Liquidity risk in money market spreads

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Abstract

The most widely used gauges of the money market liquidity conditions reflect both credit and liquidity risk. In this paper, we put forward two approaches to infer the liquidity component. A first type of approach gauges the credit risk of the banks participating in the Euribor panel by first inferring their default probability from prices on own Credit Default Swap (CDS) contracts. The liquidity component is estimated as a residual. In the second approach, the liquidity component is derived along a simultaneous model estimate, where variables include unsecured inter-bank deposit rates, zero coupon yields on financial bonds, and zero coupon yields on Treasury bonds. The results presented in this paper confirm that, throughout the market turmoil, the rise in the money market spreads owed to both liquidity and credit risks, where the relative weights of these two components changed over time with credit risk becoming more and more relevant, while initially the liquidity risk accounted for the lion's share. In responding to the crisis, the ECB (and other central banks) has utilized both traditional monetary policy instruments as well as innovative tools to provide liquidity. In the wake of these measures, the liquidity risk component fell dramatically. This result may witness in favour of the effectiveness of the policies undertaken by ECB.

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1 Introduction [1]

The liquidity of a financial instrument is the elasticity of its best bid-ask quotes, as available in the market, to a new buy or sell order: the lower this elasticity, the more liquid is the instrument.[2] If the elasticity is close to nil, an intermediary can clinch even large deals without bearing relevant adverse changes in the price. The risk associated to such price changes is usually referred to as the liquidity risk.[3]

In financial markets, the price of an instrument embeds its associated liquidity risk – or, which is the same, investors are willing to pay a premium on liquid instruments – to account for diminishing trading costs and associated risks (respectively, the “trading view cost” and the “liquidity risk view”). While there is enough consensus that markets price liquidity, we do not avail, at the current stage of research, of direct methods to disentangle this contribution and the price of the related risk. It would be enough to observe that, ideally, to measure the liquidity, the researcher would have to identify any shift of the demand / supply curves induced by a new order, controlling for any other factor that could bring about price changes. This is a tall order even when one examines the data feed of regulated markets. Measuring risk is all but a new problem in finance and many approaches have been suggested. However, a separate issue remains to break down the related premium so as to identify more specifically the remuneration for (the lack of) liquidity. Finally, while as argued above there is enough consensus on the intuition underlying the liquidity risk, and this holds well enough irrespective of the features of each instrument, in practice empirical approaches are largely dependent on the data which are available for that instrument.

As a result, quite a range of applied methods, derived from basic empirical regularities to more sophisticated model-based solutions, are applied with limited overlapping when different instruments are considered.[4] The fact that many of these methods provide only pseudo-measures of the liquidity risk adds a further layer of complexity in the debate, given that these methods yield a joint representation of a number of risks and are not really focused on liquidity. Conversely, some of the “liquidity-specialised” approaches are too specific, as they capture only one dimension of liquidity itself, say its tightness as regards its resiliency.

While the concept of liquidity can be associated to any financial instrument, in official quarters the debate and empirical work has focused on the money market instruments, as even a quick glance at central banks’ reports and bulletins can confirm. The so-called Ted spread, the spread between the interest rate on unsecured inter-bank deposits at the three month maturity and the interest rate on the Treasury bills at same maturity, or equivalent spreads between the former and other “risk-free” rates, are among the most widely reported measures. In fact, such spreads reflect more than liquidity only. For instance the Euribor-Overnight Indexed Swap (OIS) spread embodies both the liquidity risk and the credit risk borne by a lender in the inter-bank market.[5]

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1 The views expressed in this paper are the author's own and do not necessarily reflect the views of Banca d'Italia. I thank seminar participants at the Bank of Italy, the workshop of European Central Bank on “Challenges to monetary policy implementation beyond the financial market turbulence” (Frankfurt am Main, 30 November – 1° December 2009) for helpful discussions and comments. The analysis in this paper has benefited from encouragement and contributions from Michele Manna.

2 In this definition, we implicitly accept that the financial agent placing the new order is a “price taker”, not a “price maker”.

3 Besides the liquidity risk, the literature recognizes also a funding liquidity risk where the latter refers to the risk that the individual financial agent may not be able to borrow enough funds, at prevailing market conditions, to meet his/her commitments (BIS, 2008). The split between liquidity and funding liquidity risks has mainly an analytical content; on the empirical ground, the two risks are often found to be closely intertwined giving rise to liquidity spiral (Brunnermeier e Pedersen, 2009).

4 By market practice, the liquidity of a government bond is inferred from the spread in the yield-to-maturity of the *on-the-run* bond and of a comparable *off-the-run* one. No such method could be applied as regards, say, money market instruments!

5 At a conceptual level, nothing changes if one uses the Libor index in lieu of the Euribor.

Against the above, there remains a feeling of a gap between the understanding of what is liquidity in a financial instrument, which arguably is a well established concept, and the elusiveness of capturing this concept into a figure. This state of affairs has led a number of leading institutions to invest in research in this field. When the current turmoil started and (il)liquidity became the buzzword, pressure mounted to avail of more satisfactory solutions.[6] Notably, a boost have received so-called indirect methods which first aim at measuring overall risk and then, in a second step, infer the liquidity component of this risk as a residual once the credit component is deducted.

Within the latter wave of research, two strands have gained momentum. A first type of works gauges the credit risk of the banks participating in the Euribor or Libor panel by first inferring their default probability from prices on own Credit Default Swap (CDS) contracts. At this stage of development, these may still be dubbed as "base indirect" methods, because in these works a number of simplifying assumptions are typically adopted. The cost of such assumptions has turned most visible when, starting from mid-2008 the resulting measure of the liquidity premium associated to the Euribor-OIS spread was worked out at hardly plausible negative levels.

In this paper, we thus put forward an alternative approach, based on the work by Longstaff, Mithal and Neiss (2005). As a distinguished (but not the only) feature of this approach compared to the works falling in the first category is that credit events are treated as the outcome of a stochastic process, and not longer as a deterministic one.

A second line of research pursued by authors does not dwell on CDS prices. In this case the liquidity component is derived along a simultaneous model, where dependent variables include unsecured inter-bank deposit rates, interest rates on financial bonds, and interest rate on government bonds. The estimated model allows us to decompose movements in Euribor rates into changes in bank debt risk premiums and changes in a factor specific to the inter-bank market that includes a liquidity premium. The basic intuition runs as follows: the inter-bank rates embodies the same credit risk of high standing financial bonds, while the two instruments differ because of the liquidity component. This approach too is tested in this paper, having as a reference the methodology described in Christensen, Lopez and Rudebusch (2009), here partly revised also to take into account the so-called *convenience yield* which sets a government bond apart from a "true" risk-free rate.

As a further by-product of these applications, the results presented in the paper offer a yardstick against which to assess the effectiveness of conventional and unconventional loosening liquidity management policies undertaken by the European Central Bank (ECB) throughout the current market turmoil.

The rest of the paper is organised as follows. Section 2 reviews the literature. Section 3 highlights the shortcomings of most standard measures of liquidity conditions in the money market. Section 4 introduces a more advanced model to estimate the liquidity premium, starting from CDS prices. Section 5 illustrates the results of a "non-CDS" model to gauge the liquidity component. Section 6 concludes.

2 The literature

The investor holding a low-liquid financial instrument should expect to bear higher trading costs in selling the instrument itself ("trading cost view"), and accept that the definition of the terms of the deal remains associated with specific risks ("liquidity risk view"). These types of issues are not mutually exclusive, although they have been tackled by the finance literature in sequence.

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[6] Examples include Bank of England (Quarterly Bulletin, 2007 Q4), Bank of Japan (Bank of Japan Review, 2008 Q2) and IMF (FSR October 2008).

The “trading cost view” develops from the assumption that by holding low-liquid instruments an investor should earn an income high enough to offset the higher trading costs. Notably, this relationship typically takes a concave shape and, in equilibrium, least-liquid instruments are purchased by investors with longer holding periods. As a result, the extra-yield associated to the low liquidity is smaller than it would be were the relationship be linear. This “view” has first been put forward and tested by Amihaud and Mendelson (1986), whose work led to a large empirical literature.

The “liquidity risk view” highlights that liquidity is priced not only because it creates trading costs, but also because it is itself a source of risk, since it changes unpredictably over time (Pastor and Stambaugh, 2003). More specifically, low-liquid instruments tend to be affected by larger swings in liquidity itself. As a result, the investor would request an extra-yield not only to remunerate the low level of liquidity but also on account of its larger variability (the higher liquidity risk).

Acharya and Pedersen (2005) examine the cost of (il)liquidity in the context of a standard generation-overlapping CAPM model (so-called liquidity adjusted *CAPM* on gross returns). These authors conclude that the premium of liquidity which is in the price of a given instrument X is higher:

1) the higher the co-variance between the illiquidity of X and that of market across the board (where the market is to be understood as the set of instruments which may be included in the portfolio), for obvious hedging purposes;
2) the lower the co-variance between the return on X and the market illiquidity. This is because investors are more willing to hold X if this yields higher returns when otherwise market conditions are dire;
3) the lower the co-variance between the illiquidity of X and overall market prices. When the market is bear and the investors’ wealth decline, then the need to easily sell an asset to gather cash may best be appreciated.

More recently, some authors have examined the possible link between the liquidity risk and the aggregated risk, unveiling a direct relationship between the two risks. Favero, Pagano and Von Thadden (2008) use data on euro area sovereign bond to show that changes in the aggregated risk affect less the price of currently less liquid bonds but comparatively more the price of those bonds which are expect to become less liquid in the future. These authors conclude that to assess correctly the liquidity effect, one ought to bear in mind also its nexus with what they define a common risk factor. Conversely, Vayanos (2004) develop a model where fund managers are hit by funds withdrawal whenever the fund’s performance falls under a given threshold. At an aggregated level, this leads to increasing liquidity *premia* when market volatility rises. One shared by-product of the two models is the enhanced flight-to-quality at times of increasing aggregated market risk.

A more specialised research approach deals with the liquidity premium in the money market; being this the central banks’ battleground, it is of little surprise that most of the works within this approach have been authored by economists affiliated with central banks and international financial institutions. Usually, an indirect method is adopted in this literature. First, a money market is selected; then, the credit risk component is somehow identified or estimated; finally, the liquidity risk component is derived as a residual. Examples are in Bank of England (2007) and in Bank of Japan (2008), which both infer the credit risk implicit in Libor rates from the CDS prices of the banks in the Libor panel.

Quite a number of works have investigated the development in the money market spreads after the onset of the crisis. Michaud and Upper (2008) suggest to break down the risk *premia* observed in the money market across different factors, which account for both the defining elements of the banks which borrow in this market segment and overall market conditions. In low frequency times series studies, the variable that best fits the risk *premia* in the inter-bank market relates to the credit risk; conversely, in high frequency settings, such as daily time series, quite a role is played by liquidity variables.

Eisenschmidt and Tapking (2009) hint that the banks' funding liquidity risk may have driven the developments in the observed money market spreads. IMF (2008) identifies in the joint default probability of the Libor panel banks, as a proxy of the systemic risk in the banking system, the main driver behind the variability in the Libor-OIS spread; a more marginal explanatory power is associated to a set of variables, which stand for different types of volatility and liquidity risks. Dudley (2008) argues that the main factor underlying the spread patterns can be uncovered in the banks' "scarcity of capital". A bank which is equipped with adequate capital is able to borrow and to lend, without hitting above its leverage target. Put it differently, for this "well-heeled" bank the shadow price of its capital is nil. Because of that, she will manage to undertake arbitrage deals exploiting the differential between the interest rate on unsecured deposits and the expected return from the roll-over of overnight deposits while hedging herself from the overnight rate variability. Conversely, the spread between the borrowing rate and the lending rate needs to offset the so-called shadow price of capital when the bank needs additional capital to lend an additional deposit. Another explanation for the widening of the Libor spreads has been proposed by Giavazzi (2008) who has put forward the notion of predatory banks. In a model of strategic behaviour amongst financial institutions there are two reasons why excess cash is not lent. Firstly, if another bank was to fail, its assets could be bought at a depressed price following it being placed into administration. Secondly, the probability of such an event occurring is endogenously determined by the amount of liquidity available in the inter-bank money markets, such that the optimal strategy may be to hoard any funds.

A further line of research has explored the effectiveness in the quantitative easing actions pursued by central banks with a view to cutting the liquidity premium implicit in inter-bank interest rates, while controlling for patterns in credit risks (Taylor and Williams, 2009, Frank and Hesse, 2009, McAndrews, Sarkar and Wang, 2008, and Wu, 2008).

Notwithstanding small differences in the fit structures, these studies arrive at substantially different results. Wu, Frank and Hesse as well as McAndrews, Sarkar and Wang identify a significant impact of liquidity measures on money market rates, while Taylor and Williams conclude that liquidity did not play a role. Christensen, Lopez and Rudebusch (2009) prove that Fed's actions were effective, adopting a multi-factorial arbitrage free representation of the yield curve on both government and financial corporate bonds as well as Libor rates.

3 The standard measures

The most widely used gauges of the money market conditions (Ted spread, Euribor/Libor-OIS spread, Euribor-Eurepo spread as well as deviations from the covered interest parity condition in the foreign exchange market) embody both liquidity and credit risks.

The Ted spread is often presented in Bank of England and ECB reports, among others. The liquidity risk component is given by the fact that the holder of the Treasury bills can always liquidate its assets before their maturities come due, which is not an option for the deposit lender. In addition, there is a credit risk component as the investor bears a much higher risk if it places its bets in the inter-bank market rather than investing in a State liability. Finally, the Ted spread is also affected by the changes in the "convenience yield" for holding Treasury securities.[7]

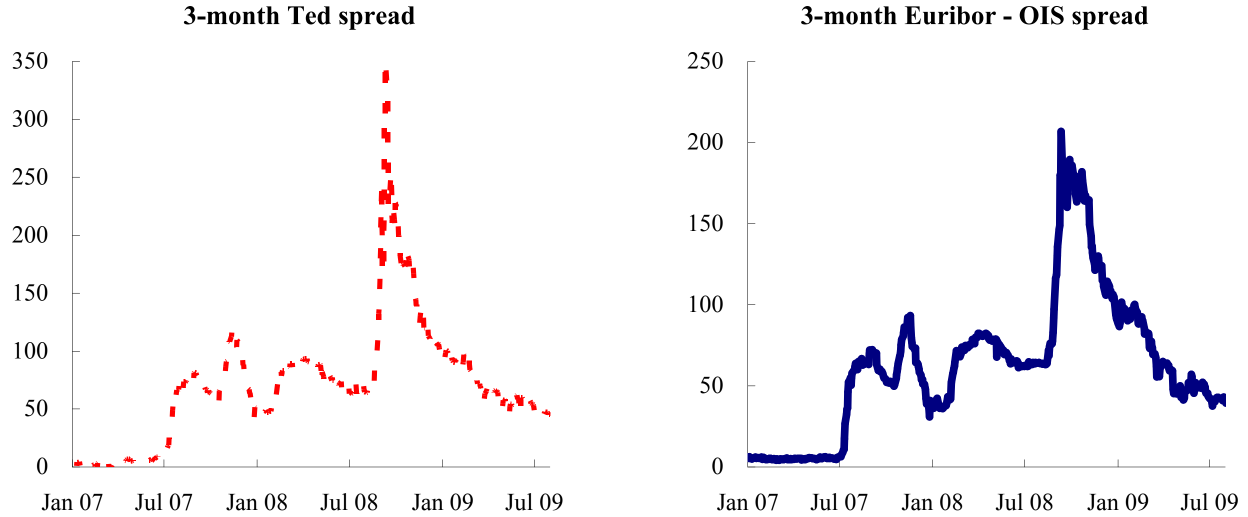
[7] More details about the "convenience yield" are in the following Section 5.

Even the Euribor-OIS (or, with only negligible differences, the Libor-OIS one), which is a darling of practitioners, is subject to similar remarks.[8] From the point of the investor, in the OIS contract there is no exchange of the principal: the straightforward implication is that in this contract the credit risk is confined to the OIS-overnight rate spread, times the notional.[9] Differently, in the unsecured inter-bank market, the lender copes with the risk that the borrower may default from his obligation altogether; in principle, the scenarios where the latter pays back only in part or at a later date than agreed cannot be ruled out. Moreover, as Eisenschmidt and Tapking (2009) show, if the lender bank considers the probability of a significant liquidity shock within the maturity of the loan to be likely and believes that it would not be able to borrow funds at a rate close to the risk-free rate (*eg.* for a shortage of high quality collateral available), then it must take into account the possibility that it may need to refinance any loan granted in the unsecured inter-bank market at a higher cost. As a result, the bank will be prepared to lend unsecured funds at a rate that also compensates for its funding liquidity risk. The Euribor-OIS is truly a measure of both the credit worthiness of the borrower as well as of the liquidity risk.

Chart 1

Money market conditions in the euro area

(*daily data; basis points*)



Another way to assess money market conditions is to look at the spread between unsecured inter-bank lending and secured inter-bank repos (repurchase agreements backed by Treasury or high quality securities) of the same maturity. In ECB reports, this spread is presented as difference between Euribor and Eurepo, where the latter is defined as the rate at which one prime bank offers funds in euro to another prime bank if in exchange the former receives general collateral from a basket of (high quality) assets. The difference between secured and unsecured lending rate may be accounted as a measure of credit risk (Taylor and Williams, 2009). However, Eisenschmidt and Tapking (2009) show that in times of liquidity problems, liquidity risk might play a lower role in (general collateral) repo markets than in unsecured market. If a bank grants a (general collateral)

8 The Euribor is defined as the rate at which euro inter-bank term deposits within the euro area are offered by one prime bank to another prime bank. The Euribor is calculated as an average of rates reported daily by a set of major banks. The banks participating in the Euribor panel are committed to report "to the best of their knowledge […] rates being defined as the rates at which euro inter-bank term deposits are being offered within the EMU zone by one prime bank to another". That is, panel banks report the rate which they assume are paid by the best banks at any given maturity.

9 The OIS is a derived contract where one side pays a fixed interest rates (the OIS rate) times the notional amount of the contract and, in turn, receives the return from rolling-over overnight in the inter-bank market the loan of same notional (where the relevant rate is the Effective Fed Funds Rate in the United States and the Eonia index in the euro area) until the OIS contract comes due. As a first approximation, the OIS rate is thought to act as measure of the average expected overnight rate which will prevail throughout the duration of the contract.

repo loan for a long term, then it receives assets eligible as collateral in repo markets for the life of the loan in exchange. It can reuse the collateral to raise funds itself in the repo market at the (lower) repo rate if it is hit by a liquidity shock before the loan matures. Moreover, as Garleanu and Pedersen (2009) show, the spread between an unsecured and secured rate is in equilibrium the shadow cost of capital of margin constrained investors and hence is correlated with liquidity risk.

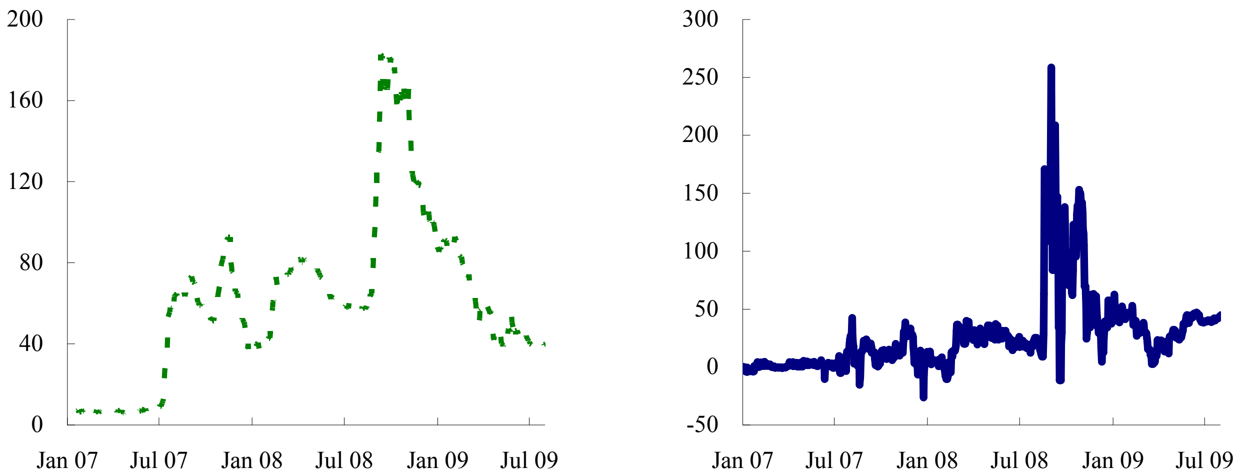
A number of papers have presented deviations from the covered interest parity condition (CIP) in the foreign exchange market as a further measure of crisis severity in the money market (Baba and Parker, 2008 and Coffey, Hrung and Sarkar, 2009). Such deviations represent an arbitrage opportunity from lending US dollars and borrowing in another currency. Coffey *et al* study the determinants of CIP deviations and show that tighter margin conditions are positively associated with CIP deviations prior to the Lehman bankruptcy, except during the period when the Fed eased margin conditions via its Term Securities Lending Facility (TSLF) program. Moreover, the central banks supply of dollars (through currency swap lines and Term Auction Facility, TAF) reduced CIP deviations during this period. As Sarkar (2009) outlines, these findings are consistent with limits to arbitrage in the foreign exchange and international money markets due to capital constraints which were eased by the central banks' supply of dollars. Alternatively, the arbitrage trade may have suddenly turned risky as previously low-risk trading counterparties became insolvent. Indeed, after the Lehman bankruptcy, counterparty risk became important determinants of CIP deviations and the central banks' supply of dollars was less effective in reducing the deviations. In general, these results indicate that both liquidity and credit risks are responsible for the failure of arbitrage but the contribution has varied over time.

Chart 2

Money market conditions in the euro area

(*daily data; basis points*)

Euribor - Eurepo 3-month spread | **Deviations from the Dollar Covered Interest Parity**



In summary, all the "relative" liquidity measures described thus far reflect therefore both credit and liquidity risks in time varying proportion. A number of papers have tried to give weights to these two risks, which while well recognized from a theoretical standpoint, are much more closely interrelated in the empirical analysis, even more so in the troubled times experienced during the market turmoil.

4 Liquidity risk premium in the money market from prices on CDS

The basic approach to infer the liquidity premium implicit in money market spreads is of an indirect type: first one infers the credit risk component in the Euribor-OIS spread (a step which typically requires a few basic assumptions); then, the liquidity risk is gauged as the difference between the spread and the estimated credit risk component. In most applications, *premia* on CDS contracts on the banks in the Euribor/Libor panel form the dataset used in step 1 of this procedure.[10] As a matter of principle, the CDS premium ought to reflect the likelihood that the reference entity defaults, the loss given default (LGD), if the credit event materializes, and the remuneration owed to the uncertainty over these two factors which cannot be measured exactly.

Abstracting from the impact of market liquidity on CDS *premia*[11], the probability that the bank acting as reference entity defaults may be gauged, in a risk-neutral setting, through a no-arbitrage condition. The model adopted in this paper applies, with some adaptations, the work by Longstaff, Mithal and Neiss (2005) for the corporate bonds. The data are 12-month CDS *premia* and 12-month money market rates. The model is fitted on daily data from January 3, 2005 to June 30, 2009. Let r_t denote the risk-free rate and λ_t the intensity of the Poisson process that drives the default probability. Moreover, let γ_t be the intensity representing the liquidity premium, that is the extra return investors seek to lend in the inter-bank market on top of the risk-free rate and of the remuneration owed for the credit risk. The three processes {r_t, λ_t and γ_t} are stochastic and mutually independent over time. The latter assumption greatly simplifies the model from an analytical standpoint. The recovery rate of the par value of the loan in the event of default (*1-w*) is assumed to be time-invariant, where *w* is the loss fraction when the credit event happens. The *risk neutral* law of motion of λ_t is:

[1] $$d\lambda = (\alpha - \beta\lambda)dt + \sigma\sqrt{\lambda}dZ_{\lambda}$$

where α, β, σ are positive scalars and Z_λ is a standard brownian process. This dynamic guarantee that any realisation of the process is always nonnegative. In these types of models, the probability of default over a period of time brief enough (from one day up to one month) is assumed to be equal to the intensity λ_t.

The *risk neutral* law of motion of the process running the intensity of liquidity is:

[2] $$d\gamma = \eta dZ_{\gamma}$$

where η is a positive scalar and Z_γ is a standard brownian process. The liquidity risk is linked to the expected dimension of the idiosyncratic and market-wide shocks that hit both the lender and the borrower. By nature, this risk takes a systemic nature (more details are in Li *et al.*, 2009), that is both the lending and the borrowing bank could be unable to sell timely and at limited cost adequate volumes of assets, without bearing losses due to adverse changes in the market price of the assets themselves.

Given the independence assumption, we do not actually need to specify the risk neutral dynamics of the risk-free rate to solve for CDS *premia* and inter-bank loan price.

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[10] In this paper, we select a sample of banks based on a criterion of availability of a own CDS contract which looks liquid enough during the period under investigation The liquidity of the contract has been assessed on the basis of the daily changes of the mid bid-ask rates. Following this procedure, only 26 of the 43 banks in the Euribor panel were eventually included in the sample. The list of such banks is in table 1

[11] The reader is referred to Longstaff, Mithal and Neiss (2005) for an assessment on how to ignore the role liquidity exerts on the mid bid-ask quote of a CDS contract.

In a CDS contract, the intensity of default can be worked out by equating the expected value of payments due by the protection buyer ("the premium side") and the expected value of the costs borne by the protection sellers if the credit event takes place ("the protection side"). Note that in CDS contracts, the buyer commits to pay quarterly instalments at times $t = t_1, t_2, \ldots, t_M$, where this obligation obviously does no longer hold when the event occurs.

At time *t* when the CDS starts to run, the expected value of the contract from the premium side is the sum of the expected stream of payments, discounted at the $r_t + \lambda_t$ rate, that is the risk-free rate adjusted by the credit risk component:

[3] $$P(cds,t,T) = E^Q\left[\sum_{i=1}^{M} \exp\left(-\int_t^{t_i}(r(s)+\lambda(s))ds\right)\times cds(t)\right]$$

where *cds(t)* is the quarterly premium and $E^Q(\ldots)$ is the expectation operator and the upper "Q" signals that such expectations are risk-neutral.

The current expected value from the "protection side" is the discounted value of expected loss at the potential default dates[12]:

[4] $$Pr(w,t,T) = E^Q\left[\sum_{i=1}^{M} \lambda(t_i)\times w\times \exp\left(-\int_t^{t_i}(r(s)+\lambda(s))ds\right)\right]$$

Assuming *P(cds,t,T) = Pr (w,t,T)* and solving for the premium *cds(t),* one obtains*:*

[5] $$cds(t) = \frac{E^Q\left[\sum_{i=1}^{M} \lambda(t_i)\times w\times \exp\left(-\int_t^{t_i}(r(s)+\lambda(s))ds\right)\right]}{E^Q\left[\sum_{i=1}^{M} \exp\left(-\int_t^{t_i}(r(s)+\lambda(s))ds\right)\right]}$$

If λ_t were not stochastic, the resulting premium would be λw. However, even when λ_t is stochastic, the premium can be interpreted as a present-value-weighted-average of $\lambda_t w$. More broadly, given the negative correlation between λ_t and $\exp(-\int_t^{t_i}(\lambda(s))ds$, the premium should be lower than the expected average value of λ_t times *w.* Given the dynamics of the process running the default intensity λ_t, closed-form solutions can be found to solve the CDS premium equation.

Likewise, it is possible to infer the value of the Euribor rate which rewards the investors both for the credit risk component, as measured from the intensity of default of each bank in the Euribor panel, and for the liquidity risk component. The quoted Euribor rate with maturity *(T-t)* is given by:

[6] $$REuribor(t,T) = \left(\frac{360}{a(t,T)}\right)\times\left(\frac{1}{NPV_t^{EUR}}-1\right)$$

where NPV_t^{EUR} denotes the net present value of an unsecured inter-bank loan and *360/a(t,T)* stands for the rule to count the days under the Euribor convention, that is *a(t,T)* is the actual number of days from *t* to *T*.

For a given bank (our representative agent), the interest rate applied to the inter-bank deposit is a function of the intensity λ_i of default of the i-bank which is borrowing the funds as well as of the γ

12 For simplicity's sake, within the model we assume that default may materialize only at the dates of payments of the quarterly instalments. In the real world, when the default occurs in the intervening period between two such dates, the protection buyer pays the instalment only *pro rata*.

intensity representing the liquidity premium. The resulting current expected value of a deposit with maturity *(T-t)* is:

$$
[7] \quad \begin{aligned} NPV_t^i &= E^Q\left[\exp(-\int_t^T (r(t)+\lambda_i(t)+\gamma(t))dt)\right]+... \\ ...&+E^Q\left[(1-w)\int_t^T \lambda_{i,t}\exp(-\int_t^{t_i}(r(s)+\lambda_i(s)+\gamma(s))ds)dt\right] \end{aligned}
$$

In [7], the first term on the right-hand side is the present value of reimbursement of the principal plus the due interest flow, the second term is the present value of recovery in the event of default. Because of the assumption of independence between r_t, λ_t e γ_t, the first term on the right-hand side of [7] can be written as:

$$
[8] \quad \begin{aligned} & E^Q\left[\exp(-\int_t^T (r(t)+\lambda_i(t)+\gamma(t))dt\right]=... \\ & E^Q\left[\exp(-\int_t^T r(t)dt\right]\times E^Q\left[\exp(-\int_t^T \lambda_i(t)dt\right]\times E^Q\left[\exp(-\int_t^T \gamma(t)dt\right] \end{aligned}
$$

Denoting with F(λ_i,T) the second expected term on the right-hand side of [8], F(λ_i,T) solves the following differential equation (Cox, Ingersoll and Ross, 1985):

$$
[9] \quad \frac{\sigma^2}{2}\lambda_i F_{\lambda_i\lambda_i}+(\alpha-\beta\lambda_i)F_{\lambda_i}-\lambda_i F-F_T=0
$$

under the boundary condition F(λ_i,0)=1. Additionally, it is possible to derive an explicit solution for F(λ_i,T) in the form of *A(T)exp*[*B(T)*λ_i] where A(t) and B(t) are the solutions of a system of ordinary differential equations (Riccati's equations).

We denote with V(γ,T) the third expected term on the right-hand side of [8]. Given the dynamic of the process of the intensity of liquidity described by [2], V(γ,T) verifies the following partial-derivatives differential equation:

$$
[10] \quad \frac{\eta^2}{2}V_{\gamma\gamma}-\gamma V-V_T=0
$$

subject to the boundary condition V(γ,0)=1. The explicit solution of V(γ,T) takes the form *C(T)exp[L(T)γ]* where C(t) e L(t) are derived by solving a system of Riccati's equations, subject to the boundary conditions C(0)=1 and L(0)=0. By the same token, from the assumption of independence, one obtains:

$$
[11] \quad \begin{aligned} & E^Q\left[\lambda_{i,T}\exp(-\int_t^T (r(t)+\lambda_i(t)+\gamma(t))dt\right]=... \\ & ...=E^Q\left[\exp(-\int_t^T r(t)dt\right]\times V(\gamma,T)\times E^Q\left[\lambda_{i,T}\exp(-\int_t^T \lambda_i(t)dt\right] \end{aligned}
$$

A short-form notation of the third term on the right-hand side of [11] is W(λ_i,T) which solves the following partial-derivatives differential equation (Duffie, Pan and Singleton, 2000):

$$
[12] \quad \frac{\sigma^2}{2}\lambda_i W_{\lambda_i\lambda_i}+(\alpha-\beta\lambda_i)W_{\lambda_i}-\lambda_i W-W_T=0
$$

subject to the boundary condition $W(\lambda_i,0)=\lambda_i$. The explicit solution of $W(\lambda_i,T)$ takes the form $exp(B(T)\lambda_i)(G(T)+H(T)\lambda_i)$ where B(t), G(t) and H(t) are derived (once again) through a system of Riccati's equations, under the appropriate boundary conditions.

The value of the premium of the CDS can be derived as:

[13]
$$cds(t)=\frac{E^Q\left[\sum_{i=1}^{M} w\times\exp(B(t)\lambda)\times D(t)\times(G(t)+H(t)\lambda)\right]}{E^Q\left[\sum_{i=1}^{M} A(t)\times\exp(B(t)\lambda)\times D(t)\right]}$$

The net present value of the loan to a given bank i, $NPV_{t,}^{i}$, is:

[14]
$$\begin{aligned}NPV_t^i &= A(T)\exp(B(T)\lambda_i)\times C(T)\times D(T)\times\exp(-\gamma T)+...\\ &...(1-w)\int_t^T \exp(B(t)\lambda_i)\times C(t)\times D(t)\times(G(t)+H(t)\lambda_i)\times\exp(-\gamma t)dt\end{aligned}$$

where λ_i e γ have been introduced above, $D(T)$ is the discount factor and here $L(T)=-T$.

The Euribor rate is defined as the rate at which euro inter-bank term deposits within the euro area are offered by one prime bank to another prime bank. The banks participating in the Euribor panel report the rate which they assume are paid by the best banks at any given maturity. Thus, the rate that a panel bank reports is not the rate at which other banks offer deposits to the reporting bank or the rate at which the reporting bank offers deposits to other banks. It is the rate at which the reporting bank believes one of the best banks offers deposits to another one of the best banks. Indeed, the (up to) 43 daily individual contributions to the one-year Euribor do not deviate much from one another: as Eisenschmidt and Tapking (2009) show, the standard deviation of individual contributions remained below five basis points even during the turmoil.

If we consider the first twenty banks in the panel which have lower intensity of default as prime banks, it is therefore plausible to estimate the Euribor rate as a simple average of the rates which would be offered to each of these banks given their intensity λ_i of default and the intensity γ of liquidity. Having previously derived the net present value of the loan offered to each bank in the Euribor panel, a simple average of rates offered to the first twenty banks provides a crude estimate of the Euribor rate:

[15]
$$REuribor(t,T)\cong\frac{1}{k}\sum_{i=1}^{k}REuribor_i=\frac{1}{k}\times\frac{360}{a(t,T)}\times\sum_{i=1}^{k}\left(\frac{1}{NPV_t^i}-1\right)$$

where k is the number of banks in the Euribor panel considered as prime banks and $REuribor_i$ is the rate which would be offered to bank i=1,..k, given its intensity λ_i of default and the intensity γ of liquidity.

In order to estimate the parameters of the processes which describe the intensity of default and of liquidity, values of the parameters α, β and σ have been generated randomly (using statistics based on the time series of the CDS *premia* to set the starting values of the random generator) for each of the n banks included in the sample; same procedure has been followed as regards the common parameter η.

Given a set of starting values for each date (day), the algorithm determines the value of λ_i which solves the [5] for each bank in the panel and the value of γ, where the latter is the fit of the model which minimises the square root of the average of the squares of the differences (the "errors") between the actual Euribor rate and the one obtained solving [15]. This procedure is applied for each date (day). Next, a new set of starting values is generated and the whole process is repeated anew. Having gone through the whole procedure 100 times, the "optimal fit" is determined as the global minimum value over the whole time series of the average square errors.

The credit risk component is worked out by imposing $\gamma = 0$ and C(T) = 1 in the optimal solutions. This allows to determine what would be the Euribor rate if there were no liquidity premium altogether. Next, the liquidity premium (non-credit component) itself is derived as a residual once such "theoretical" value of the rate is subtracted from its actual one. Because the procedure is only indirect by design and, in principle, further risks could be a play and/or there may be a joint second-order term of the credit and liquidity risks, the odds are that the resulting estimate of the liquidity premium exceeds its true value (in doing so, we implicitly assume that the covariance term takes positive sign, as in stress conditions the two basic risks are likely to be both high).

The fitting of the model proves to be highly accurate: the root mean squared error (RMSE) corresponding to the optimal solution is only 1,47 basis points, even under extremely erratic market conditions as reflected in the historically large volatility of both the CDS *premia* and of the Euribor rate.

The non-credit component (liquidity premium)

To test whether the non-credit component (the liquidity premium), which is given by simply subtracting the credit component of the spread from the total spread, reflects market liquidity as well as funding liquidity conditions, we regress the non-credit component on a number of liquidity proxies.

Data on market liquidity conditions in the money market are not easily available. For the euro money market, it is possible to compute indicators for market liquidity from prices and quantities observed on the electronic trading platform e-MID.[13] The first proxy is the bid–ask spread (in basis points) of the overnight deposits.[14] As there are very few transactions on e-MID in the segments over three-month, a number of studies (for instance Michaud and Upper, 2008) used the liquidity in the overnight market as a proxy for liquidity in term deposits. Since market liquidity in the overnight market appears to have been much less affected by the turmoil than market liquidity in the market for term deposits, the e-MID data are likely to understate the deterioration in liquidity conditions in the term market during the second half of 2007. Nevertheless, as Michaud and Upper show, these data may still provide useful information on when market liquidity was impaired.

The second proxy is given by deviation from the short-term covered interest parity (CIP) condition. This variable attempts to measure the role of U.S. dollar liquidity pressures, as many European banks with U.S. dollar assets have faced difficulties funding these positions. More specifically, soon after the turmoil began, European financial institutions increased activity to secure dollar funding to support US conduits for which they had committed backup liquidity facilities. At the same time, US financial institutions appeared to become much more cautious about lending dollars to other institutions because of their own need to preserve funds on hand. Facing unfavourable demand and supply conditions and the associated impairment of liquidity in inter-bank markets, many European institutions moved to actively convert euros into dollars through FX swaps. Deteriorating liquidity in the FX swap market likely contributed to further deviations of the FX swap market from the short-term CIP condition.

[13] According to market sources, e-MID had a share of approximately 20% of the unsecured euro money market, although this may have fallen during the turbulence.

[14] The decline in market share of the e-MID may affect the reliability of volume-based liquidity indicators but should have less of an impact on price-based measures as long as some market participants are able to arbitrage between the electronic and non-electronic markets.

Finally, as a measure of the impairment in the Euro commercial paper market we used the spread between the rate on Euro commercial paper offerings that are rated A+/P1 and with maturity 1 month and the OIS rate at the same maturity.

To estimate the relationship between these variables, a necessary first step is to verify the stationarity of the time series over the period considered (daily data from January 5, 2005 to June 30, 2009). The hypothesis of stationarity has been tested using the Augmented Dickey Fuller Test (ADF). For the independent variables and for the liquidity risk premium variable the test does not reject the null hypothesis that they follow a random walk; we conclude that the variables are integrated of order 1, I(1). The results of the regressions are shown in Table 2. The econometric analysis shows a significant positive relationship between changes of the measures of money market liquidity impairment and changes of the liquidity component of the Euribor – OIS spread. The same is true if we consider the first principal component for the money market liquidity measures previously described. When money market liquidity is drying up (i.e. values of the measures of the impairment in the money market liquidity are high), the liquidity risk is high (which would be equivalent to saying that high liquidity risk is associated with low money market liquidity conditions).

Measurement problems are greatest when it comes to assessing funding liquidity proxies. Relevant information for assessing the funding liquidity of Euribor banks would include liquidity ratios and the size of potential commitments. Unfortunately, these variables are not available on a systematic basis at a relevant frequency. However, as Drehmann and Nikolaou (2009) show, a measure for funding liquidity risk can be obtained by the spread between the submitted bid and the minimum bid rate in the Main Refinancing Operations (MROs).[15] The idea is that banks submit informed bids in the open market operations, taking into account information about future liquidity needs and future prices they have to pay to obtain it from other sources. Higher funding liquidity risk implies more aggressive bidding behaviour, in an environment with frictions in inter-bank and asset markets. Although submitted bids may not perfectly reflect the marginal value for funding liquidity, they should provide an ordinal proxy measure of funding liquidity risk.[16]

The analysis shows a significant positive relationship between the changes of the measure of funding liquidity risk and the changes of the liquidity component of the Euribor – OIS spread. When the funding liquidity risk increases, the liquidity component increases.[17]

This results supports the hypothesis that the non-credit component of the spread Euribor - OIS is due to market liquidity as well as funding liquidity conditions.

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15 This measure of funding liquidity risk is based entirely on publicly available data on the weighted average bid rate, the policy rate and the marginal rate. Nevertheless, it does not reach further than October 2008. This was the date of the last MRO before temporary changes to the auction design of the ECB were implemented. These changes involved switching from variable rate tender to fixed rate – full allotment tenders. Under the new framework only the volumes of liquidity demand are revealed but not the price, therefore one of the fundamental drivers of funding liquidity risk is shaded. As a result, this measure does not apply on the new auction design after October 2008.

16 This measure of funding liquidity risk can only be a proxy because bidding behaviour may also be influenced by other factors. The reader is referred to Ewerhart et al. (2006) and Bindseil et al (2003).

17 We use two different set of equations where the non dependent variables are market liquidity proxies and funding liquidity measure respectively, as these set of variables are strongly interrelated and affected by multicollinearity.

The liquidity risk and the ECB interventions

An examination of the evolution of risk during the crisis leads to a nuanced view of the key forces driving the crisis. Having a clear idea of the "risk context" is necessary to understand when central bank programs are likely to be effective and under what conditions the programs might cease to be effective.

As Christensen, Lopez and Rudebusch (2009) show, the provision of central bank liquidity could lower the liquidity premium on inter-bank debt through a variety of channels. On the supply side, banks that have a greater assurance of meeting their own unforeseen liquidity needs over time should be more willing to extend term loans to other banks. In addition, creditors should also be more willing to provide funding to banks that have easy and dependable access to funds, since there is a greater reassurance of timely repayment. On the demand side, with a central bank liquidity backstop, banks should be less inclined to borrow from other banks to satisfy any precautionary demand for liquid funds because their future idiosyncratic demands for liquidity over time can be met via the backstop. However, assessing the relative importance of these channels is difficult. Furthermore, any assessment of the effect of the recent extraordinary central bank liquidity provision in lowering the liquidity premium must also control for fluctuations in bank credit risk.

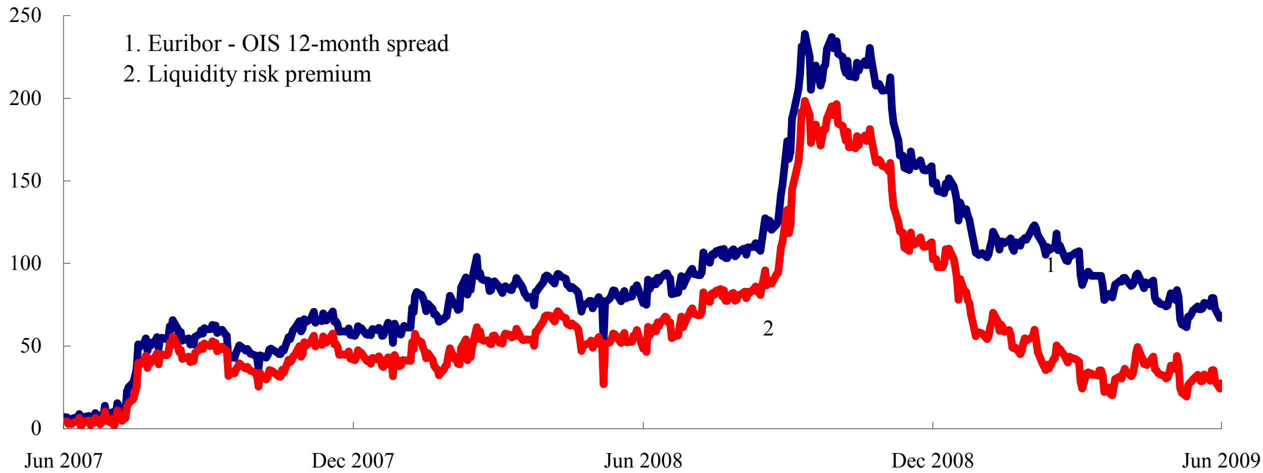
Before August 2007, the Euribor-OIS spread was fairly stable, at around three to five basis points, reflecting the fact that liquidity was flowing smoothly between borrowers and lenders, and that the probability of non-repayment of a money market loan was perceived to be low.

The developments in the Euribor-OIS spread in the early months after the onset of the crisis (Fall 2007) turn out to reflect mainly the surge in the liquidity risk while the credit risk played a lesser role; the relative weight of the latter risk increased in Spring 2008, with the Bear Stearns crisis, and accelerated as from the Lehman default. On the whole, even at the end of 2008, more than half of the spread can be explained as due to the liquidity risk, which displayed a more regular pattern and mimicked quite closely the money market spreads. The credit risk increased further in early 2009, a pattern that has come to a halt only in the following summer (Charts 3 a-b).

Charts 3 a-b

The Euribor - OIS 12-month spread and the liquidity risk premium ...

(*basis points*)



and the credit risk premium

(*basis points*)

The liquidity component fell dramatically after the enactment by ECB of unconventional loosening liquidity management policies. In the middle of 2009, it was very low compared to estimated values after the Lehman default, which could witness in favour of the effectiveness of the various policies undertaken by ECB. Conversely, even if it has partly retrenched from the high values experienced during the winter 2008-2009, the estimate of the credit risk stubbornly remained well above pre-crisis levels. Furthermore, and related to the previous observations, the financial terms applied to a loan in the money market seem to match more closely the credit worthiness of the borrower, compared to pre-Lehman conditions: would-be lenders tend to be less willing to lend to potential borrowers who are perceived as riskier and when eventually the loan takes place, a higher interest rate is charged to less credit worthy counterparties.

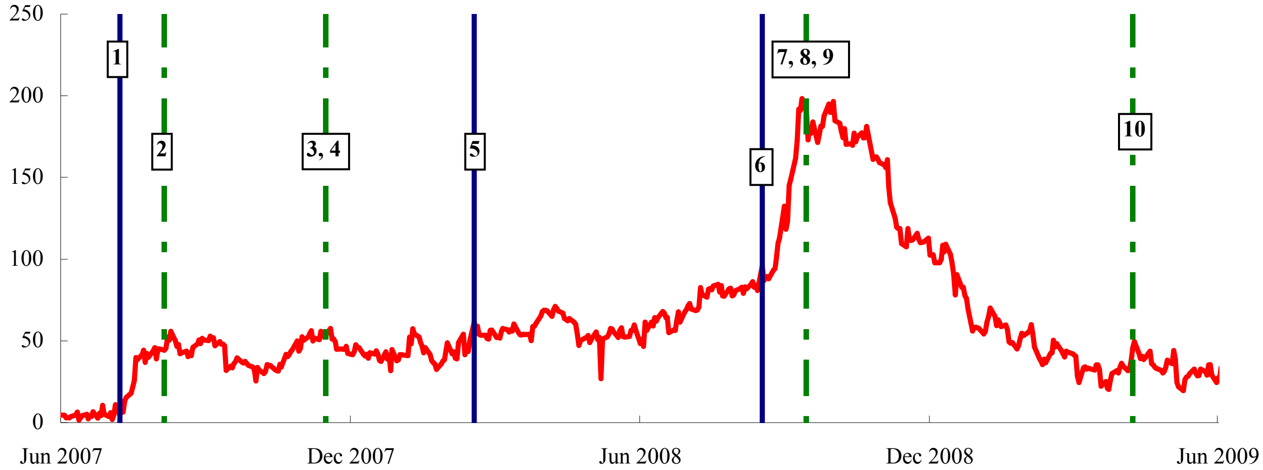
A visual analysis, is carried out in the following chart. The liquidity component, as previously calculated, is shown together with vertical solid blue lines indicating the negative events which led to such higher spreads and dotted green vertical lines indicating the most important ECB actions.

The evidence suggests that all the measures considered were followed by lower liquidity risk. Overall, this evidence could witness in favour of the effectiveness of the various policies undertaken by ECB.

Chart 4

The liquidity risk premium and relevant ECB actions

(*basis points*)



1	Start of the crisis
2	Supplementary LTROs
3, 4	US dollar liquidity-providing operations; fixed rate 2 week tender full allotment
5	JPMorgan buys Bear Stearns
6	Lehman default
7, 8, 9	Change to fixed rate tender with full allotment (prolongation in December and March beyond the end of 2009); narrowing of standing facilities corridor; expansion of the list of assets eligible as collateral
10	Announcement of 1 year tender

A word of caution is in order when assessing the estimates of the liquidity risk.

Without necessarily following any ranking or order, the following caveats should be borne in mind: first, when market conditions are especially dire, CDS *premia* could embody also some remuneration of the enhanced risk aversion (for given quantity of risk) and of the liquidity risk (see also the appendix); second, the recovery rate is set constant (a scalar) in the model, but the reader should be aware that the default probability and the loss-given-default *w* may covariate over the business cycle (this more sophisticated approach is e.g. in Altman, Resti and Sironi, 2008). If this constant constraint were loosened, in recession quarters the CDS *premia* could increase more than proportionally compared to the expected default frequency as not only the credit event becomes more likely but, in addition, if it materializes it brings about more losses.[18]

There is no simple way to determine the weight of each of these caveats, where the list is not necessarily exhaustive. When market conditions are more tense, an important factor behind the Euribor-OIS spread is usually identified in changes of the risk aversion (the price for unit of risk).

According to standard asset pricing theory, in an efficient market populated by fully rational and fully informed investors, the price of an asset as of time t should be equal to the net present value of futures pay-offs:

[18] Also, it is not clear in how far government guarantees affect the recovery rate as compared to the probability of default itself.

[16] $$p_t = E_t(m_{t+1}x_{t+1})$$

where x_{t+1} is the pay-off as of time t+1 e m_{t+1} is the stochastic discount factor. A version of [16] which is based on gross returns R_{t+1} is

[17] $$1 = E_t(m_{t+1}R_{t+1})$$

Because both the gross return R_t and the discount factor m_t are time variant, [17] can also be written as follows:

[18] $$1 = E_t(m_{t+1})E(R_{t+1}) + \text{cov}_t(m_{t+1}, R_{t+1})$$

The first term on the right-hand side of [18] measures the average return requested by risk-neutral investors. The second-term adjusts this return taking into account that, in fact, investors are risk averse. Since the gross risk free rate can be written as $R^f_{t+1}=1/E_t(m_{t+1})$, [18] becomes:

[19] $$E(R_{t+1}) - R^f_{t+1} = -R^f_{t+1}\,\text{cov}_t(m_{t+1}, R_{t+1})$$

which describes the excess expected return on a risky asset compared to a risk-free one as inversely proportional to the covariance between the expected return on the former asset (state contingent) and the stochastic discount rate. The risk premium can thus be broken down in a quantity of risk β_i which is specific of each asset *i* and the price per unity of risk which is common for all assets φ:

[20] $$E(R_{t+1}) - R^f_{t+1} = -\frac{\text{cov}_t(m_{t+1}, R_{t+1})}{\text{var}(m_{t+1})} \times \text{var}(m_{t+1}) \times R^f_{t+1} = -\beta_i \varphi\ R^f_{t+1}$$

The φ parameter can more easily be described as the risk aversion.[19] In [20] this parameter is defined as a function of the stochastic discount factor, var(m_{t+1}). In turn, the latter is the marginal rate at which the investor is willing to exchange one unit of future (uncertain) consumption with less than one unit of current consumption; as a result φ will vary with investor preferences and may change whenever investors cannot take the risks they are willing to bear, whatever is the constraint.

Over the sample examined in this paper, changes in the risk aversion have affected the dynamic in the CDS *premia* and in the money market spreads. With some simple regressions, it is possible to show that during the crisis changes of the Euribor-OIS spread have been a function of risk aversion. The risk aversion is not directly observable and the closet we can probably get is observing that, as hinted at in several empirical analyses, it is positively correlated to the implicit volatility priced in options. Approaches range from options in individual markets (such as stock exchanges or swaptions) to averages of the volatility from options traded in several markets. Same relationship hold true when a variance decomposition approach is followed. [20]

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[19] The more recent literature accepts as different the concept of risk aversion from the concept of risk appetite (or risk perception or implicit risk aversion). While the former should be constant over time, the latter should change over the cycle (Gai and Vause, 2005). In this paper, we refer more precisely to risk aversion.

[20] Additional details are available from the author on request.

5 A joint model of government bonds, financial bonds and the Euribor rate

To check the robustness of the results presented about the effectiveness of the ECB liquidity facilities in reducing inter-bank lending pressures, we use a multifactor arbitrage-free (AF) representation of the term structure of interest rates and bank credit risk. The structure adopted allows us to decompose movements in Euribor rates into changes in bank debt risk premiums and changes in a factor specific to the inter-bank market that includes a liquidity premium. This model, unlike the previously described, does not dwell on CDS prices.

The model showed in this section is an affine model with latent-state variables which is fitted on the yields of Treasury bonds, of banks' and non-bank financial firms' bonds as well as our cherished Euribor rates. The model follows on the work by Christensen, Lopez and Rudebusch (CLR, 2009) with one important add-on, besides our focus on euro data: we explicitly introduce a risk-free rate. This one differs from the so-called instantaneous interest rate on government bonds by a component which is usually referred to as "the convenience yield". As explained in Feldhutter and Lando (2008), the lower rate investor accept to earn on a government bonds depends on:

a) repo specialness due to ability to borrow money at less than the GC repo rates;
b) that Treasuries are an important instrument for hedging interest rate risk;
c) that Treasury securities must be purchased by financial institutions to fulfil regulatory requirements;
d) that the amount of capital required to be held by a bank is significantly smaller to support an investment in Treasury securities relative to other securities with negligible default risk, and
e) the ability to absorb a larger number of transactions without dramatically affecting the price.

There are several possible candidates as a proxy for risk-less rate. First, we could use data from the KfW Bankengruppe yields. As a bank owned by the Federal Republic of Germany (80 per cent) and the federal states (20 per cent), KfW Bankengruppe is a government sponsored enterprise and has an explicit and direct state guarantee (Law concerning KfW, Art.1a). Consequently, the credit risk on KfW bond issues is small. In addition, KfW issues debt in large amounts: it raises 60-70 billion in the capital markets every year and is thus the fifth-largest capital market issuer in Europe after the governments of Italy, Germany, France and the UK. The spread between KfW Bankengruppe bonds and German Treasury bonds captures the flight to liquid assets. German treasury notes are viewed by the markets as free of credit risk. However, periods of stress are often characterized by strong demand for the most liquid (on-the-run) German treasury notes.

This approach is suggested by Feldhutter and Lando (2008), who, in their analysis on US data, focus on the spread between Fannie Mae (a US government sponsored enterprise) bonds and Treasury bonds. However, these authors do not necessarily argue that the Fannie Mae yield curve is a good proxy for the risk-free yield curve; furthermore, Ambrose and King (2002) find an insignificant repo specialness effect in the ten-year Fannie Mae yield but a significant effect in shorter maturities suggesting that the short end of the Fannie Mae yield curve has stronger repo specialness effects than the long end.

As an alternative measure of the convenience yield, and the approach we adopt, is to use general collateral government repo rate. As argued by Longstaff (2000) and Liu, Longstaff and Mandell (2006), this rate is virtually a risk-less rate, since repo loans are almost always over-collateralized using Treasury securities as collateral. Furthermore, since repo loans are contracts rather than securities, they are less likely to be affected by the types of supply and demand-related “specialness” effects that influence the prices of securities. By explicitly referring to the risk-free rate, when breaking down the spread between the Euribor and the risk-free rate itself we do not need to pay attention to the “convenience yield”, which conversely is one of the elements contributing to the spread between Euribor and the Treasury bills rate (the Ted spread).

As an important value-added, the affine model allows to refer the bonds' pricing problem expressed in the general form of partial differential equations (PDE) in terms of an ordinary differential equations (ODE), corresponding to which we know closed-form solutions.

The model is fitted on weekly data from January 13, 2003 to July 29, 2009; the maturities of Treasury zero coupon bonds are 3, 6, 12, 24, 36, 60, 84 and 120 months, where the source of the data are Bloomberg and the Deutsche Bundesbank. Data of the banks' and non-bank financial firms’ zero-coupon bonds are from Bloomberg’s Fair Market Yield Curves (FMYC), where we selected the same 8 maturities described above for the government bonds. Non-government bonds are taken from four classes of rating: A and AA for debt instruments of non-bank financial firms and A, AA for debt instruments of banks. The corporate bond yield curves for the different categories are noisy, but the inclusion of several curves makes our model less sensitive to measurement errors in these curves. Euribor rates are referred to the 3, 6 and 12 months maturities; general collateral government repo rates are referred to the same 3 maturities selected for the Euribor rate, where the source of the data is Bloomberg. Overall, at each data we find in our dataset 46 observations of yields.[21]

In the model we find seven latent state variables: three of them model the dynamics of the Treasury bond yields, one describe the convenience yield; two follow the spread between the rate of corporate bonds and the risk-free rate; finally, the seventh latent state variable captures the idiosyncratic changes in the Euribor rates. Note that in CLR, one finds “only” six such variables, due to the omission of the convenience yield.[22]

Following CLR and Christensen, Diebold and Rudebusch (CDR), factor loadings on state variables have been introduced to explain the law of motion of the Treasury yields; notably, loads bind the level, the slope and the slope-change (the curvature) as derived from the standard model by Nelson and Siegel (this literature also clarifies the advantages of this loading approach compared to more standard affine models). CLR show that an arbitrage-free Nelson Siegel (AFNS) model can be readily estimated for a joint representation of Treasury, financial bonds and inter-bank yields.

Treasury yields can be described as a function of three state variables: $X_t^T=(L_t^T, S_t^T, C_t^T)$. The instantaneous rate is:

[21] $$r_t^T = L_t^T + S_t^T$$

The dynamics of the three state variables under the risk-neutral measure Q is described by the following system of equations:

[22] $$\begin{pmatrix} dL_t^T \\ dS_t^T \\ dC_t^T \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda^T & \lambda^T \\ 0 & 0 & -\lambda^T \end{pmatrix} \begin{pmatrix} dL_t^T \\ dS_t^T \\ dC_t^T \end{pmatrix} dt + \begin{pmatrix} \sigma_{L^T} & 0 & 0 \\ 0 & \sigma_{S^T} & 0 \\ 0 & 0 & \sigma_{C^T} \end{pmatrix} \begin{pmatrix} dW_t^{Q,L} \\ dW_t^{Q,S} \\ dW_t^{Q,C} \end{pmatrix}$$

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[21] The reader is referred to Feldhutter and Lando for the conversion of the Bloomberg data for financial corporate rate and Euribor rate into continuously compounded yields.

[22] CLR focus on the dynamic interactions between financial bond yields and Libor rates, so the choice of the risk-free rate is not an issue for their analysis.

where W^Q is a standard brownian process along three dimensions (that is, $W^Q \sim N(0,I_3)$). Given this structure, CDR show that the government bond zero-coupon yield with residual maturity τ at time t, $y_t^T(\tau)$, is:

$$[23] \qquad y_t^T(\tau) = L_t^T + \left(\frac{1-e^{-\lambda^T\tau}}{\lambda^T\tau}\right)\times S_t^T + \left(\frac{1-e^{-\lambda^T\tau}}{\lambda^T\tau} - e^{-\lambda^T\tau}\right)\times C_t^T + \frac{A^T(\tau)}{\tau}$$

The three factors are given exactly the same level, slope and curvature factor loadings as in the Nelson-Siegel yield curve. The term $A^T(\tau)/\ \tau$ represents an adjustment factor which is time-invariant and which changes only with the residual maturity. The volatility matrix is of a diagonal type, where CDR show that when parameters are calibrated more flexible, the out-of-sample forecast performance of the model declines.

As in Feldhutter and Lando (2008), we describe the instantaneous risk-free rate as :

$$[24] \qquad r_t^{rf} = L_t^T + S_t^T + e + CY_t$$

where the first two terms are the same we observed in the short-term rate of government bonds, while the fourth term is the convenience yield. Finally, the constant *e* is the average of CY in the risk-neutral world. We model the dynamics of the convenience yield as a "level" state variable, on the basis of the evidence on loadings of each maturity on the first principal component for the zero-coupon spread between government yields and risk-less yields:

$$[25] \qquad dCY_t = \sigma_{CY} dW_t^{Q,CY}$$

The instantaneous rate on debt securities of group *i* (banks or other financial firms) with rating *c* (A, AA) is written as:

$$[26] \quad r_t^{i,c} = \alpha_0^{i,c} + (1+\alpha_{L^T}^{i,c})\times L_t^T + (1+\alpha_{S^T}^{i,c})\times S_t^T + (\alpha_{L^S}^{i,c})\times L_t^S + (\alpha_{S^S}^{i,c})\times S_t^S + (1+\alpha_{CY^{rf}}^{i,c})\times CY_t$$

where (L_t^T, S_t^T) are the factors underlying the dynamics of the Treasury bonds and (L_t^S, S_t^S) are the factors underlying the excess yield on the corporate bonds as regards the risk-free rate. In the risk-neutral measure, the dynamics is:

$$[27] \qquad \begin{pmatrix} dL_t^S \\ dS_t^S \\ dL_t^T \\ dS_t^T \\ dC_t^T \\ dCY_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda^S & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda^T & \lambda^T & 0 \\ 0 & 0 & 0 & 0 & -\lambda^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L_t^S \\ S_t^S \\ L_t^T \\ S_t^T \\ C_t^T \\ CY_t \end{pmatrix} dt + \Sigma^S \begin{pmatrix} dW_t^{Q,L^S} \\ dW_t^{Q,S^S} \\ dW_t^{Q,L^T} \\ dW_t^{Q,S^T} \\ dW_t^{Q,C^T} \\ dW_t^{Q,CY} \end{pmatrix}$$

where Σ^S is a block diagonal matrix, as the two factors relating to the excess yield on the corporate bonds as regards the risk-free rate are independent from the four factors driving the risk-free rate. Such a structure allows to determine the coefficients of all six factors in the function which describe the corporate debt yield.

In high-rating financial institutions, this yield is about the same as the inter-bank rate (for same maturity) as both represent the cost of lending funds without guarantee to financial institutions. As such, both financial instruments share same credit risk features. However, to shed light on the idiosyncratic differences between the two, we included a seventh factor in the model. As a result, the instantaneous rate on Euribor lending is:

[28] $$r_t^{EUR} = r_t^{Fin,AA} + \alpha^{EUR} + X^{EUR}{}_t$$

The dynamics of the factor dealing with the inter-bank rate is described by:

[29] $$dX_t^{EUR} = -K_{EUR}{}^{Q} X_t^{EUR} dt + \sigma_{EUR} dW_t^{Q,EUR}$$

where this factor as well is assumed independent from the other six ones, under the risk-neutral measure.

If we denote with X_t the vector with the seven state variables, we obtain the following all-encompassing model:

[30] $$dX_t = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda^S & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda^T & \lambda^T & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_{EUR}^{Q} \end{pmatrix} (X_t) dt + \Sigma^{EUR} \begin{pmatrix} dW_t^{Q,L^S} \\ dW_t^{Q,S^S} \\ dW_t^{Q,L^T} \\ dW_t^{Q,S^T} \\ dW_t^{Q,C^T} \\ dW_t^{Q,CY} \\ dW_t^{Q,EUR} \end{pmatrix}$$

From this equation, we can derive with reference to the Euribor contracts both the result for the discount rate and for continuously compounded Euribor yield.

The relationship which links the risk-adjusted measure of probability Q with the "physical" measure of probability P goes through the market price of risk, which is time varying:

[31] $$\Gamma_t = \gamma_0 + \gamma_1 X_t$$

where γ_0 e γ_1 are, respectively, of order R^7 e R^{7x7} and include unconstrained parameters which meet the result $dW_t^Q = dW_t^P + \Gamma_t dt$.

The dynamics of the state variables can be represented in the world of physical probabilities P as:

[32] $$dX_t = K^P(\theta^P - X_t)dt + \Sigma dW_t^P$$

where K^P and θ^P are parameters of the P-world.

The representation of the model in the form space-state and the estimation methodology

We accept that yields to maturity may be subject to measurement errors. Model parameters are thus estimated with the maximum likelihood method using the Kalman filter. When the model is written in the space-state form, which is not unusual in applications of the Kalman filter, the so-called observation (or measurement) equation is:

[33] $$y_t = \begin{bmatrix} A^c \\ A^T \\ A^{RF} \\ A^{EUR} \end{bmatrix} + \begin{bmatrix} B^c \\ B^T \\ B^{RF} \\ B^{EUR} \end{bmatrix} X_t + \varepsilon_t$$

where the vector y_t is of size (*46x1*). The vector ε_t stands for the error in the measurement of the yields which follow a *N(0,R)* distribution. The so-called state (or transition) equation is a discrete version of [32]:

[34] $$X_t = e^{-Kh} X_{t-h} + \left(I - e^{-Kh}\right) \times \theta^P + \varepsilon_t$$

where $\varepsilon_t \sim N(0, \Omega_h)$ and $\Omega_h = \int_0^h e^{-Ks} \Sigma\Sigma' e^{-K's} ds$

The value h signals the frequency selected to estimate, expressed in years (*e.g.* because we work on weekly data, h=1/52).[23]

The model parameters are fitted using a maximum likelihood approach:

$$\log L(Y_N) = \sum_{t=1}^{N} \log f\left(y_t \mid I_{t-1}\right)$$

$$f\left(y_t \mid I_{t-1}\right) = \left\{ (2\pi)^{-1/2} \left| BP_{t|t-1} B^T + R \right|^{-1/2} \exp\left[-\frac{1}{2} \left(y_t - A - B\hat{S}_{t|t-1}\right)^T \left(BP_{t|t-1} B^T + R\right)^{-1} \left(y_t - A - B\hat{S}_{t|t-1}\right) \right] \right\}$$

Solving for the logarithm, the function which is to be maximised becomes (here, we follow Duffee, 2002):

$$\log L\left(Y_N\right) = -\frac{1}{2} \left| BP_{t|t-1} B^T + R \right| \left(y_t - A - B\hat{S}_{t|t-1}\right)^T \left(BP_{t|t-1} B^T + R\right)^{-1} \left(y_t - A - B\hat{S}_{t|t-1}\right)$$

where the matrices S and P represent the estimate (for time t) of respectively the expected mean and variance of the state variables. Such matrices have been worked out using optimally the information on the parameters of the "measurement" and "transition" equations as available at time t-1. Finally, S and P are revised step-wise for t which goes from 1 to N, using Kalman's algorithm.

We followed a four-steps strategy to max the likelihood function:

1) starting values of the parameters are generated, from a multivariate distribution whose variance-covariance matrix is diagonal. Mean and variances of such distribution are set as plausible values;

---

[23] The solution of the integral is: $vec(\Omega_h) = -\left((K \otimes I) + (I \otimes K)\right)^{-1} vec\left(e^{-Kh} \Sigma\Sigma^T e^{-K^T h} - \Sigma\Sigma\right)$.

2) if starting values do not meet conditions to ensure stationarity of the state variables[24], we are back to step 1 otherwise we move forward;
3) a line search algorithm[25] is used to max the likelihood function;
4) the results from step 3 are used as starting values of a derivative-based algorithm[26] to improve further the accuracy of the estimates.

This procedure is repeated 100 times. All solutions represent a maximum value for the likelihood function, each obtained from a starting value selected randomly from a multivariate probability distribution.

The results

The fit of the Treasury yields is acceptable; however, it results worse than in model of only Treasury yields (for example, CDR). The root mean squared errors (RMSE) of the fitted errors is 16 basis points (bp) on average. For the corporate bond yields, the RMSE is 27 bp on average. Overall, given the fact that we are fitting a sizeable number of corporate bond yields jointly with six state variable, the achieved fit of the corporate bond yields appears acceptable. The fit of the Euribor rates is quite good: RMSE is 13 bp on average.

The maximum likelihood parameter estimates and their asymptotic standard errors are reported in Table 3.1 and Table 3.2.

The liquidity component and the ECB interventions

During the crisis, the ECB has utilized both traditional monetary policy instruments as well as innovative tools to provide liquidity. Following CLR, the estimated model can provide a platform to assess the effectiveness of these interventions.[27] Chart 5 shows the fitted pattern of the difference of idiosyncratic factor of the Euribor rate (the "Euribor factor") relative to its historical average. The "Euribor factor" differentiates the dynamics of this rate from that of the bonds issued by financial institutions with rating AA. In the four years before the onset of the crisis, this factor displayed a fairly fast mean-reverting behaviour.

24 In order to ensure stationarity of the state variables, we impose the all the K^P eigenvalues are inside the unit circle.

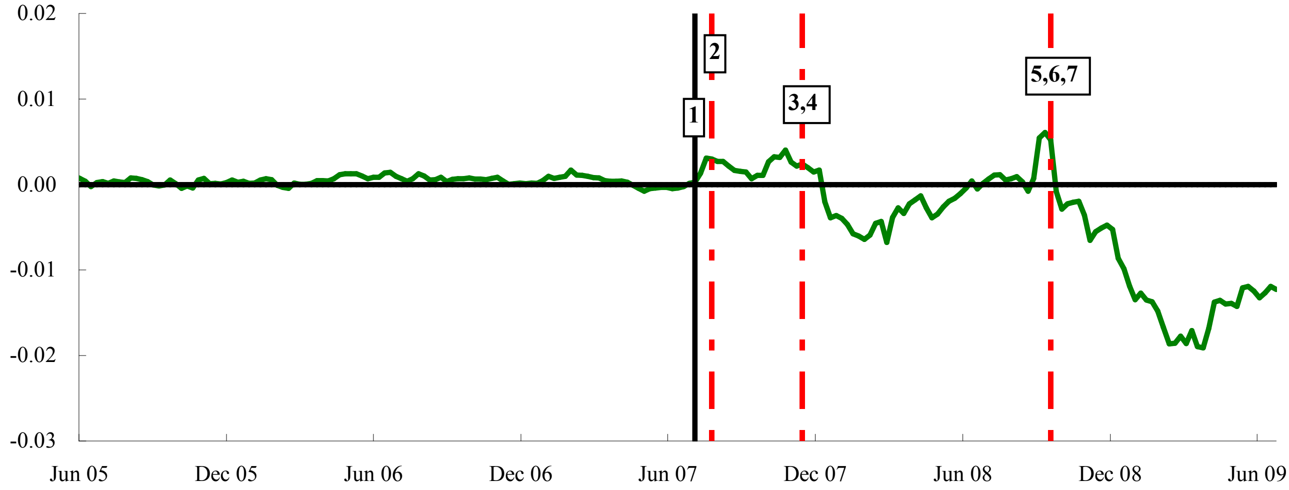
25 We used the fminsearch function in Matlab which implements the simplex method. The maximum number of the iterations is 5000; the maximum number of times the function is assessed is 10 millions; the error tolerance on x is 1e-6; the tolerance to the value of the function is 1e-6. The algorithm we used is Levenberg-Marquardt. All the other options are set to standard Matlab values.

26 We used the fmincon function in Matlab.

27 Fiscal policies have also played an important role in containing the adverse impact of the financial and economic crisis. Government support for the banking sector has represented a key element in the stabilisation of the whole financial system and the prevention of a further detrimental impact on the real economy. The measures adopted in response to the financial crisis consisted of various types of financial assistance, including government guarantees for inter-bank lending, recapitalisation of financial institutions, increased coverage of retail deposit insurance and asset relief schemes. However, in this paper we concentrate on the effectiveness of ECB interventions.

Chart 5

Difference of "Euribor factor" relative to its historical average



1	Start of the crisis
2	Supplementary LTROs
3, 4	US dollar liquidity-providing operations; fixed rate 2 week tender full allotment
5, 6, 7	Change to fixed rate tender with full allotment (prolongation in December and March beyond the end of 2009); narrowing of standing facilities corridor; expansion of the list of assets eligible as collateral

During the market turmoil, the relationship between the Euribor rate and the AA financial firms' bond yields broke down. Wu (2008) and CLR suggest that the enactment by the central banks of a number of ad hoc policies after the onset of the crisis led to a break in the money market conditions.

Notably, at the beginning of the crisis, the "Euribor factor" stood above the average; however, following the introduction of both the currency swap lines between central banks in mid December 2007 as well as of other measures[28], the factor fell below its historical average and stood at this values until September 2008, confirming that Euribor rates were lower (compared to pre-crisis levels) than the firms' bond yields. As from the Lehman default, the factor reverted and assumed values above the historical mean.

The sharp deterioration of conditions in the euro money markets after the Lehman default had important implications for the provision of refinancing by the Eurosystem to the euro area banking sector. In October 2008, the Eurosystem decided to change the tender procedure, from a variable rate with pre-gauged allotment to a fixed rate with full allotment, in which banks' bids would be satisfied in full at the fixed Main Refinancing Operation (MRO) rate. The Eurosystem considered it essential to ensure that ample liquidity was provided directly to all banks in need, given that the usual mechanism for distributing aggregate liquidity provision via the money market was seriously impaired. Moreover, the Eurosystem aimed to eliminate uncertainty about the amount of liquidity allocated to each bank. A second measure announced on October 2008 was a narrowing of the corridor formed by the rates on the two standing facilities around the MRO rate (from 200 bp to 100

[28] As an answer to the onset of the financial crisis, the Eurosystem modified the timing of liquidity supply to the banking sector during the course of the reserve maintenance period (more ample liquidity was provided at the beginning of each maintenance period, while over the course of the maintenance period the liquidity supply was gradually adjusted downwards so that by the end of each period banks continued to have, as before August 2007, a liquidity surplus of close to zero on average) and started conducting supplementary longer-term refinancing operations (LTROs) with maturities of three months, and later also six months.

bp). With the intensification of the turmoil, it was recognised that even solvent banks' ability to obtain funds in the inter-bank market was impaired, and that recourse to the standing facilities was increasingly important for banks. Finally, it was decided the expansion of the list of assets eligible as collateral for Eurosystem operations and further liquidity-providing operations in US dollars and Swiss francs.

In the wake of these measures, Euribor fell dramatically compared to AA financial bond yields, and the "Euribor factor" shifted away from its historical average.

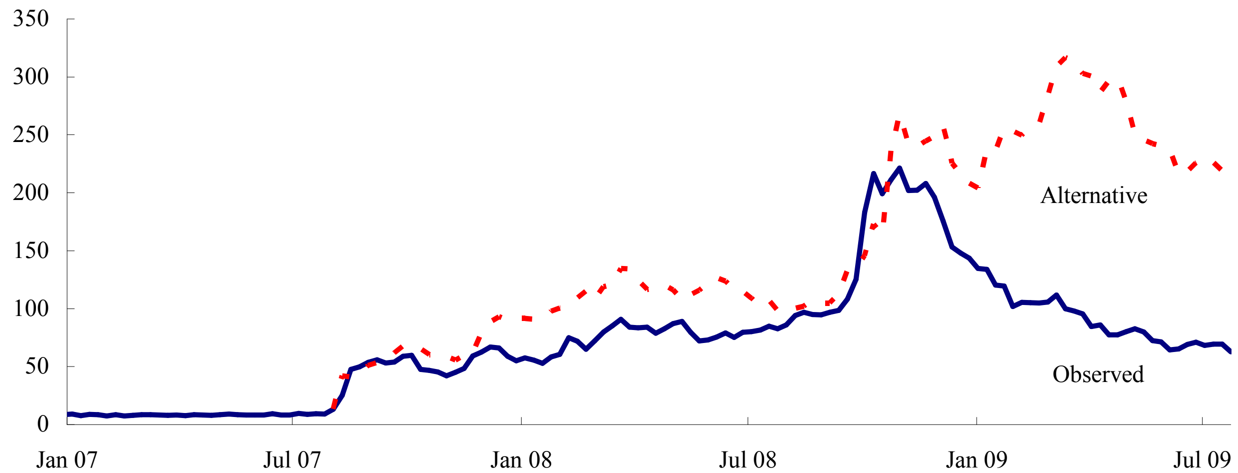
The model also offers an additional angle from which to assess the degree of effectiveness of the central banks' interventions. This is achieved through a sort of counter-factual exercise: first, within the crisis window of the sample the "Euribor factor" is set constant to its average before the loosening in monetary policies; then, one could estimate what would have been the Euribor rate (and its spread vis-à-vis the risk-free rate) if it had embodied the same liquidity risk *premia* which affected the financial firms bonds (the credit risk is deemed to be the same in the two asset classes, anyway).

This type of simulations suggest that in the sample that covers the period from August 9, 2007 to July 29, 2009, on average, the difference between the counterfactual spread ("Alternative") and the observed spread between the 12 month Euribor and the risk-free rate has been over 50 bp.[29] In the period after Lehman default, central banks did manage to lower the money market rates, compared to the dynamics of the financial firms which is used as a yardstick, by over 100 bp.[30]

Chart 6

Observed and alternative 12 month Euribor – risk free rate spread

(*basis points*)



[29] August 9, 2007 marks the start of the turmoil in financial markets and the jump in Euribor rates. An important trigger for the financial crisis and the tightening of the money markets was the announcement by the French bank BNP Paribas that it would suspend redemptions from three of its investment funds.

[30] An alternative explanation for the larger spread is the possibility of a change in the relative credit risk characteristics of the bank debt and interbank loan markets, for example, through changes in perceived recovery rates. However, CLR show (for the period that ends on July 2008) results which support the assumption of common credit characteristics across the interbank rate and bank debt rate and that this relationship did not materially change around the announcement of the central bank liquidity facilities.

6 Concluding remarks

In official quarters, the recent debate and empirical work has focused on the liquidity risk embedded in money market instruments, as even a quick glance at central banks' reports and bulletins can confirm.

As the most widely used gauges of the money market liquidity conditions embody both liquidity risk and credit risk, in this paper we put forward two approaches to infer the liquidity component in the money market spreads. A first type of approach gauges the credit risk of the banks participating in the Euribor panel by first inferring their default probability from prices on own Credit Default Swap (CDS) contracts and then is estimated the liquidity component as a residual.

In the second one, liquidity risk premium is derived along a simultaneous model estimate approach, where variables include unsecured inter-bank deposit rates, zero coupon yields on financial bonds, and zero coupon yields on Treasury bonds.

The results presented in this paper confirm that, throughout the most severe phases of the market turmoil, the rise in the Euribor-OIS spread (as in other gauges of money market conditions) owed to both liquidity and credit risks, where the relative weight of these two components changed over time with credit risk becoming more and more relevant, while initially the liquidity risk accounted for the lion's share.

In responding to the crisis, the ECB (and other central banks) has utilized both traditional monetary policy instruments as well as innovative tools to provide liquidity. In the wake of these measures, the liquidity risk component fell as a rock. These results could witness in favour of the effectiveness of the policies undertaken by central banks.

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Appendix

Decomposing the CDS spread

CDS spreads compensate investors for expected loss, but they also contain risk *premia* because of investors' aversion to default risk. With reference to the banks in the Euribor panel, the analysis shows that the estimated *premia* feature a high volatility, which is often reckoned to be an upshot of changes in the underlying risk aversion.[31] In algebraic terms, we have

[1A] $$cds(t) = \frac{E^Q\left[\sum_{i=1}^{M} \lambda(t_i) \times w \times \exp(-\int_t^{t_i} (r(s) + \lambda(s))ds)\right]}{E^Q\left[\sum_{i=1}^{M} \exp(-\int_t^{t_i} (r(s) + \lambda(s))ds)\right]}$$

In [1A], the CDS spreads are weighted averages of risk-adjusted expected losses, $E^Q(\lambda \times w)$. In principle, there may be two sources of difference between $E^Q(\lambda \times w)$ and the actual expected loss $E^P(\lambda^P \times w)$, where $E^P(\ldots)$ denotes "real-world" expectations. First, the risk adjusted intensity λ, which is the right one to price the CDS contracts, may differ from the actual intensity λ^P; the adjustment depends on the price of jump-at-default risk (JtD), that is $\lambda = \lambda^P \times (1 + \text{JtD})$, which is the compensation for the actual default of an entity and its impact on investors' wealth due to an inability to perfectly diversify credit portfolios. Second, expectations of $\lambda \times w$ are derived on probabilities adjusted to take account of investors' aversion to systemic risk S, which is the compensation for unity of volatility of the risk factors that affect the default probability.

As a result, the CDS spreads can roughly be decomposed as follows:

CDS spread ≅ expected loss ($\lambda^P \times w$) + *risk premium JtD* + *risk premium S*

which can be written also in multiplicative form:

CDS s*pread ≅ expected loss* ($\lambda^P \times w$) × *risk adjustment*

where the risk adjustment term is equal to *(1 + price of default risk)*.

This price can be interpreted as the reward for unity of expected loss and is an indicator of the investors' aversion to the default risk. Values higher than zero mean that investors demand a larger reward than what would be derived strictly from actuarial losses. As a result, both the risk adjustment term and the price of the default risk can be expressed as a ratio between the spread and the expected loss.

We adopt the following simple estimation methodology. First, we compile a measure of the risk premium as difference between the spreads and an estimation of the expected loss. The latter is obtained through the Expected Default Frequencies (EDF) by Moody's KMV as a proxy for the probability of default and assuming that loss-given-default is constant. The EDF's are derived from balance-sheet data and market values of the firm's shares in a Merton model. As the EDF data mean to measure the default probabilities over a one-year horizon, in our analysis we focused on one-year CDS.

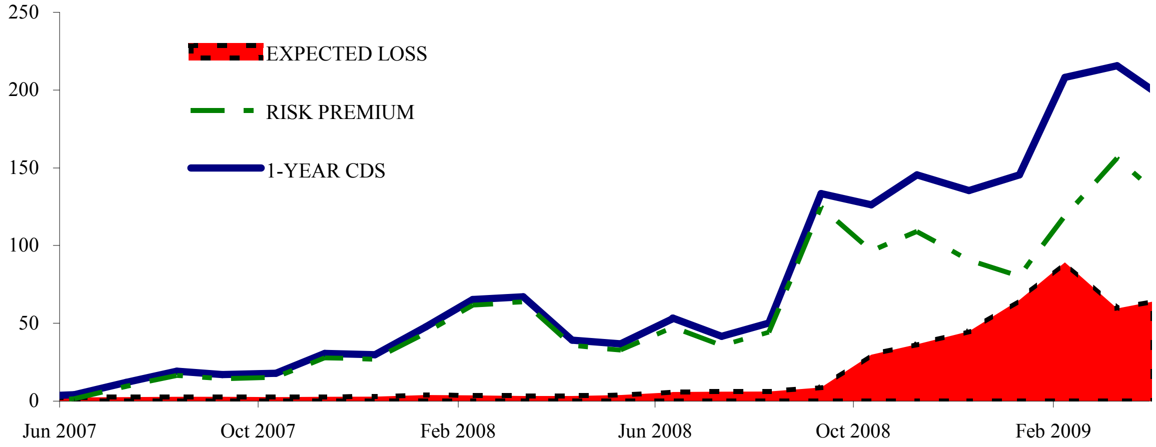
The increase in the spreads observed as from August 2007 was led, initially, by the surge in the default risk, while the component of expected loss increased only marginally in the early stages.

[31] This decomposition follows the method set out in Amato (2005).

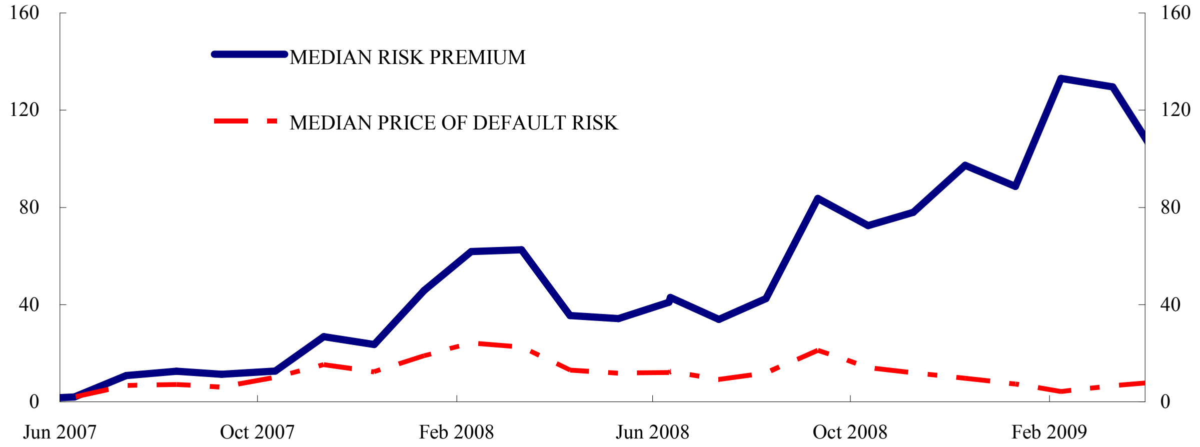
Then, as from April 2008, the former component waned, where the downward pattern continued until the early months of 2009, although the CDS spread had continued to increase. This can be explained in terms of the expected loss.

Chart A1 a-b

CDS spread decomposition



Risk premium and price of default risk



More broadly, the risk premium implicit in CDS spreads rewards the investor against the exposure to the default risk, to the liquidity risk as well as other possible non-diversifiable sources of risk.

To estimate the weight with which each of the factors explain the variance of the risk premium, we implemented a Vector Auto-Regression (VAR) model with the Cholesky method to decompose the variance. Notably, as a proxy of the liquidity risk we used, respectively, the principal component out of a set of liquidity *premia* variables collected on money, bond and stock markets as well as the swap spread; we used, respectively, an indicator of the joint probability of default[32] and the principal component derived from the CDS prices on the Euribor banks, as a proxy of the default risk. Finally, we used a measure of global risk aversion, based on averages of the implied volatility from options traded in several markets. We also added the 2 to10 years slope on the yield curve as in Campbell and Taksler (2003).

In our results, some 45% of the variance owes to factors relating to the default risk, while liquidity risk and the risk aversion weigh by 25% and 20% respectively.

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[32] As calculated from Segoviano (2006a, 2006b) and Goodhart and Segoviano (2009) and reported on IMF website.

Table 1

Banks in the Euribor panel included in the sample

BANK	COUNTRY
AMRO BANK NV	NL
INTESA SANPAOLO SPA	IT
BANCO BILBAO VIZCAYA ARG	SP
BANCO SANTANDER CNTRL HISP. SA	SP
BANK OF IRELAND	IE
BNP PARIBAS	FR
COMMERZBANK AG	DE
CREDIT AGRICOLE SA	FR
DRESDNER BANK AG	DE
FORTIS NL	NL
ING BANK NV	NL
LB.HESSEN-THURINGEN	DE
RABOBANK	NL
UNICREDITO ITALIANO SPA	IT
BARCLAYS BANK PLC	UK
HSBC BANK PLC	UK
CITIGROUP INC	USA
UBS AG	SW
WESTLB AG	DE
SOCIETE GENERALE SA	FR
DEUTSCHE BANK AG	DE
JPMORGAN CHASE & CO	USA
DANSKE BANK AS	DK
KBC GROUP NV	BE
BANCA MDP.DI SIENA SPA	IT
NORDEA BANK AB	SVE

Estimates

(Newey-West hetero/serial Consistent Estimates)

Dependent Variable	Liquidity risk component						Δ (Liquidity risk component)		
Constant	0.234***	0.091***	0.113***	0.066***	0.072***	-0.104***	0.004*	0,005*	0,005*
Bid ask spread MID	0.162***			0.300***					
Spread ECP-OIS		1.708***		1.218***					
CIP			1.159***	0.469***					
1st principal comp.					1.084***				
Funding liquidity						2.683***			
Δ Bid ask spread MID							0.523*		
Δ Spread ECP-OIS							0.240***		
Δ CIP							0.030*		
Δ1st principal comp.								0.098***	
Δ Funding liquidity									0.490***
R^2 adj.	0.16	0.79	0.63	0.85	0.77	0.73	0.27	0.08	0.16
σ^2	0.14	0.03	0.06	0.02	0.04	0.02	0.001	0.001	0.002
Freq.	daily	daily	daily	daily	daily	weekly	weekly	weekly	weekly
N. Obs.	1194	1194	1194	1194	1194	195	195	195	195

Note: The dependent variable is the liquidity component in the Euribor – OIS spread. One, two or three asterisks denote significance at the 10, 5 and 1 percent confidence level, respectively.

Table 3.1

AFNS model parameter estimates

N	variable	optimum value	standard deviation
1	ALPHA_ZERO_FINA	0.0000	
2	ALPHA_ZERO_FINAA	0.0034	0.0017
3	ALPHA_ZERO_BNKA	-0.0011	0.0013
4	ALPHA_ZERO_BBNKAA	-0.0026	0.0014
5	ALPHA_ZERO_TREAS	0.0000	
6	ALPHA_ZERO_REPO	0.0126	0.0027
7	ALPHA_ZERO_LIBOR	0.0014	0.0004
8	ALPHA_UNO_FINA_LS	1.0000	
9	ALPHA_UNO_FINA_SS	1.0000	
10	ALPHA_UNO_FINA_LT	-0.0375	0.0220
11	ALPHA_UNO_FINA_ST	-0.0455	0.0291
12	ALPHA_UNO_FINA_CT	0.0000	
13	ALPHA_UNO_FINA_CY	0.0151	0.0084
14	ALPHA_UNO_FINA_LIB	0.0000	
15	ALPHA_UNO_FINAA_LS	0.7258	0.3629
16	ALPHA_UNO_FINAA_SS	0.9049	0.3935
17	ALPHA_UNO_FINAA_LT	-0.0529	0.0311
18	ALPHA_UNO_FINAA_ST	-0.0261	0.0158
19	ALPHA_UNO_FINAA_CT	0.0000	
20	ALPHA_UNO_FINAA_CY	0.0296	0.0166
21	ALPHA_UNO_FINAA_LIB	0.0000	
22	ALPHA_UNO_BNKA_LS	0.8986	0.3907
23	ALPHA_UNO_BNKA_SS	1.0409	0.6939
24	ALPHA_UNO_BNKA_LT	-0.0552	0.0212
25	ALPHA_UNO_BNKA_ST	-0.0140	0.0061
26	ALPHA_UNO_BNKA_CT	0.0000	
27	ALPHA_UNO_BNKA_CY	0.0136	0.0151
28	ALPHA_UNO_BNKA_LIB	0.0000	
29	ALPHA_UNO_BNKAA_LS	0.6386	1.0644
30	ALPHA_UNO_BNKAA_SS	0.7441	0.6201
31	ALPHA_UNO_BNKAA_LT	-0.0085	0.0106
32	ALPHA_UNO_BNKAA_ST	-0.0472	0.0315
33	ALPHA_UNO_BNKAA_CT	0.0000	
34	ALPHA_UNO_BNKAA_CY	0.0387	0.0276
35	ALPHA_UNO_BNKAA_LIB	0.0000	
36	ALPHA_UNO_T_LS	0.0000	
37	ALPHA_UNO_T_SS	0.0000	
38	ALPHA_UNO_T_LT	1.0000	
39	ALPHA_UNO_T_ST	1.0000	
40	ALPHA_UNO_T_CT	0.0000	
41	ALPHA_UNO_T_CY	0.0000	
42	ALPHA_UNO_T_LIB	0.0000	
43	ALPHA_UNO_RF_LS	0.0000	
44	ALPHA_UNO_RF_SS	0.0000	
45	ALPHA_UNO_RF_LT	1.0000	
46	ALPHA_UNO_RF_ST	1.0000	
47	ALPHA_UNO_RF_CT	0.0000	
48	ALPHA_UNO_RF_CY	1.0000	
49	ALPHA_UNO_RF_LIB	0.0000	

The “grey” colour denotes constrained parameters.

Table 3.2

AFNS model parameter estimates

N	variable	optimum value	standard deviation
50	K1_1	0.8133	0.2804
51	K2_1	0.6271	0.0871
52	K3_1	0.6329	0.1862
53	K4_1	0.9534	0.2577
54	K5_1	0.2133	0.0350
55	K6_1	-0.8213	0.4563
56	K7_1	0.5235	0.7478
57	K1_2	0.0000	
58	K2_2	-0.5123	0.2846
59	K3_2	-1.1267	1.6096
60	K4_2	0.4953	0.2607
61	K5_2	1.2312	0.3078
62	K6_2	-0.1236	0.0537
63	K7_2	0.3756	0.1295
64	K1_3	0.0000	
65	K2_3	0.0000	
66	K3_3	0.3463	0.1283
67	K4_3	0.5468	0.3645
68	K5_3	-0.2587	0.0663
69	K6_3	0.5457	0.3816
70	K7_3	0.6897	0.4057
71	K1_4	0.0000	
72	K2_4	0.0000	
73	K3_4	0.0000	
74	K4_4	0.8678	0.4132
75	K5_4	1.4123	0.7846
76	K6_4	-0.3456	0.2033
77	K7_4	0.6324	0.3328
78	K1_5	0.0000	
79	K2_5	0.0000	
80	K3_5	0.0000	
81	K4_5	0.0000	
82	K5_5	0.7568	0.2293
83	K6_5	0.6125	0.8750
84	K7_5	0.7486	0.4404
85	K1_6	0.0000	
86	K2_6	0.0000	
87	K3_6	0.0000	
88	K4_6	0.0000	
89	K5_6	0.0000	
90	K6_6	-1.4321	0.9547
91	K7_6	0.5672	0.3151
92	K1_7	0.0000	
93	K2_7	0.0000	
94	K3_7	0.0000	
95	K4_7	0.0000	
96	K5_7	0.0000	
97	K6_7	0.0000	
98	K7_7	0.8654	0.4862
99	MU_1	0.0398	0.0085
100	MU_2	0.0326	0.0197
101	MU_3	-0.0247	0.0224
102	MU_4	0.0368	0.0368
103	MU_5	0.0454	0.0146
104	MU_6	0.0361	0.0328
105	MU_7	0.0246	0.0273
106	SIGMA1_1	0.0011	0.0003
107	SIGMA2_2	0.0342	0.0074
108	SIGMA3_3	0.0632	0.0071
109	SIGMA4_4	0.0257	0.0035
110	SIGMA5_5	0.1160	0.0149
111	SIGMA6_6	0.1215	0.0934
112	SIGMA7_7	0.0655	0.0126
113	KQ2_2	0.6209	0.2300
114	KQ4_4	0.5383	0.0995
115	KQ4_5	-0.5383	0.0995
116	KQ5_5	0.5383	0.0995
117	KQ7_7	0.8035	0.4229