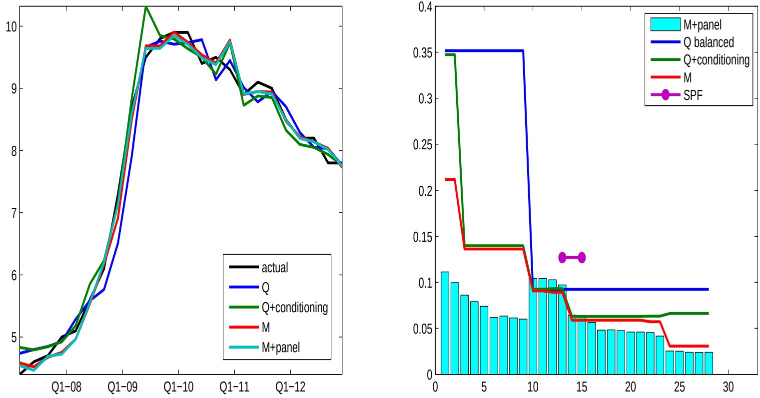

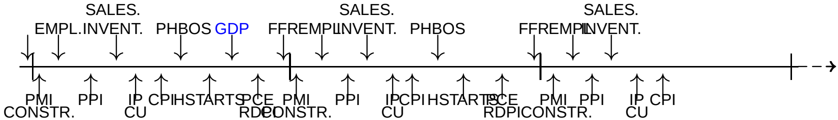

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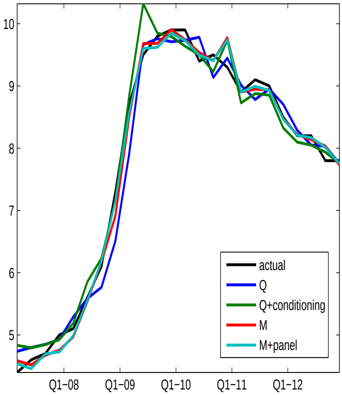

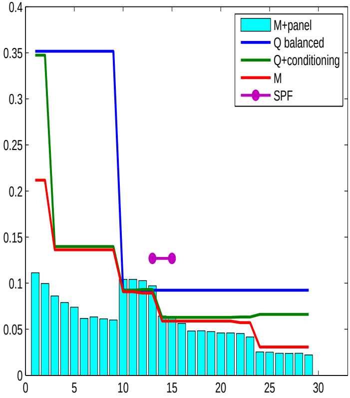

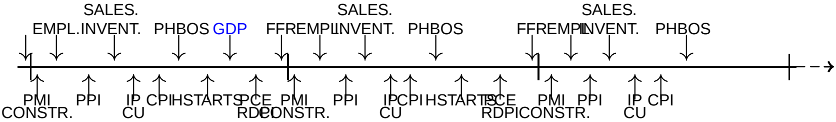

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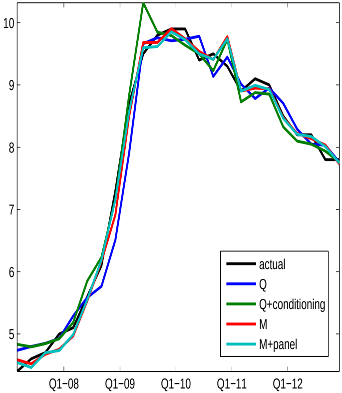

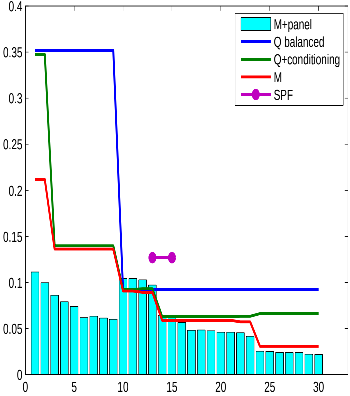

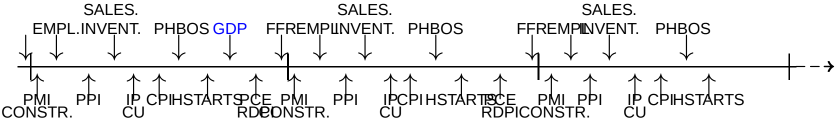

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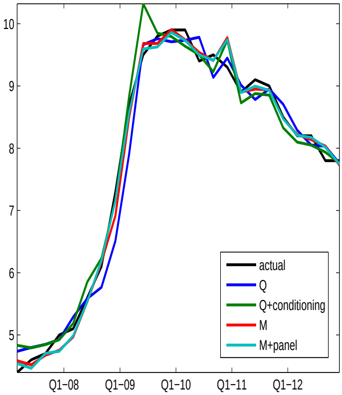

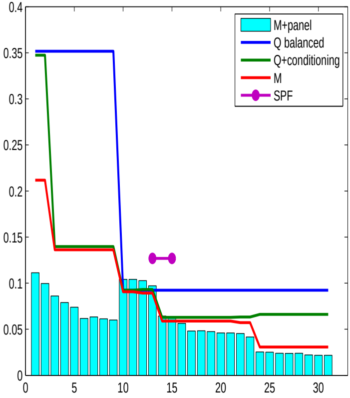

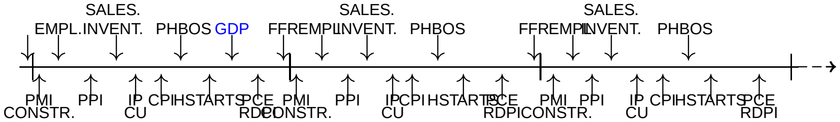

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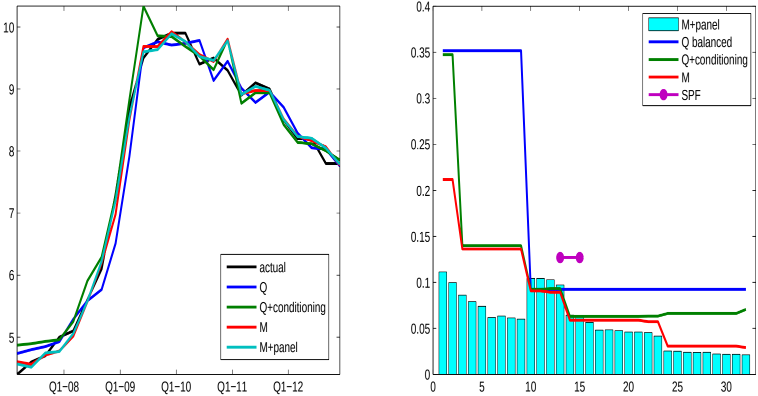
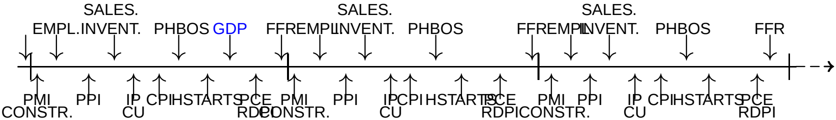
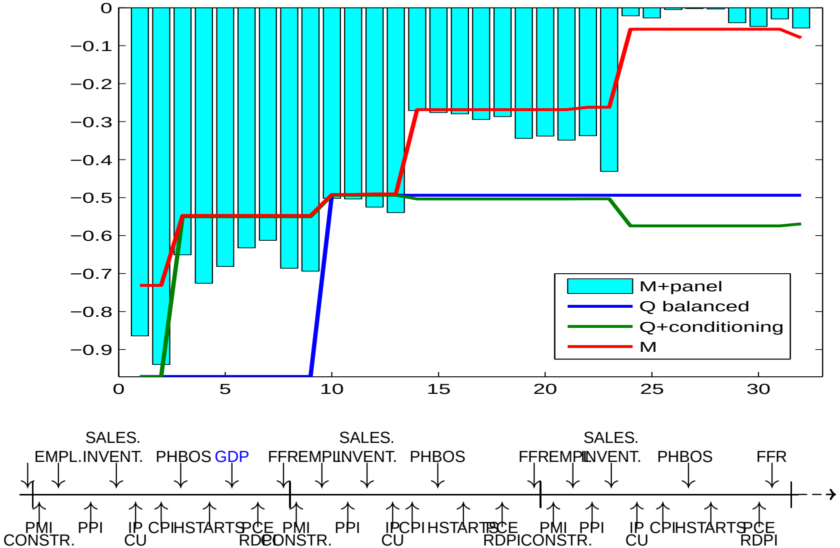
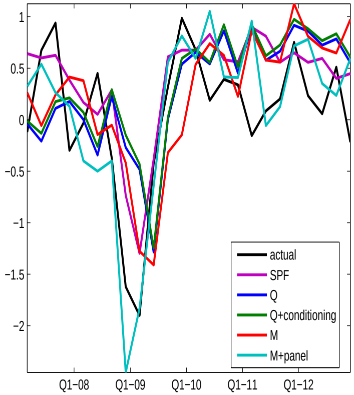


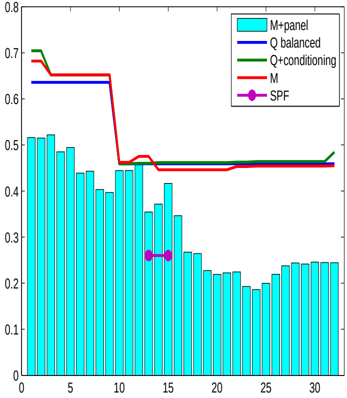


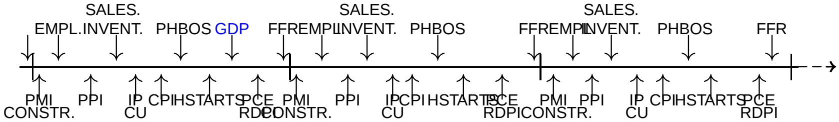
Log score of the nowcast of unemployment



Nowcasting GDP growth





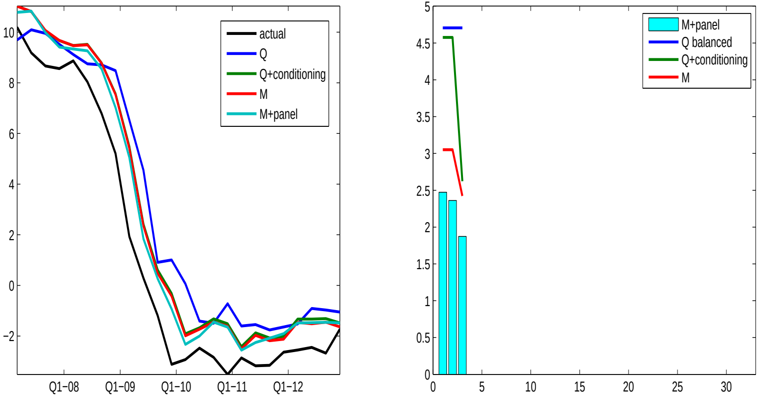


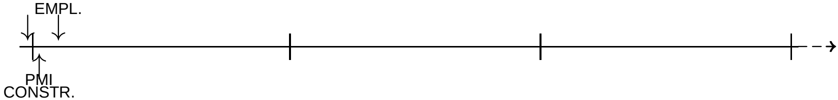
Taking advantage of the structure

Obtain **real-time estimates** of concepts such as output gap, TPF growth, natural rate... (unobserved, model-based, economic concepts)

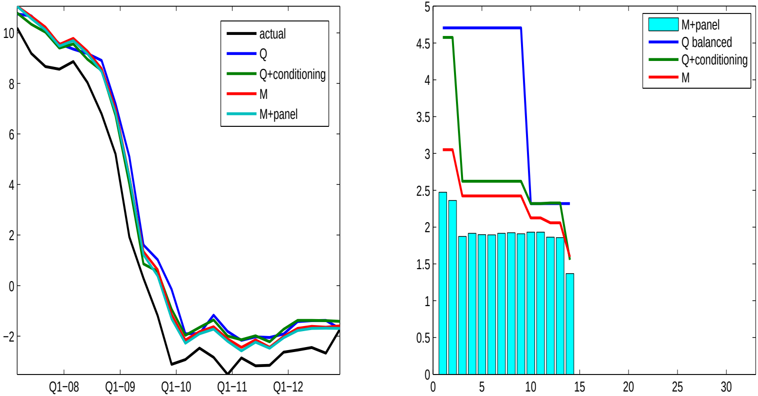
We chose the GSW because of the more “sensible” output gap

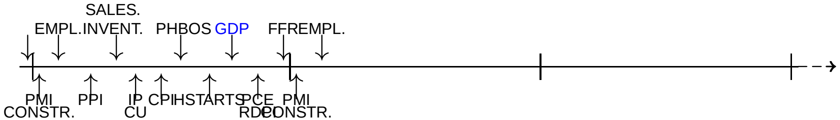
Nowcasting the output gap



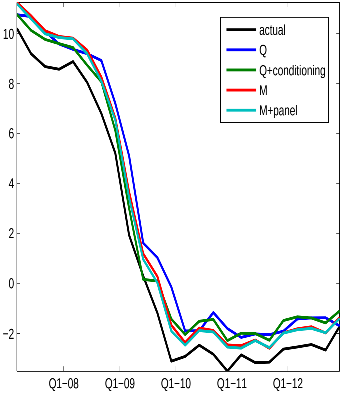


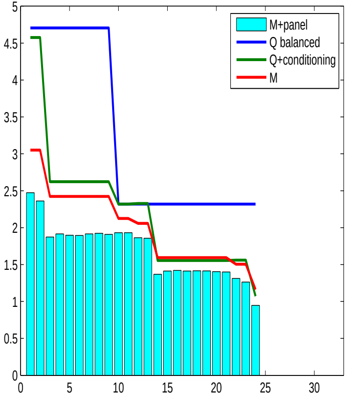
Nowcasting the output gap

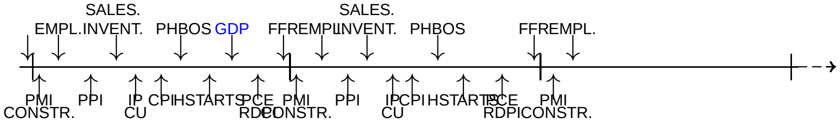




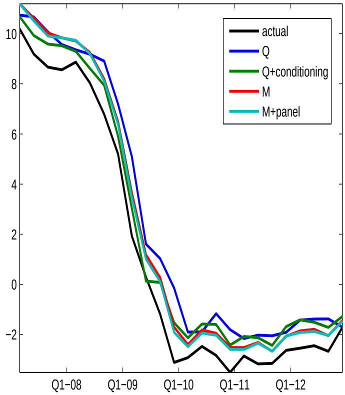
Nowcasting the output gap

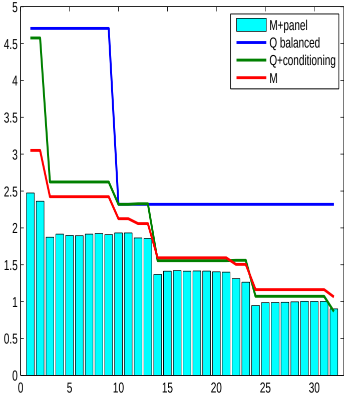


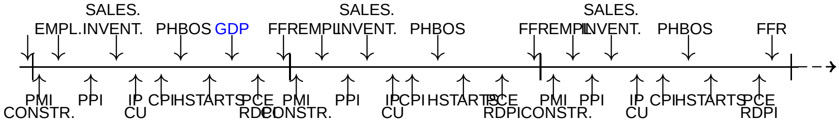




Nowcasting the output gap







Conclusions

- We define a mapping from a quarterly model to its real monthly counterpart
- to get a “mixed frequency” DSGE model
- which we bridge with timely economic indicators
- This allows us to:
  - update storytelling at each release
  - assess the impact of the news on our stories
  - analyze how the structural shocks propagate through the auxiliary variables.

From quarterly to monthly model

If the $\mathcal{T}_\theta$ is diagonalizable, *i.e* i.e $\mathcal{T}_\theta = VDV^{-1}$,

$\Rightarrow$ then the cube root of $\mathcal{T}_\theta$ can be obtained as

$$\mathcal{T}_\theta^{\frac{1}{3}} = VD^{\frac{1}{3}}V^{-1},$$

At most 3^n cube roots. How many have real coefficients?

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \quad \Rightarrow \quad A^{\frac{1}{3}} = \begin{bmatrix} a_{11}^{\frac{1}{3}} & 0 & 0 \\ 0 & a_{22}^{\frac{1}{3}} & 0 \\ 0 & 0 & a_{33}^{\frac{1}{3}} \end{bmatrix}$$

$\Rightarrow$ A has $3^3 = 27$ cube roots, but only one of them has real coefficients.

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At most 3^n cube roots: e.g. GSW (21 states, 16 nonzero eigenvalues) there are more than $3^{16} = 43046721$ cube roots.

1. We can characterize all the cube roots of the matrix and verify which have real coefficients.
2. Equivalent to:
   - real elements of $D \Rightarrow$ select their real cube root.
   - elements of D that are complex conjugate $\Rightarrow$ choose the cube root which is characterized by less oscillatory behavior, i.e. the cube root with smaller argument.

If the model is not diagonalizable $\Rightarrow$ roots are zero or infinite.

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The p_{th} root of a matrix

- If $\mathcal{T}_\theta$ does not have eigenvalues on the negative real axis, then there is a unique p_{th} root of $\mathcal{T}_\theta$, whose eigenvalues lie in the wedge $\{z : -\frac{\pi}{p} < arg(z) < \frac{\pi}{p}\}$
  - If the eigenvalue is real, then we are choosing the real root.
  - If the eigenvalue is complex, then we are just selecting the root that lies in the same quadrant of the eigenvalue we are considering.
- If $\mathcal{T}_\theta$ is real then the cube root defined above will be real.

What are the monthly and quarterly variables?

E.g.: Philadelphia Business outlook Survey

	$x^i_{t_m}$	$X^i_{t_m}$	$X^i_{t_q}$
⋮			
September	x^i_{Sep}	$\frac{1}{3}(x^i_{Sep} + x^i_{Aug} + x^i_{Jul})$	$\frac{1}{3}(x^i_{Sep} + x^i_{Aug} + x^i_{Jul})$
October	x^i_{Oct}	$\frac{1}{3}(x^i_{Oct} + x^i_{Sep} + x^i_{Aug})$	NaN
November	x^i_{Nov}	$\frac{1}{3}(x^i_{Nov} + x^i_{Oct} + x^i_{Sep})$	NaN
December	x^i_{Dec}	$\frac{1}{3}(x^i_{Dec} + x^i_{Nov} + x^i_{Oct})$	$\frac{1}{3}(x_{Dec} + x^i_{Nov} + x_{Oct})$
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We want to use the information available also in October and November!

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