Central Counterparties*

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Abstract

Central counterparties (CCPs) have increasingly become a cornerstone of financial markets infrastructure. We present a model where CCPs are necessary to implement efficient trade when trades are time-critical, liquidity is limited and there is limited enforcement of trades. We then show that – when collateral is sufficient to avoid default – profit-maximizing CCPs "overcollateralize" trades relative to user-oriented CCPs and, hence, are less efficient. When collateral is not covering all default exposure, profit-maximzing CCPs can be efficient as they guarantee trading despite allowing for some default. User-oriented CCPs to the contrary might not be efficient due to a hold-up problem, avoiding default at the cost of less trade.

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1 Introduction

Since the 1990s central counterparties (CCPs) have become more and more commonplace as a cornerstone of financial market infrastructure. The central role of CCPs is to novate contracts. In the novation process, the original contract between a buyer and a seller is extinguished and replaced by two new contracts; one between the buyer and the CCP, and another one between the seller and the CCP. For example, clearinghouses that serve as a CCP interpose themselves as the legal counterparty for trades carried out on formal security exchanges and more recently also in over-the-counter (OTC) markets.[1]

In assuming responsibility for the terms of the trade CCPs become exposed to the obligation to fulfill the terms of the original contract between a seller and a buyer even though one of these parties defaults on its obligations – or, in other words, *replacement cost risk*.[2] Novation thus concentrates default risk in the hands of a single institution, the CCP. As a consequence, there is the potential of large-scale disruptions in financial markets, if this risk is not properly controlled for (see for example Bernanke (1990)).

This motivates a closer review of the incentives for CCPs to manage their risk efficiently. Our goal is to study in particular how governance structures shape such incentives.[3] CCPs operate currently under two main governance structures. The first structure is a mutual structure, where the CCP operates in the interest of its users, which sometimes (but not necessarily) coincides with the ownership of the CCP by its users. We will refer to such institutions as user-oriented. The second type of CCPs is operated on a for-profit basis, rather than optimizing the provision of services for the majority of its users. Traditionally, CCPs were user-oriented institutions, but lately CCPs have started to demutualize and have switched their objective toward profit-maximization.

Our paper makes three important and novel contributions. First, we provide a formal model, where CCPs arise endogenously to enable trade. Second, we investigate optimal collateral policies and

[1]For a detailed overview of CCP services and recent developments in this area, see Ripatti (2004).

[2]More formally, replacement cost risk is defined as "the risk that a counterparty to an outstanding transaction for completion at a future date will fail to perform on the settlement date. This failure may leave the solvent party with an un-hedged or open market position or deny the solvent party un-realized gains on the position. The resulting exposure is the cost of replacing, at current market prices, the original transaction." (BIS (2003).

[3]The Committee on Payment and Settlement Systems (CPSS) recently issued recommendations for CCPs that address key issues such as employing transparent and prudent risk management techniques, and the design of appropriate governance structures that balance the risk and the benefit from trading (see CPSS (2004)). See also Russo, et al. (2004).

how these are influenced by the CCP's governance structure. Third, we point out that for-profit CCPs lead to more default than user-oriented CCPs, but may be desirable for users in the face of a hold-up problem among them. This leads to several testable implications about how the market environment and the characteristics of users determine the governance structure and, hence, risk management practices.[4]

We develop a framework where a CCP arises endogenously in order to implement the efficient level of trade. The model features agents with a random need to trade a risky security. The structure of markets and preferences of traders are such that (i) trades have to be carried out by a specific time (i.e., trades are time-critical), (ii) trades cannot be fully and immediately settled at that time (i.e., there is limited liquidity) and (iii) traders have an opportunity to renege on their obligations (i.e., there is a problem of enforcing the terms of the trade). We show that these elements rule out a delivery-vs.-payment (DvP) mechanism which can lead to the impossibility of trade.[5]

We introduce a CCP as a costly collateral facility that can store collateral before trading, guaranteeing the terms of trades against the collateral posted and returning it after the settlement of trades. We show that such a facility can enable trade in face of the trading imperfections outlined above. However, the operation of the collateral facility involves a replacement cost risk for the CCP, as it guarantees the terms of the trade in the event of some trader's default. The CCP controls this risk through collateral policies. It can employ margin calls on individual transactions to secure its exposure. It can also require agents – independently of and prior to their trading needs – to make distributions to a default fund. Using this fund as an insurance pool, the CCP can mutualize its losses due to replacement cost risk among members.

In our environment, collateral policies differ remarkably across governance structures. If a CCP maximizes profits, it will prefer the default fund, thus maximizing revenue from obtaining collateral independent of trading needs. To the contrary, a user-oriented CCP minimizes collateral costs for users by requiring preferably only margin calls. This imposes the cost of managing default risk on traders, i.e. directly on the source of the risk. In the absence of default, the volume of trading in our model is not affected by collateral policies. This leads then to profit-maximizing CCPs *over-collateralizing* trades, which is more costly for market participants.

[4]Starting from Telser (1981), the literature on collateral policies of clearinghouses has concentrated narrowly on risk management issues per se, but has not looked at how these issues are related to the organization of such clearinghouses (see e.g. Knott and Mills (2004) for an overview of this literature).

[5]Fleming and Garbade (2005) have provided evidence that – depending on market conditions – settlement failures for purely strategic reasons are quite common in some financial markets.

When default is possible, there is a trade-off between the volume of trading and the cost of default. To take this into consideration, we introduce an aggregate shock that increases the risk associated with the security. Whenever traders do not have sufficient collateral to cover all exposure from trading when the risk on the security increases, there will be default. In order to cover its losses, the CCP must then resort to mutualizing the default fund *independent* of its governance structure. This leads to a trade-off between realizing the gains from trading at the expense of incurring losses on the users' contributions to the default fund.

The governance structure, however, shapes this trade-off as follows. When the overall gains from trade are large relative to the overall losses from default for people that have no trading needs (non-traders), it is efficient to allow for trades even if the aggregate shock occurs. A user-oriented CCP maximizes the welfare of a majority of users. If non-traders are in the majority, the user-oriented CCP will avoid shifting losses to the default fund, since non-traders would pay a cost. Hence, once risk increases, the user-oriented CCP shuts down trade and avoids the costs of default for the majority of its users despite the overall net gains for all users – a classic hold-up problem. A profit-oriented CCP to the contrary has still an incentive to allow for trade, since its revenue is strictly increasing in collateral posted and since users bear the costs of default. A profit-oriented CCP thus avoids the problem that non-traders hold up traders.

This implies that under such circumstances only a profit-maximizing CCP can commit to implement the efficient volume of trade in case of an increase in default risk while a user-oriented cannot. Even though there is default associated with some trades, having a for-profit CCP can then be welfare maximizing ex-ante. While profit-oriented CCPs follow a more costly collateral policy when default risk is low, they enable efficient trading in times with high default risk. We therefore predict that users prefer profit-maximizing CCPs in situations where the hold-up problem is severe and the market environment sufficiently risky.

The remainder of the paper is organized as follows. The next section lays out the basic environment. Then, we show that a CCP is essential to obtain an efficient level of trade if liquidity is limited, trade is time-critical and there is limited enforcement. Section 4 derives the optimal collateral policies of user- and profit-oriented CCPs. Section 5 introduces aggregate risk leading to default on trades and explains how different governance structures shape the trade-off between trading volume and default risk. Here we characterize the circumstances, when a hold-up problem among users gives rise to an advantage for profit-oriented CCPs. The last section outlines testable implications of our theory and discusses how our model can be used to address further important questions regarding financial market infrastructure.

2 Basic Environment

The economy has a measure one of identical agents and has four periods denoted by $t = 0, 1, 2, 3$. In period $t = 0$, all agents are endowed with x_0 units of an infinitely divisible good we call cash. Agents face uncertainty with respect to their preferences and an additional endowment. Specifically, at $t = 1$ an agents is a *trader* with probability π and conditional on being a trader, with equal probability he is a *seller* or a *buyer*. With probability $1 - \pi$, agents are *non-traders*.

Sellers receive an indivisible security as an endowment at $t = 1$. The security has a random pay-off $E[x_s]$ in cash which is realized in period $t = 2$. For simplicity, we assume that the pay-off is publicly observable and equal to x_h or x_ℓ with equal probability. Buyers receive a fixed additional amount of cash x_2 in period $t = 2$. Non-traders do not receive any additional endowment. We assume that both, agents' types and the security's pay-off are public information.

Preferences are described over cash available at $t = 3$ which we denote by c_3. Sellers and non-traders are risk-averse with preferences being described by a utility function u that is strictly increasing, continuously differentiable and strictly concave. Buyers, however, are risk-neutral with linear utility in cash. Hence, agents face the risk ex-ante of having a random endowment while being risk averse.

Finally, we assume that agents cannot commit, i.e. there is limited commitment. More precisely, we assume that agents cannot be forced to keep any promises they make at a later point in time. This implies that agents cannot promise to give up cash or securities at a later stage. We abstract from reputation issues, as we are looking at a static environment.

It is useful to give a short interpretation of our simple environment. Broadly, agents can be seen as financial market participants that either trade for their own account or are intermediaries that trade for other people not formally modelled. These financial markets participants have random trading needs that are not known ex-ante. Furthermore, trading needs are set-up in the model to mirror the essentials of a futures trade: risk-averse traders would like to sell the risk they face to risk-neutral traders. Hence, sellers can be seen as hedgers, while buyers are speculators. The timing expresses then the different stages of such a trade. In $t = 1$, people learn whether they have a need to trade in a risky security or not. At $t = 2$, the security matures and in the last period consumption occurs after the obligations of a trade have been settled. In the next section, we look at the detrimental effects of limited commitment on trade and how a CCP can mitigate these.

3 The role of a CCP

3.1 Efficient Allocation of Risk and Impossibility of Trade

The main problem in this economy is to allocate risk efficiently between buyers and sellers. Such an allocation of risk by trading securities against cash can only take place after the agents have learned their type in $t = 1$. However, once the pay-off of the securities is known at $t = 2$, trading is not possible anymore. As the security's pay-off has been realized and is public information, all agents value the security at its cash payoff. Hence, trading is *time-critical* in the sense that trades have to take place at $t = 1$.

Sellers and buyers want to engage in a trade at $t = 1$ as long as it is individually rational to do so. Given a price p, sellers would like to sell their security if

$$u(x_0 + p) > \frac{1}{2}[u(x_0 + x_h) + u(x_0 + x_\ell)]. \tag{3.1}$$

Buyers would agree to buy the security as long as the price compensates them for taking on the risk, i.e., if

$$x_0 - p + x_2 + \frac{1}{2}(x_h + x_\ell) > x_0 + x_2 \tag{3.2}$$

or, equivalently, if

$$\frac{x_h + x_\ell}{2} > p. \tag{3.3}$$

There are two possibilities for such a trade: (i) a spot trade where the security is exchanged against cash at $t = 1$ and, (ii) a long-term contract where cash is transfered conditionally on the securities pay-off at $t = 2$. Due to *limited commitment*, however, nobody can promise to hand over cash at $t = 2$. Hence, a long-term contract cannot allocate risk and the only possibility is a spot trade.

In such a trade, buyers can at most pay their endowment of cash x_0 at $t = 1$. If

$$u(2x_0) < \frac{1}{2}[u(x_0 + x_h) + u(x_0 + x_\ell)], \tag{3.4}$$

sellers require, however, a payment of $p > x_0$ to sell their security at $t = 1$. A spot trade becomes then impossible, as buyers do not have sufficient cash to fully pay for the security at $t = 1$ (*limited liquidity*). Hence, any form of trade is impossible for allocating risk efficiently between risk-neutral buyers and risk-averse sellers, with autarky being the only feasible allocation in the economy. We summarize this discussion in the next proposition.

Proposition 3.1. *Let* $(x_h + x_\ell)/2 > x_0$*. If* $u(2x_0) \geq (1/2)[u(x_0 + x_h) + u(x_0 + x_\ell)]$*, the security is traded at* $t = 1$ *at some price* $p \leq x_0$*. Otherwise, there is no trade at* $t = 1$ *and, hence, no transfer of risk between sellers and buyers.*

From now on we will assume that spot trades at $t = 1$ with partial settlement in cash at $t = 2$ are impossible. This boils down to the following assumption.

Assumption 3.1. $u(2x_0) < (1/2)[u(x_0 + x_h) + u(x_0 + x_\ell)]$

For the remainder of our analysis, we will also make no further assumptions about how possible trading prices are formed, other than that the trading price falls into the interval $(p_{min}, \frac{x_h+x_l}{2})$, where $p_{min} > x_0$ is defined as the price that makes sellers indifferent between trading at $t = 1$ or, equivalently,

$$u(x_0 + p_{min}) = \frac{1}{2}[u(x_0 + x_h) + u(x_0 + x_\ell)]. \tag{3.5}$$

All prices in this interval lead to an efficient allocation of risk with the surplus distributed across sellers and buyers according to a particular price p. We discuss next whether a collateral facility can achieve a transfer of risk between sellers and buyers.

3.2 Central Counterparties - A Collateral Facility

We introduce now a technology that functions as a collateral facility. It allows agents to secure possible trades by posting cash as collateral that is returned partially or in full after trades have been settled. Agents can post collateral f at $t = 0$ and collateral m at $t = 1$. We call f a contribution to the *default fund* and m a *margin call.* The difference between these two types of collateral is that margin calls can condition on an agent's type (m_b for buyers and m_s for sellers) while the default fund cannot. Collateral bears a fee $\phi \geq 0$ per unit of cash posted. Once collateral is posted, it cannot be returned to agents prior to period $t = 3$.

The collateral facility then enables long-term contracts by solving the commitment problem for agents. Agents give up collateral against the promise that their trades will settle. If trades are settled, i.e., when agents do not default, collateral is returned to the person that has posted it minus the fee ϕ at $t = 3$. If there is default, the collateral facility retains all of the collateral, but still has to fulfill the obligations of the trade against any party that has not defaulted. In other words, the collateral facility *novates* the trade: it becomes the counterparty to any trade and takes on all the default risk in exchange for collateral.

When enabling such a long-term contract, the collateral facility faces two constraints: first, it has to prevent default, and second, the amount of collateral that can be posted is limited. Having posted collateral, sellers need an incentive to give up the security at $t = 3$ against cash p. For low pay-offs x_ℓ of the security, sellers have no incentive to default, as they receive cash when trades are settled ($p > x_\ell$). If the security has a high pay-off x_h, they do not default if and only if

$$u(x_0 + p - \phi(m_s + f)) \geq u(x_0 + x_h - (m_s + f)) \tag{3.6}$$

or

$$(1 - \phi)(m_b + f) \geq x_h - p. \tag{3.7}$$

Similarly, if the securities pays x_h buyers receive cash from sellers when trades settle ($x_h > p$). Hence, they do not have an incentive to default. If the security pays x_ℓ, buyers do not default at $t = 3$ as long as

$$x_0 + x_2 + x_\ell - p - \phi(m_b + f) \geq [x_0 + x_2 - (m_b + f)], \tag{3.8}$$

which is equivalent to

$$(1 - \phi)(m_b + f) \geq p - x_\ell. \tag{3.9}$$

Collateral that can be posted at the facility is restricted in three ways. First, agents cannot post more than the liquid funds they have available before and at the time of trading,

$$m_i + f \leq x_0. \tag{3.10}$$

for $i = s, b$. Second, agents need incentives to trade at $t = 1$ taking into account the costs of collateral ϕ and that some of these costs are sunk for any contribution to the default fund prior to trading. Sellers will trade as long as

$$u(x_0 + p - \phi(m_s + f)) \geq \frac{1}{2}[u(x_0 + x_h - \phi f) + u(x_0 + x_\ell - \phi f)] \tag{3.11}$$

while buyers will acquire the security at $t = 1$ for any price p satisfying

$$\frac{x_h + x_\ell}{2} - \phi(m_b + f) > p - \phi f. \tag{3.12}$$

Third, agents contribute to the default fund at $t = 0$ only if their expected surplus from being able to trade next period is positive. This implies that given any margin calls m, contributions f to the default fund are restricted by

$$\frac{1}{2}\pi \left[u(x_0 + p - \phi(m_s + f)) + (x_0 + x_2 + E[x_s] - p - \phi(m_b + f))\right] + (1-\pi)u(x_0 - \phi f) \geq V_{aut} \quad (3.13)$$

where $V_{aut} = (1-\pi)u(x_0) + \frac{1}{2}\pi \left[E[u(x_0 + x_s)] + (x_0 + x_2)\right]$ is the expected utility when there is no trade.

For determining whether a collateral facility can enable a long-term contract, we consider first the case where posting collateral is not costly ($\phi = 0$). In this case, the expected gains from trade are always strictly positive and for any price $p \in \mathcal{P} = (p_{min}, \frac{x_h + x_\ell}{2})$ sellers and buyers have an incentive to trade at $t = 1$. A long-term contract is then possible, if agents have enough cash available to secure the trade. By equations (3.7) and (3.9), buyers do not have an incentive to default whenever sellers do not have an incentive, since $p < \frac{x_h + x_\ell}{2}$. The incentives for sellers to default are the smallest at $p = \frac{x_h + x_\ell}{2}$. Hence, if

$$x_0 > \frac{x_h - x_\ell}{2}, \quad (3.14)$$

there exists some price p such that there is no default and, hence, trade takes place at $t = 1$. We have shown the following.

Proposition 3.2. *(First-best) Let $x_0 > \frac{x_h - x_\ell}{2}$. If collateral costs are zero ($\phi = 0$), setting collateral equal to $m + f = x_0$ for buyers and sellers enables trade at some price $p \in (p_{min}, \frac{x_h + x_\ell}{2})$ and rules out default.*

If costs of collateral are strictly positive ($\phi > 0$), there is a trade-off between the benefits of allocating risk efficiently through trade and the costs of the collateral facility. Even though a first-best cannot be achieved anymore, being able to trade is still beneficial to agents ex-ante provided costs are not too high. Given a collateral policy (m, f), agents will then be willing to incur the costs $\phi(m + f)$ when posting the required collateral in order to be able to trade.[6]

Proposition 3.3. *(Second-best) There exists $\bar{\phi}$ such that for all $\phi \in [0, \bar{\phi}]$ there is a collateral policy (m, f) where all agents participate at $t = 0$, there is trade at $t = 1$ at some price $p \in (p_{min}, \frac{x_h + x_\ell}{2})$ and no default at $t = 3$.*

Proof. See Appendix. □

The collateral facility can be interpreted as a Central Counterparty Clearinghouse. It enables a long-term contract that – through the use of collateral – establishes a delivery-vs-payment (DvP)

[6] We defer a full characterization of feasible collateral policies to the next section.

mechanism on each side of the trade at the settlement stage. The settlement of trades is in net terms using the collateral posted by the parties of the trade.[7] Potential costs for the clearinghouse of a settlement failure are associated with the replacement cost risk stemming from the trade. The remainder of the paper has now the objective to explain some observed differences in collateral schemes across different governance structures for CCPs.

4 Governance Structure

The governance of an institution consists of two aspects: (i) the allocation of control or ownership rights and (ii) the institution's objectives. We concentrate here exclusively on the second aspect and abstract from issues regarding ownership. A CCP can be operated under two general objective functions. A *user-oriented* CCP chooses its collateral policies to maximize the utility of the majority of its users at any point in time. A *profit-oriented* institution maximizes profits from its collateral policies.

In our framework, one can think about the agents as the participants and users of the CCP deciding on its objective function at period $t = 0$. Since we abstract from ownership considerations, the owners can then either be seen as the agents themselves or some outside party. When setting the objective function, the agents (or users) choose the best governance structure for the CCP in terms of their ex-ante welfare.[8] The cost ϕ is interpreted as a fee charged to users. Fees provide the necessary revenue for owners of the CCP to cover operating costs and possibly a required rate of return. These fees, however, cannot be used to cover default losses. The governance structure is thus simply given by the objective function of the CCP and can be seen as basic instructions by owners to a manager that runs the CCP on behalf of the owners. The manager itself has the expertise to set collateral policies.

We assume that the CCP rationally expects what the price for trading will be and takes these prices as given when choosing its collateral policy. This implies that the CCP cannot influence prices at which trades occur.[9] Furthermore, we do not allow the CCP to default itself and assume that the

[7] To allow for direct cash settlement beyond netting, one could extend the analysis to a weaker version of limited commitment where some fraction of cash can be seized from defaulting parties. In our set-up, this could proxy for reputation effects that partially mitigate commitment problems.

[8] In the case of outside ownership (e.g. by a financial exchange or a financial institution) this corresponds to a competitive situation when setting up the CCP.

[9] Our framework could easily incorporate price uncertainty, but we leave this for future work.

CCP cannot cross-subsidize across agents to cover the costs of default. Hence, in this section default is ruled out and we restrict attention to collateral policies that rule out default.[10] The purposes of this section is to describe how collateral policies (m, f) differ across the two governance structures when there is no default.

4.1 User-oriented CCP

The CCP can recover default losses only directly by seizing the remaining collateral (after cost ϕ) from the defaulting agent. It must thus rule out default by requiring sufficient collateral from traders to secure its exposure. The CCP also decides on default fund contributions prior to agents discovering their trading needs. Hence, when requiring margins the default fund contribution has been already made and is sunk. The CCP will then set margin calls for traders given a contribution f to the default fund such that it just enables trade without default.

Even though margin calls only affect traders, a user-oriented CCP has no incentive to change its policy once agents learn whether they are traders at $t = 1$. Non-traders are not affected by margin calls and given f, the choice of margin calls affects traders at $t = 1$ in the same way as in period 0. Hence, a user-oriented CCP chooses its collateral policy at $t = 0$ to maximize the ex-ante expected utility of users and has no incentives to deviate from this policy at the trading stage $t = 1$. Taking into account that agents must have an incentive to participate in the CCP and to trade, the problem is then given by

$$V = \max_{(m_s, m_b, f)} \quad \frac{1}{2}\pi \left[u(x_0 + p - \phi(m_s + f)) + (x_0 + x_2 + E[x_s] - p - \phi(m_b + f))\right] + (1 - \pi)u(x_0 - \phi f)$$

subject to

$$x_0 \geq m_s + f \geq \frac{1}{1 - \phi}[x_h - p] \tag{4.1}$$

$$x_0 \geq m_b + f \geq \frac{1}{1 - \phi}[p - x_l] \tag{4.2}$$

$$x_0 + x_2 + E[x_s] - p - \phi(m_b + f) \geq x_0 + x_2 - \phi f \tag{4.3}$$

$$u(x_0 + p - \phi(m_s + f)) \geq E[u(x_0 + x_s - \phi f)] \tag{4.4}$$

$$V \geq V_{aut} \tag{4.5}$$

The first two constraints, equation (4.1) and (4.2), require that collateral is sufficient to deter default

[10] The CCP simply enables trading without default to allocate risk between buyers and sellers. Any further reallocation, however, (for example by distributing default losses) is beyond the scope of the CCP.

of sellers and buyers respectively. The remaining constraints require participation at every stage: Buyers and sellers must have an incentive to trade given the costs of posting collateral (interim participation constraints (4.3) and (4.4)) and all agents need to have an incentive to participate in CCP arrangements initially (ex-ante participation constraint (4.5)). We restrict attention to set of costs ϕ such that for a given p there is at least some feasible collateral policy. Proposition 3.3 ensures that this set is non-empty for all $p \in \mathcal{P}$ and is given by an interval $[0, \phi_{max}(p)]$.

Since collateral is costly, a user-oriented CCP optimally chooses the cheapest collateral policy available that rules out default. There are then two considerations. First, total collateral should never exceed the CCP's default exposure. Second, only margin calls should be used with the default fund contributions set as low as possible ($f = 0$), since non-traders incur a sunk cost.

Proposition 4.1. *A user-oriented CCP sets* (m_s^*, m_b^*, f^*) *such that*

$$m_s^* + f^* = \frac{1}{1-\phi}(x_h - p) \tag{4.6}$$

$$m_b^* + f^* = \frac{1}{1-\phi}(p - x_l). \tag{4.7}$$

$f^* > 0$ *if and only if at least one interim participation constraint is binding.*

Proof. See Appendix. □

A user-oriented CCP would like to use only margin calls due to the sunk costs involved in using the default fund. However, this might not be feasible, if buyers or sellers lack an incentive to trade given the size of the margin call. This will be the case if – for a given price p – collateral costs are so high that the surplus from trading would be negative for either buyers or sellers. The CCP has then to use contributions to the default fund in order to cover its exposure while allowing for trade. Even then, under weak additional restrictions on the agent's utility functions, a user-oriented CCP uses the default fund as little as possible.[11]

Proposition 4.2. *(i) For any* $p \in \mathcal{P}$*, there exists a cut-off value* $\bar{\phi}(p)$ *such that* $f^* = 0$ *if and only if* $\phi \in [0, \bar{\phi}(p)]$.

(ii) Suppose the coefficient of absolute risk aversion is monotone. Then, for all $\phi \in [\bar{\phi}(p), \phi_{max}(p))$*, we have* $m_s > 0$ *and* $f < x_0$.

[11] The size of the default fund f relative to margin calls is determined by which of the participation constraints (4.3)-(4.5) is the most binding. This in turn depends on the degree of risk aversion for sellers. To obtain a sharp characterization, we have assumed that the coefficient of absolute risk aversion is monotone.

Proof. See Appendix. □

If collateral costs are above $\bar{\phi}$, the CCP relies on a default fund in order to decrease margin calls and thereby to increase the volume of trade. In this situation, trading at $t = 1$ still generates some surplus from allocating endowment risk despite high collateral costs. However, given a price p, either buyers or sellers have only an incentive to trade, when some of these costs are sunk at the time of the trading decision. Posting a default fund involves deadweight losses for non-traders. Still, users participate in the CCP and post a default fund, whenever the ex-ante expected benefit from trading outweighs the expected deadweight loss.

4.2 For-profit CCP

A profit-maximizing CCP never has an incentive to allow for default. Since default costs cannot be born by non-defaulting participants, the CCP cannot recover any losses from default. Its profits from a collateral policy are then given by $\phi f + \frac{1}{2}\phi\pi(m_s + m_b)$. When choosing a policy, the CCP must still give agents an incentive to contribute to the default fund and post margins. Hence, the participation constraints are identical to the ones of a user-oriented CCP. This implies that a profit-maximizing and a user-oriented CCP face the same feasible collateral policies.

To maximize its revenue, a CCP will require that users post collateral up-front as default fund contributions rather than margin calls. When collateral costs are sufficiently low for a given price p, having only a default fund $f = x_0$ is feasible and achieves the most revenue for a CCP. This is due to the fact that all agents contribute all their cash to the default fund. However, the deadweight loss of posting a default fund contribution is increasing in collateral costs. As costs increase, the CCP has to use margin calls and possibly reduce the total amount of collateral posted to provide incentives to participate and to trade. It will, however, always require a positive contribution to the default fund.

Proposition 4.3. *(i)For all $p \in \mathcal{P}$, there exists $\underline{\phi}(p) > 0$ such that a profit-oriented CCP sets $f = x_0$ and $m_s = m_b = 0$ for all $\phi \in [0, \underline{\phi}(p)]$.*

(ii) If the coefficient of absolute risk aversion is monotone, $0 < f < x_0$ for all $\phi \in [\underline{\phi}(p), \phi_{max}(p))$, where $\underline{\phi}(p) < \bar{\phi}(p)$ for all $p \in \mathcal{P}$.

Proof. See Appendix. □

We are now in a position to compare the welfare achieved by different governance structures when there is no default. The main result in this section is then that a user-oriented CCP strictly dominates a profit-oriented CCP in welfare terms for any cost of collateral ϕ and for any price $p \in \mathcal{P}$. When there is no default, a profit-oriented CCP has incentives to *over-collateralize* relative to a user-oriented CCP thereby lowering welfare. This result is immediate, since revenue is increasing in the collateral posted giving profit-oriented CCPs to require as much collateral as possible. However, there are two distinct channels for this result. First, profit-oriented CCPs require more collateral if collateral costs are small. Second, independent of collateral costs, a profit-oriented CCP always requires agents to contribute to the default fund upfront. To the contrary, a user-oriented CCP relies on the default fund only if necessary, i.e, when prices are sufficiently skewed to erase surplus from trading for buyers or sellers. Then a default fund is necessary to enable trade.

Proposition 4.4. *A user-oriented CCP yields a strictly higher utility for users than the profit-oriented CCP for all $\phi \in (0, \phi_{max})$, since the latter sets total collateral $2f + m_b + m_s$ and/or the default fund f strictly higher than a user-oriented CCP for any $\phi \in (0, \phi_{max}(p))$ and all $p \in \mathcal{P}$.*

Proof. See Appendix. □

A CCP that acts in the interest of the majority of its users should always welfare dominate a profit-oriented institution. The last result confirms this intuition, since ex-ante all users are identical and at the trading stage there is no incentive for a user-oriented CCP to change its initial collateral policy in the absence of default. These results, however, depend crucially on the fact that default never occurs. There is then no trade-off between allowing for trading at the expense of some default. If some default occurs, the CCP has to cover its losses from default by seizing collateral from non-defaulting parties. This drives a wedge between agents in terms of the benefits and the costs of trading at $t = 1$. In the next section, we show that a profit-oriented CCP can be a better choice ex-ante in situations where at the trading stage a majority of agents with small losses from defaults hold up a minority of agents that would benefit from trading.

5 Risk, Default and the Efficient Volume of Trading

We incorporate default into the model by introducing an aggregate shock that increases the risk associated with the security's pay-off. The economy can be one of two states when trading occurs at $t = 1$, a high risk and a low risk state occurring with probability η and $1 - \eta$ respectively. These shocks are public information at $t = 1$. In the low risk state, the security still returns x_ℓ or x_h with probability 1/2. In the high risk state, however, the risk in the security's pay-off increases as a mean-preserving spread of the original pay-off, i.e., the pay-off satisfies $\hat{x}_h - \hat{x}_\ell > x_h - x_\ell$. Furthermore, whenever the economy is in a risky state, we assume that trading needs in the economy decline, i.e., there are only $\hat{\pi}$ traders in the economy with high risk, where $\hat{\pi} < \frac{1}{2}$.[12] Finally, we assume that the trading price of the security is not influenced by the aggregate shock and that traders cannot post sufficient collateral to fully secure the trade when the aggregate shock is realized.

Assumption 5.1. *(Insufficient collateral)* $\hat{x}_h - p > (1 - \phi)x_0$ *and* $p - \hat{x}_\ell > (1 - \phi)x_0$.

This section analyzes collateral policies for different governance structure and, in particular, the incentives for a CCP to allow for trade after the aggregate shock has been realized. The main goal is to show that there are situations where only a profit-oriented CCP has the right incentives ex-post to allow for trade in case the economy has experienced an increase in default risk. We then provide examples, where it is ex-ante beneficial to choose profit maximization as a governance structure.

5.1 Mutualized Default Funds

Since the aggregate state is unknown at $t = 0$, the default fund is identical across states. Margin calls, however, are set after the aggregate shock has been observed and, hence, may differ across states. We will denote by $\hat{f}$ the amount participants are required to pledge to the default fund (independently of the aggregate shock) and $\hat{m}_i$ as the margin call in case of the aggregate shock.

In case of the high risk state, Assumption 5.1 implies that a CCP cannot avoid default once it allows for trade. There is not enough collateral to fully secure trades. If the security returns $\hat{x}_h$ ($\hat{x}_\ell$), sellers (buyers) will always default, but buyers (sellers) will not. As the CCP novates the trade, this implies that in the high risk state the CCP always incurs losses. Depending on the pay-off of the security, the loss per defaulting seller is given by $\Gamma_s = \hat{x}_h - p - (1 - \phi)(\hat{f} + \hat{m}_s)$ and per defaulting buyer by $\Gamma_b = p - \hat{x}_\ell - (1 - \phi)(\hat{f} + \hat{m}_b)$. To cover these losses, the CCP has to seize some collateral beyond the one posted by the defaulting traders. As is current practice, we assume that only the default fund can be used to finance default losses, and not the margin call of other traders. In other words, the default fund is *mutualized*.[13]

[12] We do allow the number of traders to differ across the two aggregate states. One would expect that $\hat{\pi} \geq \pi$ in an economy with higher risk.

[13] We abstract here from the question of why CCPs only use the default fund and not the margin calls of other agents (or other transfer schemes) when there is default.

Furthermore, we assume that the user-oriented CCP first charges as much as possible of its losses to traders, before it takes recourse to the default fund contributions of non-traders.[14] Taking into account collateral costs, there is $(1-\phi)\hat{f}$ cash available for the CCP in the default fund to finance its losses. Since there are $\hat{\pi}/2$ traders, total losses in the high risk state are given by $\frac{\hat{\pi}}{2}\Gamma_i$ for $i = s, b$. Non-traders have to cover some of the losses if $\frac{\hat{\pi}}{2}[\Gamma_i - (1-\phi)\hat{f}] > 0$. Hence, losses covered by the non-traders default fund contribution are equal to a fraction

$$\delta_i^{UO} = \max\left\{0, \frac{\hat{\pi}/2}{(1-\hat{\pi})(1-\phi)\hat{f}}\left[\Gamma_i - (1-\phi)\hat{f}\right]\right\} \tag{5.1}$$

of their default fund contribution.

Turing to profit-orientation, we assume that the CCP requires agents to participate equally in covering losses from default. Then, in the high risk state, the CCP seizes a fraction equal to

$$\delta_i^{PO} = \frac{\hat{\pi}/2}{(1-\hat{\pi}/2)(1-\phi)\hat{f}}\Gamma_i, \tag{5.2}$$

of the default fund from buyers and non-traders if sellers default ($i = s$) and from sellers and non-traders if buyers default ($i = b$).

5.2 Default and the Volume of Trade

Revenues for a profit-oriented CCP are increasing in total collateral posted. Hence, provided it can cover its losses, the CCP has always an incentive to allow for trade at $t = 1$ *independent* of the state of the economy. However, the loss sharing rule δ_i^{PO} must be compatible with the incentives to trade in the high risk state. Given the collateral policy $(\hat{f}, \hat{m}_i, m_i)$, buyers and sellers have a surplus from allocating risk in the high risk economy despite the default costs if

$$\frac{1}{2}(x_0 + x_2 + \hat{x}_h - p - \phi(\hat{m}_b + \hat{f}) - (1-\phi)\delta_s^{PO}\hat{f}) + \frac{1}{2}(x_0 + x_2 - (\hat{m}_s + \hat{f})) \geq x_0 + x_2 - \phi\hat{f} \tag{5.3}$$

and

$$\frac{1}{2}u(x_0 + p - \phi(\hat{m}_s + \hat{f}) - (1-\phi)\delta_b^{PO}\hat{f}) + \frac{1}{2}u(x_0 + \hat{x}_h - (\hat{m}_s + \hat{f})) \geq E[u(x_0 + x_s - \phi\hat{f})] \tag{5.4}$$

respectively, where the shares δ_i^{PO} are described by equation (5.2). The loss sharing rule for a profit-oriented CCP redistributes losses across *all* non-defaulting agents. Hence, default losses for non-defaulting traders decrease with the number of traders. Provided the number of traders in the

[14] As will become clear later, differences in treating default fund contributions are chosen to have the least favorable scenario for a profit-oriented CCP.

high risk state is sufficiently small and collateral is not too costly, the option value of default for traders outweighs the costs of bearing some of the costs. Hence, there is still an incentive to trade, even at the highest default fund contribution $\hat{f} = x_0$. This policy, however, maximizes the CCPs revenue.

Proposition 5.1. *If collateral costs ϕ and the number of traders $\hat{\pi}$ is sufficiently small, the revenue maximizing collateral policy ($\hat{f} = x_0, \hat{m}_i = 0, m_i = 0$) and a loss sharing rule (5.2) enables trade in the high risk state despite default.*

Proof. See Appendix. □

A user-oriented CCP acts in the interests of the majority of its users at any point in time. Does such a governance structure give the CCP also proper incentives to enable efficient trade when the economy is in the high risk state? Since non-traders are in the majority ($\hat{\pi} < 1/2$) and have no benefits from trading, the answer depends on whether non-traders have to bear some of the default losses from trade or, equivalently, whether $\delta^{UO} = \max_{i=s,b} \delta_i^{UO} > 0$.

If the total exposure of a trade exceeds *total* collateral by traders, non-traders have to bear some costs of default. Hence, for our purpose, a sufficient condition for $\delta^{UO} > 0$ is

$$\max_{s=h,\ell} |\hat{x}_s - p| > 2(1-\phi)x_0, \tag{5.5}$$

which is independent of the number of traders in the economy. For $\hat{\pi} < \frac{1}{2}$, this implies that a majority of users loses from trading at $t = 1$. In this situation, a user-oriented CCP will not allow for trade to protect most users from suffering losses, by requiring margin calls in excess of $\hat{f} - x_0$. This is summarized as follows.

Proposition 5.2. *Suppose $\max_{s=h,\ell} |\hat{x}_s - p| > 2(1-\phi)x_0$. If $\hat{\pi} < \frac{1}{2}$, a user-oriented CCP sets margin calls $\hat{m}_i > \hat{f} - x_0$ for $i = b, s$ and there is no trade (and, hence, no default) in the high risk state.*

This establishes under weak restrictions, that the volume of trade in the high risk state is higher with a profit-oriented CCP, but at the cost of some default. The advantage of a profit-oriented CCP is that it solves a hold-up problem inherent in user-oriented institutions, where a majority of users with small losses holds up a minority with large benefits.

5.3 Ex-ante Welfare Comparison

We provide now some numerical examples that show that it can be indeed in the interest of users to prefer a profit-oriented CCP at $t = 0$. This can only be the case when two circumstances arise. First, the *aggregate* net benefits from trading in the high risk state must be positive taking the collateral policies across governance structures as given. Otherwise, there would not be a benefit from trade in the high risk state *ex-ante*, i.e., before the uncertainty about trading needs is resolved. Second, as we have shown in Section 4 that profit-oriented CCPs employ more costly collateral policies, the aggregate net benefits must compensate users for the additional costs of collateral in the low risk state. For our examples, we assume that the profit-oriented CCP employs the most cost inefficient collateral policy $\hat{f} = x_0$, while the user-oriented CCP employs the least costly policy $\hat{f} = 0$. Hence, user-oriented CCP only rely on margin calls. Provided costs ϕ are low enough, we have shown above that such policies are feasible ex-ante.

Before the uncertainty about trading needs is resolved, aggregate benefits from trading conditional on the high risk state are given by

$$\begin{aligned}\hat{V}^{PO} &= \frac{1}{2}\hat{\pi}\left[\frac{1}{2}u((1-\phi)x_0 + p) - (1-\phi)\delta_b^{PO}x_0) + \frac{1}{2}u(\hat{x}_h)\right] \\ &+ \frac{1}{2}\hat{\pi}\left[\frac{1}{2}(\frac{1}{2}x_2 + \frac{1}{2}((1-\phi)x_0 + x_2 + \hat{x}_h - p - (1-\phi)\delta_s^{PO}x_0)\right] \\ &+ (1-\hat{\pi})\left[\frac{1}{2}u((1-\phi)x_0 - \delta_b^{PO}x_0) + \frac{1}{2}u((1-\phi)x_0 - \delta_s^{PO}x_0)\right].\end{aligned} \tag{5.6}$$

Proposition 5.2 implies that the aggregate benefits from no trading under a user-oriented CCP that only requires margin calls are given by

$$\hat{V}^{UO} = \frac{\hat{\pi}}{4}\left[u(x_0 + \hat{x}_h) + u(x_0 + \hat{x}_\ell)\right] + \frac{\hat{\pi}}{2}\left[x_0 + x_2\right] + \frac{1-\hat{\pi}}{2}u(x_0). \tag{5.7}$$

Hence, we have to ensure that $\hat{V}^{PO} > \hat{V}^{UO}$. Clearly, for the low risk state $V^{UO} > V^{PO}$ due to the inefficient policy of a profit-oriented CCP. Provided the high risk state is sufficiently likely, however, users prefer then a profit-oriented CCP.

We resort next to some numerical examples to show that profit-oriented CCPs can indeed welfare dominate user-oriented CCPs due to the hold-up problem. In these examples, we will express the gains from profit-oriented CCPs as functions of the two parameters $(\eta, \hat{\pi})$ for different values of risk when there is default $(\hat{x}_\sigma)$ and different degrees of risk aversion. To do so, we choose a CRRA utility function which is parameterized by the coefficient of risk aversion $\sigma \in (0, \infty)$. All other parameters are kept fixed and are chosen as shown in the following table.

π	ϕ	x_0	x_2	x_h	x_l	p
0.9	0.01	0.65	1	1.5	0.5	0.9

Table 1: Parameter Values

For the two examples we present one can verify that a user-oriented CCP optimally sets $\hat{f} = 0$, hence uses only margin calls and does not allow for trade once the aggregate shock has been realized. To the contrary, in these examples, it is optimal for a profit-oriented CCP to set $\hat{f} = x_0$, as collateral costs are sufficiently low. Furthermore, choosing $\pi > \frac{1}{2} > \hat{\pi}$ maximizes welfare differences among governance structures in the low risk state.

5.3.1 Example 1:

The first example uses $\sigma = 0.5$ and the mean-preserving spread is given by $(\hat{x}_h, \hat{x}_l) = (2.5, -0.5)$ as a benchmark case. Figure 1 shows three graphs that exhibit (i) the net gains from choosing a profit-oriented CCP at $t = 0$ as a function of η and $\hat{\pi}$; (ii) the critical value η_{crit} for a given degree of heterogeneity $\hat{\pi}$ such that for all $\eta > \eta_{crit}$ it is optimal to choose profit-orientation; and (iii) the gains from profit-orientation for the extreme case that $\eta = 1$, i.e. that only the high risk state occurs.

We do not show any graphs here for different values of $\hat{x}_\sigma$. However, as risk (in form of the mean-preserving spread) decreases, the critical value η_{crit} increases and the second graph shifts monotonically to the right, eventually rendering profit-orientation sub-optimal irrespective of parameters $(\eta, \hat{\pi})$. The next example, however, shows that such comparative statics need not be monotone.

5.3.2 Example 2:

In this example, we increase the coefficient of risk aversion to $\sigma = 2$ and analyze how the gains from profit-orientation change as the mean-preserving spread $\hat{x}_\sigma$ increases from 2.8 to 3. Figure 2 shows that for the lowest value we consider profit-orientation is never optimal.

Once the spread increases, profit-orientation is more attractive, the more likely the aggregate shock occurs. However, the critical value η_{crit} is *not* monotone in $\hat{\pi}$. When $\hat{\pi}$ is close to 0.5, the costs from the buyers' default when $s = \ell$ are high for sellers relative to their surplus from trading. Hence, the benefits from profit-orientation are declining. This is shown for the special case of $\eta = 1$ in Figure

3(c).

This effect diminishes, if the spread increases further as shown in the last figure. As before, if the number of traders converges to zero, there are no ex-ante benefit of profit-orientation anymore. This is due to the fact that in the high risk state the probability of being a non-trader is too high. Bearing even small costs of default with high probability in this state renders the expected surplus from trading unimportant.

6 Conclusion

Our paper studies the most important cornerstone of modern financial market infrastructure, CCPs. There are three main contributions. First, we demonstrate that a CCP is essential for enabling financial trades by managing liquidity and default problems. Second, we characterize differences in collateral policies across governance structures (user- vs. profit-oriented) of such institutions. Finally, we show that profit-orientation avoids certain hold-up problems and that it can be the better governance structure for the users of a CCP by implementing the efficient trade-off between the volume of trade and overall default risk.

The last two results lead to several testable implications. Abstracting from the market environment, user-oriented CCPs should employ less costly collateral policies. This should be reflected in the size of overall collateral required and in the composition of collateral posted (default fund vs. margin calls). Despite being more expensive, profit-oriented CCPs should be encountered in "thin" markets with high price volatilities (or risk), where trading needs are highly uncertain and infrequent for any participant. Such markets are likely to be derivative markets and over-the-counter (OTC) segments rather than "cash" markets and formal exchanges.[15] Finally, controlling for the market environment, there should be no significant differences in default rates and volume of trade. We plan in our next step to test these implication of our paper empirically by using information on the market environment, governance structures and risk management instruments of CCPs.

It is also important to address in the future some questions that are left unanswered by our analysis. For example, Bernanke (1990) points out that the major advantages of a CCP are liqudity provision, redistribution of default risk and ensuring the anonymity of traders. We only incorporate the first two in our analysis, but do not allow for heterogeneity among traders. Novation typically helps to

[15] See Kroszner (1999) for a historical comparison of infrastructure development for formal exchanges and OTC markets.

alleviate information problems that arise from different qualities of counterparties by ensuring the anonymity of counterparties in trading. Such heterogeneity among traders could easily be introduced in our model to understand the change in CCPs from mere default covering collateral facilities to modern institutions that enable exchange in electronic trading environments.

We also do not study the optimal rules how default losses are distributed across members of the CCP. Here, it is also necessary to investigate the effects of mutualizing the default fund on non-defaulting members. This is important to address a third question, systemic risk. Defaults by a large number of members in a financial market crisis could trigger more default when default fund contributions are seized. Furthermore, CCPs default themselves in reality on their obligations and members are then often required to cover these losses.[16] Such events cause major disruptions of financial markets and warrant the question whether particular governance structures are responsible for their occurrence.

7 Appendix

Proof of Proposition 3.3:

Proof. Let the collateral policy be given by $f = 0$ and

$$m \begin{cases} \frac{1}{1-\phi}(x_h - p) & \text{for sellers} \\ \frac{1}{1-\phi}(p - x_\ell) & \text{for buyers.} \end{cases} \tag{7.1}$$

There exists ϕ_1 such that for $\phi < \phi_1$, $m \leq x_0$ for sellers (and, hence, for buyers). For such a collateral policy neither sellers nor buyers default and collateral costs are minimized. Since u is concave there also exists ϕ_2 such that for all $\phi \in [0, \phi_2]$ equations (3.11) and (3.12) both are satisfied. Hence, traders prefer to trade for all $\phi \leq \phi_2$. Finally, all agents participate at $t = 0$ for such ϕ, since $f = 0$. Setting $\bar{\phi} = \min\{\phi_1, \phi_2\}$ completes the proof. □

Proof of Proposition 4.1:

Proof. A user oriented CCP would like to minimize m_s, m_b and f. Since the default fund bears a higher weight than margin calls in its objective function, it would like to increase f only as a last resort to enable trade. Therefore the default constraint will always bind. Otherwise, as $f + m_i > 0$

[16] Hill, et. al. (1999) describe several examples of CCP failures in the 1980s and 1990s.

it is possible to lower m_i, for $i = s, b$. This relaxes the participation constraints and increases the objective function of the CCP.

Now suppose $f > 0$, but no interim participation constraint is binding. Then, $f - \epsilon$ and $m_i + \epsilon$ for $i = s, b$ is feasible for $\epsilon > 0$ sufficiently small and increases utility ex-ante. This is a contradiction.

Suppose next some interim participation constraint is binding, but $f = 0$. Since $f = 0$, interim participation constraints are given by

$$E[x_s] - p \geq \phi m_b \tag{7.2}$$

$$u(x_0 + p - \phi m_s) \geq E[x_0 + x_s] \tag{7.3}$$

with some strict equality. But since default constraints hold with equality, this implies that the participation constraints do not bind. A contradiction. □

Proof of Proposition 4.2:

Proof. To prove the first statement, fix $p \in \mathcal{P}$. Set $f = 0$ and let m_i be defined by the binding default constraints. Define $\bar{\phi}$ as the highest value of ϕ that satisfies

$$E[x_s] - p - \phi m_b \geq 0 \tag{7.4}$$

$$u(x_0 + p - \phi m_s) \geq E[u(x_0 + x_s)] \tag{7.5}$$

$$\tfrac{1}{2}\pi \left[u(x_0 + p - \phi m_s) + E[x_s] - p - \phi m_b\right] \geq \tfrac{1}{2}\pi E[u(x_0 + x_s)]. \tag{7.6}$$

If $\phi > \bar{\phi}$, either no collateral policy is feasible or any feasible collateral policy has $f > 0$. If $\phi \leq \bar{\phi}$, there is a feasible collateral policy with $f = 0$.

For the second part, consider first the case when the coefficient of absolute risk aversion is non-increasing. We first show that

$$u(x_0 + p - \frac{\phi_{max}}{1 - \phi_{max}}(x_h - p)) \geq E[u(x_0 + x_s - \frac{\phi_{max}}{1 - \phi_{max}}(x_h - p))]. \tag{7.7}$$

Suppose not. Since $m_s + f \geq \frac{1}{1-\phi}(x_h - p)$, we cannot increase the LHS by lowering collateral. Furthermore, setting $f > \frac{1}{1-\phi}(x_h - p)$ to lower the RHS increases the difference, since this decreases the wealth level before risk $(x_0 + \frac{\phi_{max}}{1-\phi_{max}}(x_h - p))$ and we have non-increasing absolute risk aversion. Hence, at ϕ_{max} there does not exist a feasible collateral policy. A contradiction.

Suppose now $\phi \in (\bar{\phi}, \phi_{max})$. This increases the wealth level before risk and by non-increasing absolute risk aversion increases the difference. Hence, for all these values of ϕ the constraint is slack for $f > 0$ and $m_s = 0$. Since

$$\frac{1}{1-\phi}(x_h - p) > \frac{1}{1-\phi}(p - x_\ell) \tag{7.8}$$

we have that the optimal f can at most be equal to the RHS. Hence, $m_s > 0$.

Consider next the case where the coefficient of absolute risk aversion is non-decreasing. Suppose $f = x_0$ is optimal. Since $u(x_0 + p) > E[u(x_0 + x_2)$ for all $p \in \mathcal{P}$, we also have that

$$u((1-\phi)x_0 + p) > E[u((1-\phi)x_0 + x_s)] \tag{7.9}$$

for all $\phi \in [0, 1]$. But then, reducing $f = x_0$ by $\epsilon > 0$ small enough and increasing m_i by the same amount is feasible. Hence, $f = x_0$ cannot be optimal. A contradiction. □

Proof of Proposition 4.3:

Proof. If $f = x_0$ the buyer's default and interim participation constraints never bind. There are at most three constraints that can be violated: the seller's default constraint, the seller's interim and ex-ante participation constraints:

$$x_0 \geq \tfrac{1}{1-\phi}[x_h - p] \tag{7.10}$$

$$u((1-\phi)x_0 + p) \geq E[u((1-\phi)x_0 + x_s)] \tag{7.11}$$

$$\tfrac{1}{2}\pi\left[u((1-\phi)x_0 + p) + (x_0 + x_2 + E[x_s] - p - \phi x_0)\right] + (1-\pi)u((1-\phi)x_0) \geq V_{aut}. \tag{7.12}$$

The first and the third are fulfilled if and only if ϕ lies in a positive interval from 0. For the second one, note that

$$u(x_0 + p) > E[u(x_0 + x_s)]. \tag{7.13}$$

For the second statement, suppose first the coefficient of absolute risk aversion is non-increasing. Define $\tilde{\phi}$ such that

$$u((1-\tilde{\phi})x_0 + p) = E[u((1-\tilde{\phi})x_0 + x_s)] \tag{7.14}$$

If there is no interior value of ϕ that satisfies this equation we define $\tilde{\phi} = 1$. For ϕ larger than $\tilde{\phi}$, the inequality is violated. Finally, suppose the coefficient of absolute risk aversion is non-decreasing. Then, $u((1-\phi)x_0 + p) > E[u((1-\phi)x_0 + x_s)]$ for all ϕ. Hence, there also exists an interval such that this inequality is satisfied.

Finally, suppose $f = 0$. Then, the ex-ante participation constraint does not bind for any $\phi < \phi_{max}$. Hence, it is feasible to set $f = \epsilon > 0$ for ϵ sufficiently small. $\square$

Proof of Proposition 4.4:

We first establish the following lemma.

Lemma 7.1. *The ex-ante participation constraint never binds for a user-oriented CCP for $\phi \in (0, \phi_{max})$. However, the ex-ante participation constraint is binding for a profit-oriented CCP, whenever $f < x_0$.*

Proof. Suppose $\phi \in (0, \phi_{max})$. Let (f^*, m_s^*, m_b^*) be an optimal collateral policy for a user-oriented CCP. If $f^* = 0$, the interim participation constraints are not binding. Since there is a strictly positive surplus from trade for any $p \in \mathcal{P}$, the ex-ante participation constraint has then a strict inequality.

Let $f^* > 0$ and suppose that the ex-ante participation constraint holds with strict equality. Some of the interim participation constraint must be binding, since otherwise one could lower f^* by a sufficiently small $\epsilon > 0$ and increase m_i^* $i = s, b$ by the same amount. Suppose first, the buyer's participation constraint is binding. Then, $m_b > 0$ as $E[x_s] > p$. Consider now $\phi + \epsilon > \phi$. Then, m_b must be lower and, hence, the default fund f has to increase in order to satisfy the default constraint of buyers. Then, the ex-ante participation constraint must be violated for $\pi > 0$ unless $\phi = \phi_{max}$, which is not possible. Suppose next that the seller's participation constraint binds at ϕ. Consider $\phi + \epsilon > \phi$. In order to fulfill the ex-ante participation constraint, ϕf has to decrease. By the corollary, the seller's default constraint always binds and, hence, $m_s + f$ have to increase. This implies that for any feasible $\tilde{m}_s$ and $\tilde{f}$ at $\phi + \epsilon$ we have

$$E[u(x_0 + x_2 + (\phi + \epsilon)\tilde{f})] > u(x_0 + p - (\phi + \epsilon)(\tilde{m}_s + \tilde{f}). \tag{7.15}$$

Hence,$\phi + \epsilon$ is not feasible or, equivalently, $\phi + \epsilon > \phi_{max}$. A contradiction.

We now prove the second statement. First, let $m_i^* + f^* \leq x_0$ for all $i = s, b$ with $f^* < x_0$ be the optimal collateral policy. Suppose that the ex-ante participation constraint does not bind. If none of the interim participation constraint bind, we can lower m_i^* by $\epsilon > 0$ sufficiently small and increasing f^* by the same amount is feasible, since it relaxes the interim participation constraint. If $m_i^* = 0$ for some i, we can increase f^* directly. Since this increases the objective function, we obtain a contradiction.

Next, suppose the seller's interim participation constraint binds. Then, the same argument as before applies. Finally, suppose that the buyer's PC at $t = 1$ binds. Then, $m_b^* > 0$. Again the argument for the other cases applies. □

The proof of the proposition is then a direct consequence of Lemma 7.1.

Proof. This follows directly from Lemma 7.1 and the fact that $f^* = x_0$ only for $\phi < \underline{\phi}$ for the profit-oriented CCP. In the latter case, we always have $f^* = 0$ for the user-oriented CCP, since $\underline{\phi} < \bar{\phi}$. □

Proof of Proposition 5.1:

Proof. Since $\hat{f} = x_0$, buyers prefer to trade if

$$\frac{1}{2}(x_0 + x_2 + \hat{x}_h - p - \phi x_0 - (1-\phi)\delta_s^{PO} x_0) + \frac{1}{2}x_2 > x_0 + x_2 - \phi x_0, \tag{7.16}$$

which is equivalent to

$$\frac{1}{2}(\hat{x}_h - p - (1-\phi)\delta_s^{PO} x_0) > \frac{1}{2}(1-\phi)x_0. \tag{7.17}$$

Replacing the expression for δ_s^{PO} and Γ_s, we obtain that buyers prefer to trade if

$$[(1-\phi)x_0 - (\hat{x}_h - p)][1 - \frac{\hat{\pi}/2}{(1-\hat{\pi}/2)}] < 0. \tag{7.18}$$

Hence, by assumption 5.1, buyers always prefer to trade when the economy is in the high risk state.

Using the definition of δ_b^{PO}, sellers will prefer to trade whenever

$$\frac{1}{2}u((1-\phi)x_0 + p - \frac{\hat{\pi}/2}{1-\hat{\pi}/2}\Gamma_b) + \frac{1}{2}u(\hat{x}_h) \geq \frac{1}{2}u((1-\phi)x_0 + \hat{x}_h) + \frac{1}{2}u((1-\phi)x_0 + \hat{x}_\ell). \tag{7.19}$$

Since $\hat{x}_s$ is a mean-preserving spread, it follows that

$$u(x_0 + p - \phi x_0) \geq E[u((1-\phi)x_0 + x_s)] \geq E[u((1-\phi)x_0 + \hat{x}_s)], \tag{7.20}$$

for $\phi \in [0, \underline{\phi}(p)]$. By assumption 5.1, $\hat{x}_h > (1-\phi)x_0 + p$ and for sufficiently low $\hat{\pi}$, we have

$$\frac{1}{2}u((1-\phi)x_0 + p - \frac{\hat{\pi}/2}{1-\hat{\pi}/2}\Gamma_b) + \frac{1}{2}u(\hat{x}_h) \geq u(x_0 + p - \phi x_0). \tag{7.21}$$

□

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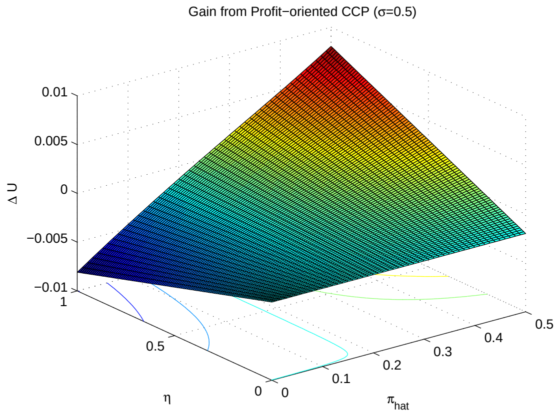
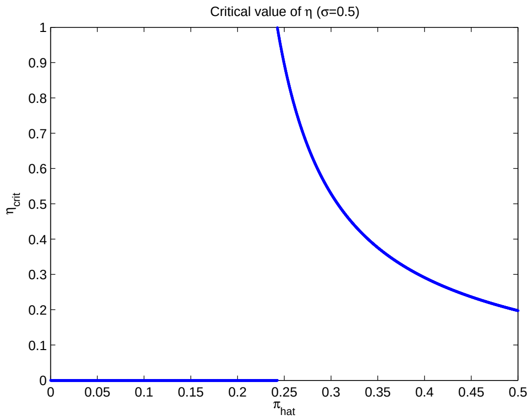
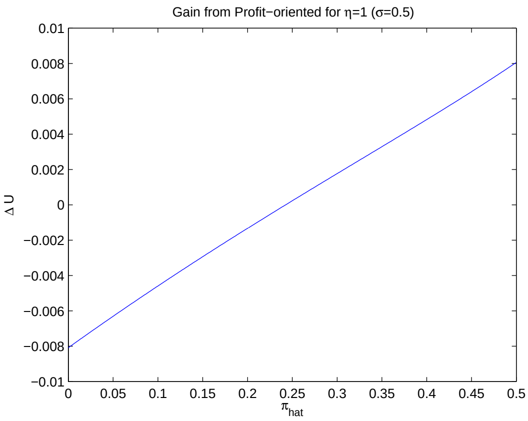


(a) Ex ante gains

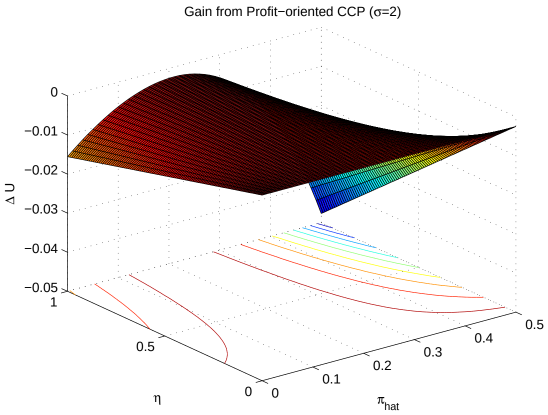


(b) Critical value for η

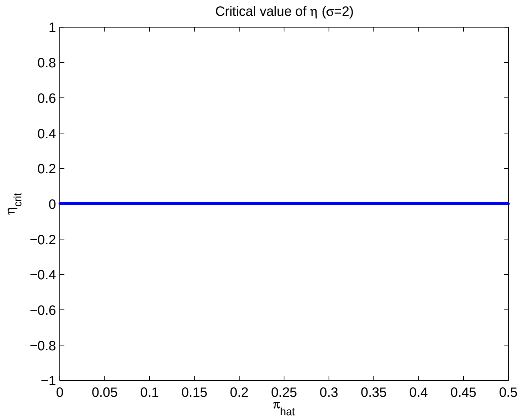


(c) Net gain for $\eta = 1$

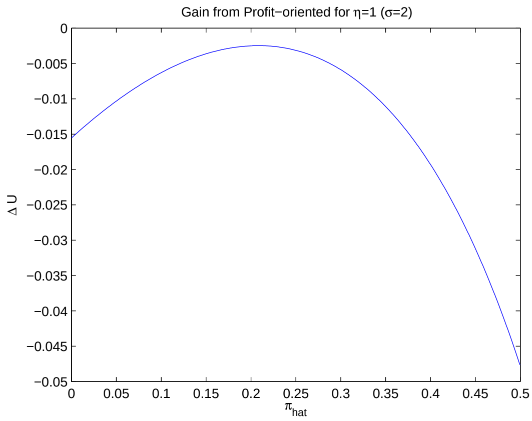
Figure 1: Benchmark case: $\sigma = 0.5$



(a) Ex ante gains

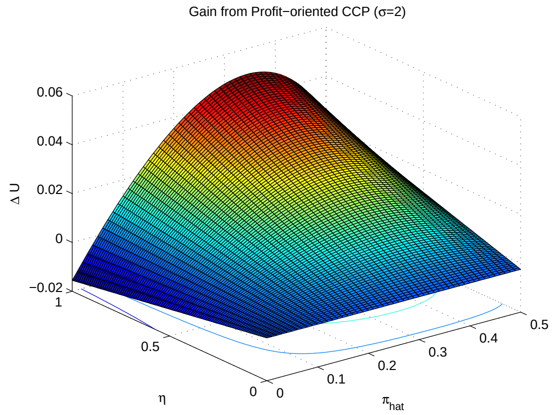


(b) Critical value for η

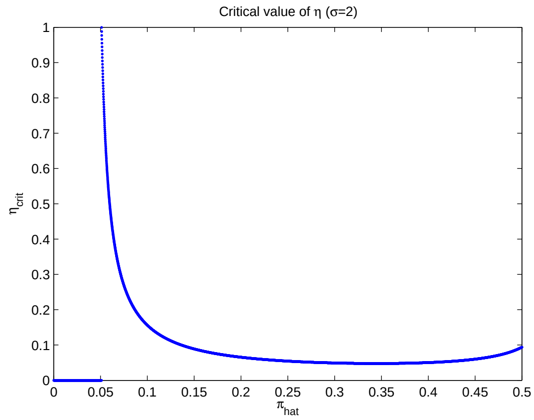


(c) Net gain for $\eta = 1$

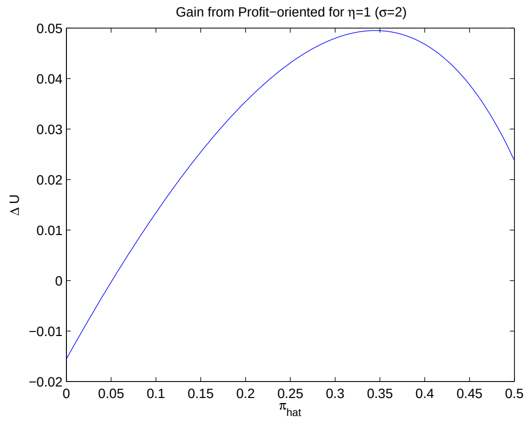
Figure 2: Mean-preserving Spread 2.8 with $\sigma = 2$



(a) Ex ante gains

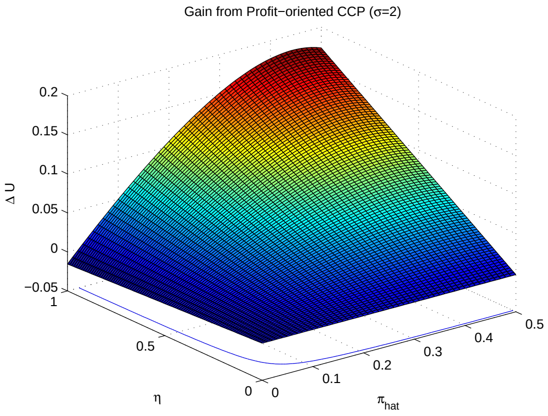


(b) Critical value for η

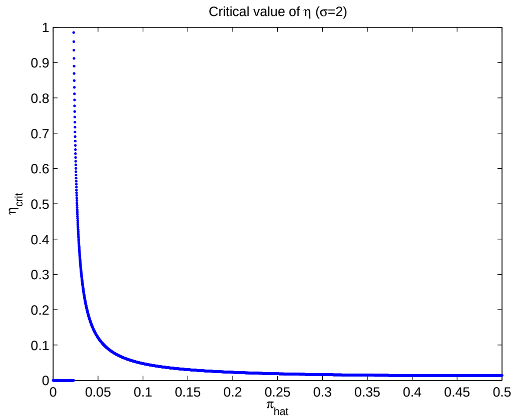


(c) Net gain for $\eta = 1$

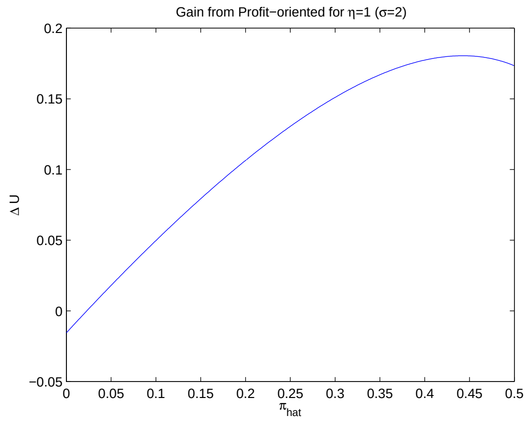
Figure 3: Mean-preserving Spread 2.9 with $\sigma = 2$



(a) Ex ante gains



(b) Critical value for η



(c) Net gain for $\eta = 1$

Figure 4: Mean-preserving Spread 3 with $\sigma = 2$