Downward Wage Rigidities and Optimal Monetary Policy in a Monetary Union

Stephan Fahr and Frank Smets^{*} European Central Bank

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Abstract

This paper analyses the effects downward nominal and real wage rigidities have on the transmission of shocks in a monetary union with optimal monetary policy. Downward rigid wages imply higher adjustment costs for wage cuts but lower costs for wage increases compared to a symmetric situation. If wages are downward nominal rigid aggregate volatility leads to grease inflation with a strictly positive inflation rate and a right skewed response of wages and prices inducing more responsive monetary policy following a negative productivity shock. These effects are stronger the more flexible prices are, confirming the strong interdependence of wage and price changes, but are absent in case of symmetric adjustment costs or real rigidity. With regions differing in the type of rigidity terms of trade always improve for the real rigid country over the medium run, independently of the direction of the shock. Following asymmetric shocks relative price adjustments dominate over wage asymmetry leaving responses nearly unaltered to symmetric wage adjustments. We find small magnitudes of deflationary pressures with a second-order approximation due to concavities in the utility function, leading to a monetary policy tightening after an asymmetric shock.

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Key words: Monetary Union, Optimal nflation, Downward wage rigidity, asymmetric adjustment costs, non–linear dynamics.

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1 Introduction

Flexible labour markets are essential for the well-functioning of the European Monetary Union (EMU). In the absence of nominal exchange rate adjustments, a flexible response of wages to both symmetric and asymmetric shocks is important to ensure a smooth and efficient economic adjustment within a monetary union. Although various labour market reforms have taken place in a number of euro area countries over the past few decades, ten years after EMU labour markets are still characterised by large differences in collective bargaining and other labour market institutions as, for example, described by Du Caju et al. (2008c). The aim of this paper is to analyse the implications of one particular friction, downward wage rigidities, in a monetary union. More particularly, we focus on the transmission of productivity shocks within a monetary union characterised by labour markets with differences in the degree of downward rigidities either in nominal or real wages and on the implications of such rigidities for optimal monetary policy.

Recent empirical research has highlighted the presence of downward wage rigidities in a large number of countries. Dickens et al. (2007) summarise the findings of the so-called International Wage Flexibility project, which uses micro-economic wage data to investigate the extent to which nominal and real wages are downwardly rigid across countries. Investigating the shape and skewness of wage change distributions, this paper finds clear evidence of both nominal and real downward wage rigidity in a number of countries. The extent of downward nominal versus real wage rigidity differs, however, across countries and is related to differences in labour market institutions such as the presence of indexation, the incidence of unemployment benefits and other labour market regulations. Within the context of the Eurosystem Wage Dynamics Network Du Caju et al. (2008a) confirms and updates some of these findings, quantifying the extent of downward wage rigidity across a number of European countries. Within EMU, one interesting case characterised by prevalent downward real wage rigidity is Belgium, which still has a system of widespread indexation of wages to changes in consumer prices (see Du Caju et al. (2008b)). Another case characterised more by downward nominal wage rigidity is Portugal, where wage cuts for job stayers are prohibited by law (see Portugal (2008)). Although the micro evidence indicates the existence of downward wage rigidities, it is not obvious to what extent these rigidities are also relevant for employment adjustment over the business cycle at a more aggregate level.

Most studies using microeconometric data are restricted to job stayers, while adjustments of labour costs could take place through wage adjustments of new hires and/or compositional changes in the age and skill composition over the business cycle. Holden and Wulfsberg (2007a, 2007b) analyse wage changes at the industry level for 19 OECD countries over the period 1973–1999 and confirm the existence of both nominal and real downward wage rigidities of different extent across different countries even at this more aggregate level. In this paper we address the following questions: How does the monetary union and its regions adjust to symmetric productivity shocks in the presence of asymmetric wage adjustments? How do asymmetric shocks influence wages and prices in the union? How should monetary policy take these asymmetries into account? What are the effects on steady-state inflation? With respect to the latter question, our analysis relates to the debate on the greasing effects of inflation initiated by Tobin (1972). In his Presidential address, he argued that inflation helps to improve the adjustment within the labour market as it reduces the necessity for nominal wage cuts.

To address these questions, we set up a two-region monetary union DSGE model with costs of adjusting prices and wages in the spirit of Rotemberg (1982). Following Kim and Ruge–Murcia (2007), the rigidities can be symmetric or asymmetric for upward and downward adjustments. Moreover, in view of the empirical evidence for some of the euro area countries, we also allow for real wage rigidity, in which case nominal wages are indexed to current inflation. Using a closed-economy model estimated on US data, Kim and Ruge–Murcia (2007) find that a low, but positive inflation objective does grease the wheels of the economy in the face of downward nominal wage rigidity and that price adjustments compensate for the lower flexibility in the wage setting. We extend their model to a currency area along the lines of Benigno (2004), Gali and Monacelli (2008) and Poilly and Sahuc (2008).

Our main contribution is therefore that we extend the analysis of asymmetric wage adjustment to a two-country set-up and to include real rather than nominal wage rigidity. This allows us to investigate how asymmetric wage adjustment affect the transmission process within the monetary union. Regarding the modelling of the labour market, we follow the monopolistically competitive model of Erceg et al. (2000), which is different from the recent focus on frictional labour markets as, for example, analysed in Poilly and Sahuc (2008), Abbritti and Mueller (2007), Andersen and Seneca (2007), Campolmi and Faia (2006), or Christoffel et al. (2008). As a result, we ignore matching frictions that govern the transition from unemployment to employment and focus exclusively on rigidities in wage adjustment. Our paper is also related to Fagan and Messina (2008), who use a flexible menu cost model to match wage change distributions at the micro level in a number of countries and investigate the implications for optimal steady-state inflation.¹ Finally, in terms of the monetary policy analysis, our paper is related to the literature on optimal monetary policy using second-order approximations of the objective function as in Benigno and Woodford (2006), but our methodology uses the second-order perturbation methods as, for example, those by Schmitt-Grohe and Uribe (2005) and Darracq Paries et al. (2007).

¹Another related paper is Benigno and Ricci (2008), who explicitly address downward wage rigidity in a different type of model, but focus on the resulting non–linearities of the Philips curve at low inflation levels.

In the rest of this paper, we first outline the monetary union model with costly wage and price adjustments in Section 2. Section 3 discusses the baseline calibration. The results are described in Section 4 and Section 5 contains the main conclusions. Three of those are worth highlighting here. First, downward nominal wage rigidity leads to a positively skewed response in nominal wage changes and a sizeable positive optimal steadystate inflation rate. This effect is stronger the lower the degree of price rigidity. To the extent that higher price stickiness in the euro area than in the United States is due to greater price rigidity as found by Altissimo et al. (2007), this would suggest a lower optimal inflation rate in the euro area. The greasing effects of inflation vanish if wages are either indexed (real wage rigidity) or if wage adjustment costs are symmetric. Second, regarding the dynamic responses to productivity shocks in the face of downward wage rigidities, we find that beyond the direct effects on wage changes also price changes are altered and become asymmetric, highlighting the strong link between prices and wages. In addition, when real wage adjustments are costly and asymmetric, the effects on hours worked are also highly asymmetric. In the case of two regions with different types of asymmetries (nominal and real), terms of trade effects are sizeable with the more rigid-wage region always adjusting with higher prices for both positive and negative shocks. Third, in the simple version of the two-country model the main burden of adjustment in response to asymmetric technology shocks is through relative price adjustments. Asymmetries in the way nominal wages are set in the two regions affect the transmission to a much smaller degree than the terms of trade channel governed by relative prices. Nevertheless, the model reveals that asymmetric shocks bear potential deflationary pressures for the union as a whole. To what degree other setups for international risk sharing and asymmetries in price adjustments lead to more profound effects in current account remains an area for further research.

2 The Model

We present the model in four parts. The first part regards the aggregation of goods into consumption bundles and the definitions for prices and terms of trade. The second and third part state the maximisation problem households and firms face taking into account that both types of agents are monopolistic suppliers of labour and goods respectively. The fourth part, finally, defines the monetary policy rules and the equilibrium. The model follows Benigno (2004) and Poilly and Sahuc (2008) for the setup of the monetary union, Erceg et al. (2000) for the labour markets and Kim and Ruge–Murcia (2007) regarding adjustment costs for prices and wages.

2.1 Aggregation and prices

The economy consists of a continuum of infinitely lived households distributed on the unit interval [0, 1] and living in two regions, home \mathcal{H} and foreign \mathcal{F} . Households [0, n)

live and work in region \mathcal{H} and households [n, 1] live in the foreign region \mathcal{F} , labour is immobile across regions. The households consume Dixit–Stiglitz aggregates of domestic and imported goods and supply labour services monopolistically. For goods produced in the two regions we use subscript \mathcal{H} and \mathcal{F} respectively, and for consumption we use no index for the home country and a star for the foreign country (C and C^*) while area-wide variables take the index w (C^w).

The composite consumption bundles C_t , C_t^* in the two regions consist of home and foreign produced goods with an elasticity of substitution of 1:

$$C_t = \frac{(C_{Ht})^{\eta} (C_{Ft})^{1-\eta}}{\eta^{\eta} (1-\eta)^{1-\eta}} \quad \text{and} \quad C_t^* = \frac{(C_{Ht})^{\eta^*} (C_{Ft})^{1-\eta^*}}{(\eta^*)^{\eta^*} (1-\eta^*)^{1-\eta^*}} ,$$
(1)

 η, η^* are the degree of preference for goods produced in region \mathcal{H} for consumers in the home and foreign region respectively, while the preference for foreign produced goods in the consumption bundle is $(1 - \eta)$ or $(1 - \eta^*)$. If $\eta > \frac{1}{2} \left(\eta^* < \frac{1}{2}\right)$ the region is characterized by a home bias. From cost minimisation of (1) the demand for the composite home and foreign produced goods by the representative consumers in each region is

$$C_{Ht} = \eta \left(\frac{P_{Ft}}{P_{Ht}}\right)^{1-\eta} C_t \text{ and } C_{Ft} = (1-\eta) \left(\frac{P_{Ft}}{P_{Ht}}\right)^{-\eta} C_t,$$
(2)
$$C_{Ht}^* = \eta^* \left(\frac{P_{Ft}}{P_{Ht}}\right)^{1-\eta^*} C_t^* \text{ and } C_{Ft}^* = (1-\eta^*) \left(\frac{P_{Ft}}{P_{Ht}}\right)^{-\eta^*} C_t^*,$$

where P_{Ht} and P_{Ft} are respectively the prices for home and foreign produced bundles and the law of one price holds between regions. Each of the consumption goods C_{Ht} and C_{Ft} is itself a composite of goods produced monopolistically by the firms in the two regions. The aggregators for the region–specific goods earn

$$C_{Ht} = \left[\left(\frac{1}{n}\right)^{\frac{\mu-1}{\mu}} \int_0^n (c_t(h))^{1/\mu} dh \right]^{\mu} \quad \text{and} \quad C_{Ft} = \left[\left(\frac{1}{1-n}\right)^{\frac{\mu-1}{\mu}} \int_n^1 (c_t(f))^{1/\mu} df \right]^{\mu},$$
(3)

where μ is the price mark-up, identical across regions, implying an elasticity of substitution of $\sigma = \mu/(\mu - 1)$ between different goods, and *n* characterizes the size of the region. For a given level of consumption we can define through cost minimisation the demand towards single home and foreign firms:

$$c_t(h) = \frac{1}{n} \left(\frac{p_t(h)}{P_{Ht}}\right)^{-\frac{\mu}{\mu-1}} C_{Ht} \text{ and } c_t(f) = \frac{1}{1-n} \left(\frac{p_t(f)}{P_{Ft}}\right)^{-\frac{\mu}{\mu-1}} C_{Ft},$$

where the corresponding price levels P_{Ht} and P_{Ft} are defined as the unit price for a home and foreign consumption bundle

$$P_{Ht} = \left[\frac{1}{n}\int_{0}^{n} p_{t}\left(h\right)^{\frac{1}{1-\mu}}\right]^{1-\mu} \quad \text{and} \quad P_{Ft} = \left[\frac{1}{1-n}\int_{n}^{1} p_{t}\left(f\right)^{\frac{1}{1-\mu}}\right]^{1-\mu}.$$

The consumption price index is determined by the cost-minimisation problem of the household when splitting expenditure between home and foreign produced goods. We assume that the law of one price holds, $p_t^F(h) = p_t^H(h) = p_t(h)$ and $p_t^F(f) = p_t^H(f) = p_t(f)$, implying that the price for the bundles C_{Ht} and C_{Ft} is identical across regions. While producer prices are equated between regions, the utility-based consumer price index (CPI) is not necessarily identical across regions due to the home bias of consumers. The CPIs in the home and foreign country, P_t and P_t^* respectively, are

$$P_t = P_{Ht}^{\eta} P_{Ft}^{1-\eta} \quad \text{and} \quad P_t^* = P_{Ht}^{\eta^*} P_{Ft}^{1-\eta^*}.$$
 (4)

In addition to the preference parameters η , η^* the size of the two countries is important for determining demand for home and foreign produced goods. With equally sized regions the case for perfect risk sharing with consumption equation across regions is $\eta = \eta^* = 1/2$. In the case of different sized countries this condition generalizes to $\eta = \eta^* = n$, where the economic size is equated to the preference structure and perfectly identical consumption patterns prevail as found by Benigno (2004). The demand for home and foreign produced goods as a function of preference and the region's size is

$$c_t(h) = \frac{\eta}{n} \left(\frac{p_t(h)}{P_{Ht}}\right)^{-\frac{\mu}{\mu-1}} T^{1-\eta} C_t \quad \text{and} \quad c_t(f) = \frac{1-\eta}{1-n} \left(\frac{p_t(f)}{P_{Ft}}\right)^{-\frac{\mu}{\mu-1}} T^{-\eta} C_t,$$
$$c_t(h) = \frac{\eta^*}{n} \left(\frac{p_t(h)}{P_{Ht}}\right)^{-\frac{\mu}{\mu-1}} T^{1-\eta^*} C_t^* \quad \text{and} \quad c_t(f) = \frac{1-\eta^*}{1-n} \left(\frac{p_t(f)}{P_{Ft}}\right)^{-\frac{\mu}{\mu-1}} T^{-\eta^*} C_t^*.$$

In the case of identical producers in each region total demand depends on preferences by the home and foreign households, η and η^* , the region's size n, the level of consumption in the two regions and the terms of trade defined as the relative price of foreign to home prices, $T_t = P_{Ft}/P_{Ht}$:

$$c_{t}(h) = \frac{1}{n} \left[\eta T_{t}^{1-\eta} C_{t} + \eta^{*} T_{t}^{1-\eta^{*}} C_{t}^{*} \right]$$

$$c_{t}(f) = \frac{1}{1-n} \left[(1-\eta) T_{t}^{-\eta} C_{t} + (1-\eta^{*}) T_{t}^{-\eta^{*}} C_{t}^{*} \right]$$
(5)

The aggregation structure is as follows: the goods from the monopolistic producers in the home and foreign country, $c_t(h)$ and $c_t(f)$ respectively, are aggregated to countryspecific bundles C_{Ht} and C_{Ft} using (3). Households aggregate home and foreign bundles to consumption goods C_t and C_t^* by (1) and decide on the respective expenditure for home and foreign goods and the demands for the single producers is the aggregate demand from the two regions stated by (5) following the cost minimisation procedure by households.

Terms of trade and international risk sharing. The terms of trade are the relative prices of foreign to home produced goods

$$T_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P_{Ft}^*}{P_{Ht}^*},$$

where the second equality holds due to the law of one price. They represent an index of competitiveness and play a crucial role in the transmission of asymmetric shocks in the two regions. We can express the CPI (4) in the two regions employing the terms of trade:

$$P_t = P_{Ht} T_t^{1-\eta} \quad \text{and} \quad P_t^* = P_{Ft} T_t^{-\eta^*}, \tag{6}$$

linking in this way the consumer prices to the producer prices.

The real exchange rate, the ratio of relative consumption price level $RER_t = P_t^*/P_t$, can be expressed as ratio of the home and foreign producer prices using (4)

$$RER_t = \frac{P_t^*}{P_t} = \left(\frac{P_{Ft}}{P_{Ht}}\right)^{\eta - \eta^*} = T_t^{\eta - \eta^*}$$
(7)

If the home and foreign region are characterised by the same degree of preference for the good produced in the home country, $\eta = \eta^*$, the real exchange rate is constant independently of the terms of trade and its level is indeterminate. In that case Purchasing Power Parity (PPP) holds between the countries.

2.2 Households

The economy consists of a continuum of infinitely lived households distributed on the unit interval [0, 1] and living in two regions, home \mathcal{H} and foreign \mathcal{F} . Households [0, n) live and work in region \mathcal{H} and households [n, 1] live in the foreign region \mathcal{F} and they are immobile across regions. Households consume Dixit–Stiglitz aggregates of domestic and imported goods and supply labour services monopolistically following Erceg et al. (2000). All households have identical preferences and initial endowments. Furthermore we assume a complete set of state-contingent assets which allows for perfect risk sharing allowing us to use the representative household for each region. As the household's problem is identical for the two regions we describe here only one region to save on notation. The representative household for home and foreign region maximises

$$E_0 \sum_{t=0}^{\infty} \beta^{t-\tau} \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{h_t^{1+\chi}}{1+\chi} \right],$$
(8)

where C_t represents the consumption bundle consisting of home and foreign goods as defined in (1) with ρ being the inter-temporal elasticity of substitution for consumption goods, h_t represents hours worked by the household and χ the disutility of work. The sequence of household's budget constraints in real terms is

$$C_t + \frac{B_t - I_{t-1}B_{t-1}}{P_t} = \frac{W_t h_t}{P_t} (1 - \Phi_t) + \frac{D_t}{P_t} \quad \text{for } t = 1, 2...$$
(9)

Households purchase a composite consumption good C_t and acquire non-contingent bonds B_t that earn nominal interest rates I_t . Labour income stems from working h_t hours for a nominal wage rate W_t net of wage adjustment costs Φ_t , dividends D_t are paid from firms to the households being owners of firms. We also assume access to a full set of state–contingent assets, but do not explicitly model it.

The household is a monopolistic supplier of labour services following Erceg et al. (2000), which are aggregated for each country in a Dixit–Stiglitz fashion

$$h_{Ht} = \left[\left(\frac{1}{n}\right)^{\frac{\theta-1}{\theta}} \int_0^n \left(h_t\left(h\right)\right)^{1/\theta} dh \right]^{\theta} \quad \text{and} \quad h_{Ft} = \left[\left(\frac{1}{1-n}\right)^{\frac{\theta-1}{\theta}} \int_n^1 \left(h_t\left(f\right)\right)^{1/\theta} df \right]_{(10)}^{\theta},$$

with *n* representing the relative size of the home region and θ the mark-up in the monopolistic labour market or alternatively $\theta/(\theta-1)$ being the elasticity of substitution for differentiated labour services, identical across regions.

The wage adjustment costs Φ_t incurred by the household take the form of a linex cost function following Kim and Ruge–Murcia (2007) and are a function of nominal or real wage changes:

$$\Phi_t\left(\Omega_t^{ind}\right) = \frac{\phi}{\psi^2} \left\{ \exp\left[-\psi\left(\Omega_t^{ind} - 1\right)\right] + \psi\left(\Omega_t^{ind} - 1\right) - 1 \right\},\tag{11}$$

with $\Omega_t^{ind} = \frac{W_t}{(\Pi_t^{ind})^{\nu} W_{t-1}}$ being the wage increase of indexed wages. The cost function is a continuous and differentiable function in wage inflation. The parameter ϕ determines the degree of asymmetry, while ψ determines the degree of convexity of the function, if $\psi = 0$ the specification nests the standard quadratic case as in Rotemberg (1982). In addition to adjustment costs with respect to nominal wages it incorporates also real wage rigidity determined by $\nu \epsilon [0, 1]$. Nominal downward wage rigidity, characterised by $\nu = 0$, implies higher costs for nominal wage cuts as opposed to wage increases, while downward real wage rigidity ($\nu = 1$) implies asymmetric costs if wage growth is stronger or smaller than the inflation measure used for indexation. Figure (9) presents a visual impression for a symmetric, an asymmetric adjustment cost function with and without indexation Π_t^{ind} .

The functional specification bears two main advantages for our purpose besides including the quadratic special case: first, it includes symmetric and asymmetric adjustment costs within a single functional form to be altered smoothly governed entirely by the parameter ϕ , and second, it is differentiable for all positive real numbers, especially around the point of zero wage inflation, i.e. $W_t/W_{t-1} = 1$ and includes the possibility of indexation to inflation measures.

The extent of downward nominal and real rigidities in the euro area has been documented by the International Wage Flexibility Project in Dickens et al. (2007) and the Eurosystem Wage Dynamics Network, e.g. Du Caju et al. (2008b). The euro area faces downward nominal and real wage rigidities due to explicit wage indexation either to national inflation (e.g. Belgium) or to an index of inflation and wage inflation in the home and in neighbouring countries. In order to account for real wage rigidities we allow the inflation rate Π_t^{ind} to which indexation takes place to be the area-wide rate $\Pi_t^w = P_t^n (P_t^*)^{1-n}$ in order to capture the process of taking other countries into the process of indexation.

2.2.1 Household optimization

The household maximises (8) under condition (9) and (11) with respect to consumption C_t , bonds B_t and wages W_t .

$$\lambda_t = u'(C_t) = C_t^{-\rho} \tag{12}$$

$$\frac{\lambda_t}{P_t} = \beta I_t E_t \frac{\lambda_{t+1}}{P_{t+1}} \tag{13}$$

$$\frac{\eta_t}{P_t} \left[\frac{\theta}{\theta - 1} h_t \left(1 - \Phi_t^i \right) + \left(\frac{W_t}{W_{t-1}} \right) (\Pi_t^w)^{-\nu} h_t \Phi_t' \right]$$

$$= h_t \left(1 - \Phi_t \right) + \left(\frac{\theta}{\theta - 1} \right) \frac{h_t^{1+\chi}}{W_t} + \beta E_t \left[\frac{\lambda_{t+1}}{P_{t+1}} \left(\frac{W_{t+1}}{W_t} \right)^2 \left(\Pi_{t+1}^{ind} \right)^{-\nu} h_{t+1} \Phi_{t+1}' \right]$$

The first equation states the valuation of consumption in form of the Lagrange multiplier, the second equation states consumption Euler equation in nominal terms for households and the last equation states wage equation which we can simplify to

$$\frac{\Omega_t}{\left(\Pi_t^{ind}\right)^{\nu}}\Phi_t' = \left[\theta\frac{h_t^{\chi}}{\lambda_t w_t} - (1 - \Phi_t)\right] / (\theta - 1) + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t}\frac{h_{t+1}}{h_t}\frac{\Omega_{t+1}^2}{\left(\Pi_{t+1}^{ind}\right)^{\nu}}\Phi_{t+1}'\right], \quad (14)$$

where $\Omega_t = W_t/W_{t-1}$ is the nominal wage growth in the region and Π_t^{ind} is the inflation rate used for indexation. We use a specification with contemporaneous inflation. Indexation is often not a purely backward looking phenomenon, but also inflation expectations are taken into account. To synthesize the backward and forward–looking element we simplify and settle for a contemporaneous specification.

In order to better understand the equation for wage growth we state the case of flexible wages:

$$\frac{W_{t}}{P_{t}} = w_{t} = \theta h_{t}^{\chi} / u' \left(C_{t} \right) = \theta \ MRS_{t}$$

Wages are a mark–up θ over the marginal rate of substitution because households supply differentiated labour services. In the case of convex adjustment costs, households face the loss of the adjustment cost Φ_t and wages become rigid as households have an incentive to smooth wage adjustments over time.

2.2.2 Fundamental risk sharing condition

The capital markets are complete, the internationally traded bond links the Euler equations (12) of the two regions. From this arbitrage condition we obtain a relationship between the real exchange rate and the consumption levels

$$RER_{t} = \psi_{0} \frac{u'(C_{t}^{*})}{u'(C_{t})} = \psi_{0} \left(\frac{C_{t}^{*}}{C_{t}}\right)^{-\rho},$$
(15)

where $\psi = RER_0 \frac{u'(C_0^*)}{u'(C_0)} = \frac{P_0^*}{P_0} \frac{u'(C_0^*)}{u'(C_0)}$ is a constant including only initial conditions for asset holdings. With PPP, $\eta = \eta^*$, the real exchange rate is $RER_t = \psi_0$ and the marginal utilities of consumption are equated up to a constant. For all other cases changes in real exchange rates reflect differing consumption levels in the two countries. Complete markets therefore do not lead to identical consumption, but to an efficient consumption differential stemming from the home bias by consumers. We assume $\psi_0 = 1$ to simplify the analysis in the following. In the presence of home bias relative consumption levels are linked to the relative output in the two regions and relative price levels reflect output of the two regions. We will return to this relationship as it serves as important propagation mechanism in the currency union with complete markets.

2.3 Firms

The final good in each region is an aggregate of differentiated products, each produced by a continuum of firms distributed on the unit interval $j \in [0, 1]$. Each firm produces with labour as unique input with decreasing returns to scale and stochastic technology in the two countries

$$y_t = x_t A_t h_t^{1-\alpha}.$$

The firm's objective is to maximise profits in real terms over an infinite horizon subject to the labour market imperfections. The firm value to be maximised is

$$E_{\tau} \sum_{t=\tau}^{\infty} \frac{\Lambda_{\tau,t}}{P_t} \left[(1-\Gamma_t) P_t^i y_t - \int_0^1 W_t^n h_t(n) dn \right],$$

where $\Lambda_{\tau,t} = \beta^{t-\tau} \frac{\lambda_t}{\lambda_{\tau}}$ is the firm's stochastic discount factor consisting of the household's discount factor β and the Lagrange multiplier λ_t . The per period profits consist of revenue from goods sold to the households net of price adjustment costs Γ_t that may differ between countries and net of wage payments to the employed workers, where h_t represents aggregate labour. All firms act individually, but are identical, so we ignore the superscript i in the following.

The demand for labour of the two countries is obtained from cost minimisation of (10)

$$h_t(h) = \frac{1}{n} \left(\frac{w_t(h)}{W_{Ht}}\right)^{-\frac{\theta}{\theta-1}} h_{Ht} \quad \text{and} \quad h_t(f) = \frac{1}{1-n} \left(\frac{w_t(f)}{W_{Ft}}\right)^{-\frac{\theta}{\theta-1}} h_{Ft}$$

where h_{Ht} and h_{Ft} denote the composite labour service from home and foreign workers, $w_t(h)$, $w_t(f)$ are the real wages for the single labour services and W_H and W_F are the nominal wage indices for the composite labour service defined as

$$W_{Ht} = \left[\frac{1}{n} \int_0^n (W_t(h))^{\frac{1}{1-\theta}} dh\right]^{1-\theta} \text{ and } W_{Ft} = \left[\frac{1}{1-n} \int_n^1 (W_t(f))^{\frac{1}{1-\theta}} df\right]^{1-\theta}.$$

Firms maximize with respect to the product price and hours worked while taking into account the demand function from households (5), the labour aggregator (10) and the price adjustment cost function:

$$\Gamma_t \left(\frac{P_t}{P_{t-1}}\right) = \frac{\gamma}{\varsigma^2} \left(\exp\left[-\varsigma \left(\frac{P_t}{P_{t-1}} - 1\right)\right] + \varsigma \left(\frac{P_t}{P_{t-1}} - 1\right) - 1 \right).$$
(16)

Price adjustment costs have the same functional form as wage adjustment costs for households (11). We stick to the same specific form in order to keep price and wage adjustments comparable in the degree of convexity. The parameter γ affects the convexity of the cost function while ς affects its asymmetry. As our focus here is on the effects of asymmetric wages, we consider only symmetric price adjustments, implying $\varsigma = 0$.

2.3.1 Firm optimization

The firm's first order conditions are composed of demand for labour and the price setting decision for final goods. The labour demand equates marginal product of labour an extra hour worked to the real wage

$$MPL_t = (1 - \alpha) x_t h_t^{-\alpha} = w_t = \frac{W_t}{P_t}$$

and the optimal pricing equation of the firm takes the form

$$\Pi_t \Gamma'_t = \left[\mu \frac{h_t W_t}{(1-\alpha) P_t C_t} - (1-\Gamma_t) \right] / (\mu - 1) + E_t \left[\Lambda_{t,t+1} \frac{C_{t+1}}{C_t} \frac{P_{i,t}}{P_{t+1}} \Pi_{t+1}^2 \Gamma'_{t+1} \right].$$
(17)

The price equation (17) includes the case of flexible prices if $\Gamma'_t = \Gamma'_{t+1} = 0$, i.e. if $\gamma = 0$

$$P_t = \mu \frac{W_t}{(1-\alpha) Y_t/h_t} = \mu M C_t.$$

Prices are set as a mark-up over nominal marginal costs^2 . The price setting equation (17) includes the costs for price adjustments Γ_t within an inter-temporal condition for smoothing price adjustments due to the convex nature of the linex cost function.

2.3.2 Resource constraint

Production in the economy is used for two different purposes, either it is demanded by households in the two regions for consumption or it is used for wage and price adjustments by the continuum of firms and households. These adjustment costs lead to a loss in aggregate value added. Consumption within the union is therefore production by all firms for each country net of price and wage adjustment costs, Γ , Φ respectively. The net supply of goods is

$$C_{Ht} = n \left[x_{Ht} A_{Ht} h_{Ht}^{1-\alpha} \left(1 - \Gamma_t^H \right) - \frac{W_{Ht}}{P_{Ht}} h_{Ht} \Phi_{Ht} \right]$$
(18)

$$C_{Ft} = (1-n) \left[x_{Ft} A_{Ft} h_{Ft}^{1-\alpha} \left(1 - \Gamma_t^F \right) - \frac{W_{Ft}}{P_{Ft}} h_{Ft} \Phi_{Ft} \right].$$

²With flexible prices no adjustment costs Γ_t are incurred by the firm and hence we use the equality $C_t = Y_t$ in this case.

At the same time total consumption from home and foreign regions of size n and (1 - n) is obtained from preferences of the consumers and the size of the two regions.

$$C_{t} = C_{t} + \frac{1-n}{n}C_{Ft} = \eta T_{t}^{1-\eta}C_{t} + \frac{1-n}{n}\eta^{*}T_{t}^{1-\eta^{*}}C_{t}^{*}$$

$$C_{Ft} = \frac{n}{1-n}C_{t} + C_{t} = \frac{n}{1-n}(1-\eta)T_{t}^{-\eta}C_{t} + (1-\eta^{*})T_{t}^{-\eta^{*}}C_{t}^{*}$$
(19)

By combining resource constraint (19) with the fundamental risk sharing equation (15) and the relationship between terms of trade and the real exchange rate (7) we can state an important equality for relative levels of consumption and prices:

$$\frac{C_{Ht}}{C_{Ft}} = T_t \frac{\eta + \eta^* RER_t^{1-1/\rho}}{1 - \eta + (1 - \eta^*) RER_t^{1-1/\rho}}$$
(20)

For the special case of log utility ($\rho = 1$), equally sized regions (n = 0.5) and symmetric countries ($\eta = 1 - \eta^*$) we obtain

$$\frac{C_{Ht}}{C_{Ft}} = T_t = \frac{P_{Ft}}{P_{Ht}}.$$

The relative levels of consumption are inversely related to the relative producer prices of the two regions generating a tight link between output and prices. Especially in the case of asymmetric productivity shocks, the international risk sharing condition with the evolution of the terms of trade is an important channel for the adjustment process.

2.3.3 Shocks

The model exhibits two types of technology shocks. The aggregate technology shock affects both regions alike and follows an AR(1) process. The asymmetric shock is a perfectly negatively correlated shock between the regions. This specification allows us to analyse symmetric and asymmetric shocks entirely separately. The autoregressive structure is identical to the aggregate shock

$$A_t = exp(\epsilon_{at})A_{t-1}^{\chi_a},$$

$$x_{Ht} = exp(\epsilon_{ht}) (x_{Ht-1})^{\chi_h},$$

$$x_{Ft} = x_{Ht}^{-1},$$

where $\chi_a = \chi_h$. The innovations ϵ_{at} and ϵ_{ht} are iid and drawn from a normal distribution, $\mathcal{N}(0, \sigma_a), \mathcal{N}(0, \sigma_h)$ respectively.

2.4 Monetary policy

We assume an optimal monetary policy setting interest rates for the monetary union as a whole. The optimal Ramsey policy takes the first order conditions of households and firms into consideration when maximising its objective function

$$\max E_{\tau} \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left[n \left(\frac{C_t^{1-\rho}}{1-\rho} - \frac{h_t^{1+\chi}}{1+\chi} \right) - (1-n) \left(\frac{(C_t^*)^{1-\rho}}{1-\rho} - \frac{(h_t^*)^{1+\chi}}{1+\chi} \right) \right].$$
(21)

The optimisation attributes the relative size of the regions as weights in the aggregation of home and foreign utility. The constraints taken into consideration are the first order conditions of the household for wage setting (14) and the firm's price setting equation (17) as well as the resource constraints (18) and (19). The monetary policy optimizes with respect to a sequence of the variables $\{I_t, \Pi_t^H, \Pi_t^F, \Omega_t^H, \Omega_t^F, C_t, C_t^*, h_t, h_t^*, w_{Ht}, w_{Ft}\}_{t=0}^{\infty}$.

3 Calibration

The model employs different data sources for the calibration to the euro area. We distinguish three types of parameters, those that we take as standard in the literature, others that match euro area volatility and a third set dealing with the asymmetry and persistence in wage and price adjustments. The model is calibrated to quarterly data. The two regions have identical size n = 0.5 in order to concentrate on the wage adjustment mechanism. All parameters are resumed in table 1.

Param.	Value	Description
α	0.36	Capital income share
β	0.992	Discount factor, annual rate of 3.3%
ρ	1.1	Inter–temporal elasticity of substitution, reflecting near log–utility
χ	1.5	Labour supply elasticity, mode in Smets and Wouters (2003)
μ	1.2	Product market mark–up, implies an elasticity of 6, RRR2008.
θ	1.4	Labour market mark–up, implies an elasticity of 3.5, RRR2008.
n	0.5	Size of home country
$\eta = 1 - \eta^*$	0.8	Home bias of the home country, Poilly and Sahuc (2008)
$\gamma^H=\gamma^F$	45	Convexity in price adj.cost function, implies price autocorr. of 0.48
$\phi^H = \phi^F$	45	Convexity in wage adj.cost function, implies wage autocorr. of 0.71
$\varsigma^H = \varsigma^F$	0	Asymmetry in the price adjustment cost function: no asymmetry
$\psi^H=\psi^F$	0,800	Asymmetry parameter in the wage adjustment cost function
$ u_H, u_F$	0, 1	0 no wage index. (for eign), 1 full indexation area-wide inflation (home)
$\rho_a=\rho_h$	0.92	Autocorr. of both technology shocks, estimation by RR-R2008
σ_a	0.01	Std. deviation of aggregate technology
σ_h	$\frac{1}{2}\sigma_a$	Std.deviation of asymm. technol., cross–country correlation is 0.66

Table 1: The table reports calibrated parameter values for identical countries and symmetric adjustment costs.

Preferences. The household's discount factor is set to $\beta = 0.992$, reflecting a real interest rate of 3.2%. The elasticity of inter-temporal substitution is $\rho = 1.1$ and the

elasticity of labour disutility takes the value $\chi = 1.5$ both from estimations by Smets and Wouters (2003) and in line with common other calibrations. The mark-up for goods is $\mu = 1.2$, or alternatively the elasticity of substitution between goods is 6. For labour services, instead, we apply a mark-up of $\theta = 1.4$, or an elasticity of substitution of 3.5 also used in the estimation by Rabanál and Rubio-Ramirez (2008). Both regions are characterised by the same degree of home bias in consumption preferences, namely $\eta = 0.8$ $(\eta^* = 0.2)$ following Poilly and Sahuc (2008).

Production. Production consists exclusively of labour input characterised by decreasing returns to scale with an elasticity for labour of 0.64, i.e. $\alpha = 0.36$. The aim is to reproduce increasing marginal costs when expanding production through more hours worked.

Price and wage rigidity. Regarding price and wage rigidities in the two regions we first calibrate these by using symmetric adjustment costs for prices and wages and address the asymmetry in a second step. The cost functions for wage and price adjustment (16), (11) consist of one parameter describing the convexity of the cost function (γ for prices and ϕ for wages) and a second parameter specifying the asymmetry between upward and downward adjustments (ς for prices and ψ for wages). The latter set of parameters is fixed to 0 in the first step. We calibrate the convexity identically between countries ($\gamma^H = \gamma^F$ for prices and $\phi^H = \phi^F$ for wages), in this way we can focus on wage asymmetry as the sole difference between regions. Using the estimations by Rabanál and Rubio-Ramirez (2008) for the euro area, we replicate their findings on price and wage persistence by setting γ^H and γ^F . Obviously there is a strong interplay in persistence of price rigidities and nominal wages to be taken into account and the presence of aggregate and asymmetric shocks reduces inflation persistence in our model compared to Erceg et al. (2000) which already has difficulties matching empirical inflation persistence. Tables 2 and 3 in the appendix summarize different moments of the baseline symmetric and asymmetric calibration.

The parameter ruling asymmetric adjustments to wages is ψ . Symmetric wage setting is characterised by $\psi = 0$ while we set $\psi = 800$ for the asymmetric case, generating in this way the skewness of wage-change distributions for nominally rigid and real rigid countries. Instead of fitting the model to a specific value of skewness for some country, we want to highlight the implications of asymmetric wage adjustments on other variables and on the transmission of productivity shocks. The International Wage Flexibility Project summarized by Dickens et al. (2007) has demonstrated that nominal and real wage rigidities wide spread in the euro area. The studies on downward nominal and real wage rigidities by Holden and Wulfsberg (2007a) and Holden and Wulfsberg (2007b) using industry data for 19 OECD countries between 1973 and 1999 gives us a reference point on the extent of nominal and real wage rigidities. The euro area may be characterised consisting of a large number of countries with downward real rigidity (Austria, Belgium, Germany, the Netherlands as well as Finland), some with downward nominal rigidity (Denmark, Sweden, Portugal and Italy) and few countries with no apparent downward rigidity (Spain, France and Greece, UK and Ireland). The effects downward rigidities have on other variables within the model become visible mainly in the mean and skewness

The parameter responsible for generating real wage rigidities is ν that appears in the cost function for wage changes (11). Nominal wages are rigid with $\nu = 0$, while $\nu = 1$ reflects a situation of full contemporaneous wage indexation with area-wide inflation.

Shock size. In order to match volatility and persistence of output we can vary volatility and persistence of symmetric and asymmetric shocks in the union. We set persistence of both shocks to be identical, $\rho_a = \rho_h$ and set the standard deviation of the asymmetric shocks to be half that of the aggregate shock $\sigma_h = \frac{1}{2}\sigma_a$. Due to the fact that asymmetric shocks imply a perfectly negatively correlated shock between the two countries, setting the standard deviation in this way makes the difference between the two countries of same magnitude as a symmetric shock although the transmission is very different. With our choice of volatility we obtain a cross-correlation of output between the regions of 0.66. The asymmetric shocks strongly affect price volatility as it is the main adjustment channel for asymmetric shocks, reducing thereby the inflationary persistence induced by aggregate shocks.

3.1 Computational solution

The model has been set up to analyse the asymmetric behaviour within a monetary union to symmetric and asymmetric productivity shocks. In order to adequately address this question the solution algorithm needs to account for such asymmetry. We use a second– order Perturbation implemented in Dynare to approximate the solution around the steady state following the algorithm by Schmitt-Grohe and Uribe (2004). The Ramsey optimization employs the algorithm implemented in Dynare by Lombardo and Sutherland (2007), which itself is similar to the one by Levin et al. (2005). In addition we counter–checked steady state results using the implementation by Ondra Kamenik in Dynare++ which uses alternative solution procedures.³

Simulations with second-order approximations exhibit the possibility of explosive paths due to the existence of an approximation-induced second steady state which is unstable. In order to make sure to remain on the stable path we implement in Dynare the pruning algorithm proposed by Kim et al. (2008). The code for the pruning algorithm is made available on the Dynare website and is readily implementable in other models that use simulations in a second-order approximation⁴.

³All these codes can be found on the Dynare website http://www.cepremap.cnrs.fr/dynare/ .

 $^{^4{\}rm The}$ pruning code can be found in the Forum section under "DYNARE contributions and examples". The exact URL is

http://www.cepremap.cnrs.fr/juillard/mambo/index.php?Itemid=95&page=viewtopic&t=1669

4 Results

4.1 Steady state effects

Asymmetric wage adjustment costs reflecting downward wage rigidity may have effects on steady state values. Tobin (1972) stated the importance of a positive inflation rate to ease real wage adjustments due to the presence of nominal illusion and an aversion for nominal wage cuts. The International Wage Flexibility project has found a large extent of downward wage rigidity at the microeconomic level for job stayers, see Dickens et al. (2007). Indeed, Bewley (2007) has found substantial resistance for downward nominal wage adjustments from managers for its detrimental impact on motivation by employees. For a more detailed discussion on possible adjustment margins at the sectoral level beyond merely wages for job-stayers see Holden and Wulfsberg (2007a).

The results are presented for our reference model in which the monetary union consists of a home region with downward real rigid wages and a foreign region with downward nominal rigid wages., in all other terms the two regions are identical. We then also present special cases changing the degrees and types of rigidities in order to isolate specific mechanisms. In our model the positive greasing effects of inflation are generated through the asymmetric adjustment cost structure for nominal wages as already pointed out in Kim and Ruge–Murcia (2007). A positive inflation rate, although permanently causing wage and price adjustment costs, bears the advantage that negative shocks do not generate as large costs for downward nominal wage adjustments as they would at lower steady state inflation rates. The greasing effect is therefore of precautionary nature to reduce adjustment costs after negative shocks compared to a zero inflation steady state.

Table 2 states steady state wage and price inflation for our reference calibration and other specifications. The reference setup for the euro area includes one region with asymmetric real wage adjustment costs and a second region with nominal asymmetries, while prices are characterized by symmetric adjustment costs for both countries. The choice for this specific combination is driven by the findings of Holden and Wulfsberg (2007b) of a large extend of nominal and real downward rigidity within OECD countries including many euro area countries. At this stage, and especially due to the fact that we consider only productivity shocks as drivers of fluctuations, the calibration is indicative of the effects, but nevertheless it is helpful to understand how rigidities in one region affect the other region and the monetary union as a whole.

The baseline model with nominal and real downward rigidities exhibits positive inflation rates of 4.7 basis points per quarter, implying an annualised rate of 0.19%. Inflation volatility remains relatively modest compared to wage inflation, recall that aggregate technology shocks have a 1% standard deviation and asymmetric shocks have half that size. The region with real asymmetric wage rigidity is characterised by higher volatility in nominal wage changes than the nominal rigid region. An interesting finding is that the right skewed distribution in wage adjustments also shifts the distribution of price adjustments

q-to-q	Mean /	u (%)	S	Std. dev	σ. σ (%)		Skev	vness γ_1	
Model	$\Pi = \Omega$	Ι	П	Ω_H	Ω_F	Ι	П	Ω_H	Ω_F	Ι
Baseline	0.047	0.84	0.28	0.97	0.64	0.29	1.66	1.45	2.0	-0.04
${}_{ ext{symm.nom.}}\psi=0, u=0$	0.00	0.78	0.17	0.29	0.29	0.25	0.04	0.02	-0.01	0.02
Asymm.nom. $\psi=$ 800, $ u=0$	0.085	0.089	0.24	0.54	0.54	0.28	2.06	2.26	2.24	0.44
Asymm.real $\psi=$ 800, $ u=1$	0.00	0.79	0.30	1.1	1.0	0.30	1.21	1.37	1.42	-0.40
Asymm.nom.w/ flex p., $\gamma=1$	0.81	1.61	1.2	1.9	1.9	0.23	2.82	2.35	2.35	0.84
Asymm.nom.w/ rigid p, $\gamma=100$	0.039	0.84	0.16	0.33	0.33	0.27	1.52	2.39	2.30	0.38

Table 2: Steady state effects on mean, standard deviation and skewness of quarter-to-quarter price and wage inflation and optimal interest rates for different model specifications. The baseline model is characterised by real asymmetric wage adjustments in the home country, and nominal adjustments in the foreign country. The other models consider identical regions with symmetric, and asymmetric adjustment costs.

seen in the results for the skewness.

To compare different alternative model specifications, we compute the same set of steady state values for a monetary union with identical regions with either symmetric $(\psi = 0)$ or asymmetric $(\psi = 800)$ adjustment costs either in nominal $(\nu = 0)$ or in real $(\nu = 1)$ terms. The last two rows highlight the effects of more flexible $(\gamma = 1)$ or more rigid $(\gamma = 100)$ prices, the reference model employed $\gamma = 45$ for both regions.

Grease inflation is most important for asymmetric nominal wage adjustment costs and relatively flexible prices. If adjustment costs are symmetric, either in real or nominal terms, positive and negative shocks generate the same degree of costs and therefore no benefits are obtained from a positive as opposed to a zero inflation rate. Likewise, if adjustment costs are asymmetric with respect to real wages, i.e. indexed wages to current inflation, higher inflation does not reduce adjustment costs for negative shocks as the indexation does not ease real wage adjustments. Therefore no positive inflation bias is expected for this case.

In addition, the relative costs for wage and price adjustments are important (last two rows). If prices are strongly rigid due to larger convexities in adjustment costs, a positive inflation rate in steady state generates substantial costs which may outweigh the precautionary motive and cancel the inflationary bias. Instead, the greasing effect is largest for strongly asymmetric nominal wages combined with low rigidity in prices. With regions characterised by different types of asymmetric wage adjustments, i.e. if only one region exhibits asymmetric costs, the effects are smaller. No effect is present if adjustments for both countries are either symmetric, asymmetric in real terms or a combination of the two.

Beyond the effects on the first moment, our asymmetric wage adjustments affect second and third moments. Especially volatility in wage inflation increases with the degree of asymmetry in wage adjustments in either nominal or real terms. The reason being that asymmetry implies higher costs for downward adjustments but lower costs for wage

	$AR \ 1$			AR 4				
Model	П	Ω_H	Ω_F	Ι	П	Ω_H	Ω_F	Ι
Baseline	0.57	0.46	0.45	0.78	-0.14	-0.19	-0.11	0.44
Symm.nom.	0.47	0.69	0.69	0.87	-0.10	0.12	0.12	0.62
Asymm.nom.	0.58	0.45	0.45	0.79	-0.03	-0.07	-0.07	0.51
Asymm.real	0.59	0.48	0.47	0.79	-0.20	-0.20	-0.19	0.36
Asymm.nom.w/flex p	0.005	-0.02	-0.02	0.91	-0.01	-0.012	-0.011	0.71
Asymm.nom.w/rigid p	0.66	0.62	0.62	0.86	0.07	0.04	0.05	0.57

Table 3: Autocorrelations for one and four quarters

increases compared to the symmetric adjustment cost function as depicted in figure 9. This implies that nominal wages increase by large magnitudes in response to positive productivity shocks and hence increase the volatility.

The shape of the ergodic distribution of wage and price adjustments is also strongly affected by downward rigidity as found in the value for skewness. Especially nominal wage changes and to a lower degree price changes become skewed due to asymmetric adjustment costs. The asymmetric wage adjustments affect the shape of interest rate responses only in the case of nominal asymmetric rigidities combined with relatively flexible prices. If asymmetric rigidity prevails for real wages, interest rates are skewed in the other direction dampening the aggressive response to inflation and being more accommodative due to the detrimental effects on employment as seen in the impulse response functions below. Results reported in the appendix reveal that wages and hours exhibit skewness in opposite directions and are smaller. Real wages are positively skewed like nominal wage changes and inflation, whereas hours worked are left skewed (negative values for skewness). As will become clearer with the analysis of impulse responses, real wages increase strongly after positive shocks but with only small movements in hours worked. In the opposite case of a negative impact on real wages, these resist to fall which implies that employment drops by a large magnitude leading to the type of shape observed.

The effects of asymmetries on the persistence of variables is mixed. While on the one hand the stronger rigidity induces more persistence especially for real wage rigidity, the asymmetry shortens persistence at longer horizons. Table 3 reports coefficients of autocorrelation at one and four lags indicating no large variations except for the situation with more flexible prices.

The steady state results depend importantly on the volatility of shock sizes. In linear models shock volatility affects exclusively the volatility of variables but has no effects on their mean or skewness while the shock size affects both of them. We are especially interested how the greasing effect varies with the volatility of aggregate shocks and the relative size of the asymmetric to the aggregate shock. Figure 1 depicts the relationship between volatility in technology shocks and the inflation and nominal interest rate for the



Figure 1: Effect of volatility on mean inflation and nominal interest rate. Aggregate standard deviation is varied from zero to 4% and at the same time asymmetric shocks are jointly increased in size.

baseline model⁵. As expected, steady state inflation increases due to the aforementioned precautionary motive, but the greasing effect is not linear in the increase in volatility and the increase in interest rates though parallel to the one by inflation is less strong, reflecting that the welfare based optimal rule takes the detrimental real effects on hours and output into consideration.

In results not reported here the surprising finding is that larger asymmetric shocks to the two regions lead to lower steady state inflation. As will become clearer with the impulse response functions two mechanisms are at play. The first one relates to the symmetric adjustment cost function of prices. As prices play a crucial role in the risk sharing process (see the fundamental risk sharing condition (15)) and this is more important with larger shocks, the positive effects of greasing are no longer of first order. Instead, the symmetric adjustment of prices dominates the influence of asymmetric wage adjustments. The second reason for decreasing steady state inflation is inherent to deflationary pressures from asymmetric shocks which only appear thanks to the second order approximation. The three different sources for these effects are, first, concavity in the disutility of working having non-linear effects on the wage as this is a mark-up over the marginal rate of substitution, second, decreasing returns to scale in the production function affecting differently an increase in hours worked or a decrease, and third, the degree of risk aversion in the utility of consumption which affects the risk sharing mechanism across countries and the way consumption is transferred intertemporaly.

⁵In this exercise we vary both types of volatility: aggregate productivity shocks become larger and at the same time asymmetric shocks are increased proportionally in order not to distort the shock composition.

4.2 Dynamic effects

Asymmetric adjustments generate very different dynamic responses to opposite shocks. We compute the impulse responses to an aggregate and an asymmetric productivity shock. The impulse response functions presented depict the dynamics after the economy has been hit by 4 different shock sizes to aggregate productivity, either a positive or negative shock of either one or two standard deviations. We compute all four different types of shocks to illustrate the asymmetric behaviour for positive and negative impulses especially if these are large in comparison to general shocks. If a large impulse shocks the economy the asymmetries become more apparent and the non-symmetric dynamics come into play. In order to force the model to explore that region the impulse size needs to be larger than the general level of fluctuations, this is achieved by using 2σ shocks. A negative shock of such size may occur only with a 2.3% probability, nevertheless it reveals the mechanism of the non-linear response in an exaggerated manner.

To analyse the response to an aggregate productivity shock we use a model with fully symmetric regions and symmetric nominal wage adjustment costs as reference to understand the mechanisms in a two region monetary union with an optimal monetary policy response under commitment. The impulse responses are stated for a quarterly time frame in percentage deviations to a shock of one or two positive and negative standard deviations (percentage points for nominal variables). We first discuss aggregate productivity shocks and thereafter asymmetric shocks.

4.2.1 Aggregate shocks

Following an aggregate shock the two regions have identical adjustments and do not exhibit any relative wage or price differentials and therefore no terms of trade effects. For simplicity we report only the responses in the home region, the foreign region's response being identical. In the case of symmetric adjustment costs nominal wages and prices share the burden to adjust real wages to the increased productivity. Nominal wage inflation increases in both regions and price inflation falls, in this way real wages increase. The observed increasing nominal wages reflect higher productivity, and decreasing prices relate to lower marginal costs incurred by firms. The presence of rigid prices and nominal wages leads to rigid real wages which exhibit a strong and persistent hump-shaped adjustment pattern. Optimal nominal interest rates are reduced after a positive shock to counter deflationary pressures and are persistent although the response of price inflation is shortlived. Note that interest rates respond strongly after a negative productivity shock to counter inflationary pressures, while response to a positive shock is more muted. Hours worked combines substitution and wealth effects leading initially to an increase in hours worked.

Adding downward nominal wage rigidity implies higher adjustment costs for nominal wage cuts and lower costs for wage increases. This hinders wage deflation and shifts the



Figure 2: Response in percentage deviations (for nominal variables in percentage points) to positive and negative shocks of one and two standard deviations to an aggrgate technology in a model with identical countries and symmetric nominal wage rigidity. Parameter values are: $\gamma = \phi = 45, \psi = 0, \nu = 0, \chi = 1.5, \rho = 1.1$.

adjustment margin for real wage adjustments towards price inflation. Figure 3 illustrates the response to different sized shocks in a monetary union characterized by identical regions with downward nominal wage rigidities and restates the results of Kim and Ruge–Murcia (2007). As expected the response of wage inflation is muted for negative shocks compared to positive shocks and instead price inflation helps to make the real wage adjustment. With a positive shock, price deflation is muted because nominal wages adjust easily, instead after a negative shock inflation is very strong to compensate for the downward nominal wage rigidity to make the adjustment to real wages. Price inflation induces monetary policy to firmly react by increasing the interest rate more strongly than in the symmetric case of figure 2 while it is even more muted for negative shocks. The response of output is altered only marginally, while hours worked decline strongly in the case of real wage rigidity reflecting the highly detrimental effects of high wages on employment.

To further understand the quantitative effects of different types of rigidities figure 4 reports the response of different variables to a negative shock of two standard deviations to aggregate technology. All situations combine identical countries with either flexible or rigid wages with symmetric adjustment costs, downward rigid wages with nominal and real rigid wages.

When moving from flexible to nominal rigid models and to the model with downward



Figure 3: Response in percentage deviations (for nominal variables in percentage points) to positive and negative shocks of one and two standard deviations to an aggrgate technology shock in an economy with identical regions and asymmetric nominal wage rigidity. Parameter values are: $\gamma = \phi = 45, \psi = 800, \nu = 0, \chi = 1.5, \rho = 1.1$.

real wage rigidity, the response of wage inflation becomes more and more positive while at the same time the response of price inflation magnifies to compensate for the inability of wage adjustments. The adjustment of real wages becomes more and more difficult and they even increase in the case with real wage rigidity which leads to highly detrimental effects on hours worked and creates persistence in the response of output. Although the overall responses change strongly in nominal and real variables, the optimal monetary policy exhibits similar responses in all four models except for the situation of real rigidities. Here, optimal monetary policy fights inflation at the beginning in a more muted fashion and is generally less aggressive to take into account the welfare effects of the lower amount of employment.

At this stage we are ready to analyse our reference scenario of a monetary union consisting of the home region with downward real rigidities and the foreign region with downward nominal rigidities. Figure 5 illustrates the response to an aggregate shock in such a monetary union. The two regions characterised by different types of asymmetries lead to terms of trade effects even for symmetric shocks. The home region with real rigid wages is characterised by a more positive response of both wage and price inflation: after a negative shock, wage inflation remains positive due to indexation and price inflation

Comparing response to a large negative shock to aggregate productivity



Figure 4: Response in percentage deviations (for nominal variables in percentage points) to a negative aggregate technology of two standard deviations. The bold line characterizes a response in a model with wage flexibility ($\phi = 0, \psi = 0, \nu = 0$,), the dotted line with symmetric adjustment costs ($\phi = 45, \psi = 0, \nu = 0$,), the dashed line a response with downward nominal rigidity ($\phi = 45, \psi = 500, \nu = 0$,) and the starred line with real wage rigidity (wage indexation, $\phi = 45, \psi = 800, \nu = 1$).

is strong to reduce real wages, while these effects are smaller in the downward nominal (foreign) rigid region. For a positive shock wage inflation is larger for the real rigid region and price deflation is more muted. In both cases prices in the real rigid country are larger than in the nominal rigid region which leads to persistent terms-of-trade effects, compensated only in the long–run.

For negative shocks the response of hours worked is highly negative for the real rigid region, but is not strongly positive after a positive shock highlighting again a strong asymmetry in response due to the concavity in the labour part of the utility function determined by the parameter χ . This illustrates that productivity increases do not lead to an increase in hours worked, while negative shocks combined with real wage rigidity are highly detrimental for employment. Finally, the response of monetary policy remains similar to situations with identical regions: a strong movement after negative shocks to counter inflationary pressures and a muted response for positive technology shocks.

Baseline model: Aggregate productivity shock



Figure 5: Response to different sized aggregate technology shocks in a monetary union with downward nominal wage rigidities in the home region and downward real rigidities in the foreign region.

4.2.2 Asymmetric shocks

The asymmetric shock is modelled as a perfectly negatively correlated shock to the home and foreign region, i.e. a zero-sum productivity shock. In this way we assess the transmission of this shock within the union without being affected by productivity gains or losses at the union level. In addition, in a linear world such an asymmetric shock leads to no aggregate effects, but through asymmetries in the transmission stemming from the second order approximation aggregate variables may nevertheless be affected.

Comparing the different impulse responses, the most striking feature is the fact that relative prices carry the biggest burden for the adjustment across regions, while wage adjustments play a minor role. Hereby the international risk sharing equation (15) combined with near to log-utility and a Cobb-Douglas aggregator of home and foreign goods creates a strong tie between terms of trade, i.e. the relative price of the two regions, and net production of the two regions as seen in (20). This strong link is an important adjustment mechanism, and the relative wage setting between the regions has a much smaller impact on the dynamics. Figure 6 illustrates the strong response of terms of trade and small adjustment in wage inflation. Due to these ties even asymmetric wage adjustments have only a minor impact as their impact is of smaller size than the symmetric adjustment of



Identical regions with symmetric wage adjustment costs: response to an asymmetric shock

Figure 6: Response in percentage deviations (for nominal variables in percentage points) to positive and negative shocks of one and two standard deviations to an asymmetric technology shock in a model with identical countries and symmetric nominal wage rigidity. Parameter values are: $\gamma = \phi = 45, \psi = 0, \nu = 0, \chi = 1.5, \rho = 1.1$.

prices.

Even in the case of purely asymmetric shocks between the regions, optimal monetary policy does have a role in such an environment. The concavity of disutility of work leads to different sized effects between the regions leading always to deflationary effects. Indeed, due to the fact that households set wages as mark-up over the marginal rate of substitution, wages increase by less in one region than they decrease in the other region leading to area-wide deflation and making an increase in the interest rate optimal in order to postpone and smooth consumption of households. With perfectly functioning capital markets, asymmetric shocks have beneficial effects for the union as a whole. The optimal response by the monetary authority is to increase interest rates to shift consumption intertemporaly

By comparing the response in four different models (flexible wages, symmetrically nominal rigid, nominal and real downward rigid) for an asymmetric shock affecting the home country negatively, the importance of the risk sharing mechanism is further underlined. A negative shock at home and a positive shock in the foreign region lead to an initial increase



Comparing response to 2σ neg. asymmetric shock in home region

Figure 7: Response in percentage deviations (for nominal variables in percentage points) to a negative asymmetric technology shock of two standard deviations in the home region. The bold line characterizes a response in a model with wage flexibility ($\phi = 0, \psi = 0, \nu = 0$,), the dotted line with symmetric adjustment costs ($\phi = 45, \psi = 0, \nu = 0$,), the dashed line a response with downward nominal rigidity ($\phi = 45, \psi = 500, \nu = 0$,) and the starred line with real wage rigidity (wage indexation, $\phi = 45, \psi = 800, \nu = 1$).

in wage inflation at home and negative wage inflation in the foreign country, an opposite effect to aggregate shocks with rigid wages this effect is muted, but the direction remains intact. Terms of trade are beneficial to the home country to such a degree that real wages in the negatively affected region increase for flexible wages. Contrasting this, prices react nearly identically to all specifications, indicating the strength of the international transmission through relative prices, which are tightly linked to production in the two regions. With higher wage rigidity the deflationary pressures are more persistent, also monetary policy reacts more firmly.

The international channel in our version is highly stylized, a richer version might give stronger pass–through from wage asymmetries onto price adjustments. Interestingly output increases only with a delay for nominally and real rigid wages reflecting the strong decline in hours worked.

With this understanding for asymmetric shocks we now turn to our reference model consisting of a real downward rigid home region and a nominally downward rigid foreign

Baseline model: Asymmetric productivity shock



Figure 8: Response to asymmetric technology shocks of different sizes in a monetary union with real (home) and nominal (foreign) downward rigid wages. The shocks are of one and two standard deviations, both positive and negative.

region. The effects are again dominated by the terms of trade effects, while the asymmetric wage adjustments play a minor role. For the international channel to be of higher asymmetric impact it is necessary to alter either the type of risk sharing, the aggregation of home and foreign consumption bundles or even introduce asymmetric adjustments of prices., but the empirical evidence for the last possibility is scarce.

4.3 Sensitivity analysis

A core parameter for our results is the degree of asymmetry in wage adjustments, specified by ψ . We therefore undertake a sensitivity analysis for this represented in figure 10 in the appendix. Nominal wage inflation changes in a highly non-linear way, the skewness increases strongly from zero to above two for values between 0 and 400 and is flat thereafter. While the skewness of prices increases linearly, the skewness in interest rates become steeper once the wage inflation skewness has reached its upper level. This indicates that from that point on, real effects are taken into consideration more strongly. This is also highlighted by the fact that skewness in real wages picks up at roughly the same level. Overall the sensitivity analysis indicates that asymmetric wage adjustments first influence purely the nominal side, and at higher asymmetries it also has effects on real wages. The response of hours worked is an exception in this respect, was it is negatively skewed, complementing thereby real wage changes.

A similar picture to the changes induced by asymmetric wage adjustments stems from volatility in technology shocks as seen in figure 11. The larger these become, the larger the skewness of the variables but the relationship is not linear for nominal wages and prices. Again, nominal wages face an early satiation level, but now price adjustments also tend to reach a maximum level of skewness if volatilities are raised further. Instead, interest rates, real wages and hours worked exhibit linear relationships for the ranges of volatility considered.

5 Conclusion

Downward nominal and real wage rigidities introduce distortions in the adjustment of nominal and real wages to technology shocks. In this paper we highlighted to what degree these rigidities become important in a monetary union and have identified three distinct areas, the effects on steady state, the response to an aggregate shock with potential effects on terms of trade and finally the distortions following an asymmetric technology shock.

Downward nominal wage rigidities lead to a positively skewed response in nominal wage changes and a sizeable positive optimal inflation rate. This effect is stronger the lower price rigidity. The greasing effects of inflation vanish if wages are either indexed (real wage rigidity) or if adjustment costs are symmetric.

Regarding the dynamic responses of downward wage rigidities we find that beyond the direct effects on wage changes also price changes are altered and become asymmetric, highlighting the strong link between prices and wages. In addition for real wage adjustments, the effects on hours worked are highly asymmetric. In the case of two regions with different types of asymmetries (nominal and real), terms of trade effects are sizeable with the more rigid region for wages adjusts for positive and negative shocks always with higher prices. To what degree other setups for international risk sharing and asymmetries in price adjustments lead to more profound effects in current account remains a further area of research.

The effects of asymmetric technology shocks between regions of a monetary union have been at the heart of debates regarding the optimality of currency areas. The simple version presented shifts the main burden of adjustment towards relative price adjustments of the concerned regions. Asymmetries in the way nominal wages are set in the two regions affect the transmission to a much smaller degree than the terms of trade channel governed by relative prices. Nevertheless, the model reveals that asymmetric shocks bear potential deflationary pressures for the union as a whole stemming from non-linear production and utility functions implying a tightening response of optimal monetary policy in times of deflationary price adjustments. These results are made possible due to the non-linear approximation and require further investigation in future projects.

From here, different strands of research may be further fruitful to investigate. A project

already started addresses the question how asymmetric wage changes affect wages and employment within a matching model. Due to the opposite effects on skewness on wages and hours worked a matching framework may transmit and magnify this effect through the delay in re-matching workers that have become unemployed. Another promising path would be the analysis of a richer risk sharing scheme for the transmission of asymmetric shocks.

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6 Appendix

6.1 Summary of all equations for the Ramsey optimisation problem

Euler equation for home consumers

$$C_t^{-\rho} = \beta I_t E_t \left[\frac{C_{t+1}^{-\rho}}{\Pi_{t+1}} \right]$$

Wage setting behaviour for the two countries

$$\frac{\Omega_t^i}{\left(\Pi_t^i\right)^{\nu}} \Phi_t^\prime \left(\frac{\Omega_t}{\left(\Pi_t^i\right)^{\nu}}\right) = \frac{\theta \frac{h_t^{\lambda}}{\eta_t w_t} - \left(1 - \Phi_t^i\right)}{\theta - 1} + E_t \left[\Lambda_{t,t+1} \left(\Pi_{t+1}^i\right)^{1-\nu} \Omega_{t+1}^{i2} \frac{h_{t+1}}{h_t} \Phi^\prime \left(\frac{\Omega_{t+1}^i}{\left(\Pi_{t+1}^i\right)^{\nu}}\right)\right] \quad \text{for } \mathcal{H}, \mathcal{F}$$

Price setting behaviour

$$\Pi_t^i \Gamma_t' \left(\Pi_t^i \right) = \frac{\mu \frac{h_t w_t}{C_t (1-\alpha)} - \left(1 - \Gamma_t^i \right)}{\mu - 1} + E_t \left[\Lambda_{t,t+1} \frac{C_{t+1}}{C_t} \Pi_{t+1}^i \Gamma' \left(\Pi_{t+1}^i \right) \right] \quad \text{for } \mathcal{H}, \mathcal{F}$$

Adjustment cost functions

$$\Phi^{i}\left(\frac{\Omega_{t}}{\Pi_{t}^{\nu}}\right) = \frac{\phi^{i}}{\left(\psi^{i}\right)^{2}} \left(\exp\left[-\psi^{i}\left(\frac{\Omega_{t}}{\Pi_{t}^{\nu}}-1\right)\right] + \psi^{i}\left(\frac{\Omega_{t}}{\Pi_{t}^{\nu}}-1\right) - 1\right) \text{ for } \mathcal{H}, \mathcal{F}$$

$$\Gamma_{t}^{i}\left(\Pi_{t}\right) = \frac{\gamma^{i}}{\left(\varsigma^{i}\right)^{2}} \left(\exp\left[-\varsigma^{i}\left(\Pi_{t}-1\right)\right] + \varsigma^{i}\left(\Pi_{t}-1\right) - 1\right) \text{ for } \mathcal{H}, \mathcal{F}$$

Risk sharing condition

$$RER_t = \left(\frac{C_t^*}{C_t}\right)^{-\rho}$$

Terms of trade and Real exchange rate

$$\frac{TOT_t}{TOT_{t-1}} = \frac{\Pi_{Ft}}{\Pi_{Ht}}$$
$$RER_t = TOT_t^{\eta - \eta^*}$$

Resources

Demand

$$C_{Ht} = \frac{1}{n} \left(\eta T_t^{1-\eta} C_t + \frac{1-n}{n} \eta^* T_t^{1-\eta^*} C^* \right)$$

$$C_{Ft} = \frac{1}{1-n} \left(\frac{n}{1-n} (1-\eta) TOT^{-\eta} C + \frac{1-n}{n} (1-\eta) TOT^{-\eta^*} C^* \right)$$

Production

$$C_{Ht} = n \left(x_{Ht} A_t h_{ht}^{1-\alpha} (1-\Gamma_{ht}) - w_{ht} h_{ht} \Phi_{ht} \right)$$

$$C_{Ft} = (1-n) \left(x_{Ht}^{-1} A_t h_{ft}^{1-\alpha} (1-\Gamma_{ft}) - w_{ft} h_{ft} \Phi_{ft} \right)$$

Inflation and wage inflation

$$\Pi_{wt} = \Pi_{Ht}^{n} \Pi_{Ft}^{1-n}$$
$$\Pi_{t} = \Pi_{Ht}^{\eta} \Pi_{Ft}^{1-\eta}$$
$$\Pi_{t}^{*} = \Pi_{Ht}^{\eta^{*}} \Pi_{Ft}^{1-\eta^{*}}$$
$$\Omega_{Ht} = \frac{w_{Ht}}{w_{Ft-1}} \Pi_{t}$$
$$\Omega_{Ft} = \frac{w_{Ht}}{w_{Ft-1}} \Pi_{t}^{*}$$

Exogenous Shocks

$$A_t = exp(\epsilon_{At})A_{t-1}^{\chi_A}$$
$$x_{Ht} = exp(\epsilon_{xt})x_{Ht-1}^{\chi_A}$$
$$x_{Ft} = x_{Ht}^{-1}$$

7	Complete s	set of	steady	state	results
	1				

	MEAN	STD. DEV.	VARIANCE	SKEWNESS	KURTOSIS
Omh	1.000466	0.009697	0.000094	1.451838	6.618199
Omf	1.000466	0.006402	0.000041	2.005283	8.279504
DeltaOm	1.000003	0.005945	0.000035	-0.575690	3.720151
nint	1.008448	0.002859	0.000008	-0.041994	0.938270
Pieh	1.000464	0.003531	0.000012	1.173966	3.612728
Pief	1.000465	0.002960	0.000009	0.962228	2.949655
DeltaPi	1.000016	0.005615	0.000032	0.002783	0.077628
Pie	1.000463	0.002789	0.000008	1.658581	5.700446
wh	0.271811	0.007609	0.000058	0.340451	0.888204
wf	0.271553	0.007037	0.000050	0.211962	0.338850
hh	0.948553	0.011587	0.000134	-1.239523	3.204403
hf	0.948812	0.008555	0.000073	-0.599040	0.783560
yh	0.483922	0.016581	0.000275	0.025561	0.219304
yf	0.483339	0.015489	0.000240	0.076640	0.039538
ТОТ	1.001511	0.028710	0.000824	0.104978	0.055097

Table 4: Mean of moments for 20,000 simulations of a monetary union with home country characterised by asymmetric real wage adjustment costs and foreign country with asymmetric nominal wage adjustments, the reference model.

	1	2	3	4	5
Omh	0.462887	0.090435	-0.115346	-0.190210	-0.181152
Omf	0.454732	0.134378	-0.037252	-0.106305	-0.117588
DeltaOm	0.415854	0.030600	-0.171815	-0.229748	-0.200614
nint	0.781709	0.606053	0.498146	0.435631	0.403944
Pieh	0.591948	0.258139	0.023925	-0.119592	-0.178458
Pief	0.588939	0.270925	0.060876	-0.058332	-0.119961
DeltaPi	0.627212	0.344929	0.147413	0.019379	-0.062495
Pie	0.577397	0.233688	-0.000301	-0.135751	-0.187755
wh	0.958403	0.877960	0.788172	0.704835	0.633430
wf	0.971919	0.916930	0.851745	0.785530	0.722568
hh	0.827189	0.647842	0.476472	0.328032	0.203602
hf	0.705823	0.468477	0.289168	0.160443	0.065191
yh	0.933215	0.858603	0.780778	0.705343	0.637085
yf	0.928352	0.851878	0.774939	0.701703	0.635786
тот	0.980395	0.937294	0.880887	0.818737	0.755746

Table 5: Mean of autocorrelation coefficients at different horizons for 20,000 simulations for the reference monetary union.

	MEAN	STD. DEV.	VARIANCE	SKEWNESS	KURTOSIS
Omh	0.999926	0.002891	0.000008	0.021720	0.008386
Omf	0.999926	0.002915	0.000008	-0.014384	-0.050767
DeltaOm	1.000001	0.001443	0.000002	0.007625	-0.060275
nint	1.007825	0.002531	0.000006	0.015331	-0.008223
Pieh	0.999925	0.002298	0.000005	0.017287	-0.011278
Pief	0.999923	0.002277	0.000005	0.056808	0.011998
DeltaPi	1.000011	0.005098	0.000026	0.042067	-0.034439
Pie	0.999923	0.001701	0.000003	0.042560	0.019418
wh	0.271934	0.006838	0.000047	0.076733	0.021884
wf	0.271871	0.006975	0.000049	0.162049	-0.161210
hh	0.949189	0.007034	0.000049	-0.126845	-0.007436
hf	0.949185	0.007114	0.000051	-0.096924	-0.026460
yh	0.484384	0.015536	0.000241	0.085375	0.051319
yf	0.484237	0.015830	0.000251	0.163345	-0.116695
ТОТ	1.000703	0.027298	0.000745	-0.008836	-0.068379

Table 6: Mean of moments for 20,000 simulations of a monetary union with identical countries characterised by symmetric nominal wage adjustment costs.

	1	2	3	4	5
Omh	0.690121	0.441412	0.256387	0.120648	0.028555
Omf	0.693963	0.447550	0.259318	0.123127	0.026895
DeltaOm	0.469768	0.151741	-0.018652	-0.109188	-0.147754
nint	0.866651	0.766915	0.688866	0.623423	0.567071
Pieh	0.537180	0.242137	0.062320	-0.039696	-0.094991
Pief	0.530546	0.230195	0.060504	-0.034998	-0.078788
DeltaPi	0.615598	0.344051	0.164030	0.041264	-0.037633
Pie	0.467893	0.149118	-0.021444	-0.100895	-0.126850
wh	0.983607	0.946620	0.897897	0.843466	0.787274
wf	0.983801	0.948570	0.902165	0.850206	0.796354
hh	0.603205	0.329994	0.154063	0.036318	-0.036237
$\mathbf{h}\mathbf{f}$	0.610391	0.337305	0.156618	0.035029	-0.043157
yh	0.930530	0.856639	0.784333	0.716515	0.654292
yf	0.932789	0.862055	0.793479	0.729077	0.669975
ТОТ	0.982255	0.943370	0.892485	0.835856	0.777766

Table 7: Mean of autocorrelation coefficients at different horizons for 20,000 simulations of a monetary union with identical countries characterised by symmetric nominal wage adjustment costs.

	MEAN	STD. DEV.	VARIANCE	SKEWNESS	KURTOSIS
Omh	1.009449	0.015863	0.000252	2.724699	10.779835
Omf	1.009449	0.015885	0.000252	2.721046	10.803240
DeltaOm	1.000004	0.002803	0.000008	-0.010747	0.046166
nint	1.017561	0.002085	0.000004	1.123537	1.944813
Pieh	1.009414	0.013367	0.000179	2.540413	10.176627
Pief	1.009415	0.013419	0.000180	2.545092	10.219930
DeltaPi	1.000084	0.012824	0.000164	0.059925	0.047617
Pie	1.009407	0.012828	0.000165	2.867286	12.125629
wh	0.271454	0.008497	0.000072	0.018557	0.016054
wf	0.271485	0.008272	0.000068	0.071986	0.075218
hh	0.949377	0.002433	0.000006	-0.037880	-0.047899
hf	0.949401	0.002501	0.000006	0.024470	-0.164703
yh	0.483212	0.015229	0.000232	0.024296	-0.020341
yf	0.483278	0.014777	0.000218	0.085016	0.052890
ТОТ	1.000347	0.032228	0.001039	0.066997	-0.099889

Table 8: Mean of moments for 20,000 simulations of a monetary union with identical countries both characterised by asymmetric nominal wage adjustment costs.

	1	2	3	4	5
Omh	-0.018597	-0.011961	-0.007576	-0.011589	0.002089
Omf	-0.019300	-0.011881	-0.007194	-0.010710	0.000721
DeltaOm	-0.028189	-0.038168	-0.051986	-0.029206	-0.021995
nint	0.914736	0.838760	0.772417	0.711855	0.654947
Pieh	0.004938	0.004404	-0.004972	-0.003606	-0.005443
Pief	-0.000567	-0.004927	-0.005480	-0.003388	0.001194
DeltaPi	-0.034767	-0.037846	-0.051474	-0.028927	-0.021563
Pie	0.005485	0.003104	-0.001083	-0.001194	-0.000371
wh	0.921111	0.847831	0.781359	0.720602	0.664452
wf	0.914454	0.836965	0.769190	0.707029	0.648854
hh	0.917304	0.841290	0.772927	0.712211	0.655834
hf	0.923544	0.851574	0.784911	0.725692	0.671472
yh	0.921548	0.848666	0.782241	0.721766	0.665912
yf	0.914065	0.836463	0.768588	0.706549	0.648467
ТОТ	0.920799	0.847037	0.779225	0.719587	0.664486

Table 9: Mean of autocorrelation coefficients at different horizons for 20,000 simulations of a monetary union with identical countries characterised by asymmetric nominal wage adjustment costs.



Figure 9: Different specifications of adjustment cost curves. The dotted line represents a symmetric adjustment cost function as used for prices, the continuous line depicts an asymmetric cost function as used for nominal asymmetric wage adjustments and the dashed-dotted line is an asymmetric adjustment cost function as used for asymmetric real wages with an underlying indexation of 2%.



Figure 10: Sensitivity analysis for the parameter ψ governing the degree of asymmetry in the wage adjustment cost function.



Figure 11: Sensitivity analysis for volatility. The graphs depict an increase in the standard deviation for aggregate shocks from 0 to 0.04 with a parallel increase in asymmetric shocks.