Macroeconomic Uncertainty, Difference in Beliefs, and Bond Risk Premia

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ABSTRACT

In this paper we study empirically the implications of macroeconomic disagreement for the time variation in bond market risk premia. If there is a source of heterogeneity in the belief structure of the economy then differences in beliefs can affect equilibrium asset prices, and the dynamics of disagreement may generate a source of predictable variation in excess bond returns. Using survey data on macroeconomic forecasts of fundamentals spanning interest rates, real aggregates and inflation variables at different horizons we propose a new empirically observable proxy to aggregate macroeconomic disagreement and find a number of novel results. First, consistent with a general equilibrium model, heterogeneity in beliefs affects the price of risk so that disagreement regarding the real economy, inflation, short, and long ends of the yield curve explain excess bond returns with $\overline{R}^2$ between 22%- 29%. Second, we show that macroeconomic disagreement is exogenous to time t price innovations. Third, disagreement not only contains information on expected bond returns, but also contains significant information on expected changes in future interest rates: the combination of disagreement and the forward slope predict 1-year changes in the spot rate with an $\overline{R}^2$ equal to 38%. Fourth, we show that the information contained in agents' belief structure of the economy is different from that contained in macroeconomic aggregates. Finally, lending economic support to the findings of Cochrane and Piazzesi (2005), we find that disagreement about inflation, real GDP, and short term interest rates are highly correlated the return forecasting factor, but that disagreement is only partially spanned by the yield curve, containing 'unspanned' and 'above' components important for bond pricing.

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Introduction

This paper investigates the empirical implications of macroeconomic disagreement for the time variation in bond market risk premia. When moving from single agent to heterogeneous agent models several important properties of asset prices change. If there is a source of heterogeneity in the belief structure on the endowment process then differences in beliefs can affect the stochastic discount factor, thus equilibrium asset prices. This is important since the dynamics of macroeconomic disagreement may become a source of predictable variation in excess bond returns. A growing body of evidence indicates that heterogeneity plays an important role in a variety of settings, including equity, foreign exchange, and derivative markets, however, little is known about its affect on bond markets. In this paper we test the link between macroeconomic disagreement and expected returns of bonds using the BlueChip data set of macroeconomic forecasts which allows us to distinguish real, nominal, and monetary components as well as to construct a direct term structure of differences in beliefs.

The literature studying the term structure of interest rates is truly vast. Traditional reduced-form and structural models have provided significant insights that have improved our understanding of the dynamics of interest rates and are used in a number of applications, including risk management, trading, and monetary policy. At the same time, however, the literature has highlighted several empirical regularities that are difficult to reconcile with traditional homogeneous economies with no frictions. First, long term bond yields appear too volatile to accord with standard representative agent models (Shiller (1979)). Second, model implied sharpe ratios appear difficult to reconcile with those observed in the data (Duffee (2011)). Third, while unconditional excess returns on bonds are close to zero, there appears to be a large degree of predictability in returns by several yield curve factors (Fama and Bliss (1987) , Cochrane and Piazzesi (2005)). In addition, term structures contain information on future term structures in a direction and magnitude that is difficult to explain within some classes of models (Campbell and Shiller (1991)). Fourth, expected excess bond returns can take both positive and negative signs which cannot be captured by the completely affine class since compensation for risk is given as a constant multiple of factor volatility. Relaxing this restriction Duffee (2002) proposes an 'essentially affine' extension that allows for a richer specification for the price of risk. However, while the essentially affine class can better match some salient features of the data, they are unable to match at the same time first and second moments of yields. Furthermore, in their canonical form, essientially affine models imply that primitative shocks underlying the economy are perfectly spanned by the yield curve so that macroeconomic aggregrates contain no incremental information useful for bond pricing. And finally, interest rate dynamics display unspanned stochastic volatility: bond portfolios appear unable to hedge interest rate derivatives, thus suggesting some form of market incompleteness. It appears that the set of state variables driving volatility is not the same set driving yields. The solutions posed in the literature can be roughly sorted into three strands: i) statistical models which include either extensions to the price of risk (essentially affine, extended affine, quadratic models) or extensions to the state space (time-varying covariances, Wishart or multi-frequency dynamics) ; ii) reduced form economic models which either use new econometric methods for measuring state variables (dynamic factor analysis, least absolute shrinkage and selection operator (lasso) approaches), or introduce observable information (monetary policy shocks extracted from high frequency data, spanned and/or unspanned risk factors); or iii) structural macro-finance

models that include a richer preference structure (habit formation, ambiguity aversion, or recursive preferences). In this paper we begin a new line of research by focusing on heterogeneity in expectations and in doing so investigate the existence of new information unspanned by either yields or macro aggregates. This approach is important since we study the idea that homogenous macro term structure models are incapable of fully explaining the regularities in the term structure, and provide the first set of empirical results relating investor heterogeneity to bond market risk premia.

One of the first results showing the potential role played by heterogeneous beliefs is discussed in Harris and Raviv (1993) , who developed a model of speculative trading based on difference of opinion in which investors receive common information but differ in the way in which they interpret information[1]. All investors in their economy agree on the nature of the information, be it positive or negative, but disagree on its importance. They show that the heterogeneity in beliefs has important implications for asset prices. Similar settings have been employed by Detemple and Murthy (1994) and Zapatero (1998) in the context of a continuous time economy. Buraschi and Jiltsov (2006) consider a general equilibrium economy with Bayesian learning and show how heterogeneous beliefs affect the equilibrium stochastic discount factor and become a source of variation for option prices and trading volumes. As disagreeing agents engage in risk sharing, prices of options are affected in equilibrium and even small changes in the differences in beliefs can generate an option-implied volatility smile and help to explain the dynamics of option prices. Disagreement can be shown to arise for agents (even when they posses common priors) due to different degrees of confidence on the data generating process. In times of high economic uncertainty agents who observe noisy realisations of the dividend process disagree on the precision of the empirically generated probability measure and form different opinions regarding the model of expected future cash flows. Therefore, higher economic uncertainty is directly linked to difference in beliefs. Scheinkman and Xiong (2003) couple the assumption of overconfidence with short-selling constraints and show that these two assumptions can lead to equilibrium asset price bubbles. More recently, Xiong and Yan (2010) provide a theoretical treatment of bond risk premia in a heterogeneous agent economy. The authors develop a model of speculative trading in which two types of investors hold different beliefs regarding the central bank's inflation target. In the model, the inflation target is unobservable so investors form inferences based on a common signal. Although the signal is actually uninformative with respect to the inflation target heterogeneous prior knowledge causes investors to react differently to the signal flow. Investor trading drives endogenous wealth fluctuations that amplify bond yield volatilities and generates a time varying risk premium. They provide a calibration exercise and show that a simulation of their economy can reproduce the Campbell and Shiller (1991) regression coefficients and the tent shaped linear combination of forward rates from Cochrane and Piazzesi (2005). Our results build on those by Xiong and Yan (2010) as no empirical evidence on this topic is yet available in the literature.

The structure of the paper is as follows. In the section below we present a motivating example that frames our discussion and focuses on the particular channel of interest studied here. Next, with intuition at hand, we move on to test empirically the effects of macroeconomic disagreement. Using survey data from BlueChip on a set of macroeconomic forecasts on variables spanning inter-

[1]Recent equilibrium treatments of heterogeneity in beliefs include Bhamra and Uppal (2011); David (2008); Gallmeyer and Hollifield (2008) and Buraschi, Trojani, and Vedolin (2010)

est rates, real aggregates and inflation variables, we construct an empirically observable proxy for differences in beliefs in order to investigate whether disagreement affects the stochastic discount factor in a way that generates significant implications for the dynamics of term premia. Our proxy for uncertainty is therefore based on the cross-sectional dispersion in beliefs of professional economists. Moving to the empirical sections we test the content of macro economic disagreement for time variation in the bond market risk premium, expected future interest rates, time-variation in the slope of the forward curve, and the spanning properties of the cross-section of bond yields.

First, casting our empirical efforts in the context of classical predictability regressions we derive a number of novel results. Fama and Bliss (1987) show that on a 1-year horizon the forward-spot spread tracks expected excess returns but has no forecasting power for changes in the short rate. Campbell and Shiller (1991) reach a similar conclusion by testing the predictive content contained in the slope of the yield curve for long and short dated yield changes. Both authors find evidence of a time-varying risk premium, for example, Fama and Bliss (1987) obtain R^2 ranging between 5% and 14% when forecasting 1-year holding period returns on $n-$year bonds from the n year forward-spot spread. The inclusion of disagreement in multivariate forecasting regressions is striking: we find that disagreement on inflation, real GDP, short and long ends of the yield curve are individually and jointly statistically significant (using robust Hansen-Hodrick standard errors), with economically large point estimates, forecasting bond returns at an annual horizon with $\overline{R}^2$ ranging between 21% and 28%. The loading on real disagreement is positive, which is consistent with a risk-sharing model in which agents trade insurance written on consumption growth that induces a positive time-varying risk premium on long term bonds. Disagreement regarding inflation lowers expected excess returns: assuming a positive real-nominal covariance, agents who are pessimistic regarding consumption growth also expected inflation to decline. For these agents nominal bonds insure against high marginal utility states and therefore, in equilibrium, demand higher prices and lower expected returns. The signs on disagreement about short term and long term interest rates are positive and negative, respectively, which again is consistent with a risk sharing argument. We address the robustness of these findings using quarterly lags of disagreement of disagreement, estimating Hodrick (1B) standard errors, and running 'reverse regressions' of 1-period returns on a lagged summation of right hand variables, and find the results persist.

Second, investigating the relationship between cross-sectional heterogeneity and the slope of the forward curve we control for disagreement in classic Fama-Bliss term premia and complimentarity regressions. Projecting monthly T-Bills returns on the the forward-spot spread we find R^2's ranging between 3% and 8%. The addition of macro disagreement more than doubles this forecasting power while the loadings and statistical significance of the forward-spot spread is largely unaffected, indicating that disagreement contains information on short run predictability that is orthogonal to the slope forward curve. We emphasise that both long and short run predictability should not be confused with evidence of market inefficiency: we present our findings in the context of a special class of rational expectations models that include multiple agents with subjective expectations; therefore, the evidence presented here suggests traditional tests of single agents models are mis-specified. Moving to the Fama-Bliss complementarity regressions we project n-period changes in the spot rate on disagreement after controlling for the n-period forward-spot spread and find an interesting result. Disagreement not only contains information on expected bond

returns, but also contains significant information on expected changes in future interest rates. For example, at the 2 to 4 month horizon real and monetary components enter significantly at 5% after controlling for the forward-spot spread and add between 3% and 7% in terms of R^2 with respect to a univariate regression including the forward-spot spread alone. At an annual horizon the power of macro disagreement is more pronounced increasing the R^2 from 12% to 38% and, again, real and monetary components enter significantly at the 5% level or more. The marginal increase in forecast power is greatest for horizons closest to the 12-month mark after which the power of disagreement tails off. Consistent with our motivating example, the sign on the slope coefficient on real disagreement is negative for all horizons. Shocks to real disagreement increase state prices and drive down contemporaneous yields, and given the persistence of disagreement, high disagreement today forecasts high disagreement tomorrow, so that shocks to real disagreement today predict lower interest rates tomorrow.

Third, we present a set of results related to the macro finance literature of the term structure of interest rates and the link between macro-economic activity and macro-economic disagreement. A large body of empirical results show that bond returns are predictable by pure financial indicators such as forwards spreads, yield spreads, or latent yield curve factors, but for a long time the macro-finance literature had limited success in providing a convincing understanding of which macro variables risk premia vary with. There are some obvious reasons for this tension. Firstly, macroeconomic aggregates are measured with relatively large measurement error; secondly, some key macro states may be latent, and thirdly, model mis-specification may point us too far from the truth. Recently, Ludvigson and Ng (2009b) address the difficulties associated with measurement error by utilising factor analysis for large data sets. They estimate factors which have strong predictive content for excess bond returns, explaining 26% of the one-year-ahead variation in returns and, importantly, containing information unspanned by the cross-section of yields. Such findings are ruled out by traditional affine term structure models where yield inversion reveals all state variables relevant for bond pricing. In order to test the distinction between macro-disagreement and macro-activity we build an aggregate disagreement proxy by taking linear combinations of real, nominal, and monetary components from the right hand variables in the above discussed multivariate regressions. We also include a macro-activity factor constructed in the spirit of Ludvigson and Ng (2009b). As in Ludvigson and Ng (2009b), macro-activity indeed forecasts bond excess returns with $\overline{R}^2$ between 20% and 26% with statistically significant t-statistics at all maturities. More interestingly, in multivariate regressions using both macro-disagreement and macro-activity we find that both factors contain significant marginal predictive power for excess bond returns and are individually statistically significant at all maturities. Both cross-sectional disagreement and the risk associated with economic fundamentals matter for bond pricing, forecasting bond returns with $\overline{R}^2$ between 23% and 32%. Summarising, while disagreement and risk are both related to the business cycle, they contain different information regarding expected returns, which is confirmed in multivariate regressions of bond returns after controlling for liquidity risk.

The final set of results are related to the spanning properties of macroeconomic disagreement. Lending economic support to the findings of Cochrane and Piazzesi (2005) we find that disagreement regarding inflation, real GDP, and short term interest rates explains a large proportion of

the time-variation in the forward rate return forecasting factor with an $\overline{R}^2$ statistic of 43% in a contemporaneous linear projection. Our findings suggest that disagreement is related to the shape of the yield curve so that cross-sectional price information reveals the properties of the stochastic discount factor which are affected by disagreement. Information content embedded in the cross-section of yields is then time varying with respect to variables it forecasts and reveals information on expected returns. However, disagreement is only partially spanned by the yield curve in the sense that important components of disagreement orthogonal to the first 5 principle components of yields contain economically and statistically important information on expected returns. In a return predictability regression including the unspanned components of disagreement as right hand variables we find that disagreement about inflation and real GDP are unimportant but that disagreement about short and long ends of the yield curve are statistically significant at the 5% level, forecasting bond returns with an $\overline{R}^2$ of between 20% and 25%. Furthermore, exploring the relationship between the time-series dynamic of yields and disagreement we project the hidden risk premium factor from Duffee (2011) on unspanned components and find that disagreement about short term interest rates is statistically significant at the 1% level with an $\overline{R}^2$ statistic of 7%. Finally, using information orthogonal to a space spanned by both the cross-section and time-series dynamics of yields we discover an 'above' component linked to disagreement about long term interest rates which retains economically important forecasting power for expected returns.

Summarising, we show that a key determinant for bond risk premia is the joint cross-sectional disagreement surrounding the real economy, inflation and monetary policy, and that disagreement embeds important information about expected returns that is not captured solely by the forward curve, or embedded in macro aggregates. Far from suggesting the existence of arbitrage opportunities our results show that the time variation in expected returns is connected to macro-disagreement. The times when returns are highest are exactly the times when it is hardest to profit from them: when disagreement is high, in order to profit from higher expected returns agents must be confident, must bet against the opinion of others, and must do so exactly at the time when the wrong decision is most painful.

I A Motivating Example

To frame the empirical discussion which follows we begin from a simple economic model that demonstrates the potential role played by macroeconomic disagreement in bond markets. Consider a finite horizon economy evolving in discrete time populated by two types of investors, an optimist and a pessimist, who have binomial type beliefs regarding the future state of the endowment process (figure 1). The pessimist believes that the probability of low consumption states is larger than the probability of high consumption states, and vice-versa for the optimist. Each agent receives a stochastic endowment, $\epsilon_{t+1} = \epsilon_t \ \eta_{t+1}$, where $\{\eta_{t+1}\}_{t=0}^{T-1}$ is a set of i.i.d random variables which determines the drift of the endowment process:

$$\eta_{t+1} = \begin{cases} u & \text{with prob. } \pi_u^i \\ d & \text{with prob. } \pi_d^i. \end{cases} \tag{1}$$

Endowment shocks then follow a Bernoulli process, X_t, with parameter π_u: $\eta_{t+1} = uX_{t+1} + d(1 - X_{t+1})$. Denoting n the number of draws, and k the success count of this process, the beliefs

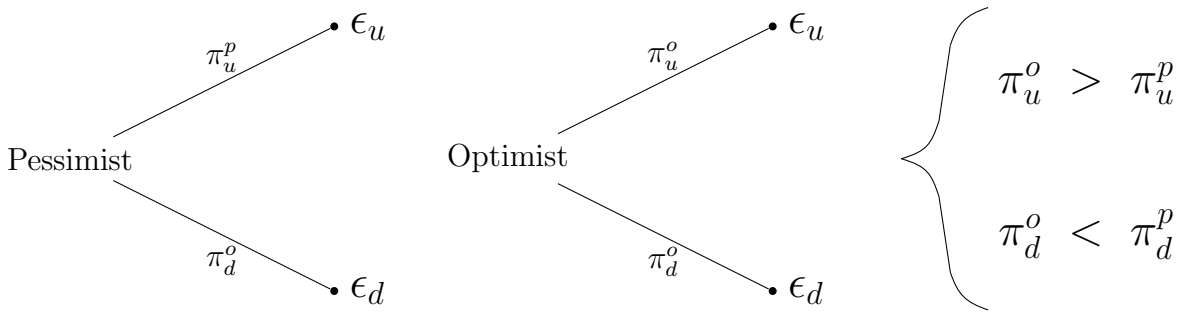


Figure 1 – Belief Structure.

The first period probabilities of a multi-period binomial heterogeneous economy.

regarding future states (n, k) is given by $Q^i(n, k)$ (figure 2, appendix). In each period, investors receive an endowment and choose optimal consumption plans by solving a utility maximisation problem and trading in state contingent claims:

$$\max_{q(n,k)} u(c_0^i) + E_t^{Q^i}\left(\sum_{t=1}^{T} U(c_t)\right), \tag{2}$$

$$\text{s.t} \quad c_o^i + \xi(n,k)' \cdot q(n,k) = \epsilon_0^i, \tag{3}$$

$$c^i(n,k) = \epsilon^i(n,k) + q^i(n,k). \tag{4}$$

where $\xi(n, k)$ and $q(n, k)$ are equilibrium price and demand, respectively, for Arrow-Debreu securities that pays \$1 in state (n, k).

[Insert figure 2 about here.]

Markets are complete which means the optimisation can be carried out using martingale or dynamic programming techniques. Solving state-by-state we obtain:

$$\xi(n,k) = \beta Q^i(n,k)\frac{u'(c(n,k))}{u_0'^i)} = \beta Q^i(n,k)\left(\frac{c^i(n,k)}{c_0^i}\right)^{-\gamma}, \tag{5}$$

where the last equality assumes power utility: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. In this economy individual consumption plans (and hence state prices) depend on the initial wealth and subjective probabilities used by agents to maximise their lifetime utility.

Definition 1. *An equilibrium is a state price density and consumption process (c_t^i) such that each agent solves his optimisation problem and markets clear, i.e., $c^p(n,k) + c^o(n,k) = \epsilon(n,k) \quad \forall\ n, k$.*

Pessimists in this economy believe the likelihood of recessions occurring to be larger than the optimists. Equilibrium is therefore supported by risk sharing whereby agents trade a set of state contingent contracts. Optimists insure pessimists against recessions by transferring marginal utility in down states but do so only at a premium by receiving addtional marginal utility in up

states. Thus, in recessions optimists are hit twice, not only are they poor, but they must also compensate pessimists for ex-ante signed contracts. Although agents have subjective probabilities regarding the growth rate of the endowment process, in equilibrium, ex-ante expected marginal utilities $E_t U'$ must equate:

$$Q^o(n,k)\left(\frac{c^o(n,k)}{c_0^o}\right)^{-\gamma} = Q^p(n,k)\left(\frac{c^p(n,k)}{c_0^p}\right)^{-\gamma}, \tag{6}$$

rearranging we have the consumption of the optimist in terms of consumption of the pessimist and the disagreement between them:

$$c^o(n,k) = \omega DiB(n,k)^{1/\gamma} c^p(n,k), \tag{7}$$

where $\omega = \frac{c_o^o}{c_o^p}$ and $DiB(n,k) = \frac{Q^o(n,k)}{Q^p(n,k)}$. We can then write the consumption of both optimists and pessimists in terms on the aggregate endowment and disagreement:

$$c^p(n,k) = \frac{\epsilon(n,k)}{1+\omega DiB(n,k)^{1/\gamma}}. \tag{8}$$

Since $DiB(n,k) < 1$, for low states we see that $c^o(n,k) < c^p(n,k)$ so that pessimists consumes relatively more in down states, and recessions are more expensive for optimists with respect to a homogeneous setting. Moreover, higher levels of disagreement increase the demand for insurance which is only supplied at increasingly large premiums. Equilibrium consumption now not only depends on realisations of the endowment process but also on the dynamics of disagreement. Finally, from equation (5) the price of an Arrow-Debreu security for state (n,k) is given by:

$$\xi(n,k) = \beta Q^p(n,k)\left(\frac{c_0^p + c_0^o DiB(n,k)^{1/\gamma}}{\epsilon(n,k)}\right)^{\gamma}. \tag{9}$$

A period k default free discount bond is then a security that pays a single dollar in each state of the world, i.e.,

$$p^{(n)} = \frac{1}{(1+y^{(n)})^n} = \sum_{n=1}^{2^k} \xi(n,k), \tag{10}$$

which directly depends on $DiB(n,k)$. Although the motivating example is just a simple generalisation of a standard general equilibrium Lucas economy, it reveals some interesting implications for the role that disagreement might take in bond markets. In an economy with heterogeneous agents ex-ante socially optimal portfolio allocations drive a wedge between the ex-post consumptions. The equilibrium stochastic discount factor is now affected by two components: The first related to aggregate endowment shocks (macroeconomic activity) and the second related to differences in beliefs (macroeconomic disagreement). Two effects emerge.

The first effect is on the shape of the yield curve. Since the term structure of consumption allocation between the two agents depends on the term structure of disagreement, then the term structure of interest rates must reveal information on the term structure of disagreement. To appreciate the potential importance of this link, in Figure 3 we show the result obtained by calibrating the previous binomial economy using different belief structures. Firstly, Figure 3(a),

computes bond prices for short term disagreement. Agents disagree mostly on the endowment in period 1 and disagree on subsequent periods at decreasing levels so that by period 5 agents are homogeneous in their beliefs. A belief structure of this type could be interpreted in a setting for which the Radon-Nikodym derivative reverts to a steady state level in the long run. When we compute the implied yield curve, we find that this term structure of disagreement implies an increase in disagreement at the short end which drives up state prices, eventually increasing the slope of the yield curve. Next, we analyze the opposite case. We consider the case in which agents agree on period 1 fundamentals but disagree at increasing levels up to the last period: the term structure of disagreement is increasing. Figure 3(b) summarizes the result. This second belief structure can flatten the long end for small shocks to disagreement and may even generate a humped term structure for larger levels of disagreement. Thus, the term structure of interest rates may depend crucially on the term structure of disagreement.

[Insert figure 3 about here.]

Treasury bond markets provide a convenient testing ground for disagreement since the absence of a cash flow channel means that bond prices are simply the conditional expectation of the stochastic discount factor:

$$p_t^{(n)} = E_t \left[M_{t,t+1} M_{t+1,t+2} \dots M_{t+n-1,t+n} \cdot 1 \right], \tag{11}$$

and the stochastic discount factor is a function of disagreement, i.e. $M_{t+j,t+j+1}(DiB(t+j, t+j+n))$. The second effect is on bond risk premia. In the setting suggested earlier, disagreement regarding fundamentals is a priced risk factor because of its role on the stochastic discount factor. Thus, disagreement may also become a source of correlation between the stochastic discount factor and asset price innovations, which is important since the dynamics of DiB_t can become a source of predictable variation in the price of risk and help explain the dynamics of expected excess returns on bonds. If we define $rx_{t,t+1}^{(n)}$ as the n-period bond excess return, standard arguments imply that:

$$E_t \left[rx_{t,t+1}^{(n)} \right] = -R_f \ \mathrm{cov}(M_{t,t+1}(DiB(t, t+1)), rx_{t,t+1}^{(n)}). \tag{12}$$

A shock to disagreement regarding the real economy today increases, in absence of other market frictions and distributional effects, the marginal utility of the representative agent in bad states tomorrow, $M_{t,t+1}$. At the same time disagreement impacts positively on bond prices today through the channel laid out in equations 9 and 10, reducing $rx_{t,t+1}^{(n)}$, and inducing a negative covariance between excess returns on long term bonds and marginal utility, i.e. $\mathrm{cov}(M_{t,t+1}(DiB(t, t+1)), rx_{t,t+1}^{(n)}) < 0$. Therefore, investors demand a positive compensation for holding these securities. A simple simulation of the previous economy confirms the result.

The economic mechanism we have just discussed relies on a risk sharing argument in a setting with no other frictions. A related stream of the literature argues that differences in beliefs can affect asset prices even in absence of risk sharing if agents are subject to short selling constraints. This second approach has been used to explain the divergence of asset prices from fundamentals within a rational framework (rational bubbles). These arguments find their main inspiration from the Miller (1977) conjecture: when opinions diverge in a market with binding short-sale constraints

(or other reasonable frictions) only opinions of optimistic investors will be revealed since investors holding pessimistic valuations cannot sell securities. Difference of opinion therefore result in inflated prices and lower expected returns. Theoretical models of this type include, for example, Scheinkman and Xiong (2003) who develope a structural model with overconfident risk neutral agents who engage in speculative trading. Notice, however, that while in both cases differences in beliefs affect expected excess returns, the testable predictions on expected excess returns are opposite for (neo-classical) risk-sharing and (behavioural) friction models.

The U.S Treasury bond market provides an excellent testing ground to identify the empirical link between expected returns and heterogeneity and to disentangle these two competing theories of asset prices. In the following empirical sections, we study the following set of testable hypotheses $\mathcal{TH}_1$ to $\mathcal{TH}_3$:

$\mathcal{TH}_1$: The slope coefficient in a predictive regression of expected excess bond returns on disagreement about real economic innovations is *positive*:

> In heterogeneous economies, agents who are more optimistic about the endowment process provide insurance to pessimistic agents, therefore, shocks to disagreement affect the equilibrium distribution of consumption. In good states optimists are relatively better off but in bad states optimists are hit twice as hard: not only is consumption growth low but they must give up marginal utility to pessimists therefore real disagreement is a priced risk factor that should generate increased expected returns. Models assuming short-selling constraints and appealing to Miller (1977) conjecture predict instead a negative slope coefficient.

$\mathcal{TH}_2$: The slope coefficient in a predictive regression of expected excess bond returns on disagreement about inflation is *negative*.

> The impact of nominal shock and the associated disagreement depends on the correlation between real and nominal bonds. If the covariance between real consumption growth and nominal shocks is positive, then investors who are pessimistic regarding the real economy expect inflation to be lower. For these investors, nominal bonds are good hedges against bad states of economy and they will demand nominal bonds to insure against high marginal utility states. Thus, the risk premium associated to nominal disagreement is negative: shocks to disagreement about inflation should increase demand and forecast lower expected excess returns ex-ante for nominal bonds.

$\mathcal{TH}_3$: A component of macroeconomic disagreement can affect bond markets as an *unspanned risk factor*.

> From equation 9, an increase in date t disagreement regarding the endowment process in period n reduces yields on $p(t, n)$ bonds, but, as conjectured above, at the same time raises expected returns on these bonds. This offsetting effect on yields and risk premia implies that disagreement may be priced yet not necessarily revealed through in the cross-section of yields. Yield curve inversion would then not be enough to reveal the priced dynamics of heterogeneity and could be 'hidden' in the sense of Duffee (2011). More generally, heterogeneity may drive a wedge between the dynamics of the conditional first moment first and second

moments of the discount factor and create the possibility that bond prices do not completely span all risk factors. Below we test the marginal predictive content of disagreement after conditioning on the date t cross-section of yields, and therefore test the hypothesis that disagreement is a spanned risk factor.

To summarise, in classical return predictability regressions, risk-sharing based rational expectations models predict a positive sign for the slope coefficient on real macroeconomic disagreement and a negative sign for the slope coefficient on inflation disagreement, while behavioural friction based arguments predict both slope coefficients to be negative. We leave to the empirical study the task of providing us with insights on these important questions.

II Data

A Bond Data

In order to compute default free bond yields, prices and forward rates we use the $1-6$ month T-Bills and $1-5$ year maturity Fama-Bliss zero coupon bonds from CRSP for the sample period December 1985 to December 2009. We choose to use unsmoothed prices for two reasons: a) multicolinearity is known problem in smoothed data; and b) later we will study the return forecasting factor of Cochrane and Piazzesi (2005) for which CRSP is the relevant data source. The dataset consists of end-of-month observations from post-war U.S Treasuries and is updated regularly to provide the most recent data. We introduce notation along the lines of Cochrane and Piazzesi (2005) by defining the date t log price of a n-year discount bond as:

$$p_t^{(n)} = \text{log price of n-year zero coupon bond.} \tag{13}$$

The yield of a bond, the known annual interest rate that justifies the bonds price is given by:

$$y_t^{(n)} = -\frac{1}{n} p_t^{(n)}. \tag{14}$$

The date t 1-year forward rate for the year from $t+n-1$ and $t+n$ is:

$$f_t^{(n)} = p_t^{(n)} - p_t^{(n+1)}. \tag{15}$$

The log holding period return is the realised return on an n-year maturity bond bought at date t and sold as an $(n-1)$-year maturity bond at date $t+12$:

$$r_{t,t+12}^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)}. \tag{16}$$

Excess holding period returns are denoted by:

$$rx_{t,t+12}^{(n)} = r_{t,t+12}^{(n)} - y_t^{(1)}. \tag{17}$$

Finally, the same letters without the superscript indicate a vector of the respective variable, i.e., the vector of yields on 1 - 5 year maturity bonds is given by $y_t = \left[y_t^{(1)} y_t^{(2)} y_t^{(3)} y_t^{(4)} y_t^{(5)}\right]$.

B Disagreement Data

In order to test the effects of heterogeneity we look directly at market participants' expectations of future fundamentals. Survey data provides a rich source to learn how agents form beliefs about economic variables, unfortunately, econometric analysis of survey data is littered with challenges. For example, few sources exist with large sample periods or appropriate frequencies, survey data is known to contain biases due to frequency and timing of responses, and systemic biases in forecasts have been shown (see Capistrán and Timmermann (2009)) to exist rationally under the assumption of asymmetric loss. Furthermore, if the objectives of forecasters and end users differ principal-agent issues may arise. BlueChip Economic Indicators does, however, provide expectations data which is less likely to suffer from principle agent issues and, importantly, allows a simple aggregation procedure (discussed below) that mitigates problems associated with the frequency of responses. BlueChip carry out monthly surveys of professional economists from leading financial institutions and service companies regarding a large set of economic fundamentals covering real, nominal, and monetary variables. Exact timings of the surveys are not published but in general the survey is conducted over the first two days of the beginning of each month and mailed to subscribers on the third day [2]. Our empirical analysis is therefore not affected by biases induced by overlapping observations of returns and responses. Forecasts are available for the period Jul 1984 - December 2009 and cover:

1. Real: Real GDP, Disposable Personal Income, Non-residential Fixed Investment, Unemployment, Industrial Production, Corporate Profits, Housing Starts, Auto/Truck Sales.
2. Nominal: Consumer Price Inflation, Nominal GDP.
3. Monetary: 3 Month Treasury Rate, 10 year Treasury, AAA corporate Bond.

Furthermore, for each variable two types of forecast are made:

1. Short-Term: an average for the remaining period of the current calendar year;
2. Long-Term: an average for the following year.

For example, in July 2003 each contributor to the survey made a forecasts for the percentage change in total industrial production for the remaining two quarters of 2003 (6 months ahead), and an average percentage change for 2004 (18 months ahead). The December 2003 issue contains forecasts for the remaining period of 2003 (1 month ahead) and an average for 2004 (13 months ahead). The moving forecast horizon induced a seasonal pattern in the survey which we adjust for using an X-12 ARIMA filter [3]. On average 51 respondents are surveyed for short term forecasts

[2] An exception to the general rule was the survey for the January 1996 issue when non-essential offices of the U.S. government were shut down due to a budgetary impasse and at the same time a massive snow storm covered Washington, DC: www.nytimes.com/1996/01/04/us/battle-over- budget-effects-paralysis-brought-shutdown-begins-seep-private-sector.html. As a result, the survey was delayed a week.

[3] X-12-ARIMA is an iterative procedure that carries out a multiplicative decomposition of time series into trend and seasonal components using block moving average filters. Alternative, model based seasonal adjustments were carried out using SEATS/TRAMO. Since our time series for disagreement is relatively large, and the method and definitions of collection have remained constant, there is no noticeable difference between the procedures.

and 49 for long term forecasts with standard deviations of 1.6 and 3.3 respectively. Figures 4 and 5 plot the distributions and time series properties of respondent numbers which show that only on rare occasions are survey numbers less than 40 and no business cycle patterns are visible.

[Insert figure 4 and 5 about here.]

Macroeconomic disagreement, shown in figures 6 and 7, is then proxied by cross-sectional mean-absolute-deviation (MAD) in forecasts. Summary statistics for macroeconomic disagreement are given in tables II.

[Insert figure 6 and 7 about here.]
[Insert table II about here.]

Eye-balling figure 6 we observe a general decline in the level of disagreement since the 1980's and economically interesting periods characterised by large spikes. Figure 7 displays similar characteristics, although the decline in uncertainty is more pronounced. Swanson (2006) makes a similar observation using a different measure of cross-sectional uncertainty on the same data set. The purpose of Swanson (2006), quite unrelated to the focus of this paper, was to study the effect of central bank transparency with respect to private sector interest rate forecasts. Using various measures of forecast accuracy the author shows that since the 80's private sector agents have a) improved projections of the federal funds rate; and b) are more unanimous (cross-sectionally) in forming expectations. In unreported results we find that not only are agents more unanimous regarding interest rate forecasts but are more unanimous regarding real, nominal, and monetary elements of the economy.

Table III reports the cumulative variance explained by successive principles components from an eigenvalue decomposition of the unconditional covariance matrix. We find that disagreement spikes in bad states of the world and that a term structure of disagreement exists in the sense that short and long term disagreement are not the same process. Supporting this claim we find 51% of the variance of short term disagreement is explained by the first principle component as opposed to 79% of the variance for long term disagreement. Figure 8 plots the first principle component of long and short term disagreement. Casual comparison shows that while in general there exists a large degree of co-movement, at times long and short disagreement behave somewhat independently. Furthermore, the fact that in both cases a single component explains such a large proportion of total variance is further support for a common learning mechanism by which agents discover the true state of fundamentals. In other words, it appears that shocks to disagreement are pervasive and distributed throughout real, nominal, and monetary dimensions of the economy suggesting the formation of beliefs contain a systematic component.

[Insert table III about here.]
[Insert figure 8 about here.]

In order to construct a constant 1-year maturity disagreement measure, we take a weighted average of the short and long term forecasts from the BlueChip survey. Figure 9 gives a visual explanation to the construction of these measures. Let j be the month of the year, so that $j = 1$ for January and $j = 1, 2..12$. A constant maturity disagreement is formed taking as weight $(1-\frac{j}{12})$,

for the short term disagreement (the remaining forecast for the same year), and $\frac{j}{12}$, for the long-term disagreement (the forecast for the following year). As an example, in April each year the approximate 1-year difference in belief is constructed from 9/12th's of the short term forecast and 3/12th's of the long term forecast. The exception to this rule is the construction of disagreement regarding real GDP. Since real GDP is currently only reported quarterly we imposed a restriction on the implied updating process of agents by revising 1-year disagreement in a quarterly fashion. For example, in January, February and March of each year disagreement is taken as the short term forecast, while in April, May and June, disagreement is taken as 3/4's of the short term forecast and 1/4 of the long term forecast. The same reasoning does not apply to the GDP deflator which lends itself to estimation in 'real time'. Specifically, the GDP Price Index is computed from a fixed-weight of date t prices from a changing basket of date $t+1$ goods. Further support for quarterly construction of disagreement regarding Real GDP is given by Patton and Timmermann (2008) who show empirically that agents face an asymmetric measurement problem when forecasting real GDP growth versus inflation.

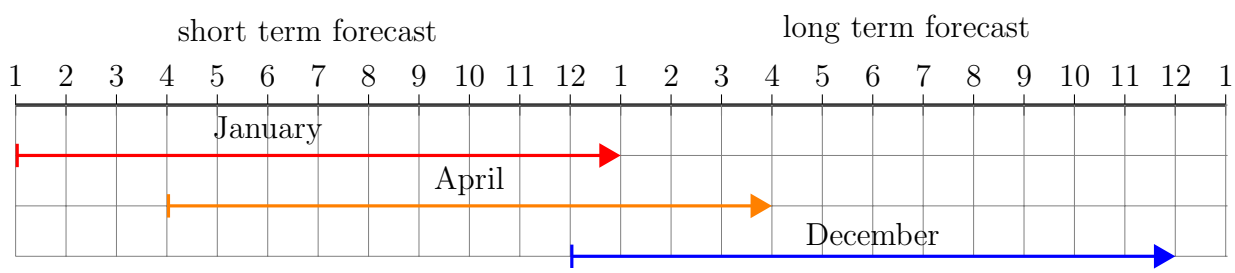


Figure 9 – Constant Maturity Disagreement

Diagram illustrating the construction of the constant maturity disagreement measures built from a moving weighted average of long term and short term disagreement measures.

C Macro Data

Dynamic macroeconomic theory suggests a small set of common factors are responsible for co-movement of a large set of economic and financial time series. However, until recently the search for these factors has been carried out with limited success. Empirical studies usually find that bond returns are predictable by financial variables such as term spreads, default spreads, or dividend yields, but that the forecasting power of fundamentals such as inflation or consumption is poor. Table I documents the difficulties of predicting bond returns from the levels and second moments of supposedly key economic variables, that is, inflation and industrial production growth[4]. The $\overline{R}^2$ in this macro predictability exercise show that just 2% to 7% of the variation in 1-year excess returns on 2 to 5-year bonds is forecastable ahead of time. Worse, while industrial production growth does contribute significantly to this predictability both the level of inflation and

[4] Monthly observations of macro variables are generally quite volatile , largely due to measurement error, which hide the true state of the underlying process. Following Granger and Newbold (1986) and Joslin, Priebsch, and Singleton (2009) we construct filtered series based on an exponentially weighted average of past observations from the conditional 1-step head forecasts from a fitted ARMA(1,1) model. We then form volatility estimates by fitting a GARCH(1,1) to the residuals of the filtered series.

Table I – Macro Predictability Example

Return predictability regression of annual excess returns on bonds on the levels and volatility of inflation and consumption growth:

$$rx_{t,t+12}^{(n)} = \alpha + \beta_1 INF_t + \beta_2 \sigma(INF_t) + \beta_3 IPG_t + \beta_4 \sigma(IPG_t) + \varepsilon_{t+12}^{(n)}.$$

INF is a simple exponential monthly difference model for CPI All Urban Consumers: Non-Durables (SA). IPG is an exponential monthly difference model for Industrial Production and Capacity Utilization, All Major Industry Groups (SA). The volatility of INF and IP are estimated from a GARCH(1,1) model fitted to the respective residuals. Sample Period: 1986.1 - 2010.1. A constant is included in all regressions but omitted from the table. t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. ***'s indicate the usual significance levels.

Maturity	INF_t	$\sigma(INF_t)$	IPG_t	$\sigma(IPG_t)$	$\overline{R}^2$
$rx_{t,t+1}^{(2)}$	0.01	-0.01^{***}	-0.02	-0.00	0.07
	(0.80)	(-0.76)	(-3.42)	(-0.01)	
$rx_{t,t+1}^{(5)}$	0.02	-0.02	-0.03	0.00	0.02
	(0.50)	(-0.28)	$(-1.84)^{*}$	(0.14)	

the volatility of inflation and industrial production growth are insignificant, at odds with classical understanding of risk compensation for bonds[5].

The limited success of linking the macro economy to term premia led researchers to explore alternative empirical routes to pin down the state variables priced in bond markets. For example, Ang and Piazzesi (2003) estimate a VAR with identifying restrictions derived from the absence of arbitrage and find the combination of macro and yield curve factors improves performance over a model including yield factors only. Diebold, Rudebusch, et al. (2006) incorporate macro variables, specifically a real, nominal and monetary policy instrument, into a statistical model of the term structure of the Nelson-Siegel type. Interpreting the traditional level, slope and curvature factors the authors find a bidirectional dynamic link between the macro economy and yields but that the effect of fundamentals on yields is stronger than yields on fundamentals. Further evidence of the potential role of macroeconomic factors is discussed in Rudebusch and Wu (2008), Joslin, Priebsch, and Singleton (2009), and Duffee (2011). However, while these recent ideas have matured and deepened our understand of bond markets, the ability of macro factors to explain the dynamics of the term structure remains limited.

In order to control for the state of the economy we adopt the procedure of Ludvigson and Ng (2009b) who estimate macro-activity factors using static factor analysis on a large panel of macroeconomic data. The panel used in our estimation is an updated version of the one in Ludvigson and Ng (2009a), except that we exclude price based information in order to interpret

[5]It should be noted that the success of linking bond returns to the macroeconomy is largely dependant on the sample period considered; for example, whether the monetary policy experiment (79/83) or the recent financial crisis (08/10) are included or excluded.

this variable as a pure macro factor and allow clearer distinction between information contained in agents' beliefs from that contained in macroeconomic aggregates [6]. After removing price based information and from the panel we end up with a 102 macro series. In the following sections we test the explanatory power of disagreement after controlling for macroeconomic activity, which we denote M_t.

III Learning about Bond Risk Premia

A The Role of Disagreement

We focus our empirical efforts on symmetric disagreement measures regarding real and nominal variables, and augment this set with natural candidates that potentially reveal information on disagreement regarding monetary policy. Specifically, we focus on four disagreement measures which are supposed to capture inflation, real, and monetary macroeconomic uncertainty: *i) the implicit GDP deflator, ii) Real GDP, iii) the 3 month Treasury bill rate, and iv) the 10 year Treasury note rate* [7]. The expectations hypothesis says that nothing should forecast excess returns, or alternatively, in expected return regressions the factor loadings on right hand variables should not be different from zero. If the risk premium is time-varying, however, we can expect deviations and the existence of predictability. To investigate this link, we first run an unrestricted regressions of 1 year excess returns from 2 - 5 year maturity bonds on the above four disagreement measures:

$$rx_{t,t+12}^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} DiB_t^{INF} + \beta_2^{(n)} DiB_t^{RGDP} + \beta_3^{(n)} DiB_t^{LR} + \beta_4^{(n)} DiB_t^{SR} + \varepsilon_{t+12}^{(n)}. \tag{18}$$

Table IV documents that all disagreement measures are statistically significant in the above expected return regression with slope coefficients which are increasing in magnitude with bond maturity, indicating a larger change in the term premium for longer maturity bonds given a shock to any one factor. In terms of predictable variation, disagreement forecasts excess returns with $\overline{R}^2$'s ranging from 21% to 28%. The results of the predictive regressions are striking: for 2-year bonds the t-statistics for the slope coefficient on the short interest rate (real GDP, long interest rate, and inflation) is equal to 3.95 (2.50, -5.22, and -2.09) respectively. The signs of the slope coefficient on disagreement regarding real GDP and the short rate is positive, while the signs of the slope coefficient on disagreement about inflation and the long rate are negative. Consistent with the rational expectations example above, shocks to the belief structure on the endowment process, DiB_t^{RGDP}, increase the demand for insurance against bad states by pessimists, which is supplied by optimists in exchange for a premium which maps increases in bond risk premia. Shocks to disagreement about inflation, DiB_t^{INF}, forecast lower expected returns, as expected. Repeating the intuition, if the covariance between real and nominal shocks is positive, investors who are pessimistic regarding consumption growth tomorrow also expected inflation to be lower. For these investors nominal bonds insure against high marginal utility states and, in equilibrium, are willing

[6] Examples of price variables removed include: S&P dividend yield, the Federal Funds (FF) rate; 10 year T-bond; 10 year - FF term spread; Baa - FF default spread; and the dollar-Yen exchange rate. A small number of discontinued macro series were replaced with appropriate alternatives or dropped.

[7] While one may worry about the inclusion of persistent interest rates as right hand variables, *disagreement* about interest rates is not that persistent (see table II). In constructing disagreement about long term rates we use forecasts of AAA rated corporate bonds before 1996 since 10 year Treasury rate forecasts are unavailable.

to accept higher prices and lower expected returns [8]. Referring to testable hypothesis $\mathcal{TH}_1$ and $\mathcal{TH}_2$, we find that the slope coefficient on disagreement regarding real and nominal components is consistent risk-sharing rational expectations based arguments as opposed to a behavioural friction based stories.

Disagreement regarding short term rates, DiB_t^{SR}, enters with a positive loading. This can be understood in terms of the relative demands for long versus short term bonds by optimists versus pessimists. Holding expectations of the long rates constant, those agents who have negative expectations of the short end (pessimists), expect the term structure to steepen and so demand short term bonds and supply long term bonds. Conversely, if bonds are in zero net supply, those agents who have positive expectations of the short end (optimists) supply short term bonds and demand long term bonds. In bad states of the world pessimists profit while optimists make a loss; thus, in equilibrium, pessimists pay a premium for short term bonds, while optimists buy long term bonds at a discount. Shocks to disagreement about the short end therefore increase risk premia on long-short spread trades. Finally, the slope coefficient on disagreement about long rates, DiB_t^{LR}, is negative. Larger disagreement about the long end makes those agents holding negative expectations about future long term rates demand long-term bonds, and since for these agents bonds are positively correlated with marginal utility, they are a good hedge, and therefore demand a negative risk-premium.

Although the above arguments presents one possible explanation for the empirically observed signs on disagreement, it should be noted that the equilibrium implications of heterogeneity are highly complex, depending on many factors, including agents preferences, the representative's risk aversion, prudence, elasticity of intertemporal substitution, and the the relative on the proportion of optimists versus pessimists. A discussion of this last channel is given in Xiong and Yan (2010) where the term premium depends on agents' wealth weighted average belief (which determines the Pareto optimal set) regarding an unobservable inflation target. Specifically, in their economy the yield spread between long and short dated Treasuries is positive (negative) when agents' wealth weighted average belief is dominated by the pessimistic (optimistic) agent so that the loading on inflation uncertainty takes both positive and negative signs. The sign of the general equilibrium effect is, ultimately, an interesting empirical question.

[Insert table IV about here.]

A.1 Multi-horizon returns

The above results suggest a substantial proportion of excess returns to bonds is predictable ahead of time using cross-sectional informational available in agents expectations. However, the information set used to condition expected returns was constructed with a priori knowledge (approximate 1-year maturity disagreement measures were built) of the forecasting horizon and one may worry

[8] In our sample period, the unconditional correlation coefficient between smoothed estimates of inflation and industrial production growth, as in table I, is 0.20. However, real-nominal covariance can switch sign, as noted by Campbell, Sunderam, and Viceira (2009), who estimate a negative correlation immediately following the recession of 2001 and again in the run up to the recent 2007-09 financial crisis.

that the observed statistical power of macro disagreement is an artefact of its construction. In order to rule out this possibility we run forecasting regressions across multiple horizons from 1 to 24 months and evaluate the size and shape of the observed statistical power. Figure 10 plots the $\overline{R}^2$'s for 1, 3, 6, 12 and 24 month return horizons for a long-short portfolio of bonds. In computing returns for horizon h we follow the literature and approximate the price of an $(n-h)$-year bond as $\exp(-(n-h/12)\ y^{(n)})$. The long side of the portfolio invests \$1 equally across $2, 3, 4$ and 5 year bonds funded by a short position in the appropriate risk free rate, which for the 1, 3, and 6 month horizons are taken from the CRSP Fama TBill structures; for annual return horizon it is the 1-year Fama-Bliss discount bond; and for the 2-year horizon it is proxied as a date t position the the 1-year discount bond and a date t forward contract for borrowing between dates $t+1$ and $t+2$. The obvious exception to the bond portfolio is the 24-month return which invests a long position in the $3, 4$ and 5 year bonds only. The R-square plot displays an intuitive term structure of predictability given the constant maturity construction of disagreement. Beginning at the 1-month horizon the $\overline{R}^2$ is upward sloping, increasing from 6% to 12% at the 3-month horizon, further increasing from 15% to 24% at the 6-month horizon, peaking at 24% for the annual horizon, and tailing off to 17% at the 24-month horizon. To put these findings in perspective, in unreported results we find the addition of the variance risk premium[9], a factor studied elsewhere in context of short run predictability regressions adds no predictive power, in terms of R^2, after controlling for disagreement disagreement. These results are interesting since they lend evidence to the intuition from the stylised model which suggests that if heterogeneity matters for asset pricing it does so at a finite future horizon, corresponding to the horizon implied by the risk sharing agreement due to disagreement. Leaving aside predictable variation (R^2's) the statistical significance of disagreement at multiple horizons is an issue which we address in depth in the following section.

[Insert figure 10 about here.]

A.2 Robustness: Hodrick Errors, Reverse Regressions, and Exogeneity

A standard approach in the predictability literature relies on compounding returns and conducting significance tests of explanatory variables using overlapping observations. It is well known, however, that the use of overlapping returns in not innocuous from a statistical point of view. Compounding returns induces an MA(12) error structure under the null of no predictability which must be corrected for during estimation. In the above we conducted tests of return predictability using a robust GMM generalisation of Hansen and Hodrick (1983) with an 18-lag Newey-West correction. Inference hinges critically on the the choice of standard errors, serial correlation in returns, persistence in explanatory variables, and in-sample correlation between unobserved innovations of left and right hand variables. While most researchers agree that risk premia are time-varying the size of the observed predictability is a topical question. The arguments against a 'large' predictable component in asset returns are summarised by Ang and Bekaert (2007) in the space of stock returns or Wei and Wright (2010) in the space of bond returns. Ang and Bekaert

[9]The variance risk premium is defined as the difference between the risk-neutral and objective expectations of realized variance. Risk-neutral expectation of variance is measured as the end-of-month VIX-squared and the objective expectation of variance is computed from a linear forecast of past realised variance extracted from high frequency data. See Zhou (2010).

find that the evidence of long horizon predictability using Hansen-Hodrick or Newey-West errors disappears once robust correction of heteroskedasticity and autocorrelation is conducted, while Wei and Wright argue that long-horizon predictive regressions using overlapping observations induce serious size distortions even after correction. Both sets of authors advocate use of an alternative inference procedure proposed by Hodrick (1992). In the following we repeat the above predictability tests following Hodrick's specification for return predictability regressions.

Denoting the parameter vector of equation 18 by $\theta = (\alpha, \beta^{(n)}$, the asymptotic distribution of the OLS estimator can be derived using Hansen's generalised method of moments under the assumption of heteroskedasticity and serially correlated errors. It can be shown that $\sqrt{T}(\hat{\theta}-\theta) \sim N(0, \Omega)$ where $\Omega = Z_0^{-1} S_0 Z_0^{-1}$, $Z_0 = E(x_t x_t')$, with $x_t' = (1, DiB_t)$. Hodrick recommend reversing the overlapping return regression under the assumption of covariance-stationarity to compute alternative estimates for both S_0 and the numerators in θ.

First, with regards to the estimation of S_0, rather than summing $\epsilon_{t,t+k}$ serially correlated and heteroskedastic disturbances in the future, Hodrick standard errors sum $x_t x_{t-j}'$ observations of explanatory variables in the past. The unconditional spectral density estimate is

$$S_0 = \sum_{j=-k+1}^{k+1} E[u_{t+k} x_t u_{t+k-j} x_{t-j}'], \tag{19}$$

where $u_{t+k} = (\epsilon_{t+1} + \cdots + \epsilon_{t+k})$ and ϵ_{t+1} is the one step ahead serially uncorrelated forecast error. Then, we can re-write the expectation as

$$E(u_{t+k} x_t u_{t+k-j} x_{t-j}') = E\left[\sum_{h=1}^{k} \epsilon_{t+h} x_t \sum_{i=1-j}^{k-j} \epsilon_{t+i} x_{t-j}'\right] \tag{20}$$

$$= E\left[\sum_{h=1}^{k-j} \epsilon_{t+h}^2 x_t x_{t-j}'\right] = E\left[\epsilon_{t+1}^2 \sum_{i=0}^{k-j-1} x_{t-i} x_{t-j-i}'\right]. \tag{21}$$

Repeating the above over all j the alternative estimator of the spectral density is given by $S_0 = E\left[\left(\epsilon_{t+1} \sum_{i=0}^{k-1} x_{t-i}\right)\left(\epsilon_{t+1} \sum_{i=0}^{k-1} x_{t-i}'\right)\right]$. The Hodrick standard errors (1B) can then be used to compute test-statistics from the point estimates of the overlapping regressions.

Second, Hodrick also proposes an alternative estimator for point estimate of θ. The numerator of the estimator $\hat{\beta}^{(n)}$ is a covariance. Hodrick suggests to re-write the numerator in the overlapping regression coefficients by lagging the right hand side variables as opposed to implementing the traditional projection of the future overlapping returns on time t observations. Covariance stationarity should lead to the same result:

$$cov(r_{t,t+k}, x_t) = cov(r_t + \ldots r_{t+k}, x_t) \tag{22}$$

$$= \sum_{j=0}^{k-1} cov(r_{t+j}, x_t) = cov(r_t, \sum_{j=0}^{k-1} x_{t-j}). \tag{23}$$

The last term is the numerator of the slope coefficient of a regression of 1-period returns on a lagged summation of right hand variables. Long ($\hat{\beta}^{(n)}$) and short ($\hat{\gamma}$) horizon regression coefficients are therefore linked by the relation:

$$\hat{\beta}_k = V_o^{-1} cov(r_{t,t+k}, x_t) = V_0^{-1} V_n \hat{\gamma}. \tag{24}$$

where V_o is the parameter covariance matrix from the overlapping regression and V_n is the parameter covariance matrix from the non-overlapping regression. Therefore, a necessary and sufficient condition to reject the null of no-predictability using overlapping annual horizon returns is that the loading on a 12-period lagged sum of past disagreement measures be different from zero in a monthly forecasting regression. Following Wei and Wright (2010) we call this the '*reverse regression*'.

In addition to testing the robustness of the above findings we also investigate the extent to which macroeconomic disagreement is exogenous to time t price innovations. One may worry that heterogeneity in beliefs might be correlated with contemporaneous return volatility which cause agents to disagree. In this case, disagreement would map risk premia associated with some other unobserved fundamental factor. In this case, date $t \rightarrow t+h$ returns and date t disagreement would be correlated by construction and no causal interpretation could be attached. We proceed to study the exogeneity issue by lagging disagreement and projecting annual excess returns on a quarterly lag (3-periods) of real and inflation disagreement and date t disagreement regarding monetary variables. Next, we repeat this test with a quarterly lag of all disagreement measures. Allowing for different lags in DiB allow us to address the common co-movement (thus potential collinearity) of the right hand side variables and break the date t linear dependency. Secondly, it allows us to test whether macro and monetary disagreements are independently exogenous.

Considering Panel B in which all right hand variables are quarterly lagged we obtain $\overline{R}^2$'s ranging between 15% and 18% with Hansen-Hodrick t-statistics which are significant at the 5% level or higher for DiB^{RGDP}, DiB^{LR}, and DiB^{SR}, and significant at least at the 10% level for DiB^{INF}. Consistent with findings of previous authors we find that Hansen-Hodrick standard errors are considerably smaller than Hodrick (1B) standard errors. Considering Panel A Hansen-Hodrick standard errors strongly reject the null of no predictability for all disagreement measures, while this result disappears for inflation disagreement once Hodrick (1B) standard errors are employed. For example, the t-statistic on DiB^{INF} for a 2-year bond drops from -2.32 to -1.31, while for DiB^{RGDP} on the same bond the t-statistic drops from -3.57 to -2.26. Scanning the t-statistics for all bonds returns one finds a similar pattern. Moving to Panel B we consider a quarterly lag of all right hand variables. With respect to Panel A, we observe the same pattern in test statistics only now we find that monetary disagreement is no-longer significant. The return predictability results in Panel B that utilise lagged explanatory variables and Hodrick(1B) standard errors are a tough test of the role of disagreement. Still, we find that real disagreement retains its statistical significance, disagreement regarding the long rate remains significant for 2 and 3-year bond returns, and the $\overline{R}^2$'s remain between 15% and 18%. When moving from Panel A to B, lagged disagreement on the short rate is no longer significant under Hodrick (1B) standard errors but remains significant under Hansen-Hodrick standard errors. This could be due to two reasons: a) monetary disagreement is, in fact, endogenous, or b) that the common co-movement in different

DiB measures generates a collinearity issue in the data matrix $(X'X)^{-1}$. We pursue this idea by dropping disagreement about inflation and the short end, and test the significance of the disagreement regarding the real economy and the long end using the reverse regression approach outlined above.

Table VII address the question of the exogeneity and statistical significance of real and long term disagreement by running the reverse regression outlined above. We consider projections of 1-period returns on a $3, 6, 12$ and 24 month lagged summation of disagreement measures corresponding to implied forecasting horizons discussed in the previous section. Mindful of the so-called 'Richardson's Critique' who argues that interpretation of such results should take into account correlation in the test statistics, we estimate each lag simultaneously and test the hypothesis that loadings on DiB^{RGDP} and DiB^{LR} are jointly different from zero. Considering the loadings on real disagreement, the t-statistics are significant at the 5% level for $3, 6$ and 12 lags (t-stat: 2.38, 2.00, and 2.50) implying that real disagreement contains substantial information on expected bond returns for horizons up to 1-year. Similarly, disagreement regarding the long end of the yield curve is significant at the 5% level for return horizons up to 1-year (t-stat - -2.19,-2.26, and -2.05). At the 24-month horizon both real and long term disagreement lose some significance, although a joint test across lags strongly rejects the null of no predictability with asymptotic $\chi^2(4)$ values of 25.09 and 20.76, respectively.

A.3 The Dynamics of Disagreement

We now utilise a VAR specification for real, nominal, and monetary disagreement in order to study the dynamics of disagreement in the context of the above forecasting regressions. Consider a monthly first order VAR containing disagreement about inflation, the real economy, long term and short term interest rates: $DiB_t = \Phi DiB_t + \varepsilon_t$. Parameter estimates and test statistics for this above system are reported in table V while figure 11 plots the orthogonalised impulse response function. In moving from figure 11 (a) to (d) we find that the disagreement process is highly persistent: *a 1-standard deviation shock to each equation results in a permanently increase in the level of disagreement over the following 12 months.* Panel (a) allows contemporaneous correlation between shocks to disagreement regarding inflation, and shocks to disagreement about real and monetary components. The initial shock to inflation disagreement is contemporaneously uncorrelated with real disagreement while there is large subsequent dynamic correlation. Moving to panel (b) we find the impact of a shock to real disagreement is again permanent over 12-months and impacts on monetary disagreement measures with equal magnitude. Panels (c) and (d) plot the impulses of shocks to disagreement about long term rates and short term rates, respectively. The message from these plots is that the disagreement process is highly persistent, dynamically correlated, giving support to the long horizon predictive power reported above. An additional significant feature is that the level of disagreement is almost identical in the long run whichever shock is considered, suggesting long run co-movement (cointegration). Both observations are key since together they hint at the existence of a systematic factor that drives all disagreement measures. This is important, since it could have been the case that DiB_t^{INF} and DiB_t^{RGDP} behave idiosyncratically, in which case macroeconomic disagreement could be diversified and would not affect the equilibrium discount factor. In fact, we find the opposite, disagreement appears to be systematic factor, which explains the rejection of the null of no predictability.

[Insert table V about here.]
[Insert figure 11 about here.]

B Disagreement and the Forward Curve

B.1 Term Premia

Fama (1984) and Fama and Bliss (1987) study the information content in forward rates regarding future interest rates and expected returns by deriving testable restrictions on expected bond returns and expected interest rate changes. The authors uncover small but significant predictability in both short (1-month) and long (1-year) horizon term premia from projecting excess returns on n-period bonds on n-period forward-spot spreads. They also run complementary regressions that show forward rates contain surprisingly little forecasting power for interest rates at horizons up to 1 year (excess returns are predictable so that interest rates are not) but that forecasting power builds up for longer maturity changes so that forward-spot spreads forecast changes in the short rate 2 - 4 years ahead. We refer the reader to the cited papers for a detailed explanation of the testable restrictions but summarise the intuition here. Fama and Bliss conclude that bond returns are predictable because forward spreads embed information about time variation in term premia, thus compensation for risk, which masks the forecasting power of forward rates for expected yield changes. Given the above documented forecasting power of macroeconomic disagreement, and the given intuition that factor loadings on disagreement depend on the 'term structure of disagreement', it is prudent to ask whether cross-sectional uncertainty in agents expectations simply proxies for macro economy risk premia already embedded in the slope of the forward curve.

The original Fama and Bliss (1987) paper covered the sample period 1964.1 - 1985.12, and showed the slope of the forward curve contains important information on expected bond returns; for example, in their sample the forward-spot spread forecasts bond returns with R^2 between $5\% - 14\%$ with statistically significant loadings at all maturities. Updating the sample period to cover 1964.1 - 2010.1 the results and learning point regarding the slope of the forward curve are unchanged. However, limiting the sample to that overlapping our observations for disagreement reveals that the forward-spot spread has virtually no explanatory power for bond risk premia (we find R^2 between $0\% - 2\%$). First, we run regressions of 1-month returns on $2-6$-month T-Bills on all disagreement measures (real, nominal, and monetary) after controlling for the forward-spot spread to show that macro uncertainty contains 'high' frequency information on term premia and that this information does not simply proxy for, or is a better measured realisation of, the information already embedded in the slope of the forward curve:

$$rx_{t,t+1}^{(n)} = \alpha + \beta_0 \left(f_{t-1}^{(n)} - y_{t-1}^{(1)} \right) + \beta_1 DiB_t + \varepsilon_{t+1}^{(n)}. \tag{25}$$

[Insert table VIII about here.]

It is mechanically true that forward spreads must forecast expected returns or changes in spot interest rates - it is an accounting identity. For the monthly holding period returns documented in table VIII the forward curve contains economically and statistically significant information on expected returns. In univariate regressions the forward spot spread contributes R^2's between 3%

and 8%. The addition of macro disagreement factors more than doubles this forecasting power indicating that disagreement contains information on bond returns orthogonal to the slope of the forward curve. Furthermore, the sign on the factor loadings are consistent with the annual holding period return regressions above. Considering the statistical power of disagreement for short-run predictability, disagreement regarding the 10-year Treasury and the short rate enter significantly, while real and nominal disagreement do not. Since agents continually observe yield changes it is reasonable to assume updating of beliefs regarding these variables occurs continuously, whereas large revisions to inflation and real disagreement plausibly only occur at monthly or quarterly frequencies or less; thus disagreement regarding monetary variables may be more important for short run predictability [10]. The results presented here should not be confused with evidence of market inefficiency or irrationality. In fact, the learning point is quite the opposite, these results should be interpreted as a positive result in the context of a special class of rational expectations models that include multiple agents with subjective expectations; thus traditional tests of single agents models are mis-specified.

B.2 Interest Rate Forecasts

We now run the Fama-Bliss complementarity regressions of the n-period changes in the spot rate on n-period forward-spot spread, including a range of forecast horizons from 1-month to 4 years:

$$y_{t+n}^{(1,12)} - y_t^{(1,12)} = \alpha + \beta_1 DiB_t^{INF} + \beta_2 DiB_t^{RGDP} + \beta_3 DiB_t^{LR} + \beta_4 DiB_t^{SR} + \psi \left(f_t^{(n)} - y_t^{(1,12)} \right) + \varepsilon_{t+n}. \tag{26}$$

[Insert table IX about here.]

Table IX reports two sets of results: monthly changes in the 1-month rate, or yearly changes in the 1-year rate. For both sets as the forecast horizon is increased the power of the forward-spot spread builds up. For example, for changes in the 1-year rate as the forecast horizon is extended from 1 — 4 years ahead the R^2 increases from $12\% - 65\%$. Panel B reports regressions that include macroeconomic disagreement after controlling for the information contained in the forward-spot spread. Considering the monthly forecasts, real and monetary disagreement enter significantly at the 5% level for all horizons while disagreement regarding inflation appears unimportant. In moving from Panel A to Panel B, at a 4-month horizon the R^2 increases from 12% to 28%. Considering the forecasts at annual horizons, the statistical significance of disagreement peaks at between 12 and 24 months as one would expect given the 12-month constant maturity structure and the persistence of the disagreement process as implied by the above VAR. Furthermore, the marginal increase in forecast power is greatest for horizons closest to the 12-month mark. Finally, consistent with our motivating example, the slope coefficient on real disagreement is negative for all horizons. Shocks to heterogeneity regarding the real economy increase state prices which in turn drives down *contemporaneous* yields. Given the persistence of disagreement, high disagreement

[10] Mueller, Vedolin, and Zhou (2011) provide additional evidence of a short run predictable component to bond market risk premia. For example, the authors find that the combination of a market variance risk premium factor and the return forecasting factor from Cochrane and Piazzesi (2005) predict 1-month excess returns on 2-5 month Treasury bills with R^2's between 8% and 17%

today forecasts high disagreement tomorrow, and thus increases in real disagreement today predict lower interest rates tomorrow [11].

IV Macro Activity or Macro Disagreement?

A A Single Factor Representation for Disagreement

The results above suggest that expected excess returns are time-varying and that a component of this time variation is correlated with changes in macroeconomic disagreement. These results, however, could be subsumed by other more traditional risk factors that have been discussed in the recent fixed-income literature. It is our interest, therefore, to study in greater detail the marginal contribution of the difference in beliefs after controlling for other potentially important risk factors. To this end, we follow Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009b) and construct a single state variable that embeds information relevant for bond pricing by taking linear combinations of real, nominal and monetary measures of disagreement, where the weights are found from the fitted values in a regression of average excess return (across maturity) on factors:

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t,t+12}^{(n)} = \beta_0 + \beta_1^{(n)} DiB_t^{INF} + \beta_2^{(n)} DiB_t^{RGDP} + \beta_3^{(n)} DiB_t^{LR} + \beta_4^{(n)} DiB_t^{SR} + \bar{\varepsilon}_{t+1}, \tag{27}$$

$$\overline{rx}_{t,t+12} = \beta^\top DiB_t^i + \bar{\varepsilon}_{t+1}. \tag{28}$$

[Insert table X about here.]

The DiB_t state variable is then constructed according to $DiB_t = \hat{\beta}_0 + \hat{\beta}_1 DiB_t^{INF} + \hat{\beta}_2 DiB_t^{RGDP} + \hat{\beta}_3 DiB_t^{LR} + \hat{\beta}_4 DiB_t^{SR}$. Empirically, we find that macroeconomic disagreement peaks before recessions or during periods of financial distress, when the volatility of future expected cash flows surges and it becomes difficult to infer the correct model of the economy. The 2008 credit crisis provides a recent illustration of the effects of disagreement. Other notable examples include the Saving & Loans Crisis (1990), the Asian and Russian Crisis (1997-1999), and Black Monday (1987). We also find that events related to monetary policy, such as changes in the federal funds target rate affect the dynamics of disagreement.

[Insert figure 12 and 13 about here.]

B Dissecting Macro-Disagreement from Macro-Activity

Given the above explanatory power of macroeconomic disagreement for bond returns we test the extent to which the information contained in agents' belief structure of the economy is different from that contained in macroeconomic aggregates. We also control for liquidity shocks since,

[11] For an interesting study of the impact of disagreement on bond yields, see Pasquariello and Vega (2007), who show derive a theoretically link between trading activity and heterogeneous speculators, and identify empirical measures of dispersion that amplifies the impact of order flow on yield changes.

as documented by Krishnamurthy and Vissing-Jorgensen (2007), U.S Treasury bonds contain a significant liquidity premium behaving in some respects akin to money. We run the following regressions:

$$rx_{t,t+12}^{(n)} = \phi^{(n)} DiB_t + \gamma^{(n)} M_t + \rho^{(n)} L_t + \varepsilon_{t+12}^{(n)}, \tag{29}$$

where DiB_t is constructed from the linear combination of disagreement factors described above, M_t is a macro-activity factor (see 'Macro Data' section for details on its construction), and L_t is a liquidity factor.[12] A test of the conditional predictive content of disagreement amounts to asking whether ϕ is different from zero after controlling for macro-risk and liquidity.

[Insert table XIabout here.]

As shown by Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009b), the R^2's in a restricted regression of this type are consistent with an unrestricted regression. Standard errors are corrected for potential autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction for overlap. Compounding returns induces an MA(12) error structure under the null that 1-year holding period returns are non-forecastable. We follow previous authors and account for the possibility of a singular or near-singular covariance matrix with an 18-lag Newey-West correction.

Table (XI) summarises the results. For each bond maturity, row (a) reports the unconditional explanatory power of disagreement for bond returns. The loadings on disagreement are significant at all maturities increasing from 0.49 on a 2-year bond to 1.39 on a 5-year bond. A single aggregate disagreement factor containing information on real, nominal, and monetary dimensions of the economy explains 1-year holding period returns with R^2 between $22\% - 29\%$. To test the hypothesis that disagreement contains information on returns above that contained in macro fundamentals we run multivariate regressions with both disagreement and a macro-activity factor. Consistent with the findings in Ludvigson and Ng (2009b), we find (row b) that macro-activity forecasts excess returns with $\bar{R}^2$ between 20% and 27% with statistically significant t-statistics at all maturities with comparable loadings to those on disagreement. This is consistent with models in which the price of risk increases in bad state of the worlds and suggest that the Ludvigson and Ng (2009b) adjusted procedure provides an effective proxy to capture the time-variation that helps to explain the dynamics of bond risk premia. More importantly, however, in multivariate regressions with risk and disagreement added, at the same time, as explanatory variables we find that both factors contain marginal predictive content for bond returns with $\bar{R}^2$ between 27% (5-year bonds) and 36% (2-year bonds). We also find that, after controlling for these two factors, liquidity risk is statistically insignificant. Table XII addresses the question of economic significance. According to the model expected excess returns are highly variable; for example, 5-year bond returns averaged 2.3% above the risk free 1-year bond return but with a standard deviation of 2.5%. Furthermore, disagreement contributes an economically significant proportion to this variation. A 1-standard deviation shock to disagreement raises expected returns on 5-year bonds by 1.3% compared with

[12]The factor L_t is the liquidity factor studied in Fontaine and Garcia (2008) who estimate a no-arbitrage dynamic term structure model that captures the so-called 'on-the-run premium', that is, the differences between prices of recently issued bonds and prices of otherwise similar but older bonds.

1.2% for a 1-standard deviation shock to the macro-activity factor. This finding suggests that both disagreement and macroeconomic activity matter for bond expected excess returns.

[Insert table XII about here.]

Consistent with economic theory, we find that bond excess returns are countercyclical, i.e., they are high in recessions when investors are more risk averse. Both disagreement and macro factors are countercyclical with a smoothed correlation coefficient with industrial production growth of -40% and -55%, respectively. The macro activity index is high in recessions, capturing the risk of co-movement of state variables that impact negatively on consumption or wealth. Disagreement is also high in recessions and spikes in periods of distress. The intuition is straight forward. During recessions or financial crises expected future cash flows are volatile which increases the dispersion of posterior estimates regarding true state of the economy. During these times pessimistic agents demand insurance against protracted recessions, which is only supplied at an higher premium by optimistic agents, thus increasing the risk premium on bonds.

However, although disagreement and macro activity track the business cycle, they are not the same thing. Panel A in table XIII reports numerical estimates of the spectral density for macro economic disagreement which is displayed visually in figure 14. Both disagreement and risk contain low frequency components, but while risk contains components that correspond to 4 yearly cycles, the low frequency component of disagreement lasts around a year. Furthermore, disagreement contains visible peaks at quarterly and monthly frequencies. Smoothing out the high frequency components we see (figure 15) that although both macro-disagreement and macro-activity track the business cycle, their respective time series properties are very different.

[Insert table XIII about here.]

[Insert figure 14 about here.]

[Insert figure 15 about here.]

The remaining rows of table XI report robustness checks against the possible effects of liquidity shocks[13]. Although unreported, as in Fontaine and Garcia (2008), we find that in univariate regressions shocks to liquidity predict lower excess returns for bonds with R^2's between 8% and 10% and statistical significance at the 1% level. In multivariate regressions when we include all three factors (Table XI row (d)), the statistical significance of disagreement and macroeconomic activity survives, but the liquidity risk factor is no longer significant. Finally, figure 16 plots an R^2 decomposition of the marginal forecasting power of each factor. For example, we find an R^2 of 30% when projecting excess returns on 5-year bonds on the space spanned by all factors, which approximately breaks down to 5% for liquidity, 11% for macro-activity, and 14% for disagreement.

[Insert figure 16 about here.]

[13]We have also tried including individual macro series and price variables. The main result is unaffected.

V The Information 'In', 'Not In', and 'Above' the Term Structure

An affine term term structure model is one in which interest rates are modelled as an affine (linear) function of some state vector. The main advantage of this class over their non-linear counterparts is tractability, which led to their wide adoption by both academics and practioners.[14]. In an affine term structure model there are N state variables, denoted $X_t \equiv [X^Q_{t,1}, \ldots, X^Q_{t,N}]'$, which drive the instantaneous (short) interest rate as $r_t = \delta_0 + \delta X_t$, where δ_0 is a scalar and δ is an N-vector. The dynamics for the state variables follow an affine diffusion under the equivalent martingale measure Q:

$$dX_t = \kappa^Q(\Theta^Q - X_t)dt + \Sigma\sqrt{S_t}dB^Q_t, \tag{30}$$

where B^Q_t is an N-vector of independent standard brownian motions, and K^Q and Σ are $N \times N$ matrices. The matrix S_t is diagonal with ith element $S_{ii,t} = \alpha_i + \beta_i' X_t$, where β_i is an N-vector and α_i is a scalar. Denoting the time t price of a default free zero coupon bond maturing at $t+\tau$ as $P(X_t, \tau)$ we know from Duffie and Kan (1996) that bond prices are exponentially-affine:[15]

$$P(X_t, \tau) = \exp(a(\tau) + b(\tau)' X_t),$$

where $a(\tau)$ is a scalar function and $b(\tau)$ is an N-valued function. Continuously compounded yields are therefore affine in the state vector: $y(X_t, n) = -\frac{\log P(X_t,\tau)}{T} = A(\tau) + B(\tau)' X_t$, for coefficients $A(\tau) = -a(\tau)/\tau$ and $B(\tau) = -b(\tau)/\tau$. Two key ingredients for an affine term structure model are the dynamics of the short rate r under Q and the change from the equivalent martingale measure to the physical measure P. The dynamics for the pricing kernel, $\mathcal{M}$, are then written as $\frac{d\mathcal{M}}{\mathcal{M}} = -r_t dt - \Lambda_t' dB^P_t$, where B^P_t is an N-vector of independent standard Brownian motions under the physical measure and $\Lambda_t = \Lambda(Y_t, t)$ is the N-vector market prices of risk. Invoking Girsanov's theorem the dynamics of X_t under the physical measure can then be written as $dX_t = \kappa^Q(\Theta^Q - X_t)dt + \Sigma\sqrt{S_t}\Lambda_t dt + \Sigma\sqrt{S_t}dB^P_t$. From the fundamental pricing equation,

$$E_t\left(\frac{dP_t}{P_t}\right) - r_t dt = -E_t\left(\frac{dP_t}{P_t}\frac{d\mathcal{M}}{\mathcal{M}}\right), \tag{31}$$

noting that $P_t = P(X_t, \tau)$, and using Ito's lemma we obtain the instantaneous expected excess return to holding a T period bond:

$$rx^{(T)}_{t,t+dt} = -b(T)'\Sigma\sqrt{S_t}\Lambda_t. \tag{32}$$

The model is closed with a specification for the price of risk which, chronologically, resulted first in the 'completely affine' class, $\Lambda_t = \sqrt{S_t}\lambda_1$, in which expected excess returns are completely determined by factor variance. Dai and Singleton (2000) denote the admissible subfamily of completely affine models as $A_m(N)$ which are those with m state variables driving N conditional variances

[14]The seminal work of Vasicek (1977) and Cox, Ingersoll Jr, and Ross (1985) are early examples of equilibrium structural affine models. More recent affine specification are discussed by Wachter (2006). Non-linear structural models include, for example, Buraschi and Jiltsov (2007) and Porchia and Trojani (2009)

[15]A complete characterisation of multi-factor affine term structure models was provided by Duffie and Kan (1996), while Dai and Singleton (2000) discuss the structural differences and empirical strengths/weaknesses among the completely affine class and provide parameter restrictions required to ensure S_t is non-negative for all i and in all states .

S_t. Although convenient, the completely affine specification imposes significant restrictions for the link between conditional first and second moments of bond yields and expected bond returns. Specifically, elements of the state vector X_t that do not affect factor volatility (and hence bond volatility) cannot affect expected returns, thus factor variance and expected returns go hand-in-hand. To highlight the limitation of this restriction table XIV reports projections of annual holding period returns onto the conditional variance of the 3-month Treasury bill rate, which is the implied forecasting regression from a 1-factor completely affine model. As expected, although the short rate variance does contain some predictive content its empirical link is weak. Motivated by this observation, Duffee (2002) extends the completely affine class to a set of 'essentially' affine models in which the risk factors in the economy enter the market price of risk directly and not just through their factor volatilities.[16] The essentially affine price of risk is written as

$$\Lambda_t = \sqrt{S_t}\lambda^0 + \sqrt{S_t^-}\lambda^X X_t,$$

where λ^X is an $n \times n$ matrix of constants and S^- is a diagonal matrix such that $[S_t^-]_{ii} = (\alpha_i + \beta' X_t)^{-1}$ if $\inf(\alpha_i + \beta_i' X_t) > 0$ and zero otherwise. The additional flexibility of non-zero entries in S^- translates into additional state dependent flexibility for the price of risk such that the tight link between risk compensation and factor variance is broken. Duffee (2002) estimates essentially affine $A_0(3)$ and $A_1(3)$ models showing that his specification for the price of risk provides better in and out-of-sample forecasts than the corresponding specifications in Dai and Singleton (2000). For example, $A_0(3)$ models do a good job at forecasting yields and predicting excess returns but impose the unattractive restriction of constant volatility, while $A_1(3)$ models prove more accurate measures of volatility but gives up ability to fit excess returns.

[Insert table XIV about here.]

A shared characteristic of the $A_m(N)$ subfamily of affine term structure models is that the cross-section of bond yields follows a Markov structure so that all current information regarding future interest rates (and thus expected returns) is summarised in the shape of the term structure today. Linear combinations of date t bond yields thus suffice to characterise date t risk factors through so-called yield curve inversion.[17] Building on this notion Cochrane and Piazzesi (2005) show that the shape of the term structure of forward rates embeds substantial information that explains the dynamics of bond excess returns. The Cochrane-Piazzesi return forecasting factor, CP_t, is a tent-shaped linear combination of forward rates that predicts excess returns on bonds with R^2 statistics as high as 43% (in their sample period) and has been shown to capture $\sim 99\%$ of the time-variation in expected returns.[18] While the discovery of such a potent price-based

[16] Cheridito, Filipovic, and Kimmel (2007) extend even further this class to yield models that are affine under both objective and risk-neutral probability measures without permitting arbitrage opportunities.

[17] Specifically, assume N bond yields are measured without error. Then, stacking these yields into the vector $y^N = A^N + B^N X_t$, we can solve for the risk factors through inversion as $X_t = (B^N)^{-1}\left(y^N - A^N\right)$ so long as the matrix B^N is non-singular.

[18] For a detailed discussion of CP_t we refer the reader to Cochrane and Piazzesi (2005). Briefly, the single factor construction begins with projecting average excess return (across maturity) on a constant plus available forward rates: $\frac{1}{4}\sum_{n=2}^{5} rx_{t,t+12}^{(n)} = \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(1)} + \gamma_3 f_t^{(2)} + \gamma_4 f_t^{(3)} + \gamma_5 f_t^{(4)} + \bar{\varepsilon}_{t+12} = \gamma' f_t + \bar{\varepsilon}_{t+1}$. Next, the fitted regression coefficients are used as loadings in forming a linear combination of forward rates that serves as a state variable in restricted univariate and multivariate regressions: $rx_{t,t+12}^{(n)} = \beta(\gamma' f_t) + \phi X_t + \varepsilon_{t+1}^{(n)} = \beta CP_t + \phi X_t + \varepsilon_{t+12}^{(n)}$.

forecasting factor marked a watershed for the fixed income literature, questions alluding to the underlying embedded risks and economic interpretation are yet to be answered. More worryingly, recent evidence presented by Ludvigson and Ng (2009b) and Cooper and Priestley (2009) suggest that yield inversion is not enough to reveal all relevant dynamics for underlying state variables and thus crucial ingredients for affine term structure models lay outside the space of yields. Recent work along these lines is found in Duffee (2011) and Joslin, Priebsch, and Singleton (2009) who independently develop the theme of hidden factor models, or unspanned macro risk, in which time variation in macro variables orthogonal to the cross-section of yields (and thus absent from date t prices) contains substantial forecasting power for future excess returns on bonds.

What are the deeper learning points with respect to returns predictability that we present here? In a single agent Gaussian economy term structure inversion reveals the dynamics of risk factors and thus expected returns. In a multiple agent economy this isn't necessarily true even if the above holds: a) risk sharing / market clearing may generate non-affine prices; or b) $E\left[\frac{M(T)}{M(t)}g(X_t)\right]$ may not reveal all relevant dynamics of risk factors, X_t, and thus disagreement may be unspanned by the cross-section of prices yet reveal information about expected returns. A natural question to ask is which component of disagreement relevant for expected returns is revealed by the cross-section of prices (Cochrane-Piazzesi) versus the time-series of prices (Duffee; Joslin, Priebsch, Singleton). Proceeding in two steps, we first define the information set $G_1 \subseteq \sigma\left(PC(1-5)\right)$ and compute the unspanned component of DiB which is not explained by the cross-section of bond prices (the first five principal component, as used in Cochrane and Piazzesi (2005)): $\mathcal{UN}_{DiB_t} = DiB_t - P_j\left[DiB_t \middle| G_1\right]$.[19] Then, we proceed to test the content of unspanned, i.e. '*Not-In*', disagreement as follows:

$$rx_{t,t+12}^{(n)} = const + \beta_1^{(n)}\mathcal{UN}_{DiB_t}^{INF} + \beta_2^{(n)}\mathcal{UN}_{DiB_t}^{RGDP} + \beta_3^{(n)}\mathcal{UN}_{DiB_t}^{LR} + \beta_4^{(n)}\mathcal{UN}_{DiB_t}^{SR} + \varepsilon_{t+12}^{(n)}. \tag{33}$$

Second, we define $G_2 \subseteq \left[G_1 \cup \sigma(y^{(n)})\right] \setminus G_1$ where $G_2 \sim \sigma(H_t)$ is the 'Hidden' factor filtered from the time-series of prices from a 5-factor Gaussian term structure model studied in Duffee (2011).[20] Then, we estimate the component of disagreement unspanned neither by the cross-section of prices nor by information related to the hidden factor H_t. We define $\mathcal{AB}_{DiB_t} = \mathcal{UN}_{DiB_t} - P_j\left[\mathcal{UN}_{DiB_t} \middle| H_t\right]$ and test the predictive content of macroeconomic disagreement which is '*Above*' the yield curve as

$$rx_{t,t+12}^{(n)} = const + \beta_1^{(n)}\mathcal{AB}_{DiB_t}^{INF} + \beta_2^{(n)}\mathcal{AB}_{DiB_t}^{RGDP} + \beta_3^{(n)}\mathcal{AB}_{DiB_t}^{LR} + \beta_4^{(n)}\mathcal{AB}_{DiB_t}^{SR} + \varepsilon_{t+12}^{(n)}. \tag{34}$$

Panel A of table XV reports a contemporaneous projection of CP on the above studied disagreement measures. The results show that a substantial proportion of the time-variation in CP can be explained by macroeconomic disagreement with an $\overline{R}^2$ of 43% and t-statistics for the slope coefficient on the disagreement regarding inflation (real GDP, short term interest rates) equal to -1.98 (2.88, 2.78), respectively. The first learning point, then, is that time variation in the shape

[19] More specifically, G_1 is the sigma algebra (information set) generated by the eigenvalue decomposition of the unconditional covariance matrix of yields, or, alternatively, since there exists a linear mapping between yields and forward rates, G_1 is the space spanned by the return forecasting factor CP.

[20] We thank G. Duffee for providing the data on the hidden factor H_t.

of the forward curve can in part represents heterogeneity in the belief structure of the economy, thus lending economic support to the empirical results of Cochrane and Piazzesi (2005) and the theoretical results of Xiong and Yan (2010). The second learning point, reported in Panel B, shows that after controlling for information already embedded in the cross-section of prices there exists additional information in the cross-section of expectations which is informative for bond risk premia. Specifically, both disagreement regarding short and long ends of the yield curve remains significant at the 5% level or higher, with $\overline{R}^2$ between 20% and 25%, implying that a large proportion of macroeconomic disagreement relevant for bond pricing is unspanned by the yield curve. Next, Panel A of table XVI examines the time-series characteristics of unspanned disagreement: in a projection of the hidden risk premium component from Duffee (2011) on unspanned DiB measures we find that disagreement regarding short end of the yield curve is strongly statistically significant (t-stat: 3.02) with an $\overline{R}^2$ statistic of 7% [21]. The final learning point in this section is presented in Panel B of table XVI which shows the marginal impact of the 'above' component to disagreement. This particular unspanned component is specific to disagreement regarding the long end of the yield curve and is orthogonal to i) the cross-section of yields; and ii) a risk premium component embedded in the time series of yields. Still, it contains substantial information for future expected bond returns, with t-statistics significant at the 1% level, and $\overline{R}^2$ between 20% and 25%. Importantly, this component is also economically important for bond risk premia, as seen in table XVII, which shows a 1-standard deviation shock to $AB^{LR}_{DiB_t}$ lowers expected excess returns on 5-year bonds by 2.38%, which is larger that its sample mean excess return of 2.22%.

[Insert table XV about here.]

[Insert table XVI about here.]

[Insert table XVII about here.]

A Return Predictability and Financial Crises

An important question relates to the stability of the link between previously studied risk factors and the time variation of bond risk premia. Updating all datasets to include observations up to January 2010 we find that the predictive power of the return forecasting factor is more than halved in terms of $\overline{R}^2$ with respect to the numbers reported in Cochrane and Piazzesi (2005), and is furthermore subsumed in multivariate regressions including disagreement. Such findings would be discouraging for price based predictability if it weren't for the nature in which CP_t fails. Figure 17 plots a time-series of the $\overline{R}^2$'s obtained from forecasting regressions computed with a 15-year window using either CP_t or DiB_t on the right hand side. While the degree of predictable variation due to disagreement remains fairly stable over time CP_t is cut dramatically during periods of financial crisis. This statement is made clear by eye-balling the correlation between the $\overline{R}^2$ from CP_t and the level of the Federal Funds rate: note the dramatic decline in the aftermath

[21] This compares with an R^2 of 10% in a projection of H_t on the real activity factor (PC1) from Ludvigson and Ng (2009b).

of the dot-com bubble and, again, during the recent 2007-2009 financial crisis. The spanning properties of cross-sectional price information during these periods is substantially weaker, but while the extent of the change is certainly dramatic this evidence may not be contradicting the economic interpretation of the CP_t factor given by its authors. A possible explanation is that bond markets were simply not pricing the surprising policy interventions by the Fed in response to these two large and rare economic shocks. If these shocks were unforeseen, or absent from the conditional first moments of the discount factor, then this would explain why CP_t makes incorrect forecasts during these periods. Was the dot-com bubble foreseen by fixed income markets before March 10th 2000? Evidence from CP_t suggests otherwise. In the period in-between the bubbles technical burst and 24th September 2001, while the Nasdaq lost 77% of its value, the $\overline{R}^2$ of the return forecasting factor dropped from 50% to 20%. During the more recent 2007-2009 financial crisis, we observe a similar drop in R^2. Bond prices do not appear to span this event and the response of the Fed. On March 18th, 2009 the U.S. Federal Reserve Bank announced a plan to buy almost $1.2 trillion worth of Treasury bonds to help boost lending and promote economic recovery in response to 2007 financial slowdown an imminent credit crisis. It said it would start buying medium and long-term government debt and expand purchases of mortgage-related debt. The size of the intervention was targeted to make the U.S. Fed the de-facto marginal investor in the debt market, thus absorbing part of the large price of risk and injecting money in the system. This measure followed a series of previous decisions, such as starting the Troubled Asset Relief Program (TARP) programme (announced on October 3rd, 2008), which eventually took the form of the government purchasing equity capital and assets from over 600 US commercial banks, and the Term Asset-Backed Lending Facility (TALF) Repo loans programme. Over this same period the forecasting power CP_t is again cut dramatically, from 40% pre-2007 to around 10% in recent data. During 2007 and 2009 such interventions have certainly played an important role in affecting the shape of the forward curve, distorting the equilibrium price of risk and removing (by construction) the price-based information for expected excess returns contained in the forward curve.

[Insert figure 17 about here.]

Interestingly, Panel B of Figure 17 shows that during the same period the R^2 associated to DiB is stable and in excess of 30% during the financial crisis.

VI Concluding Remarks

For a long time the empirical success of linking macroeconomic fluctuations to bond market dynamics was limited. Researchers typically resorted to reduced form models which reveal latent state variables through yield curve inversion. Increasingly sophisticated models led researchers to better fit the yield curve and match time-variation in risk premia yet a deep understanding of link between bond markets and the macroeconomy remained elusive.

This paper contributes to the debate from a new perspective by studying the implications of *macroeconomic disagreement* for time-variation in bond market risk premia. We focus our attention on macroeconomic disagreement through the lens of a special class of rational expectations models in which agents make subjective assessments of the the future path of the economy but 'agree to disagree' on its outcome. Equilibrium is supported by risk sharing whereby agents

trade a set of state contingent contracts: optimistic agents insure pessimistic ones against recessions by transferring marginal utility in down states but do so in exchange for an ex-ante premium.

Using a unique dataset of professional market participants' expectations of fundamentals we build constant maturity disagreement measures covering real, nominal and monetary components of economic activity. We learn that, jointly in classical return predictability regressions, all disagreement measures are statistically and economically significant with $\overline{R}^2$ between 22% and 28%. These findings are robust to Hodrick (H1B) standard errors, reverse regression methodologies, lagged disagreement, and the link is stable over time. In an effort to disentangle the roles played by macro-disagreement versus macro-activity we estimate a pure macro-activity factor from a large panel of fundamentals. Novel to the macro-finance literature, we find the the information contained in macro aggregates is different to that contained in the cross-sectional dispersion of agents expectations, jointly forecasting excess bond returns with $\overline{R}^2$ of 39%. Furthermore, while revisiting the classic Fama-Bliss return predictability regressions we find forecast power for both spot rates and expected returns.

Exploring the spanning properties of heterogeneity, we find that disagreement regarding inflation and the real economy are spanned by the cross-section of yields, and will therefore be revealed through yield curve inversion, while disagreement about short term rates is only partially spanned in the sense that information from both the cross-section and the time-series of yields is needed to capture its relevant pricing dynamics. These findings lend some economic support to Cochrane and Piazzesi (2005) who show that the shape of the yield curve contains significant information on future bond returns in addition to that contained in the level or slope. For example, disagreement is highly correlated with the return forecasting factor with an $\overline{R}^2$ statistic of 43% in a contemporaneous linear projection. However, we find the existence of an additional component, which is linked to disagreement about long term interest rates, that appears 'above' the yield curve: it is neither spanned by the cross-section or by the time-series of yields.

A Appendix

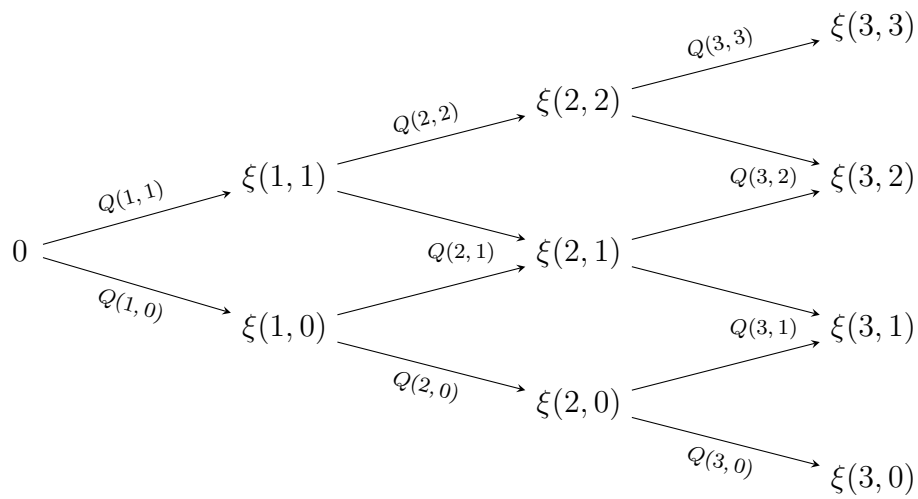
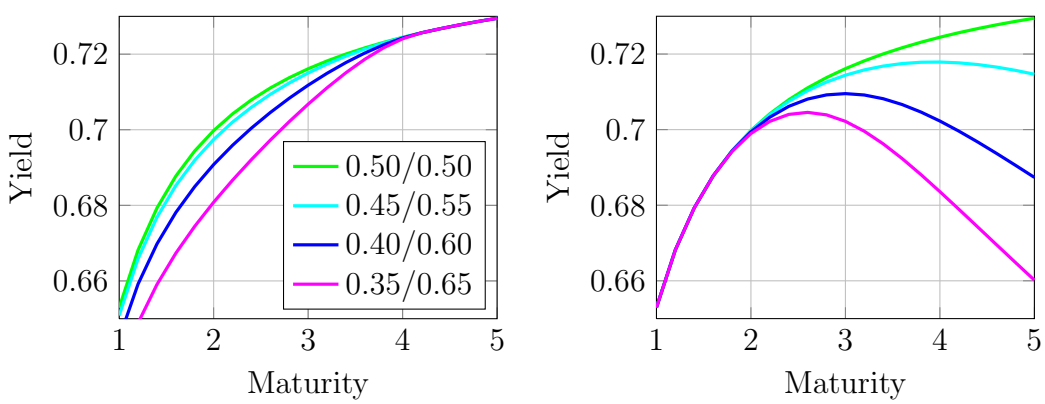


Figure 2 – State Price Tree

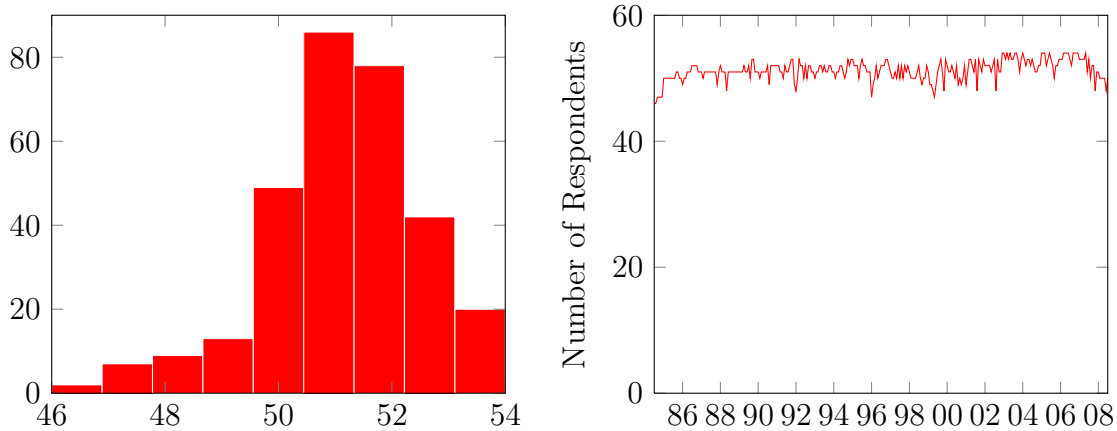
Multi-period belief structure for the binomial market model. $\xi(n,k)$ is the price of an Arrow-Debreu security that pays \$1 in state (n,k).



(a) Short term disagreement (b) Long term disagreement

Figure 3 – Term Structure of Disagreement

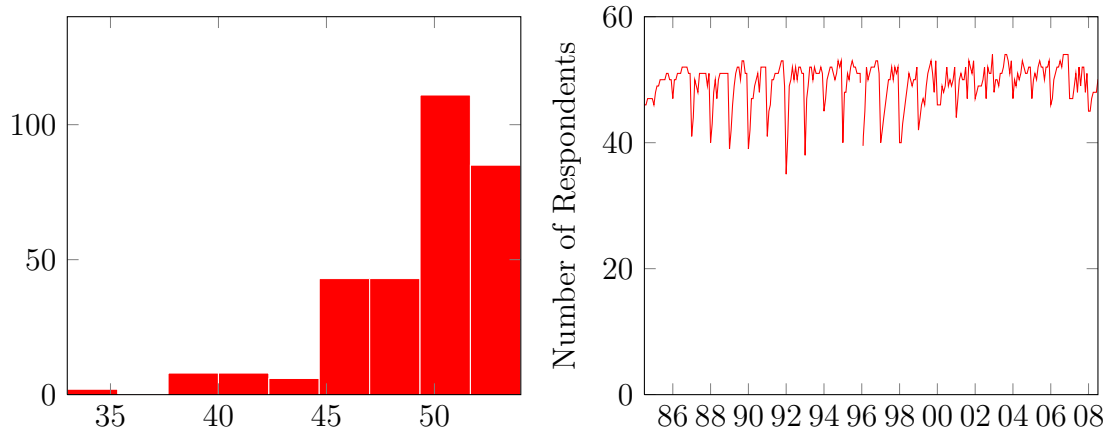
Term structures from simple heterogeneous endowment economy. The legend entries give disagreement regarding the low state, i.e., $DiB(L) = \frac{0.45}{0.55}$. Panel (a) is a static analysis of short term disagreement where agents disagree on the first period but agree on the last period. Panel (b) shows the opposite situation where agents agree on the short term but disagree by increasing amounts in the long term.



(a) Distribution of respondent numbers (b) Time Series of number of respondents

Figure 4 – Short Term Forecast Respondent Numbers

Panel (a): histogram displaying the distribution of the number of respondents for short term forecasts (average for the remaining period of the current calendar year). Panel (b): time series of number of respondents contributing to the short term forecast.



(a) Distribution of respondent numbers (b) Time Series of number of respondents

Figure 5 – Long Term Forecast Respondent Numbers

Panel (a): histogram displaying the distribution of the number of respondents for short term forecasts (an average for the following year). Panel (b): time series of number of respondents contributing to the long term forecast.

Abbreviations:

- RGDP = Real Gross Domestic Product, chained Dollars of 2000$ (NIPA, BEA).
- GDPI = GDP Chained Price Index. National Income and Product Accounts (NIPA), Bureau of Economic Analysis (BEA).
- CPI = Consumer Price Index-All Urban Consumers. Bureau of Labor Statistics (BLS).
- IP = Industrial Production (FRB).
- DPI = Disposable Personal Income, 2000$ (NIPA, BEA).
- NRI = Non-residential Investment, 2000$ (NIPA, BEA).
- UNEM = Unemployment Rate, civilian work force (BLS).
- HS = Housing Starts (BEA).
- CP = Corporate Profits, NIPA (BEA).
- AS = Total U.S Auto and Truck sales (BEA).
- SR = 3-month, secondary market, bank discount basis. Federal Reserve Board (FRB).
- LR = 10 yr constant maturity Treasury Note yield (FRB).

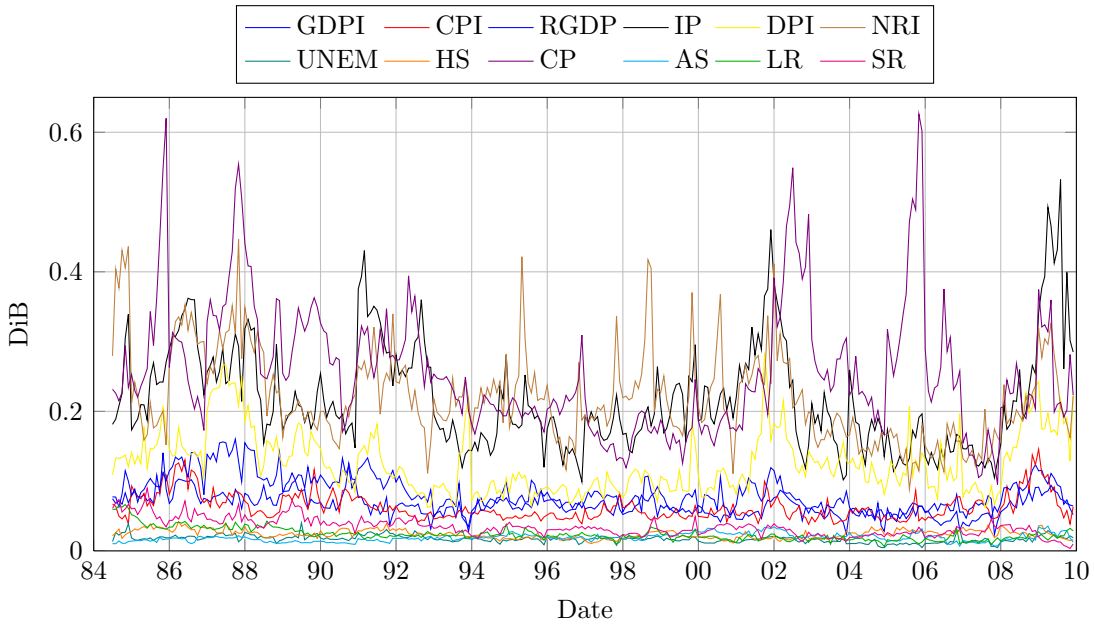


Figure 6 – Short Term Disagreement

Time series of DiB (cross-sectional mean absolute deviation) for short term forecasts discussed in section II B.

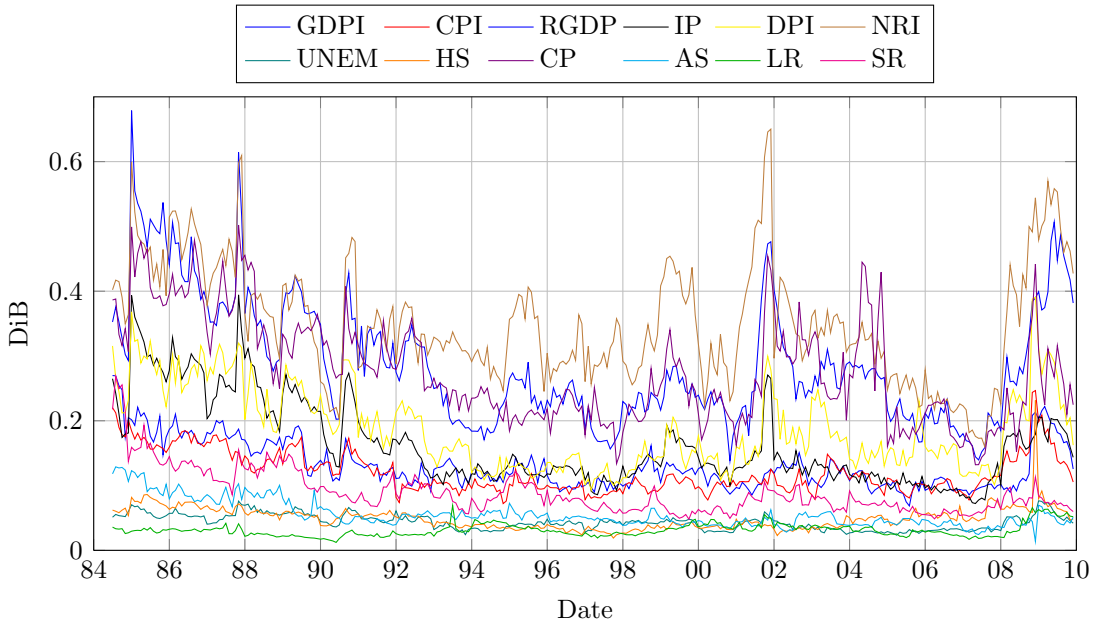


Figure 7 – Long Term Disagreement

Time series of DiB (cross-sectional mean absolute deviation) for long term forecasts discussed in section II B.

Table II – Summary Statistics

Table reports the summary statistics for mean-absolute-deviation in economist forecasts for real, nominal, and monetary components. Sample Period: 1986.1 - 2010.1. The abbreviations used in the column and row headings are above. Panel A reports constant maturity disagreement, Panel B reports short term disagreement, and Panel C reports long term disagreement. The lower ×4 rows of each panel reports the correlation matrix for each disagreement type.

	DiB_t^{INF}	DiB_t^{RGDP}	DiB_t^{LR}	DiB_t^{SR}
Panel A:				
Mean	0.25	0.21	0.25	0.27
SDev	0.05	0.07	0.08	0.08
Skew	1.21	0.77	1.16	0.90
AC(1)	0.87	0.85	0.87	0.91
DiB_t^{INF}	1.00	0.60	0.63	0.67
DiB_t^{RGDP}	0.60	1.00	0.64	0.69
DiB_t^{LR}	0.63	0.64	1.00	0.74
DiB_t^{SR}	0.67	0.69	0.74	1.00
Panel B:				
Mean	0.17	0.21	0.15	0.15
SDev	0.04	0.07	0.05	0.05
Skew	0.78	0.76	0.72	0.88
AC(1)	0.74	0.86	0.80	0.82
DiB_t^{INF}	1.00	0.40	0.49	0.41
DiB_t^{RGDP}	0.40	1.00	0.67	0.66
DiB_t^{LR}	0.49	0.67	1.00	0.64
DiB_t^{SR}	0.41	0.66	0.64	1.00
Panel C:				
Mean	0.34	0.41	0.37	0.41
SDev	0.08	0.16	0.11	0.12
Skew	1.04	1.26	1.15	0.87
AC(1)	0.88	0.93	0.87	0.91
DiB_t^{INF}	1.00	0.77	0.67	0.71
DiB_t^{RGDP}	0.77	1.00	0.70	0.74
DiB_t^{LR}	0.67	0.70	1.00	0.77
DiB_t^{SR}	0.71	0.74	0.77	1.00

Table III – Principle Component Analysis

This table reports the cumulative percentages explained by successive static principle components estimated from the eigenvalue decomposition of disagreement measures: $\text{var}[DiB] = Q\Lambda Q^{\top}$. Panel A reports results for short and long term forecasts while Panel B reports results for the constant constant maturity disagreement measure discussed in section II B. Sample Period: 1986.1 - 2010.1.

	PC1	PC2	PC3	PC4	PC5	PC6
Panel A:						
Short Term DiB	51.4	80.2	90.2	95.2	97.2	98.6
Long Term DiB	79.2	86.9	91.2	93.9	96.4	97.9
Panel B:						
Constant Maturity DiB	59.6	72.6	80.9	85.0	89.6	92.4

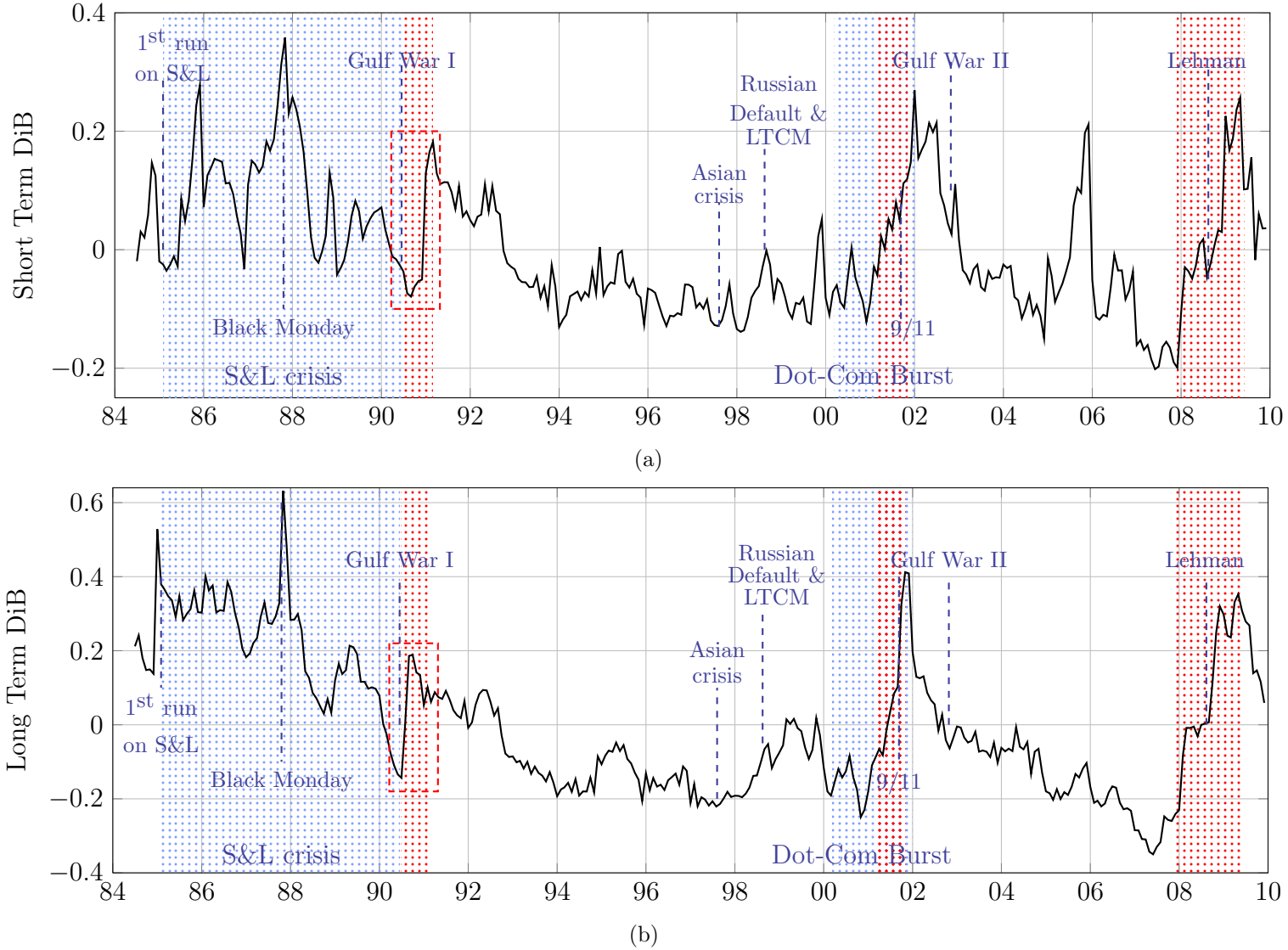


Figure 8 – 1st Principle Component

Time series of 1st principle component (PC1) from an eigenvalue decomposition of the covariance matrix of short term (top panel) and long term (bottom panel) disagreement measures listed on page 34.

Table IV – Predictability Regressions

This table reports estimates from OLS regressions of annual ($t \to t+12$) excess returns of 2 - 5 year bonds on disagreement factors:

$$rx_{t,t+12}^{(n)} = const^{(n)} + \beta_1^{(n)} DiB_t^{INF} + \beta_2^{(n)} DiB_t^{RGDP} + \beta_3^{(n)} DiB_t^{LR} + \beta_4^{(n)} DiB_t^{SR} + \varepsilon_{t+12}^{(n)},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics are computed using 18 Newey-West lags. '<>' brackets report p-values. $\bar{R}^2$ reports the adjusted R^2. Sample Period: 1986.1 - 2010.1.

Maturity	$const$	DiB_t^{INF}	DiB_t^{RGDP}	DiB_t^{LR}	DiB_t^{SR}	$\overline{R}^2$	χ^2
$rx_{t,t+12}^{(2\text{yr})}$	0.01	−0.05	0.07	−0.13	0.09	0.28	38.46
	(1.94)	(−2.09)	(2.50)	(−5.22)	(3.95)		< 0.00 >
$rx_{t,t+12}^{(3\text{yr})}$	0.03	−0.09	0.11	−0.25	0.17	0.26	33.70
	(2.12)	(−1.84)	(2.37)	(−4.68)	(3.58)		< 0.00 >
$rx_{t,t+12}^{(4\text{yr})}$	0.04	−0.14	0.13	−0.33	0.25	0.24	32.18
	(2.35)	(−1.90)	(2.11)	(−4.20)	(3.52)		< 0.00 >
$rx_{t,t+12}^{(5\text{yr})}$	0.05	−0.14	0.13	−0.39	0.30	0.21	24.97
	(2.34)	(−1.60)	(1.82)	(−3.95)	(3.36)		< 0.00 >

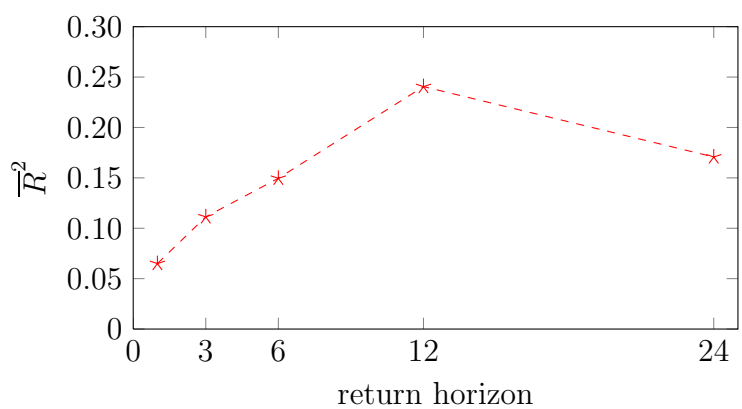
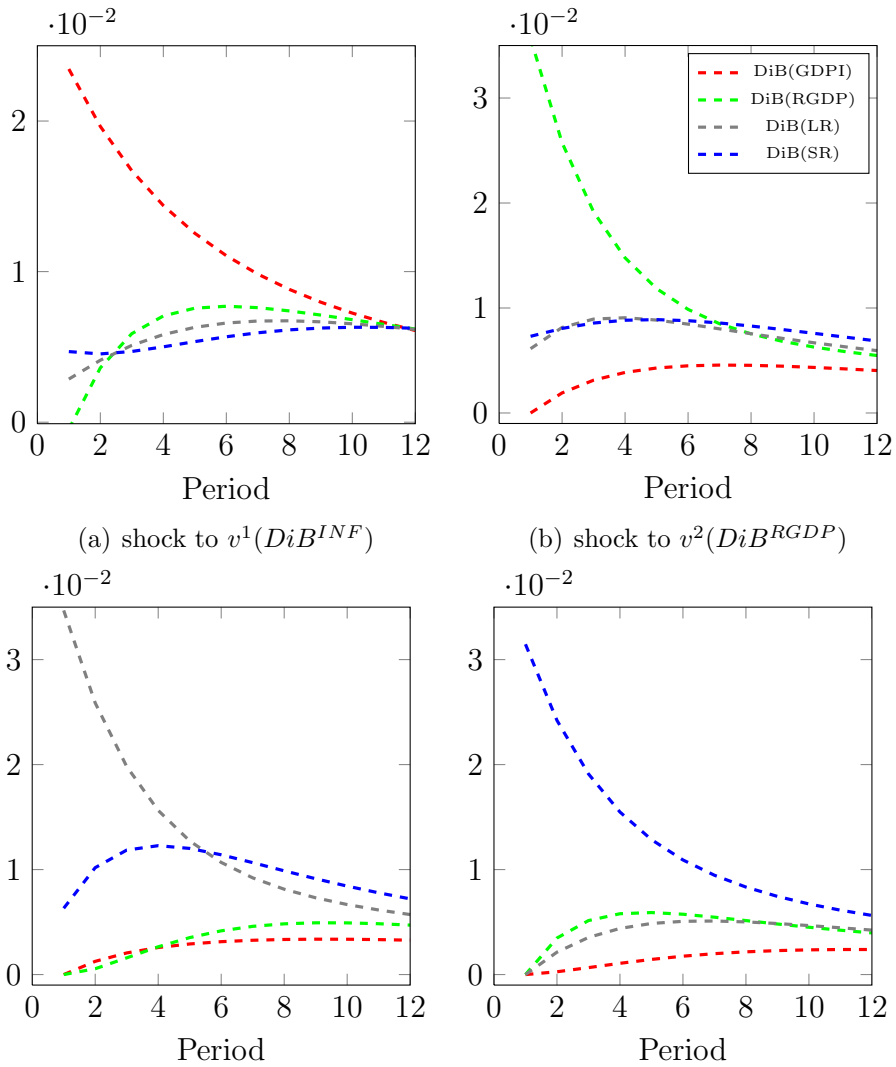


Figure 10 – Multiple Horizon $\overline{R}^2$

This figure plots the $\overline{R}^2$ for forecasting regressions over multiple horizons (1, 3, 6, 12, and 24 months). Excess returns are computed from long-short portfolios of 2 - 5 year bonds and the appropriate risk free rate:

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t,t+k}^{(n)} = \beta_0 + \beta_1 GDPI_t + \beta_2 RGDP_t + \beta_3 LR_t + \beta_4 SR_t + \bar{\varepsilon}_{t+k}.$$

Note: for the 24 month return the portfolio is long a $3, 4$ and 5 year bond and short 1-year bond plus a 1-year forward.



(a) shock to $v^1(DiB^{INF})$ (b) shock to $v^2(DiB^{RGDP})$

(c) shock to $v^3(DiB^{LR})$ (d) shock to $v^4(DiB^{SR})$

Figure 11 – Impulse Response

Orthogonalised impulse response function of the VAR of DiB_t^{INF}, DiB_t^{RGDP},DiB_t^{LR}, and DiB_t^{SR}. The original innovations (ε_t^i) are decomposed into a set of uncorrelated components (v_t^i) via Cholesky decomposition of the variance-covariance matrix Ω.

Table V – Vector Autoregression

Reports estimates of a monthly first order VAR containing disagreement on inflation(INF), real GDP ($RDGP$), 10 year Treasury yields (LR), and the 3-month short rate (SR) for the sample period: 1986.1 - 2009.1:

$$DiB_t = m + \Phi DiB_{t-1} + \varepsilon_t.$$

A robust parameter variance-covariance matrix is estimated under the assumption of heteroskedasticity and serially correlated errors. Panel C reports the Granger p-value computed from a Lagrange Multiplier test where the assumption of heteroskedasticity and serially correlated errors is maintained when estimating score covariances.

Panel A: Parameter Estimates

	m	Φ				R^2
INF	0.02	0.83	0.05	0.03	0.01	0.80
$RGDP$	0.00	0.15	0.70	-0.00	0.11	0.74
LR	0.01	0.07	0.09	0.73	0.07	0.78
SR	0.01	0.02	0.04	0.15	0.77	0.84

Panel B: t-statistics

	m	Φ			
INF	3.30	28.38	1.02	0.72	0.07
$RGDP$	-0.05	12.10	13.15	-0.05	2.40
LR	0.38	1.34	5.45	12.41	0.87
SR	0.37	0.41	0.30	10.77	13.58

Panel C: Granger p-values

	$INF(t-1)$	$RGDP(t-1)$	$LR(t-1)$	$SR(t-1)$
$INF(t)$	$\star$	0.10	0.15	0.74
$RGDP(t)$	0.01	$\star$	0.92	0.02
$LR(t)$	0.33	0.03	$\star$	0.16
$SR(t)$	0.70	0.25	0.00	$\star$

Table VI – Predictability Regressions: Robustness

Table reports estimates from OLS regressions of yearly ($t \to t+12$) excess returns of 2 - 5 year bonds on quarterly lags of disagreement measures. Reported in ( ) are t-statistics computed using robust Hansen and Hodrick (1983) with an 18-lag Newey-West correction. Reported in [ ] are alternative t-statistics computed using standard errors '1B' from Hodrick (1992). Panel A reports excess returns on a quarterly (date $t-3$) lag of macro disagreement and contemporaneous (date t) monetary disagreement, while Panel B reports excess returns on a quarterly (date $t-3$) lag of all disagreement measures.

Panel A: Maturity		$const$	DiB^{INF}_{t-3}	DiB^{RGDP}_{t-3}	DiB^{LR}_{t}	DiB^{SR}_{t}	$\overline{R}^2$
$rx^{(2yr)}_{t,t+12}$	slope	0.16	−0.12	0.14	−0.23	0.16	0.30
	GMM-t	(4.61)	(−2.32)	(3.57)	(−4.73)	(4.48)	
	H1B-t	[3.01]	[−1.31]	[2.26]	[−3.07]	[2.34]	
$rx^{(3yr)}_{t,t+1}$	slope	0.22	−0.18	0.21	−0.33	0.24	0.31
	GMM-t	(4.76)	(−2.39)	(3.84)	(−4.73)	(4.40)	
	H1B-t	[2.59]	[−1.30]	[2.10]	[−2.86]	[2.26]	
$rx^{(4yr)}_{t,t+1}$	slope	0.27	−0.27	0.25	−0.40	0.31	0.30
	GMM-t	(4.92)	(−2.54)	(3.84)	(−4.27)	(4.45)	
	H1B-t	[2.34]	[−1.41]	[1.95]	[−2.64]	[2.19]	
$rx^{(5yr)}_{t,t+1}$	slope	0.31	−0.32	0.28	−0.46	0.36	0.29
	GMM-t	(4.95)	(−2.38)	(3.78)	(−4.09)	(4.25)	
	H1B-t	[2.18]	[−1.43]	[1.79]	[−2.45]	[2.12]	

Panel B: Maturity		$const$	DiB^{INF}_{t-3}	DiB^{RGDP}_{t-3}	DiB^{LR}_{t-3}	DiB^{SR}_{t-3}	$\overline{R}^2$
$rx^{(2yr)}_{t,t+1}$	slope	0.16	−0.10	0.15	−0.17	0.08	0.18
	GMM-t	(4.00)	(−1.70)	(3.02)	(−3.14)	(2.33)	
	H1B-t	[2.99]	[−1.07]	[2.16]	[−2.02]	[1.15]	
$rx^{(3yr)}_{t,t+1}$	slope	0.22	−0.15	0.21	−0.24	0.12	0.18
	GMM-t	(4.05)	(−1.89)	(3.18)	(−3.11)	(2.55)	
	H1B-t	[2.58]	[−1.08]	[2.01]	[−1.86]	[1.11]	
$rx^{(4yr)}_{t,t+1}$	slope	0.27	−0.22	0.26	−0.29	0.16	0.17
	GMM-t	(4.13)	(−2.14)	(3.26)	(−2.73)	(2.40)	
	H1B-t	[2.33]	[−1.20]	[1.88]	[−1.65]	[1.05]	
$rx^{(5yr)}_{t,t+1}$	slope	0.31	−0.27	0.29	−0.30	0.16	0.15
	GMM-t	(4.08)	(−2.02)	(3.21)	(−2.40)	(1.91)	
	H1B-t	[2.18]	[−1.21]	[1.72]	[−1.44]	[0.89]	

Table VII – Reverse Regressions

This table reports estimates from OLS estimates of monthly ($t \to t+1$) average returns across maturity (2 - 5 year bonds) in excess of the Fama 1-month Risk Free Rate (CRSP) on a lagged summation of right hand disagreement factors according to Hodrick (1992) :

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t,t+1}^{(n)} = const^{(n)} + \beta_1^{(n)}\overline{DiB}_{t-1}^{RGDP} + \beta_3^{(n)}\overline{DiB}_{t-1}^{LR} + \varepsilon_{t+1}^{(n)},$$

where $\overline{DiB}_{t-1} = \sum_{i=1}^{h} DiB_{t-1-i}$. t-statistics, reported in ( )'s, are adjusted for heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. The $\chi^2(4)$ statistic tests the joint hypothesis that all three slope coefficients are zero. Data spans 1988.1 - 2010.1.

Lag(h)	*const*	$\overline{DiB}_{t-1}^{RGDP}$	$\overline{DiB}_{t-1}^{LR}$
3	0.00	0.02	−0.02
	(2.63)	(2.38)	(−2.19)
6	0.00	0.04	−0.04
	(2.73)	(2.00)	(−2.26)
12	0.00	0.04	−0.04
	(2.71)	(2.50)	(−2.05)
24	0.00	0.04	−0.04
	(2.74)	(1.76)	(−1.51)
$\chi^2(4)$		25.09	20.76
p-value		0.00	0.00

Table VIII – Fama-Bliss: Term Premia Regressions

Panel A reports estimates from the OLS regression of excess returns on n-month T-Bills on date t disagreement and the n-month forward-spot spread:

$$rx_{t,t+k}^{(n)} = const + \beta_1 DiB_t^{INF} + \beta_2 DiB_t^{RGDP} + \beta_3 DiB_t^{LR} + \beta_4 DiB_t^{SR} + \psi\left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+k}^{(n)}.$$

Panel B reports the corresponding regressions for n-year zero coupon bonds. t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. Only the adjusted $\bar{R}^2$ is reported. Sample Period: 1986.1 - 2010.1.

Maturity	const	DiB_t^{INF}	DiB_t^{RGDP}	DiB_t^{LR}	DiB_t^{SR}	$\left(f_t^{(n)} - y_t^{(1)}\right)$	$\overline{R}^2$
Panel A							
$rx_{t,t+1}^{(2\text{mo})}$	$-0.64e^{-3}$	$0.56e^{-3}$	$1.75e^{-3}$	$1.34e^{-3}$	$0.62e^{-3}$	0.05	0.32
	(−5.00)	(0.73)	(2.95)	(2.37)	(1.02)	(0.77)	
$rx_{t,t+1}^{(3\text{mo})}$	$-0.40e^{-3}$	$0.62e^{-3}$	$1.58e^{-3}$	$-0.21e^{-3}$	$1.35e^{-3}$	0.16	0.21
	(−2.44)	(0.85)	(2.82)	(−0.28)	(2.34)	(2.62)	
$rx_{t,t+1}^{(4\text{mo})}$	$-0.33e^{-3}$	$0.53e^{-3}$	$1.70e^{-3}$	$-1.78e^{-3}$	$2.5e^{-3}$	0.12	0.16
	(−1.51)	(0.58)	(2.17)	(−1.82)	(3.09)	(1.42)	
$rx_{t,t+1}^{(5\text{mo})}$	$-0.36e^{-3}$	$0.91e^{-3}$	$2.71e^{-3}$	$-3.23e^{-3}$	$3.15e^{-3}$	0.27	0.15
	(−1.16)	(0.71)	(2.95)	(−2.49)	(3.17)	(2.99)	
Panel B							
$rx_{t,t+12}^{(2\text{yr})}$	0.01	−0.05	0.07	−0.14	0.09	0.11	0.28
	(1.85)	(−1.93)	(2.49)	(−5.28)	(3.00)	(0.41)	
$rx_{t,t+12}^{(3\text{yr})}$	0.03	−0.08	0.10	−0.25	0.17	0.21	0.26
	(1.91)	(−1.81)	(2.26)	(−4.76)	(3.70)	(0.59)	
$rx_{t,t+12}^{(4\text{yr})}$	0.04	−0.11	0.12	−0.33	0.24	0.34	0.25
	(1.91)	(−1.82)	(1.92)	(−4.30)	(3.65)	(0.90)	
$rx_{t,t+12}^{(5\text{yr})}$	0.04	−0.14	0.12	−0.39	0.30	0.38	0.22
	(1.95)	(−1.85)	(1.59)	(−3.92)	(3.64)	(0.96)	

Table IX – Fama-Bliss: Complimentarity Regressions

This table reports estimates from the OLS regression of the n-period change in the n-period spot rate on disagreement and the forward spot spread:

$$\text{Panel A1 / A2}: y_{t+n}^{(\text{1mo/yr})} - y_t^{(\text{1mo/yr})} = const + \psi\left(f_t^{(n)} - y_t^{(\text{1mo/yr})}\right) + \varepsilon_{t+n},$$

$$\text{Panel B1 / B2}: y_{t+n}^{(\text{1mo/yr})} - y_t^{(\text{1mo/yr})} = const + \beta_1 DiB_t^{INF} + \beta_2 DiB_t^{RGDP} + \beta_3 DiB_t^{LR} + \beta_4 DiB_t^{SR} + \psi\left(f_t^{(n)} - y_t^{(\text{1mo/yr})}\right) + \varepsilon_{t+n}.$$

The sample size for the 1-year ($n = 12$) change in the 1-year spot rate is 277 while in subsequent regressions the sample size is reduced by 12 months each time the forecast horizon is extended 1 year. t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. For panel B only the adjusted $\bar{R}^2$ is reported. Sample Period: 1986.1 - 2010.1.

Horizon	$const$	$\left(f_t^{(n)} - y_t^{(1)}\right)$	R^2	$const$	DiB_t^{INF}	DiB_t^{RGDP}	DiB_t^{LR}	DiB_t^{SR}	$\left(f_t^{(n)} - y_t^{(1)}\right)$	$\overline{R}^2$
	Panel A1			Panel B1						
$y_{t+1}^{(\text{1mo})} - y_t^{(\text{1mo})}$	$-0.05e^{-3}$	0.08	0.01	$0.07e^{-3}$	$-0.07e^{-3}$	$-0.25e^{-3}$	$0.45e^{-3}$	$-0.62e^{-3}$	0.10	0.01
	(−1.61)	(1.68)		(0.69)	(−0.13)	(−0.42)	(0.82)	(−1.07)	(1.88)	
$y_{t+2}^{(\text{1mo})} - y_t^{(\text{1mo})}$	$-0.20e^{-3}$	0.37	0.12	$0.07e^{-3}$	$0.05e^{-3}$	$-1.14e^{-3}$	$1.17e^{-3}$	$-1.31e^{-3}$	0.43	0.15
	(−3.72)	(6.63)		(0.36)	(0.06)	(−2.75)	(1.70)	(−2.40)	(7.13)	
$y_{t+3}^{(\text{1mo})} - y_t^{(\text{1mo})}$	$-0.21e^{-3}$	0.42	0.10	$-0.07e^{-3}$	$0.37e^{-3}$	$-1.78e^{-3}$	$2.63e^{-3}$	$-1.93e^{-3}$	0.42	0.15
	(−3.20)	(4.89)		(−0.28)	(0.32)	(−3.04)	(2.70)	(−2.55)	(5.39)	
$y_{t+4}^{(\text{1mo})} - y_t^{(\text{1mo})}$	$-0.33e^{-3}$	0.47	0.15	$-0.08e^{-3}$	$0.08e^{-3}$	$-2.16e^{-3}$	$3.39e^{-3}$	$-2.48e^{-3}$	0.48	0.23
	(−3.58)	(6.83)		(−0.24)	(0.05)	(−2.72)	(2.50)	(−2.50)	(8.58)	
	Panel A2			Panel B2						
$y_{t+12}^{(\text{1yr})} - y_t^{(\text{1yr})}$	−0.01	0.84	0.12	−0.01	0.06	−0.07	0.14	−0.11	0.89	0.38
	(−2.04)	(2.08)		(−1.74)	(1.51)	(−2.36)	(4.85)	(−4.50)	(3.37)	
$y_{t+24}^{(\text{1yr})} - y_t^{(\text{1yr})}$	−0.02	1.32	0.28	−0.03	0.04	−0.12	0.20	−0.10	1.50	0.48
	(−3.10)	(3.57)		(−2.24)	(0.84)	(−2.55)	(5.09)	(−3.07)	(5.76)	
$y_{t+36}^{(\text{1yr})} - y_t^{(\text{1yr})}$	−0.03	1.74	0.58	−0.04	0.05	−0.07	0.14	−0.10	1.82	0.66
	(−5.09)	(6.49)		(−2.75)	(1.11)	(−1.69)	(3.45)	(−2.55)	(9.15)	
$y_{t+48}^{(\text{1yr})} - y_t^{(\text{1yr})}$	−0.03	1.63	0.65	−0.03	0.01	−0.01	0.02	−0.03	1.63	0.65
	(−6.05)	(8.20)		(−1.94)	(0.33)	(−0.43)	(0.47)	(−0.67)	(8.84)	

Table X – Single Factor Representation for Disagreement

This table reports estimates from the OLS regression of average excess return across maturity (2 - 5 year bonds) on disagreement factors:

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t,t+12}^{(n)} = \beta_0 + \beta_1 DiB_t^{INF} + \beta_2 DiB_t^{RGDP} + \beta_3 DiB_t^{LR} + \beta_4 DiB_t^{SR} + \varepsilon_{t+12},$$
$$\overline{rx}_{t,t+12} = \beta^\top DiB_t^i + \bar{\varepsilon}_{t+1}.$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics are computed using 18 Newey-West lags. '<>' brackets report p-values. $\bar{R}^2$ reports the adjusted R^2. Sample Period: 1986.1 - 2010.1.

	$const$	DiB_t^{INF}	DiB_t^{RGDP}	DiB_t^{LR}	DiB_t^{SR}	$\overline{R}^2$	χ^2
$\overline{rx}_{t,t+12}$	0.03	−0.11	0.11	−0.28	0.21	0.24	31.64
	(2.27)	(−1.84)	(2.15)	(−4.35)	(3.54)		< 0.00 >

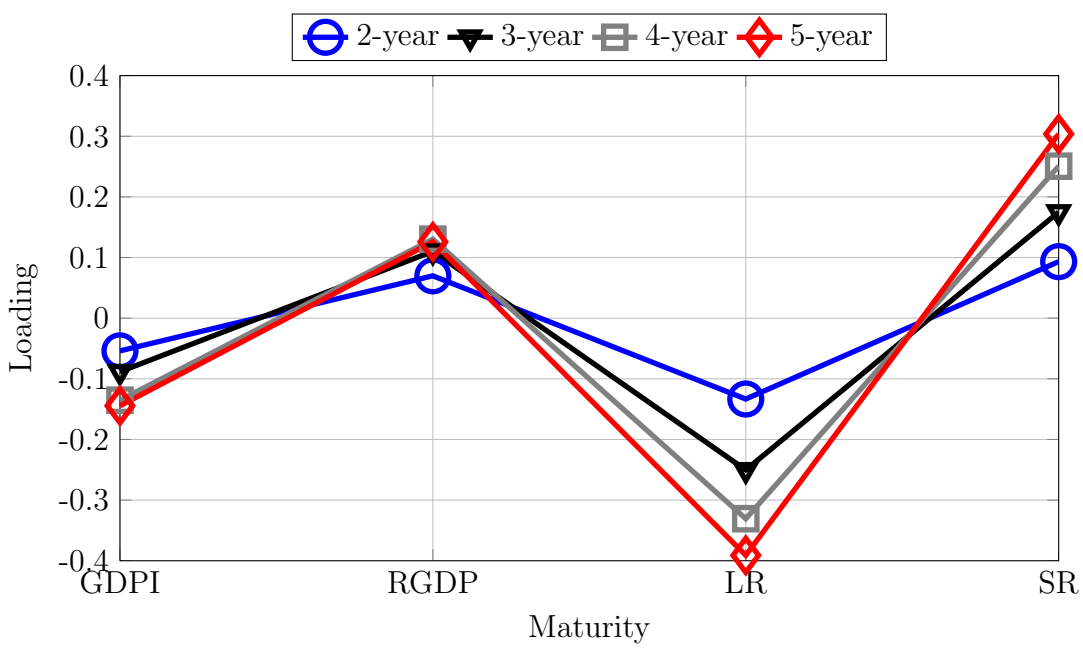


Figure 12 – Factor Loadings: DiB_t

Plot of fitted values of excess returns for 2 - 5 year maturity bonds on disagreement measures.

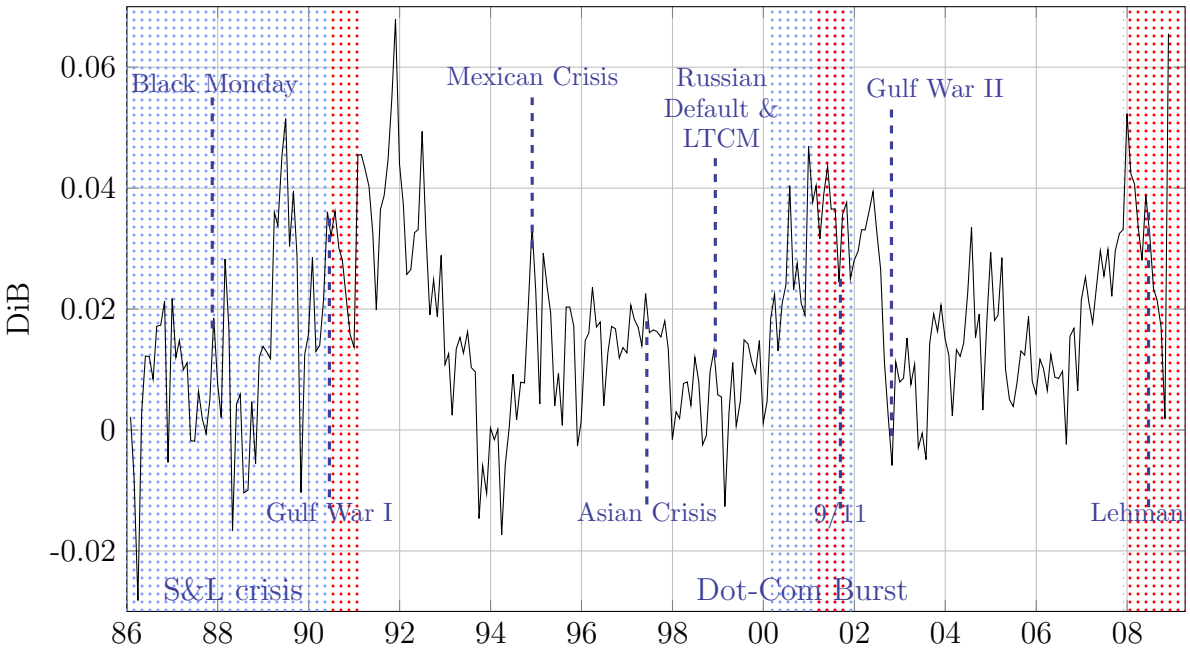


Figure 13 – Disagreement

Time series of single factor representation for macroeconomic disagreement (DiB_t). Red dotted areas are NBER recession dates.

Table XI – Predictability Regressions: Macro-Disagreement & Macro-Activity

Regression of excess holding period returns on 2 - 5 year maturity bonds on single factor construction for macroeconomic disagreement, macroeconomic risk, and liquidity:

$$rx_{t,t+12}^{(n)} = \phi^{(n)} DiB_t + \gamma^{(n)} M_t + \rho^{(n)} L_t + \varepsilon_{t+12}^{(n)}.$$

DiB is constructed from a linear combination of orthogonal disagreement factors obtained from the dispersion in analyst forecasts of economic fundamentals. M_t is a pure macro risk factor estimated using static factor analysis the spirit of Ludvigson and Ng (2009b) from a panel of 102 measures of real and nominal economic activity. L_t is a liquidity measure studied in Fontaine and Garcia (2008). t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. Only the adjusted $\bar{R}^2$ is reported. Sample Period: 1986.1 - 2010.1.

Maturity		ϕ	γ	ρ	$\overline{R}^2$
$rx_{t,t+12}^{(2\text{yr})}$	(a)	0.49			0.29
		(6.41)			
	(b)		0.49		0.27
			(5.33)		
	(c)	0.29	0.26		0.36
		(2.90)	(2.98)		
	(d)	0.32	0.32	−0.01	0.39
		(3.30)	(3.15)	(−1.33)	
$rx_{t,t+12}^{(3\text{yr})}$	(a)	0.90			0.27
		(6.52)			
	(b)		0.90		0.24
			(5.55)		
	(c)	0.55	0.47		0.33
		(2.83)	(2.87)		
	(d)	0.58	0.57	−0.02	0.36
		(2.94)	(3.48)	(−1.50)	
$rx_{t,t+12}^{(4\text{yr})}$	(a)	1.22			0.25
		(6.54)			
	(b)		1.22		0.22
			(5.43)		
	(c)	0.75	0.63		0.30
		(2.84)	(2.66)		
	(d)	0.77	0.74	−0.04	0.34
		(2.78)	(3.38)	(−1.72)	
$rx_{t,t+12}^{(5\text{yr})}$	(a)	1.39			0.22
		(6.34)			
	(b)		1.39		0.20
			(5.76)		
	(c)	0.85	0.72		0.27
		(2.65)	(2.64)		
	(d)	0.89	0.87	−0.04	0.30
		(2.55)	(3.76)	(−1.61)	

Table XII – Economic Significance

Economic significance from regression of excess holding period returns on 2 - 5 year maturity bonds on single factor construction for macroeconomic disagreement, macroeconomic risk, and liquidity:

$$rx_{t,t+12}^{(n)} = \phi^{(n)} DiB_t + \gamma^{(n)} M_t + \rho^{(n)} L_t + \varepsilon_{t+12}^{(n)}.$$

$\sigma(E)$ is the standard deviation of the model, i.e., $\sigma(\phi DiB_t + \gamma M_t + \rho L_t)$, while the remaining columns are the response to a 1-standard deviation shock to each factor.

	$E[rx]$	$\sigma(E[rx])$	$\phi\sigma(DiB)$	$\gamma\sigma(M)$	$\rho\sigma(L)$
$rx_{t,t+12}^{(2\text{yr})}$	0.82	0.86	0.48	0.46	−0.17
$rx_{t,t+12}^{(3\text{yr})}$	1.51	1.58	0.87	0.80	−0.40
$rx_{t,t+12}^{(4\text{yr})}$	2.07	2.14	1.16	1.05	−0.63
$rx_{t,t+12}^{(5\text{yr})}$	2.34	2.50	1.34	1.23	−0.72

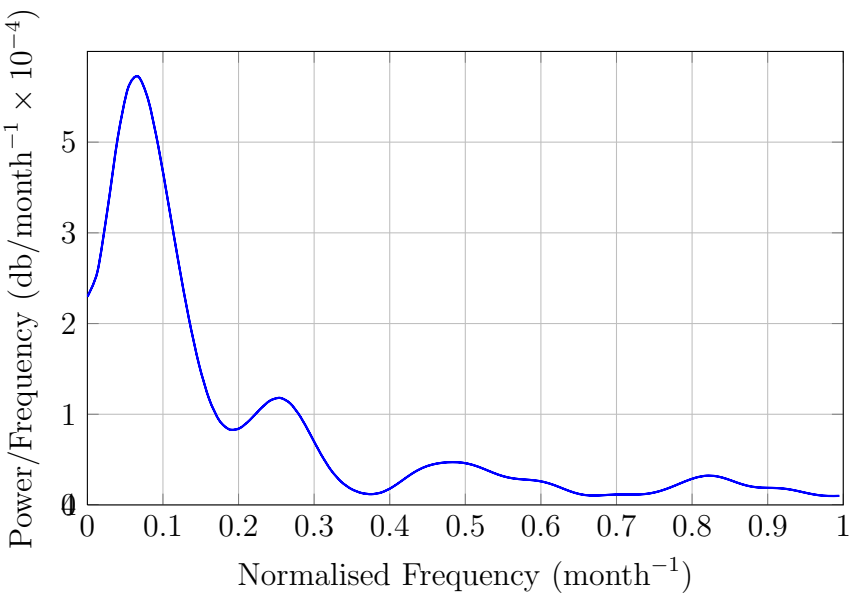


Figure 14 – Spectral Density

Spectral density estimate of Disagreement factor computed using Welch's method with a Bartlett window.

Table XIII – Spectral Density

Table documents the period, in months, of peaks obtained from a spectral density estimate computed using Welch's method with a Barteltt window for macro disagreement, DiB_t, and macro risk, M_t.

	Peak 1	Peak 2	Peak 3
DiB_t	13	4	2
M_t	50		

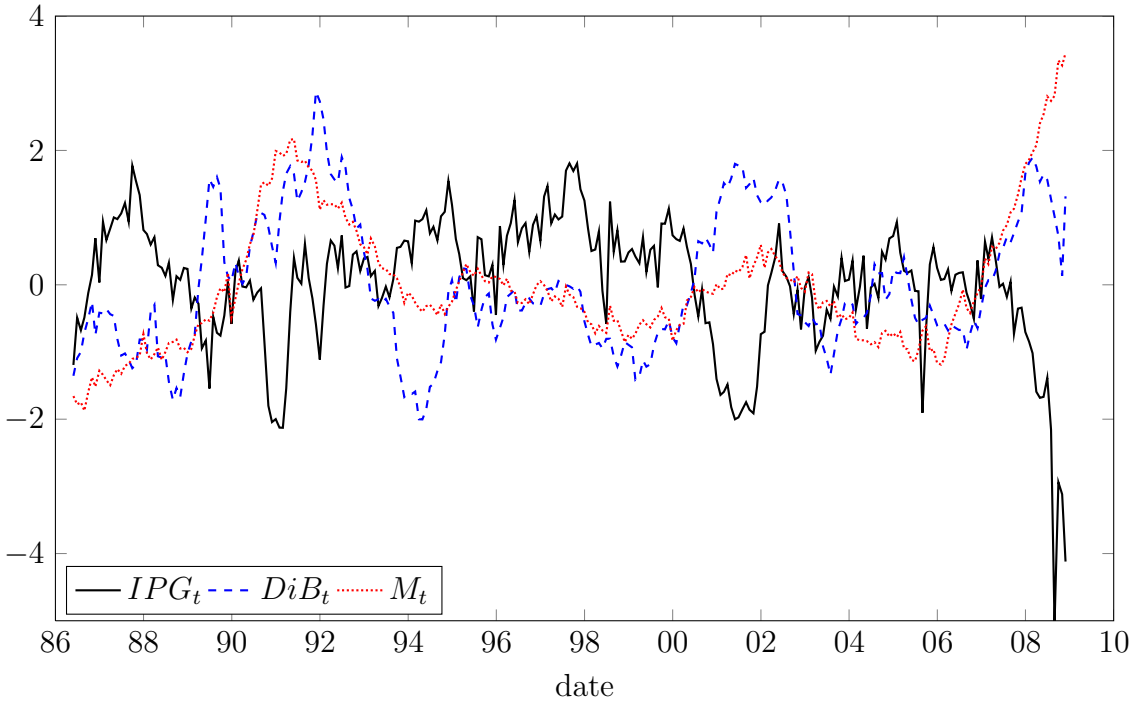


Figure 15 – Industrial Production Growth, Disagreement, and Macro-Activity

Time series plot of 6 month exponentially weighted moving average of Industrial Production Growth (IPG_t), Disagreement (DiB_t), and Macro-Activity (M_t). All series are standardized by $\frac{X_t - \mu_X}{\sigma_X}$.

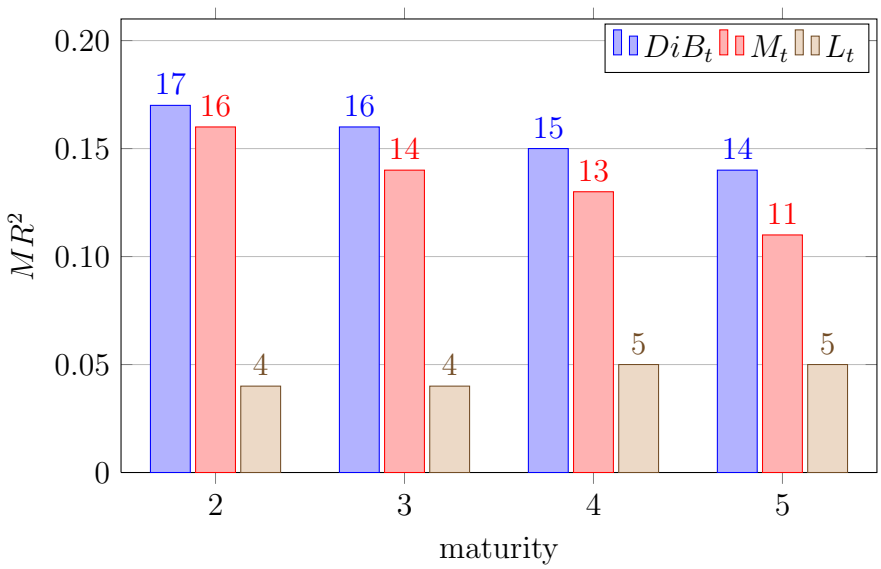


Figure 16 – R^2 **Contribution**

Marginal R^2 contribution for each factor is computed from averaging all sequences of regressions including the factor minus the same regression with the factor removed: For example, the R^2 on disagreement is found from:

$$MR^2_{DiB} = \frac{1}{4}\left(R^2\left[rx^n_{t+12} = \alpha + \sum_{i\in S} b_i x_i + \phi DiB + \varepsilon_{t+1}\right] - R^2\left[rx^n_{t+12} = \alpha + \sum_{i\in S} b_i x_i + \varepsilon_{t+1})\right]\right),$$

where S is the set of all possible combination of M_t and L_t.

Table XIV – Predictability Regressions: Short Rate Variance

OLS regression of excess return on 2 - 5 year maturity bonds on the conditional variance obtained from an GARCH(1,1) model fitted to the 3 month treasury rate:

$$\text{Panel A}: rx_{t,t+12}^{(n)} = \alpha + \beta_0 \sigma_t^2 + \varepsilon_{t+12}^{(n)},$$
$$\text{Panel B}: rx_{t,t+12}^{(n)} = \alpha + \Big(\beta_0 + \beta_1 DiB_t + \beta_2 M_t\Big)\sigma_t^2 + \varepsilon_{t+12}^{(n)},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics are computed using 18 Newey-West lags. <> brackets report p-values. For panel B only the adjusted $\bar{R}^2$ is reported. Sample Period: 1986.1 - 2010.1.

	Panel A: const. price of risk				Panel B: time-varying price of risk					
Maturity	α	β_0	R^2	χ^2	α	β_0	β_1	β_2	$\bar{R}^2$	χ^2
$rx_{t,t+12}^{(2\text{yr})}$	0.01	0.06	0.05	6.15	0.01	−0.07	2.98	1.16	0.13	8.69
	(1.66)	(2.48)		< 0.01 >	(2.19)	(−1.00)	(2.37)	(1.21)		< 0.03 >
$rx_{t,t+12}^{(3\text{yr})}$	0.01	0.13	0.06	6.37	0.01	−0.11	5.15	2.09	0.12	8.58
	1.60	(2.52)		< 0.01 >	(2.13)	(−0.83)	(2.32)	(1.17)		< 0.04 >
$rx_{t,t+12}^{(4\text{yr})}$	0.01	0.17	0.05	6.22	0.02	−0.14	6.89	2.70	0.11	10.26
	1.70	(2.49)		< 0.01 >	(2.22)	(−0.77)	(2.38)	(1.03)		< 0.02 >
$rx_{t,t+12}^{(5\text{yr})}$	0.01	0.21	0.05	6.72	0.02	−0.18	7.90	3.92	0.11	10.97
	1.58	(2.59)		< 0.01 >	(2.18)	(−0.79)	(2.37)	(1.25)		< 0.01 >

Table XV – Predictability Regressions: Unspanned Disagreement

Panel A reports a contemporaneous linear projection of the return forecasting factor (Cochrane and Piazzesi (2005)) on disagreement measures. Panel B reports estimates from projections of annual ($t \to t+12$) excess returns of 2 - 5 year bonds components of disagreement factors orthogonal to $G_1 \subseteq \sigma\left(PC(1-5)\right)$:

$$\mathcal{UN}_{DiB_t} = DiB_t - P_j\left[DiB_t \middle| G_1\right],$$
$$rx^{(n)}_{t,t+12} = const + \beta^{(n)}_1 \mathcal{UN}^{INF}_{DiB_t} + \beta^{(n)}_2 \mathcal{UN}^{RGDP}_{DiB_t} + \beta^{(n)}_3 \mathcal{UN}^{LR}_{DiB_t} + \beta^{(n)}_4 \mathcal{UN}^{SR}_{DiB_t} + \varepsilon^{(n)}_{t+12},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics are computed using 18 Newey-West lags. '<>' brackets report p-values. $\bar{R}^2$ reports the adjusted R^2. Sample Period: 1986.1 - 2010.1.

	const	DiB^{INF}_t	DiB^{RGDP}_t	DiB^{LR}_t	DiB^{SR}_t	$\overline{R}^2$	
Panel A:							
CP_t	0.01	−0.06	0.04	0.01	0.05	0.43	
	(1.48)	(−1.98)	(2.88)	(0.56)	(2.78)		
	const	$\mathcal{UN}^{INF}_{DiB_t}$	$\mathcal{UN}^{RGDP}_{DiB_t}$	$\mathcal{UN}^{LR}_{DiB_t}$	$\mathcal{UN}^{R}_{DiB_t}$	$\overline{R}^2$	χ^2
Panel B:							
$rx^{(2\text{yr})}_{t,t+12}$	0.01	−0.04	0.04	−0.14	0.07	0.25	23.08
	(4.15)	(−0.98)	(1.54)	(−4.21)	(2.29)		< 0.00 >
$rx^{(3\text{yr})}_{t,t+12}$	0.01	−0.07	0.05	−0.26	0.13	0.24	21.09
	(4.11)	(−0.87)	(1.23)	(−3.80)	(2.16)		< 0.00 >
$rx^{(4\text{yr})}_{t,t+12}$	0.02	−0.08	0.05	−0.34	0.18	0.23	19.42
	(4.13)	(−0.75)	(0.76)	(−3.45)	(2.05)		< 0.00 >
$rx^{(5\text{yr})}_{t,t+12}$	0.02	−0.10	0.03	−0.39	0.23	0.20	17.51
	(3.83)	(−0.75)	(0.47)	(−3.15)	(2.07)		< 0.00 >

Table XVI – Predictability Regressions: Above Disagreement

Panel A reports a contemporaneous linear projection of the hidden risk premium factor (Duffee (2011)) on disagreement measures. Panel B reports estimates from projections of annual ($t \to t+12$) excess returns of 2 - 5 year bonds components of disagreement factors unspanned by the cross-section of yields or the information set generated by the 'Hidden Factor' from Duffee (2011):

$$\mathcal{AB}_{DiB_t} = DiB_t - P_j\left[\mathcal{UN}_{DiB_t}\middle| H_t\right],$$
$$rx^{(n)}_{t,t+12} = const + \beta^{(n)}_1 \mathcal{AB}^{INF}_{DiB_t} + \beta^{(n)}_2 \mathcal{AB}^{RGDP}_{DiB_t} + \beta^{(n)}_3 \mathcal{AB}^{LR}_{DiB_t} + \beta^{(n)}_4 \mathcal{AB}^{SR}_{DiB_t} + \varepsilon^{(n)}_{t+12},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics are computed using 18 Newey-West lags. '<>' brackets report p-values. $\bar{R}^2$ reports the adjusted R^2. Sample Period: 1986.1 - 2009.1.

	$const$	$\mathcal{UN}^{INF}_{DiB_t}$	$\mathcal{UN}^{RGDP}_{DiB_t}$	$\mathcal{UN}^{LR}_{DiB_t}$	$\mathcal{UN}^{SR}_{DiB_t}$	$\overline{R}^2$	
Panel A:							
H_t	−0.00	−0.00	0.00	−0.01	0.02	0.07	
	(−0.73)	(−0.43)	(0.66)	(−1.09)	(3.02)		
	$const$	$\mathcal{AB}^{INF}_{DiB_t}$	$\mathcal{AB}^{RGDP}_{DiB_t}$	$\mathcal{AB}^{LR}_{DiB_t}$	$\mathcal{AB}^{R}_{DiB_t}$	$\overline{R}^2$	χ^2
Panel B:							
$rx^{(2\text{yr})}_{t,t+12}$	0.01	−0.04	0.03	−0.13	0.05	0.25	22.03
	(4.06)	(−0.88)	(1.29)	(−3.92)	(1.35)		< 0.00 >
$rx^{(3\text{yr})}_{t,t+12}$	0.01	−0.06	0.04	−0.24	0.08	0.24	21.43
	(4.02)	(−0.76)	(0.96)	(−3.50)	(1.28)		< 0.00 >
$rx^{(4\text{yr})}_{t,t+12}$	0.02	−0.07	0.03	−0.32	0.11	0.23	20.69
	(4.04)	(−0.64)	(0.50)	(−3.16)	(1.16)		< 0.00 >
$rx^{(5\text{yr})}_{t,t+12}$	0.02	−0.09	0.02	−0.36	0.13	0.20	18.84
	(3.74)	(−0.64)	(0.20)	(−2.84)	(1.15)		< 0.00 >

Table XVII – Economic Significance

Economic significance from regression of excess holding period returns on 2 - 5 year maturity bonds on above component of disagreement measures:

$$rx_{t,t+12}^{(n)} = const + \beta_1^{(n)} \mathcal{AB}_{DiB_t}^{INF} + \beta_2^{(n)} \mathcal{AB}_{DiB_t}^{RGDP} + \beta_3^{(n)} \mathcal{AB}_{DiB_t}^{LR} + \beta_4^{(n)} \mathcal{AB}_{DiB_t}^{SR} + \varepsilon_{t+12}^{(n)}.$$

$\sigma(E)$ is the standard deviation of the model, i.e., the standard deviation of the right hand side, while the remaining columns are the response to a 1-standard deviation shock to each right hand variable. Sample period overlaps that of the Hidden factor: 1986.1 - 2010.1.

Maturity	$E(Rx)$	$\sigma(E)$	$\beta_1^{(n)} \mathcal{AB}_{DiB_t}^{INF}$	$\beta_2^{(n)} \mathcal{AB}_{DiB_t}^{RGDP}$	$\beta_3^{(n)} \mathcal{AB}_{DiB_t}^{LR}$	$\beta_4^{(n)} \mathcal{AB}_{DiB_t}^{SR}$
$rx_{t,t+1}^{(2\text{yr})}$	0.79	0.71	−0.17	0.20	−0.84	0.27
$rx_{t,t+1}^{(3\text{yr})}$	1.45	1.33	−0.26	0.25	−1.57	0.50
$rx_{t,t+1}^{(4\text{yr})}$	1.99	1.82	−0.32	0.18	−2.08	0.63
$rx_{t,t+1}^{(5\text{yr})}$	2.22	2.12	−0.39	0.09	−2.38	0.78

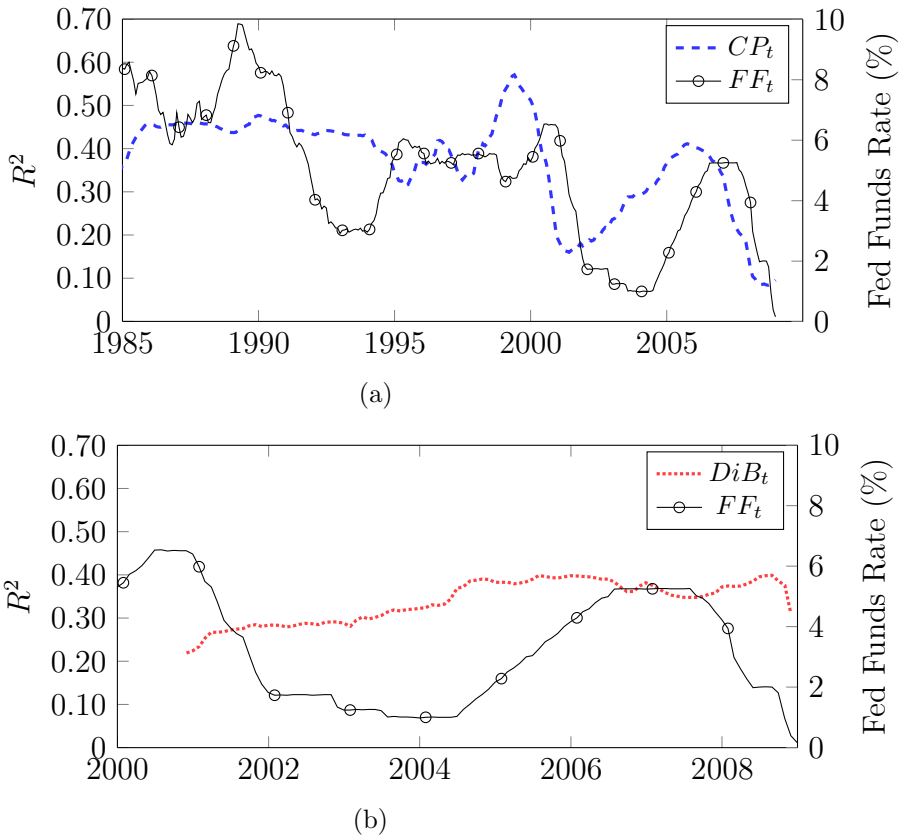


Figure 17 – Rolling R^2's

Left Axis: Rolling R^2's from return predictability regressions. R^2 statistics are computed using a 15-year rolling window of 1-year excess returns from the portfolio: $\frac{1}{4}\sum_{n=2}^{5} rx_{t,t+12}^{(n)}$. Right Axis: Effective Federal Funds rate (FF_t), H15 release. Panel (a): Cochrane-Piazzesi return forecasting factor (CP_t). Panel (b): disagreement factors (DiB_t) from section III.

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