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## Designing QE in a fiscally sound monetary union

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**Abstract**

This paper develops a tractable model of a monetary union with a sound fiscal governance structure and shows how in such environment the design of monetary policy above and at the lower bound constraint on short-term interest rates can be linked to well-known findings from the literature dealing with single closed economies. The model adds a portfolio balance channel to a New Keynesian two-country model of a monetary union. If the monetary union is symmetric and the portfolio balance channel is not active, the model becomes isomorphic to the canonical New Keynesian three-equation economy in which central bank purchases of long-term debt (QE) at the lower bound are ineffective. If the portfolio balance channel is active, QE becomes effective and we prove that for sufficiently small shocks there exists an interest rate rule augmented by QE at the lower bound which replicates the equilibrium allocation and the welfare level of a hypothetically unconstrained economy. Shocks large enough to push the whole yield curve to the lower bound require, in addition, forward guidance. We generalise these results to an asymmetric monetary union and illustrate them through simulations, distinguishing between asymmetric shocks and asymmetric structures. In general, asymmetries give rise to current account imbalances which are, depending on the degree of financial integration, funded by private capital imports or through the central bank balance sheet channel. Moreover, our findings support that at the lower bound, as long as asymmetries between countries result from shocks, outcomes under an unconstrained policy rule can be replicated via a symmetric QE design. By contrast, asymmetric structures of the countries which matter for the transmission of monetary policy can translate into an asymmetric QE design.

*Keywords:* Monetary Union, Monetary Policy, Quantitative Easing, Lower Bound.

*JEL classification numbers:* E43, E52, E61, E63.

## Non-technical summary

In response to the global financial crisis central banks of many advanced economies have adopted large-scale asset purchase programmes in order to overcome the lower bound constraint on short-term interest rates. These programmes are often labelled in a summary fashion as Quantitative Easing (QE). Yet, these programmes allow for distinct differences between countries. This paper starts out from the observation that the design of QE for monetary unions like the euro area involves specific considerations which can be linked to the unique architecture of the euro area, which consists of a single monetary policy and nineteen, currently imperfectly governed fiscal policies, predominantly decided at the national level. As stressed by the Five President's Report, the architecture of the euro area is in many dimensions still incomplete, leading to a call for urgent reforms in various policy domains, including progress towards an improved fiscal framework and better integrated financial markets. This diagnosis leads to the conclusion that *"...progress will have to follow a sequence of short- and longer-term steps, but it is vital to establish and agree the full sequence today. The measures in the short-term will only increase confidence now if they are the start of a larger process, a bridge towards a complete and genuine EMU."*

A comprehensive model-based characterisation of monetary policy options near the lower bound in the euro area should take the current incompleteness of the Economic and Monetary Union (EMU) as given. At the same time, to ensure a robust forward-looking dimension of the design, judgement will be needed as concerns possible short- and longer-term changes to the euro area architecture. To address this challenge is beyond the scope of this paper. However, it seems clear that such characterisation will not be possible without a clear view of the new steady-state configuration to be achieved in the longer-term. Motivated by this insight, this paper singles out the forward-looking dimension and explores how monetary policy could be designed once the EMU has been made more complete via reforms such that it commands, in particular, over a stable framework for the governance of national fiscal policies. Moreover, we allow, in parallel, for the possibility of a shift towards better integrated financial markets.

This paper has a conceptual focus. The main idea is to develop a tractable model of a monetary union with a sound fiscal governance structure. The paper shows how in such environment the design of monetary policy above and at the lower bound constraint on short-term interest rates can be linked to well-known findings from the literature dealing with single closed economies (which command over a stable governance structure for a single monetary policy and a single fiscal policy). The model adds a portfolio balance channel to a New Keynesian two-country model of a monetary union. The portfolio balance channel allows for imperfect substitutability between short-term and long-term debt. Moreover, domestic and foreign bonds may as well be perceived as imperfect substitutes. In each country, all debt is assumed to be held by banks, backed by deposits of households. Through this assumption deposit rates measure the relevant opportunity cost of households of holding real money balances. Moreover, monetary policy can remain effective even if the single short-term policy rate has reached its lower bound.

The main results are as follows. If the monetary union is symmetric and the portfolio balance channel is not active, the linearised model becomes isomorphic to the canonical New Keynesian three-equation economy in which central bank purchases of long-term debt (QE) at the lower bound are ineffective. If the portfolio balance channel is active, QE becomes effective and we prove that for sufficiently small shocks there exists an interest rate rule augmented by QE at the lower bound which replicates the equilibrium allocation and the welfare level of a hypothetically unconstrained economy. Shocks large enough to push the whole yield curve to the lower bound require, in addition, forward guidance.

We generalise these results to an asymmetric monetary union and illustrate them through simulations, distinguishing between asymmetric shocks and asymmetric structures. In general, asymmetries give rise to current account imbalances which are, depending on the degree of financial integration, funded by private capital imports or through the central bank balance sheet channel. Moreover, our findings support that at the lower bound, as long as asymmetries between countries result from shocks, outcomes under an unconstrained policy rule can be replicated via a symmetric QE design. By contrast, asymmetric structures of the countries which matter for the transmission of monetary policy can translate into an asymmetric QE design.

The latter finding suggests that a portfolio bias of QE, in a sense, could fix asymmetric structures. Yet, when interpreting this finding it needs to be kept in mind that an incomplete monetary union, in particular when characterised by a weak fiscal governance structure and excessive exposure of banks to their own sovereign, involves additional strategic considerations that are related, inter alia, to incentive effects and risk-sharing modalities. The various safeguards of the PSPP (Public Sector Purchase Programme) adopted by the European Central Bank in January 2015 incorporate such considerations. Incorporating strategic design issues of QE in an environment of an incomplete monetary union is beyond the scope of this paper and left for future work.

# 1 Introduction

In response to the global financial crisis central banks of many advanced economies have adopted large-scale asset purchase programmes in order to overcome the lower bound constraint on short-term interest rates. These programmes are often labelled in a summary fashion as Quantitative Easing (QE). Yet, these programmes allow for distinct differences between countries. This paper starts out from the observation that the design of QE for monetary unions like the euro area involves specific considerations which can be linked to the unique architecture of the euro area, which consists of a single monetary policy and nineteen, currently imperfectly governed fiscal policies, predominantly decided at the national level. As stressed by the Five President's Report, the architecture of the euro area is in many dimensions still incomplete, leading to a call for urgent reforms in various policy domains, including progress towards an improved fiscal framework and better integrated financial markets. This diagnosis leads to the conclusion that *"...progress will have to follow a sequence of short- and longer-term steps, but it is vital to establish and agree the full sequence today. The measures in the short-term will only increase confidence now if they are the start of a larger process, a bridge towards a complete and genuine EMU."*[1]

A comprehensive model-based characterisation of monetary policy options near the lower bound in the euro area should take the current incompleteness of the Economic and Monetary Union (EMU) as given. At the same time, to ensure a robust forward-looking dimension of the design, judgement will be needed as concerns possible short- and longer-term changes to the euro area architecture. To address this challenge is beyond the scope of this paper. However, it seems clear that such characterisation will not be possible without a clear view of the new steady-state configuration to be achieved in the longer-term. Motivated by this insight, this paper singles out the forward-looking dimension and explores how monetary policy could be designed once the EMU has been made more complete via reforms such that it commands, in particular, over a stable framework for the governance of national fiscal policies. Moreover, we allow, in parallel, for the possibility of a shift towards better integrated financial markets.

To this end, we develop a tractable model of a fiscally sound monetary union and discuss how in such environment the design of monetary policy above and at the lower bound constraint can be linked to some well-known findings from the literature dealing with single closed economies. Our model builds on the framework of Harrison (2011, 2012), who adds a portfolio balance channel in the spirit of Tobin and Brainard (1963) and Tobin (1969) to an otherwise standard New Keynesian single economy set-up. We extend this framework to a two-country model of a monetary union, by allowing for a certain degree of imperfect substitutability between domestic and foreign bonds. Otherwise our model is deliberately similar to monetary union models in the tradition of Benigno (2004). This facilitates our distinct focus on closed-form results which, starting out from a small-scale analytical core of linearised equilibrium conditions, generalise findings from the New Keynesian literature.

In the benchmark version of our model, which captures the notion of integrated financial markets, competitive banking systems in both countries hold portfolios which include short-term and long-term debt issued by the governments of the two countries.[2] The two types of debt

[1] See European Commission (2016, p. 5).

[2] In the absence of financial market integration, banking systems in the two countries operate under autarky,

are imperfect substitutes, reflecting portfolio adjustment costs.[3] The portfolios of banks are funded by deposits and rates of return on deposits are a weighted average of rates on short- and long-term debt. As concerns the degree of substitutability between domestic and foreign bonds, we assume that short-term government debt, which is linked to the standard implementation of monetary policy via a conventional interest rate rule, carries the same rate of return across countries. By contrast, long-term debt of the two countries is imperfectly substitutable, carrying country-specific returns. This feature ensures that at the lower bound, where for both countries the short-term interest rate reaches zero, there is scope to stimulate the economy via central bank purchases of long-term government debt. We refer to this type of central bank purchases of long-term government debt as QE. To establish under which circumstances QE is effective or not we consider a range of monetary union specifications and ask throughout the question whether it is possible, once the lower bound constraint on short-term rates becomes binding, to replicate via QE those outcomes that would be achieved under an unconstrained policy rule (i.e. under a conventional interest rate rule which hypothetically pretends that the lower bound constraint can be ignored). It is worth emphasising that the QE design addressed in this paper does not necessarily minimise welfare differentials across the member states of the union. We rather ask how QE needs to be designed in a monetary union to replicate the outcomes of an unconstrained *uniform* monetary policy. We perceive this as a particularly relevant benchmark since it corresponds to the outcomes of a single monetary policy that would prevail if the central bank was not constrained in its single and uniformly designed conventional instrument.

We consider in a first step a symmetric monetary union (where both countries are assumed to be identical) before we then turn to the analysis of an asymmetric monetary union. Our main results are as follows. If the monetary union is symmetric and we shut down the portfolio balance channel, our linearised model, consistent with Harrison (2012), becomes isomorphic to the canonical New Keynesian three-equation economy, in line with Woodford (2003). In this prominent reference model (with well-understood properties of interest rate rules of the Taylor-type), short- and long-term government debt are perfect substitutes. As a result, QE at the lower bound turns out to be ineffective, while forward guidance (i.e. the commitment of the central bank to keep future policy rates lower for longer when the lower bound constraint ceases to be binding) is effective, as shown by Eggertsson and Woodford (2003).[4] If the portfolio balance channel is present, however, QE becomes effective at the lower bound.[5] To verify this claim we consider a shock to the natural rate which has the potential to make the lower bound constraint binding. Specifically, we prove that for realisations of this shock of a certain magnitude there exists an interest rate rule augmented by QE at the lower bound (to be labelled, for short, as a *QE-augmented policy rule*) which replicates the equilibrium allocation and the corresponding welfare level achieved under a hypothetically unconstrained policy rule. The QE-augmented policy rule, in fact, embeds the standard interest rate rule (which can be implemented only above the lower bound) as a special case, while it allows for appropriate central bank purchases of long-term debt whenever the lower bound constraint becomes binding. For this result to hold, the magnitude of the shock must ensure that the crisis is severe enough such that the policy rate hits the lower bound constraint, yet small enough such that the longer end of the yield curve still

i.e. they can only hold domestic (and not foreign) bonds.

[3]For a similar segmentation of bond markets of different maturities, see Andrés et al. (2004).

[4]This ineffectiveness result of QE is consistent with the seminal paper by Wallace (1981).

[5]Orphanides and Wieland (2000) and Coenen and Wieland (2003) consider open economy models of single economies and show how the portfolio balance channel can be exploited by a central bank at the lower bound by switching from an interest rate rule to a monetary-base rule.

has room to manoeuvre. This qualification concerning the magnitude of the shock reflects that in the model with an active portfolio balance channel the dynamics of the IS-curve are driven by the deposit rate. This rate measures at the same time the opportunity cost of holding real money balances. Hence, the QE-augmented policy rule needs to respect that the deposit rate remains non-negative in order to avoid that deposits become dominated in return by real money balances. In other words, the non-negativity of the deposit rate is needed to maintain standard interior optimality conditions, replacing thereby the non-negativity of the policy rate from the reference New Keynesian model without the portfolio balance channel. For large shocks, this constraint for the deposit rate becomes binding. But this does not mean that monetary policy becomes ineffective, because, if not on QE, it can still rely on the forward guidance channel in the spirit of Eggertsson and Woodford (2003).

In the next step, we extend these findings to an asymmetric monetary union in which the two countries cease to be identical, either because they receive shocks to the natural rate of different magnitude ("asymmetric shocks") or they exhibit structural differences in the transmission of monetary policy ("asymmetric structures"). Four main findings emerge. First, asymmetric shocks give rise to equilibrium dynamics characterised by current account imbalances which act as a built-in device to absorb asymmetric adjustment needs of the two countries. We show that the scope for current account imbalances depends on the degree of financial market integration. In the absence of financial market integration current account imbalances will be funded through the central bank balance sheet (akin to TARGET balances in the euro area). Relative to this benchmark, financial market integration increases the scope for current account imbalances which now become predominantly funded by private capital imports and exports. Second, for sufficiently large shocks, which make the lower bound constraint binding (but do not challenge the non-negativity of the unconstrained deposit rates in both countries), we prove that there exists a *QE-augmented policy rule* which replicates the equilibrium allocations and welfare levels of the unconstrained conventional policy rule in both countries. Third, turning to illustrative simulations, we assume that the lower bound is reached in an environment characterised by *asymmetric shocks and symmetric structures.* Upon this assumption our results support a *symmetric QE-augmented policy rule*, meaning that the central bank purchases identical per capita amounts of long-term debt issued by the two governments. Intuitively, this finding reflects that the lower bound imposes a constraint on the uniform instrument of the short-term policy rate. This creates for both countries a symmetric restriction for the portfolio adjustments induced by the lower bound constraint, irrespective of potential asymmetries in the magnitude of the originating shocks. In view of the findings of Benigno (2004), it is worth stressing that this result remains unchanged if in response to asymmetric shocks the conventional policy rule attaches asymmetric weights to the two countries. Any uniformly applied policy rate creates at the lower bound a symmetric restriction for both countries, irrespective of the origin of this rate in terms of country-specific weighting schemes. Fourth, if we assume that the lower bound is reached in an environment characterised by *symmetric shocks and asymmetric structures*, our simulations indicate that *asymmetric QE purchases* are needed to replicate the outcomes of the unconstrained policy rule. Intuitively, structural differences in the transmission of monetary policy trigger an asymmetric private demand for long-term bonds, leading to different long-term rates of the two countries in response to shocks, in the unconstrained and the constrained environment. Yet, the central bank will be able to replicate the unconstrained outcomes via asymmetric purchases of long-term debt if this creates a supply pattern of privately held bonds which overturns the asymmetric demand pattern induced by the lower bound constraint and thereby restores the unconstrained deposit rates in both countries. We illustrate this finding by assuming that banking systems in the two countries, while being financially integrated, exhibit a different degree of home bias in holding government debt. Such an asymmetric structure translates into asymmetric central bank purchase volumes of long-term debt at the lower bound, favouring the country where banks are more strongly exposed towards their own sovereign. This portfolio bias of QE, in a sense, fixes asymmetric structures. Within our model, this bias is not a cause for concern since our assumption of a sound fiscal governance structure rules out that asymmetric QE purchases can come together with adverse incentive effects for governments.

Our paper connects to related literature from various perspectives. First, an incomplete monetary union, in particular when characterised by a weak fiscal governance structure, involves in addition non-trivial strategic considerations (related, inter alia, to incentive effects and risk-sharing modalities). The various safeguards of the ECB's PSPP (Public Sector Purchase Programme) adopted in January 2015 incorporate such considerations.[6] To address them in an extension of the analysis of this paper is left for future work. For generic treatments of strategic aspects see Chari and Kehoe (2008), Cooper and Kempf (2004) and Farhi and Tirole (2016).[7] Second, our model shares with Benigno (2004) the motivation to understand differences of monetary policy responses between symmetric and asymmetric monetary union specifications. Yet, in order to preserve the analytical tractability of our model in spite of the challenges arising from the lower bound constraint, we do not address fully optimal specifications of monetary policy. Instead, we take outcomes from an unconstrained interest rate rule as benchmarks and establish conditions under which these outcomes can be replicated via appropriately sized QE purchases when the lower bound becomes binding. Complementing our analysis, Bletzinger (2017) studies features of fully optimal monetary policy above the lower bound in a monetary union with portfolio adjustment costs. Third, we do not explore fiscal policy options to mitigate the lower bound constraint in asymmetric monetary unions, as done in Blanchard et al. (2017). Fourth, to operationalise the notion of a sound fiscal governance structure, we assume, for simplicity, that governments follow a credible feedback rule which preserves fiscal sustainability at the going price level, in line with the notion of a passive fiscal policy advanced by Leeper (1991). This feature allows us to abstract from the possibility of sovereign default. However, a fiscally complete monetary union may well allow for orderly procedures for the restructuring of national sovereign debt, as advocated by CEPR (2018).[8] Fifth, the idea to consider replications of unconstrained outcomes in extensions of the New Keynesian model is related to Wu and Zhang (2017). In this paper, dynamics of the IS-curve are driven by the short-term interest rate and unconstrained outcomes can be replicated when this rate is replaced by an appropriate shadow rate (which captures unconventional measures and can become negative). Eggertsson et al. (2017) consider an extension with banks and storage costs for money in which dynamics are driven by the deposit rate received by savers. Similar to our paper the ability of the central bank to improve outcomes is bounded by the deposit rate, but there is no distinction between short- and long-term assets.[9] Sixth, we do no touch on issues related to global dynamics and multiple equilibria in the vicinity of the lower bound, as done by Benhabib et al. (2001) and Mertens and Ravn (2014). Finally, our paper contributes to the large literature on the effectiveness of standard

[6]For details see the accounts of the Governing Council meeting in European Central Bank (2015).

[7]For discussions of policy aspects specific to the euro area, see Reis (2016) and Orphanides (2017).

[8]Depending on the degree of political integration, fiscally complete monetary unions can be characterised by much richer specifications. For a recent discussion, see, for example, Farhi and Werning (2017).

[9]See Brunnermeier and Koby (2017) for reversal effects in the derivation of the effective lower bound.

and non-standard monetary policies, as summarised, for example, by Woodford (2012).[10] In line with our focus, den Haan (2016) offers a summary of studies on the effectiveness of QE, distinguishing, in particular, between studies which allow for the existence of a portfolio balance channel or not.[11] Our analysis also speaks to papers which attempt to disentangle effects which can be attributed purely to forward guidance (as opposed to other channels, including the portfolio balance channel).[12]

The remainder of the paper is structured as follows. Section 2 summarises the model. Section 3 presents results for a symmetric monetary union, while Section 4 addresses an asymmetric monetary union. Section 5 closes with concluding remarks. Technical material is delegated to the Appendix.

## 2 The model

Our model extends a standard New Keynesian two-country framework of a monetary union, in line with Benigno (2004), into three dimensions.[13] First, to introduce the notion of a portfolio balance channel, we introduce banking systems in the two countries which face in their portfolio decisions imperfect substitutability between bonds of different maturities and country origin. Second, we consider richer specifications of monetary and fiscal policies, by allowing for short- and long-term government bonds and a detailed characterisation of the sources and the distribution of central bank income in a two-country monetary union. Third, we assume that the lower bound constraint on the short-term interest rate controlled by the central bank can occasionally be binding, leading to specifications of monetary policy rules which go beyond standard interest rate rules and allow for QE-type central bank purchases of government debt. In particular, the portfolio balance channel allows the central bank to conduct unconventional policies across the monetary union via purchases of long-term debt. This channel goes beyond what standard monetary policy can achieve which operates via adjustments in the single short-term interest rate which uniformly applies to all short-term debt issued by the two governments.

The model economy consists of two countries, each consisting of households, firms, banks and a government. Moreover, both countries have a common central bank which conducts monetary policy across the monetary union. Households consume domestic and imported goods, save in the form of deposits and money holdings, and work. Firms employ domestic labour for production and face nominal rigidities when setting prices. Banks act as portfolio managers of domestic households and invest their deposits in non-monetary assets, i.e. short- and long-term debt issued by the domestic and foreign governments.

The two countries feature the same general structure. For this reason, our presentation refers

[10]From a broader perspective, QE-type policies, may suffer from limitations. For example, Cúrdia and Woodford (2011) depart from the representative agent set-up and stress the disruptive role of financial frictions in the intermediation process. As a result, the analysis naturally favours targeted credit easing policies as opposed to broad-based QE-type interventions.

[11]For a rationalisation of the portfolio balance channel via safety premia associated with short-term government debt, see Krishnamurthy and Vissing-Jorgensen (2012).

[12]In particular, see the literature inspired by the forward guidance puzzle, as identified by Del Negro et al. (2012). For recent contributions, see, for example, McKay et al. (2016).

[13]We follow the modelling approach by Bletzinger (2017) who analyses fully optimal government bond purchases in a monetary union above the lower bound constraint.

mostly only to one country, labelled $N$. With the exception of the terms of trade all other variables are defined symmetrically for the other country, labelled $S$. The monetary union is populated by a continuum of identical households, with a constant share $\alpha$ living in $N$ and the remaining share $1-\alpha$ living in $S$, implying that each of the two countries is characterised by a single representative household. If not otherwise stated, variables with a country-superscript such as $x^N$ denote country per capita values of that variable. Moreover, any country-specific nominal variable $X^N$, when deflated by the country-specific consumer price index $P_c^N$, is denoted by a lower-case letter, thus $x^N \equiv \frac{X^N}{P_c^N}$. Union-wide nominal variables are deflated with the union-wide consumer price index $P_c$. The model is solved in a log-linear version around a zero-inflation steady state. Variables with a hat denote percentage deviations from steady-state values (indicated by a bar), that is $\hat{x}_t \equiv \ln(x_t) - \ln(\bar{x}) \approx \frac{x_t - \bar{x}}{\bar{x}}$. Variables with a tilde denote level deviations from steady-state values, that is $\tilde{x}_t \equiv x_t - \bar{x}$. The following subsections motivate and derive the equilibrium conditions for each agent.

## 2.1 Households

The representative household in country $N$ obtains utility from overall consumption $c^N$ and real money balances $\frac{M^N}{P_c^N}$, and disutility from hours worked $h^N$. The country-specific consumer price index is given by $P_c^N$. The optimisation problem is given by:

$$\max_{c_t^N, h_t^N, M_t^N, D_t^N} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \phi_t^N \left[ \frac{\left(c_t^N - \varsigma c_{t-1}^N\right)^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \frac{\left(h_t^N\right)^{1+\psi}}{1+\psi} + \frac{\chi_m^{-1}}{1-\sigma_m^{-1}} \left(\frac{M_t^N}{P_{c,t}^N}\right)^{1-\sigma_m^{-1}} \right]$$

The utility function exhibits constant relative risk aversion with $\sigma > 0$ determining the elasticity of inter-temporal substitution, $\varsigma \in [0,1]$ denoting the degree of habit formation in consumption, $\psi > 0$ the Frisch elasticity of labour supply and $\sigma_m > 0$ the interest elasticity of money demand. $\phi^N$ denotes a country-specific inter-temporal preference shock to the otherwise constant discount factor $\beta$. Every period, the household's disposable income consists of wage income $W^N h^N$, income earned on one-period interest-bearing deposits $R_D^N D^N$, money holdings $M^N$ carried over from the previous period and a lump-sum payment $\Gamma^N$ consisting of transfers from the fiscal authority as well as domestic profits from firms and banks (see Appendix A.2 for an exact specification of $\Gamma^N$). This income is used to finance overall consumption $c^N$ at price $P_c^N$, new money holdings and new deposits held with banks. The budget constraint for the household expressed in nominal terms is:

$$D_t^N + M_t^N + P_{c,t}^N c_t^N = R_{D,t-1}^N D_{t-1}^N + M_{t-1}^N + W_t^N h_t^N + \Gamma_t^N \tag{1}$$

The gross inflation rate of consumer prices in country $N$ is defined as $\Pi_{c,t+1}^N \equiv \frac{P_{c,t+1}^N}{P_{c,t}^N}$. Given that the labour market and the market for deposits are separated between countries, both wage and interest rates have a country-specific index $N$. Appendix A.2 lists the full set of the household's optimality conditions.[14] In the main part of the paper, in order to simplify the derivation of our theoretical results below, we focus on the interior optimality conditions without habit formation, which in linearised form are given by:[15]

$$\hat{c}_t^N = \hat{c}_{t+1}^N - \sigma(\hat{R}_{D,t}^N - \hat{\pi}_{c,t+1}^N - \hat{r}_{n,t}^N) \tag{2}$$

[14]Appendix A.2 shows the optimality conditions with habit formation in consumption. This feature creates more realistic impulse responses when illustrating our results below.

[15]The expectation operator is dropped in the log-linear equations to simplify the notation. Any variable with a $t+1$ time index is not known in period $t$ and is therefore treated as an expectation.

$$\psi \hat{h}_t^N = \hat{w}_t^N - \frac{1}{\sigma}\hat{c}_t^N \tag{3}$$

$$\hat{m}_t^N = \frac{\sigma_m}{\sigma}\hat{c}_t^N - \frac{\sigma_m \beta}{1-\beta}\hat{R}_{D,t}^N \tag{4}$$

The natural rate of interest is defined as $\hat{r}_t^N \equiv -(\hat{\phi}_{t+1}^N - \hat{\phi}_t^N)$ and follows an exogenous autoregressive process of the form

$$\hat{r}_{n,t}^N = \rho_n \hat{r}_{n,t-1}^N + \varepsilon_{n,t}^N \tag{5}$$

with $\rho_n \in (0,1)$ and $\varepsilon_{n,t}^N$ being white noise. The three optimality conditions represent the Euler condition for the optimal inter-temporal allocation of consumption, the intra-temporal optimality condition characterising the trade-off between work and consumption, and the intra-temporal optimality condition that sets the marginal rate of substitution between real money balances and consumption equal to the opportunity cost of holding money, respectively.

Following the literature on open economy models as in Obstfeld and Rogoff (1995, 2000) and monetary union versions of such models like Benigno (2004) and Ferrero (2009), the overall consumption bundle $c^N$ consumed by the household results from a two-stage Dixit-Stiglitz aggregation which allows for home bias. First, the bundle is defined as a combination of domestic and foreign (imported) consumption bundles which are, in a second step, each made up of differentiated goods produced in the respective country. The elasticity of substitution between the two countries is determined by $\eta > 0$ and the elasticity across differentiated goods within the same country by $\varepsilon > 0$. The home bias in consumption is given by the country-specific parameter $\lambda_N$.[16] The detailed derivation in Appendix A.1 confirms that consumer price inflation is simply a weighted average of producer price inflation, defined as $\Pi_{p,t+1}^N \equiv \frac{P_{p,t+1}^N}{P_{c,t}^N}$, of the two countries:

$$\hat{\pi}_{c,t}^N = \lambda_N \hat{\pi}_{p,t}^N + (1-\lambda_N)\hat{\pi}_{p,t}^S \tag{6}$$

## 2.2 Firms

In each country there exists a continuum of firms that face monopolistic competition and set the price of their differentiated product subject to a demand equation. Nominal price rigidity is introduced by means of quadratic adjustment costs (Rotemberg, 1982). Because the model does not feature capital, firms only employ labour $h(n)$, which is used in the production function

$$y(n) = ah(n) \tag{7}$$

where $a$ is an exogenous productivity parameter, used to calibrate the steady-state output. The optimisation problem for firm $n$ in country $N$ is given by

$$\max_{P_t(n)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Delta_t^N}{P_{c,t}^N}\left[P_t(n)y_t(n) - W_t^N h_t(n) - \frac{\chi}{2}\left(\frac{P_t(n)}{P_{t-1}(n)} - 1\right)^2 P_{p,t}^N y_t^N\right]$$

where $\Delta_t^N$ is a stochastic discount factor related to the marginal consumption of households who are the ultimate owners of firms. The first-order condition, as confirmed in Appendix A.2,

[16] For a symmetric specification of the home bias in consumption, we assume $1-\lambda_S = \frac{\alpha}{1-\alpha}(1-\lambda_N)$.

results in the New Keynesian Phillips curve augmented with the terms of trade, $T \equiv \frac{P^S_{p,t}}{P^N_{p,t}}$, which creates a link between the two countries:

$$\hat{\pi}^N_{p,t} = \beta \hat{\pi}^N_{p,t+1} + \frac{\varepsilon - 1}{\chi}\left[\hat{w}^N_t - \hat{a}^N_t + (1-\lambda_N)\hat{T}_t\right] \tag{8}$$

$$\hat{T}_t = \hat{T}_{t-1} + \hat{\pi}^S_{p,t} - \hat{\pi}^N_{p,t} \tag{9}$$

### 2.3 Banks

Given that banks in each country are identical and that we assume perfectly competitive financial sectors, we focus on one representative bank per country which accepts deposits from domestic households and invests them in the most profitable way. When choosing between short-term and long-term bonds (with the latter being modelled as consols) the bank faces quadratic portfolio balance costs. These costs may capture regulatory features or, alternatively, a certain preference structure of bank customers. The latter is known as preferred habitat preferences as proposed by Vayanos and Vila (2009). Without being more specific on the most suitable micro-foundation, it is important to realise that these costs break the perfect substitutability paradigm of financial assets. They are the key factor for unconventional monetary policy being able to affect the real economy through the portfolio balance channel. Moreover, holdings of long-term debt of the two countries are considered as imperfect substitutes by banks. We model this by introducing the same type of portfolio adjustment costs between domestic and foreign consols as between short- and long-term debt.

The balance sheet of the bank is made up of domestic deposits from households $D^N$ on the liability side and of short-term $B^N_{SP}$ and long-term $B^N_{LP}$ debt holdings on the asset side. In nominal terms:

$$D^N_t = B^N_{SP,t} + B^N_{LP,t} \tag{10}$$

Financial integration allows for cross-holdings of bonds issued in the two countries. Therefore, both short- and long-term private holdings consist of domestic and foreign bonds:

$$B^N_{SP,t} = B^N_{SD,t} + B^N_{SF,t} \tag{11}$$

$$B^N_{LP,t} = B^N_{LD,t} + B^N_{LF,t} \tag{12}$$

Like firms, banks face adjustment costs which imply a loss of resources, measured in terms of domestic output. The profit maximisation problem of the representative bank can be stated as a per-period optimisation problem in period $t$:

$$\max_{D^N_t, B^N_{SP,t}, B^N_{LD,t}, B^N_{LF,t}} \mathbb{E}_t[R_{S,t}B^N_{SP,t} + R^N_{L,t+1}B^N_{LD,t} + R^S_{L,t+1}B^N_{LF,t} - R^N_{D,t}D^N_t - \frac{\nu_1}{2}\left(\delta\frac{B^N_{SP,t}}{B^N_{LP,t}} - 1\right)^2 P^N_{p,t} - \frac{\nu_2}{2}\left(\frac{\omega_N}{1-\omega_N}\frac{B^N_{LF,t}}{B^N_{LD,t}} - 1\right)^2 P^N_{p,t}]$$

subject to the balance sheet identities (10), (11) and (12). Gross rates of return of the various assets between periods $t$ and $t+1$ are denoted by the corresponding values of $R$. A subscript $t$ denotes a fixed nominal rate of return which is known when the investment is decided in period $t$. Accordingly, the $t+1$ time index on country-specific long-term returns $R_L$ indicates that the bank, when investing in long-term bonds, optimises given *expected* long-term interest rates, while realised returns on long-term bonds are subject to one-off revaluation effects (as we will further clarify below). On the contrary, short-term bonds have a known return and they are considered as perfect substitutes, since their return is given by the single short-term interest rate $R_S$ set by the central bank. Let $\delta = \frac{\bar{B}^N_{LP}}{\bar{B}^N_{SP}} > 0$ denote the steady-state ratio of total private long-term bonds to total private short-term bonds. Moreover, the steady-state share of domestic long-term bonds in all privately held long-term bonds is given by $\omega_N = \frac{\bar{B}^N_{LD}}{\bar{B}^N_{LP}} \in (\frac{1}{2}, 1)$. Any deviation from these steady-state shares is costly due to the presence of quadratic portfolio adjustment costs. The parameters $\nu_1 \geq 0$ and $\nu_2 \geq 0$ determine the size of these costs for short- and long-term and domestic and foreign deviations, respectively.

### 2.3.1 Optimality conditions

The first-order interior optimality conditions, as derived in Appendix A.2, create relationships between the different rates of return and the relevant portfolio shares. In particular, extending the findings of Harrison (2012) to a two-country model, the linearised interest rate on deposits can be written as a linear combination of short- and long-term rates:

$$\hat{R}^N_{D,t} = \frac{1}{1+\delta}\hat{R}_{S,t} + \frac{\delta\omega_N}{1+\delta}\hat{R}^N_{L,t+1} + \frac{\delta(1-\omega_N)}{1+\delta}\hat{R}^S_{L,t+1} \tag{13}$$

This equation is central for our findings. The deposit rate, which is an average of the short-term policy rate and long-term rates, is from the perspective of households the relevant opportunity cost measure of holding real money balances. Thus, when conventional monetary policy in the form of the short-term interest rate is constrained due to the lower bound, the central bank can still stimulate the economy by decreasing long-term rates and thus the deposit rate. Yet, this works only as long as the deposit rate itself has not yet reached its own lower bound constraint, since households will cease to hold deposits if they become dominated in return by real money balances, leading to the constraint on implementable gross deposit rates

$$R^N_{D,t} \geq 1 \quad \forall\, t. \tag{14}$$

Moreover, the relationships between deposit and short-term rates as well as between domestic and foreign long-term rates satisfy

$$\hat{R}^N_{D,t} = \hat{R}_{S,t} + \tilde{\nu}_1\left[\hat{b}^N_{LP,t} - \hat{b}^N_{SP,t}\right] \tag{15}$$

$$\hat{R}^N_{L,t+1} = \hat{R}^S_{L,t+1} + \tilde{\nu}_2\left[\hat{b}^N_{LD,t} - \hat{b}^N_{LF,t}\right] \tag{16}$$

where we define $\tilde{\nu}_1 \equiv \frac{\nu_1\beta\delta}{\bar{b}^N_{LP}} = \frac{\nu_1\beta}{\bar{b}^N_{SP}}$ and $\tilde{\nu}_2 \equiv \frac{\nu_2\beta}{\omega_N(1-\omega_N)\bar{b}^N_{LP}}$. These optimality conditions illustrate the imperfect substitutability of the corresponding financial assets. In particular, in the spirit of Tobin and Brainard (1963), there exists a positive relationship between relative returns and relative portfolio shares of privately held bonds.

### 2.3.2 Home bias and symmetric vs. asymmetric transmission channels

In the equations presented so far the variables $\delta$, $\tilde{\nu}_1$ and $\tilde{\nu}_2$ carry no country-specific index. This can be rationalised as the outcome of a particular choice of assumptions which ensure that in the two countries monetary policy works through symmetric transmission channels, defining a benchmark pattern of *symmetric structures*.

To obtain symmetric structures, we use three assumptions. First, we assume that in the steady state the capital accounts both for short-term and long-term bonds are balanced. Second, we abstract from home bias in short-term bond holdings. This simplification helps to overcome the a priori indeterminate breakdown of private short-term bonds in the portfolios of domestic and foreign banks (in view of the identical return on these bonds). Hence, we assume that in all periods domestic short-term bonds are held according to country size, i.e. $B^N_{SD,t} = \alpha B^N_{SP,t}$. When combined with the assumption of a balanced capital account, this ensures that the steady-state per capita value of real short-term privately held debt $\bar{b}^N_{SP}$ will be the same for both countries.[17] Third, we allow for symmetric home bias in long-term bond holdings.[18] This is done via the separate parameter $\omega_N$ which fixes the steady-state distribution (while transitory deviations from this are possible via the above stated optimality conditions). For a symmetric transmission, we assume that this parameter, when correcting for country size, is equal in the two countries, i.e.

$$\alpha(1-\omega_N) = (1-\alpha)(1-\omega_S) \tag{17}$$

When combined with the assumption of a balanced capital account, this assumption ensures that the steady-state per capita value of real long-term privately held debt $\bar{b}^N_{LP}$ will be the same for both countries.[19] In sum, these features ensure that under symmetric structures the variables $\delta$, $\tilde{\nu}_1$, and $\tilde{\nu}_2$ carry no country-specific index.

By contrast, country-specific (i.e. asymmetric) transmission channels of monetary policy arise if one allows, for example, for different degrees of home bias in long-term bond holdings and relaxes the symmetry assumption (17). This implies that the variables $\delta$ and $\tilde{\nu}_2$ become country-specific (i.e. $\delta_N \neq \delta_S$ and $\tilde{\nu}_{2N} \neq \tilde{\nu}_{2S}$). The implications of such pattern of *asymmetric structures* will be addressed in Section 4.2.

## 2.4 Fiscal policy

Given the monetary union structure of our model it is important to clearly distinguish between the policy contributions coming from fiscal and monetary policy-makers. In any period, the government (i.e. the fiscal authority operating at the country level) has to fund the service of outstanding debt and lump-sum transfers to domestic households, $P^N_c \tau^N$. For simplicity, government expenditure is assumed to be zero. Similar to Harrison (2012), funding takes place through risk-less nominal one-period bonds $B^N_{SG}$ and nominal consols $B^N_{consols}$ with value $V^N$. Moreover, the government receives a certain amount of seigniorage $S^N$ from the common central bank. Consols have an infinite maturity and pay a fixed coupon of one nominal currency unit per period. Holders of consols bear a risk of capital gains or losses since $V^N$ is not known in advance.

[17] In real terms the steady-state capital account for short-term debt, assuming the same price level, will be balanced if $\alpha\bar{b}^N_{SF} = (1-\alpha)\bar{b}^S_{SF}$. The absence of home bias for short-term debt implies $\bar{b}^N_{SF} = (1-\alpha)\bar{b}^N_{SP}$ and $\bar{b}^S_{SF} = \alpha\bar{b}^S_{SP}$. Combining these expressions yields $\bar{b}^N_{SP} = \bar{b}^S_{SP}$.

[18] We do not explicitly model the home bias in long-term bonds with a CES function as we do for consumption. Yet, our choice of the steady-state share of bond holdings can be motivated with such a function in mind. For the log-linear model dynamics, using a CES formulation or not is not essential.

[19] In real terms the steady-state capital account for long-term debt, assuming the same price level, will be balanced if $\alpha\bar{b}^N_{LF} = (1-\alpha)\bar{b}^S_{LF}$. Home bias for long-term debt implies $\bar{b}^N_{LF} = (1-\omega_N)\bar{b}^N_{LP}$ and $\bar{b}^S_{LF} = (1-\omega_S)\bar{b}^S_{LP}$. Combining these expressions yields $\alpha(1-\omega_N)\bar{b}^N_{LP} = (1-\alpha)(1-\omega_S)\bar{b}^S_{LP}$. Invoking the symmetry asumption (17) implies $\bar{b}^N_{LP} = \bar{b}^S_{LP}$.

For the fiscal authority this risk only materialises if it changes the number of outstanding consols. The government's flow budget constraint in period $t$ in nominal terms is given by

$$B^N_{SG,t} + V^N_t B^N_{consols,t} + S^N_t = R_{S,t-1} B^N_{SG,t-1} + (1 + V^N_t) B^N_{consols,t-1} + P^N_{c,t} \tau^N_t$$

The outstanding nominal value of consols equals $B^N_{LG} \equiv V^N B^N_{consols}$ and the ex post nominal return is

$$R^N_{L,t} \equiv \frac{1 + V^N_t}{V^N_{t-1}} \tag{18}$$

These definitions allow us to rewrite the government budget constraint as

$$B^N_{SG,t} + B^N_{LG,t} + S^N_t = R_{S,t-1} B^N_{SG,t-1} + R^N_{L,t} B^N_{LG,t-1} + P^N_{c,t} \tau^N_t \tag{19}$$

where the different time indices associated with short-term and long-term interest rates capture the main difference between short-term and long-term debt, i.e. from the perspective of the representative period $t$ the return on outstanding short-term debt is predetermined, while the return on long-term debt is subject to one-off revaluation effects. We assume that the government will keep the real debt structure constant, using the rule:

$$\hat{b}^N_{LG,t} = \hat{b}^N_{SG,t} \tag{20}$$

Short-term government debt is the residual in the budget constraint and thus absorbs any remaining fluctuations. In order to curb these fluctuations, fiscal transfers to households follow a simple feedback rule which reacts to the short-term real interest rate and debt stock. The functional form which will be used in our simulations below is given by[20]

$$\tilde{\tau}^N_t = -\tilde{\theta} \left[ \hat{R}_{S,t-1} - \hat{\pi}^N_{c,t} + \hat{b}^N_{SG,t-1} \right] \tag{21}$$

with $\tilde{\theta} \equiv \frac{\theta \bar{b}^N_{LP}}{\delta} = \theta \bar{b}^N_{SP} > 0$, where we assume a parameterisation which supports a standard assignment, consistent with a passive fiscal policy in the sense of Leeper (1991). Equation (21) ensures that in response to a shock the debt-to-GDP ratio will over time return to its steady-state value. It is not necessary to add long-term debt to this feedback rule, because the feedback is indirectly passed on from short-term to long-term debt via rule (20).

## 2.5 Monetary policy

The common central bank controls the short-term interest rate $R_S$ which is uniform across the union, implying that both governments face identical short-term funding costs. In line with standard New Keynesian specifications, we let the short-term interest rate respond to inflation and output. More specifically, the interest rate is set by a Taylor-type rule which responds to union-wide variables. In log-linear terms the policy rule, the aggregate inflation rate and the aggregate output deviation are given by:

$$\hat{R}_{S,t} = \rho_R \hat{R}_{S,t-1} + (1 - \rho_R)(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t) + \varepsilon^R_t \tag{22}$$

$$\hat{\pi}_t = \alpha \hat{\pi}^N_{c,t} + (1 - \alpha) \hat{\pi}^S_{c,t} \tag{23}$$

$$\hat{y}_t = \alpha \hat{y}^N_t + (1 - \alpha) \hat{y}^S_t \tag{24}$$

[20] Transfers are stated as level deviations in order not to be inconsistent with steady-state levels of zero.

with $\phi_\pi, \phi_y > 0$ and where the latter two equations are derived from $P = P_c^{N\alpha} P_c^{S1-\alpha}$ and $y = \alpha y^N + (1-\alpha) y^S$ with $\bar{y}^N = \bar{y}^S$, respectively. Whereas standard monetary policy has a symmetric design, unconventional monetary policy via active central bank bond purchases of long-term debt may be conducted asymmetrically across the union. We label nominal bond purchases by the central bank as $Q$ (in reminiscence of QE), with $Q_t = \alpha Q_t^N + (1-\alpha) Q_t^S$. The central bank may then, in principle, decide to follow a country-specific purchase rule of long-term government debt. In general, such rule, when linearised, can in real terms be written as:

$$\tilde{q}_t^N = f^N(.) + \varepsilon_{q,t}^N \tag{25}$$

The function $f^N(.)$ is zero in any well-behaved steady state (in which the lower bound constraint is not binding) and could for example be a reaction function similar to the interest rate rule. The design of such a central bank purchase rule of long-term debt is of central importance for our following analysis. Therefore, we will return to it below.

In the canonical New Keynesian model of a single closed economy the way money is injected into the economy and the associated seigniorage income is typically of no importance. In particular, real money balances enter only one optimality condition (equation (4)) and under standard assumptions their optimal level can be recursively determined without feeding back on the equilibrium values of other variables. Moreover, as long as seigniorage is transferred back to households in a lump-sum fashion (either via the government budget constraint or directly) the central bank profit resulting from money creation has no economic implication as a result of the Ricardian equivalence proposition. However, in a monetary union with a single central bank which may conduct country-specific bond purchases it is necessary to model the issuance of money, the central bank's balance sheet and the distribution of seigniorage more accurately for at least three reasons. First, Ricardian equivalence makes lump-sum payments irrelevant only at the union level. Yet, the distribution of lump-sum payments between countries can affect equilibrium allocations. It is therefore necessary to know how much money is held in each of the two countries. Second, the central bank sets a single short-term interest rate which, by assumption, applies uniformly to short-term debt in both economies. With a single price, there is a shortage of instruments for simultaneous market clearing of short-term debt at the country level. Thus, there is a need for a clearing mechanism at the union level. Essentially, this is offered by the central bank's balance sheet, reflecting that the central bank stands ready to buy or sell any amount of short-term debt, wherever issued, at the single short-term interest rate which is specified by its monetary policy rule.[21] If the countries were not part of a monetary union with a uniform conventional monetary policy this channel would be the nominal exchange rate. Third, long-term government bond purchases create another source of income for the central bank in addition to conventional seigniorage income resulting from standard operations. To track these various dimensions of monetary policy, Table 1 illustrates the central bank's balance sheet of our model.

As discussed, the implementation of conventional monetary policy is supported by an appropriate amount of short-term government bond holdings of the central bank. Unconventional monetary policy in the form of QE appears as long-term government bond holdings on the balance sheet. We abstract from reserve holdings of banks. Hence, all central bank asset purchases are funded

[21] This market clearing channel, in the context of the TARGET balances recorded for the Eurosystem, has recently received attention in the literature, going back, in particular, to Sinn and Wollmershäuser (2012). We come back to this in more detail in Section 2.6.1.

| Assets | | Liabilities | |
|---|---|---|---|
| Short-term bonds | $\alpha B_{SC}^N$ | Money in circulation | $\alpha M^N$ |
| | $(1-\alpha)B_{SC}^S$ | | $(1-\alpha)M^S$ |
| Long-term bonds | $\alpha Q^N$ | | |
| | $(1-\alpha)Q^S$ | | |

Table 1: Stylised balance sheet of the central bank in our monetary union

with the issuance of money which is ultimately held by households. The balance sheet identity corresponding to Table 1 is:

$$\alpha\left(B_{SC,t}^N + Q_t^N\right) + (1-\alpha)\left(B_{SC,t}^S + Q_t^S\right) = \alpha M_t^N + (1-\alpha)M_t^S \tag{26}$$

This equation only holds at the aggregate level for reasons discussed above. There is no reason why the money in circulation in one country needs to be always identical to the amount that has initially been created through purchases of short- and long-term bonds issued in this particular country. This degree of freedom is inherent to the definition of a monetary union.

In each period, the central bank is assumed to pay out any net interest income it earns on its assets, implying that the total amount of seigniorage transferred to the fiscal authority in $N$ can be written as:

$$\begin{aligned}\alpha S_t^N =& \left(1-(1-\alpha)\mu_1\right)\left(R_{S,t-1}-1\right)\alpha B_{SC,t-1}^N + \alpha\mu_1(R_{S,t-1}-1)(1-\alpha)B_{SC,t-1}^S \\ &+ \left(1-(1-\alpha)\mu_2\right)\left(R_{L,t}^N-1\right)\alpha Q_{t-1}^N + \alpha\mu_2(R_{L,t}^S-1)(1-\alpha)Q_{t-1}^S\end{aligned} \tag{27}$$

The first two elements of the sum on the right-hand side capture conventional seigniorage income which the central bank creates by implementing standard monetary policy via purchases of short-term government bonds $B_{SC}$. The other two terms capture the central bank's return on its long-term government bond holdings $Q$. Importantly, central bank profits generated from bond purchases in one country may be distributed to the other country. The parameters $\mu_1 \in [0,1]$ and $\mu_2 \in [0,1]$ control the degree of central bank income sharing among the fiscal authorities.[22] A value of 1 means full income sharing, whereas 0 implies no sharing at all.

## 2.6 General equilibrium

In general equilibrium, the decisions of households, firms and banks need to be individually optimal and consistent with each other via market clearing at the aggregate level, taking as given the behaviour of monetary and fiscal policy-makers and the evolution of exogenous shock processes. The market clearing conditions for the goods market, the short-term bond market and the long-term bond market are:

$$y_t^N = \lambda_N\left(\frac{P_{p,t}^N}{P_{c,t}^N}\right)^{-\eta} c_t^N + (1-\lambda_S)\left(\frac{P_{p,t}^N}{P_{c,t}^S}\right)^{-\eta}\frac{1-\alpha}{\alpha}c_t^S + \Xi_t^N \tag{28}$$

$$B_{SG,t}^N = B_{SD,t}^N + \frac{1-\alpha}{\alpha}B_{SF,t}^S + B_{SC,t}^N \tag{29}$$

[22]Even though *risk sharing* has recently obtained attention in the design of QE in the euro area, we deliberately abstain from that label as there is no risk of default in our model.

$$B^N_{LG,t} = B^N_{LD,t} + \frac{1-\alpha}{\alpha} B^S_{LF,t} + Q^N_t \tag{30}$$

The full sets of equilibrium conditions, both in non-linear and in log-linear form, including the definition of the losses of real resources via price and portfolio adjustments as captured via $\Xi^N$, are listed in Appendix A.2 and Appendix A.3, respectively. These conditions can be organised around a transparent analytical core consisting of a few equations. To derive this core it is important, in particular, to keep track of the open economy dimension of the model which leads to a number of linkages between the two countries.

### 2.6.1 Current account imbalances and financial linkages

Because of the open economy dimension of the model, the value of consumption in any of the two countries does not have to be equal to the value of domestic output (net of resource losses $\Xi$). There is rather scope for current account imbalances (restricted below to be transitory). Such imbalances can be funded in different ways. In particular, assuming financially integrated markets, short- and long-term bond markets allow for cross-ownership of bonds in the two countries' banking systems, as captured by the market clearing conditions (29) and (30), implying that there is scope to finance current account deficits via private capital imports. Alternatively, such deficits can be funded via the central bank's balance sheet.

To make this precise it is helpful to realise that Walras' law implies that consumption levels in the two countries, in sum, are constrained by the *combined* resource constraint of the two countries:

$$\alpha P^N_{c,t} c^N_t + (1-\alpha) P^S_{c,t} c^S_t = \alpha P^N_{p,t} \left[ y^N_t - \Xi^N_t \right] + (1-\alpha) P^S_{p,t} \left[ y^S_t - \Xi^S_t \right] \tag{31}$$

In order to track current account imbalances between the two countries within this combined constraint, we introduce the notation

$$P^S_{p,t} \Omega^S_t \equiv P^S_{c,t} c^S_t - P^S_{p,t} \left[ y^S_t - \Xi^S_t \right] \tag{32}$$

where $\Omega^S$ denotes in real terms the per capita difference between consumption and output (net of adjustments costs) in $S$, i.e. a positive value of $\Omega^S$ corresponds to a current account deficit of $S$. When combining the private sector budget constraint (1), the budget constraint of the government (19), the seigniorage contribution to the government's budget from the central bank (27) as well as marketing clearing via (29) and (30) it is possible to decompose $P^S_p \Omega^S$ into five distinct funding channels, that is:

$$\begin{aligned}
P^S_{p,t} \Omega^S_t \quad = \quad & \frac{\alpha}{1-\alpha} \left[ M^N_t - M^N_{t-1} - (B^N_{SC,t} - B^N_{SC,t-1}) - (Q^N_t - Q^N_{t-1}) \right] \\
& + \mu_1 \alpha (R_{S,t-1} - 1) \left[ B^N_{SC,t-1} - B^S_{SC,t-1} \right] \\
& + \mu_2 \alpha \left[ (R^N_{L,t} - 1) Q^N_{t-1} - (R^S_{L,t} - 1) Q^S_{t-1} \right] \\
& + \frac{\alpha}{1-\alpha} \left[ B^N_{SF,t} - R_{S,t-1} B^N_{SF,t-1} \right] - \left[ B^S_{SF,t} - R_{S,t-1} B^S_{SF,t-1} \right] \\
& + \frac{\alpha}{1-\alpha} \left[ B^N_{LF,t} - R^S_{L,t} B^N_{LF,t-1} \right] - \left[ B^S_{LF,t} - R^N_{L,t} B^S_{LF,t-1} \right]
\end{aligned} \tag{33}$$

By construction, any current account deficit of $S$ must be matched by a corresponding current account surplus of $N$. Equation (33) states that in period $t$, whenever the value of per capita consumption in $S$ exceeds the value of per capita output in $S$ net of adjustment costs, this gap can be linked to a number of distinct sources of funding, belonging to the five rows of equation (33):

1) If there is an increase in money holdings in $N$ which exceeds the increase in central bank holdings of (short- and long-term) debt issued in $N$.

2a) If ordinary seigniorage income of the central bank (earned on short-term bond holdings) is shared and a larger amount of this income is generated from bonds issued in $N$ than in $S$.

2b) If QE income of the central bank (earned on long-term bond holdings) is shared and a larger amount of this income is generated from bonds issued in $N$ than in $S$.

3a) If markets for short-term bonds are financially integrated and banks in $N$ buy more short-term debt issued in $S$ (net of redemptions) than vice versa.

3b) If markets for long-term bonds are financially integrated and banks in $N$ buy more long-term debt issued in $S$ (net of redemptions) than vice versa.

For further reference below, we group these sources of funding into three broader channels, namely 1) the central bank balance sheet (or TARGET) channel, 2) the seigniorage channel (of both ordinary and QE-related CB income) and 3) the channel of private capital imports (both short-term and long-term). All three channels are sources of a redistribution of resources between the two countries and they can only materialise in asymmetric constellations. This can be easily verified if one adds to equation (33) the corresponding equation for $N$, i.e.

$$P_{p,t}^N \Omega_t^N \equiv P_{c,t}^N c_t^N - P_{p,t}^N \left[ y_t^N - \Xi_t^N \right]$$

which leads to the combined resource constraint of the two countries in equation (31).

The role of these three funding channels can be made consistent with the conventional insight that a current account surplus of a country corresponds to an improvement in the position of net foreign assets. Recall that savings of households in any of the two countries consist of deposits and real money balances. Deposits represent claims of households which, through the balance sheet of the banking system, are invested in a private portfolio of domestic and foreign bonds. Similarly, real money balances can be interpreted as claims of households which, through the balance sheet of the central bank, are invested in a central bank portfolio of domestic and foreign bonds. In any period $t$, if savings of households in $N$ will be more strongly invested in foreign bonds than savings of households in $S$, this will contribute to an improvement in the position of net foreign assets of $N$. The central bank channel in a monetary union is special, in the sense that it can contribute to the funding of current account imbalances for a given central bank portfolio of domestic and foreign bonds. In other words, country $N$ can run a current account surplus if the desire of its households to hold more real money balances (and to consume less) matches a corresponding desire of households in $S$ to hold fewer real money balances (and to consume more). In the euro area, this mechanism to fund current account imbalances (i.e. a possible disconnect between the distribution of money holdings across countries and the distribution of central bank holdings of assets issued in the countries) is related to the TARGET balances. Conceptually, this mechanism is of interest because it is at work even in case of poorly integrated financial systems, as to be discussed in Section 4.

### 2.6.2 Analytical core of the equilibrium conditions

This reasoning can be used to establish an analytical core of the equilibrium conditions which consists of a block similar to the canonical closed economy New Keynesian model

$$\hat{c}_t^N = \hat{c}_{t+1}^N - \sigma\left[\hat{R}_{D,t}^N - \hat{\pi}_{c,t+1}^N - \hat{r}_{n,t}^N\right] \tag{34}$$

$$\hat{\pi}_{p,t}^N = \beta\hat{\pi}_{p,t+1}^N + \frac{\varepsilon-1}{\chi}\left[\psi\hat{y}_t^N + \frac{1}{\sigma}\hat{c}_t^N + (1-\lambda_N)\hat{T}_t\right] \tag{35}$$

$$\hat{R}_{S,t} = \rho_R\hat{R}_{S,t-1} + (1-\rho_R)\left[\phi_\pi\hat{\pi}_{c,t} + \phi_y\hat{y}_t\right] + \varepsilon_{R,t} \tag{36}$$

as well as of equations determining the various rates of return[23]

$$\hat{R}_{D,t}^N = \hat{R}_{S,t} + \tilde{\nu}_1\left[\hat{b}_{LP,t}^N - \hat{b}_{SP,t}^N\right] \tag{37}$$

$$\hat{R}_{L,t+1}^N = \hat{R}_{L,t+1}^S + \tilde{\nu}_2\left[\hat{b}_{LD,t}^N - \hat{b}_{LF,t}^N\right] \tag{38}$$

$$\hat{R}_{D,t}^N = \frac{1}{1+\delta}\hat{R}_{S,t} + \frac{\delta\omega_N}{1+\delta}\hat{R}_{L,t+1}^N + \frac{\delta(1-\omega_N)}{1+\delta}\hat{R}_{L,t+1}^S \tag{39}$$

and of a law of motion of the current account (as defined in Section 2.6.1)

$$\tilde{\Omega}_t^N = \hat{c}_t^N - \hat{y}_t^N + (1-\lambda_N)\hat{T}_t \tag{40}$$

where the expressions $\tilde{\Omega}_t^N$ and $\hat{T}_t$ capture key linkages between the two countries. Additional linkages for inflation, output and terms-of-trade developments are provided by the standard auxiliary equations

$$\begin{aligned}
\hat{\pi}_{c,t}^N &= \lambda_N\hat{\pi}_{p,t}^N + (1-\lambda_N)\hat{\pi}_{p,t}^S \\
\hat{T}_t &= \hat{T}_{t-1} + \hat{\pi}_{p,t}^S - \hat{\pi}_{p,t}^N \\
\hat{\pi}_{c,t} &= \alpha\hat{\pi}_{c,t}^N + (1-\alpha)\hat{\pi}_{c,t}^S \\
\hat{y}_t &= \alpha\hat{y}_t^N + (1-\alpha)\hat{y}_t^S
\end{aligned}$$

These linkages will become relevant below when we explore the effectiveness of standard and non-standard monetary policies in symmetric as opposed to asymmetric specifications of monetary unions.

## 2.7 Calibration

We solve our model, with all equations as summarised in Appendix A.2, around a zero-inflation steady state. To simplify the exposition, in the symmetric baseline scenario both countries are assumed to be identical. Hence, the country size is $\alpha = 0.5$. In the steady state, the terms of trade equal unity and the current account is balanced. Without loss of generality, per capita output is calibrated to be unity, that is $\bar{y}^N = \bar{y}^S = 1$. The log-linear system in A.3 reveals that only a few steady-state ratios need to be chosen in order to solve the model, including the ratio of privately held long-term to short-term government bonds $\delta$, the long-term debt to output ratio $\bar{b}_{LP}^N$ and the ratio of money balances to privately held short-term bonds $\bar{m}_b$. As summarised in Table 2 and in line with euro area data, we set $\delta = 3$, $\bar{b}_{LP}^N = 0.6$ and $\bar{m}_b = 0.3$. The parameter of the fiscal transfer rule is set to $\theta = 0.0328$. This value facilitates persistent movements of government debt outside the steady state, but it assures stationarity of the model, in line with

| Parameter | Value | Description |
|---|---|---|
| $\alpha$ | 0.5 | Relative country size of *North* |
| $\lambda_N$ | 0.8 | Home bias of consumption in *North* |
| $\omega_N$ | 0.7 | Home bias of long-term bond holdings in *North* |
| $\eta$ | 1.0 | Substitutability of domestic and foreign goods |
| $\beta$ | 0.9925 | Household discount factor |
| $\sigma$ | 6.0 | Elasticity of inter-temporal substitution |
| $\varsigma$ | 0.7 | Habit formation parameter in consumption |
| $\psi$ | 2.0 | Frisch elasticity of labour supply |
| $\sigma_m$ | 1.0 | Interest elasticity of money demand |
| $\varepsilon$ | 5.0 | Elasticity of substitution across goods |
| $\chi$ | 28.65 | Price adjustment cost parameter |
| $\nu_1$ | 0.0038 | Short-long portfolio balance cost parameter |
| $\nu_2$ | 0.0127 | Domestic-foreign portfolio balance cost parameter |
| $\theta$ | 0.0328 | Adjustment parameter in the fiscal transfer rule |
| $\mu_1$ | 1.0 | Degree of income sharing from ordinary seigniorage |
| $\mu_2$ | 0.0 | Degree of income sharing from bond purchases |
| $\phi_\pi$ | 1.5 | Inflation coefficient in the interest rate rule |
| $\phi_y$ | 0.5 | Output coefficient in the interest rate rule |
| $\rho_R$ | 0.5 | Smoothing parameter in the interest rate rule |
| $\rho_n$ | 0.85 | Smoothing parameter for the natural rate shock |
| $\bar{T}$ | 1.0 | Steady-state value of the terms of trade |
| $\bar{m}_b$ | 0.3 | Steady-state ratio of money to short-term bonds |
| $\bar{b}_{LP}^N$ | 0.6 | Steady-state ratio of long-term bonds to output |
| $\delta$ | 3.0 | Steady-state ratio of long- to short-term bonds |

Table 2: Calibrated parameters of the symmetric benchmark model

Leeper (1991). Conventional monetary policy is assumed to be active in line with a standard Taylor rule (with feedback coefficients of $\phi_\pi = 1.5$ and $\phi_y = 0.5$), augmented with a smoothing term of $\rho_R = 0.5$ in order to introduce sluggishness in the interest rate consistent with Smets and Wouters (2003, 2007). The discount factor of the representative household is set at $\beta = 0.9925$, implying for all interest rates an annualised steady-state value of 3.06 percent. The remaining parameters describing household preferences are set at $\sigma = 6$, $\varsigma = 0.7$, $\psi = 2$ and $\sigma_m = 1$, in line with Christiano et al. (2010) and Gerali et al. (2010). Moreover, we set the elasticities of substitution between countries and across differentiated goods to $\eta = 1$ and $\varepsilon = 5$, respectively. The slope of the Phillips curve is calibrated to $\chi = 28.65$ as estimated by Gerali et al. (2010). As concerns the crucial parameters which determine the portfolio adjustment costs, Harrison (2012) presents an analysis for various values of these costs between short- and long-term bonds. We choose a value of $\nu_1 = 0.0038$. This value, after an appropriate transformation, is close to his baseline calibration of 0.1 which he finds to be between empirical estimates of Andrés et al. (2004) and Bernanke et al. (2004). We set the adjustment costs between domestic and foreign bonds at $\nu_2 = 0.0127$ in order to obtain meaningful spread dynamics. The home bias in consumption is calibrated such that 20 percent of consumption is imported, implying $\lambda_N = 0.8$. For long-term bond holdings we allow a stronger integration with a home bias of only $\omega_N = 0.7$. As concerns the distribution of central bank profits across the union, we assume that ordinary

[23] As indicated above, in Section 4.2 the variables $\delta$ and $\tilde{\nu}_2$ become country-specific.

seigniorage is fully shared ($\mu_1 = 1$), while seigniorage related to QE is not ($\mu_2 = 0$), broadly in line with the practice of the Eurosystem. Finally, as concerns the dynamics of the shock to the natural rate of interest we choose a standard smoothing parameter of $\rho_n = 0.85$. The value of the shock $\varepsilon_n^N$ is specifically calibrated in each of the scenarios shown below.

# 3 Symmetric monetary union

To facilitate a transparent discussion of our findings it is instructive to start with a symmetric monetary union, by assuming that the two countries are in all aspects identical and face the same shocks. By construction, a symmetric monetary union will be isomorphic to a single closed economy, characterised by a much simplified analytical core, i.e. equations (34) - (40) reduce to

$$\begin{aligned}
\hat{c}_t^N &= \hat{c}_{t+1}^N - \sigma \left[ \hat{R}_{D,t}^N - \hat{\pi}_{c,t+1}^N - \hat{r}_{n,t}^N \right] \\
\hat{\pi}_{c,t}^N &= \beta \hat{\pi}_{c,t+1}^N + \frac{\varepsilon - 1}{\chi} (\psi + \frac{1}{\sigma}) \hat{c}_t^N \\
\hat{R}_{S,t} &= \rho_R \hat{R}_{S,t-1} + (1 - \rho_R) \left[ \phi_\pi \hat{\pi}_{c,t}^N + \phi_y \hat{c}_t^N \right] + \varepsilon_{R,t} \\
\hat{R}_{D,t}^N &= \hat{R}_{S,t} + \tilde{\nu}_1 \left[ \hat{b}_{LP,t}^N - \hat{b}_{SP,t}^N \right]
\end{aligned}$$

where we have used $\hat{\pi}_{p,t}^N = \hat{\pi}_{c,t}^N$ as well as $\hat{T}_t = \tilde{\Omega}_t^N = 0$, implying $\hat{y}_t^N = \hat{c}_t^N$, i.e. output and consumption dynamics in each country become tightly linked. Moreover, we have exploited that in a symmetric constellation, by construction, $\hat{R}_{L,t+1}^N = \hat{R}_{L,t+1}^S$ needs to be satisfied.[24] This reduced analytical core of a symmetric monetary union does not only reveal the special role played by the portfolio balance channel, but it also nests the reference New Keynesian model in the spirit of Woodford (2003) via a single parameter restriction as a special case. This insight helps to analyse the effectiveness of standard and non-standard monetary policies in the general setting.

Throughout, we consider constellations in which the natural rate of interest, which follows the exogenous law of motion (5), unexpectedly experiences a negative shock. This triggers a demand-driven recession. Moreover, for sufficiently large shocks the economy will be driven to the lower bound constraint on the short-term interest rate.

## 3.1 The model without the portfolio balance channel: a special case

In line with the closed economy analysis of Harrison (2012), the only difference between the general setting and the reference New Keynesian model is the presence of more than one interest rate as a result of portfolio adjustment costs. In other words, in the special case where these costs are assumed to be absent (i.e. $\tilde{\nu}_1 = 0$), the interest rates on deposits and short-term bonds become identical and the first three equations exactly resemble the canonical three equations of the reference New Keynesian model, i.e. the IS curve, the New Keynesian Phillips curve and a conventional Taylor-type interest rate rule. These equations form a well-analysed dynamic system with the potential to determine output, inflation and the short-term interest rate. In this system, the lower bound constraint on the nominal short-term interest rate corresponds to

[24] In other words, in equilibrium there is no scope for portfolio rebalancing costs between domestic and foreign bonds. Moreover, equation (39) reduces to $\hat{R}_{D,t}^N = \frac{1}{1+\delta} \hat{R}_{S,t} + \frac{\delta}{1+\delta} \hat{R}_{L,t+1}^N$. This expression can be residually used to determine the newly contracted long-term rate $\hat{R}_{L,t+1}^N$ for given values of $\hat{R}_{D,t}^N$ and $\hat{R}_{S,t}$.

a level of zero (one) of the net (gross) rate, reflecting that the short-term interest rate measures the opportunity cost of holding real money balances.

Moreover, consider a sufficiently large shock which drives the economy to the lower bound constraint. It is well-known that in this situation unconventional monetary policy via QE-type central bank purchases of long-term debt is completely ineffective. The reason for this is that short-term and long-term assets act as perfect substitutes, implying that via the no-arbitrage condition all assets simultaneously reach the lower bound constraint. However, as shown by Eggertsson and Woodford (2003), there exists an alternative channel which can restore a certain effectiveness of monetary policy. This channel works via forward guidance and it relies on the commitment of the central bank to keep future policy rates lower for longer when the lower bound constraint ceases to be binding.

## 3.2 The model with the portfolio balance channel

In general, when portfolio adjustment costs are present (i.e. $\tilde{\nu}_1 > 0$), the implied imperfect substitutability between short-term and long-terms bonds creates spreads between interest rates as evidenced by equation (37). The deposit rate, as revealed by equation (34), becomes the single most relevant interest rate for the dynamics of the extended New Keynesian economy with portfolio adjustment costs, i.e. it drives via the consumption Euler equation the paths of consumption, savings and hours worked. Importantly, the deposit rate evolves over time as a weighted average of short- and long-term interest rates, in line with equation (13). While the deposit rate is subject to a lower bound constraint of zero (reflecting that now the deposit rate measures the opportunity cost of holding real money balances), this is mechanically no longer true for the short-term interest rate. Simple model extensions could achieve this (for example, by allowing banks to hold money as an alternative to short-term bonds), but it would be equally possible to establish a negative lower bound for the short-term interest rate (for example, by assuming that banks face some storage cost when holding money, in the spirit of Eggertsson et al. (2017)). Without loss of generality, we consider a lower bound value of zero for simple comparability with the reference New Keynesian model.

### 3.2.1 Outcomes above the lower bound

Figure 1 illustrates the effect of a negative shock to the natural rate. The shock, starting out from a steady-state constellation, is assumed to get realised in period $t = 5$. Due to the symmetry assumption, we only show responses for one country. All inflation and interest rates in Figure 1 are shown as annualised net nominal levels in percent. All other variables are presented as percentage deviations from their steady-state values. On impact, the shock favours savings at the expense of consumption. This triggers a demand-driven recession, characterised by a joint drop in output and inflation which calls, according to the conventional interest rate rule, for a reduction of the short-term nominal interest rate. By assumption the shock depicted in Figure 1 is sufficiently small such that the lower bound constraint is not reached. In response to the reduced short-term rate banks rebalance their portfolios towards long-term bonds, leading to a decline in long-term rates to be contracted from period $t = 5$ onwards. Reflecting the existence of portfolio adjustments costs, long-term rates fall by less than short-term rates. Moreover, we observe a decline in deposit rates (reflecting a weighted combination of the decline in short- and long-term rates) and monetary policy is effective in stimulating the economy to the extent that it induces a decline in the expected *real* rate on deposits (as given by the difference between the

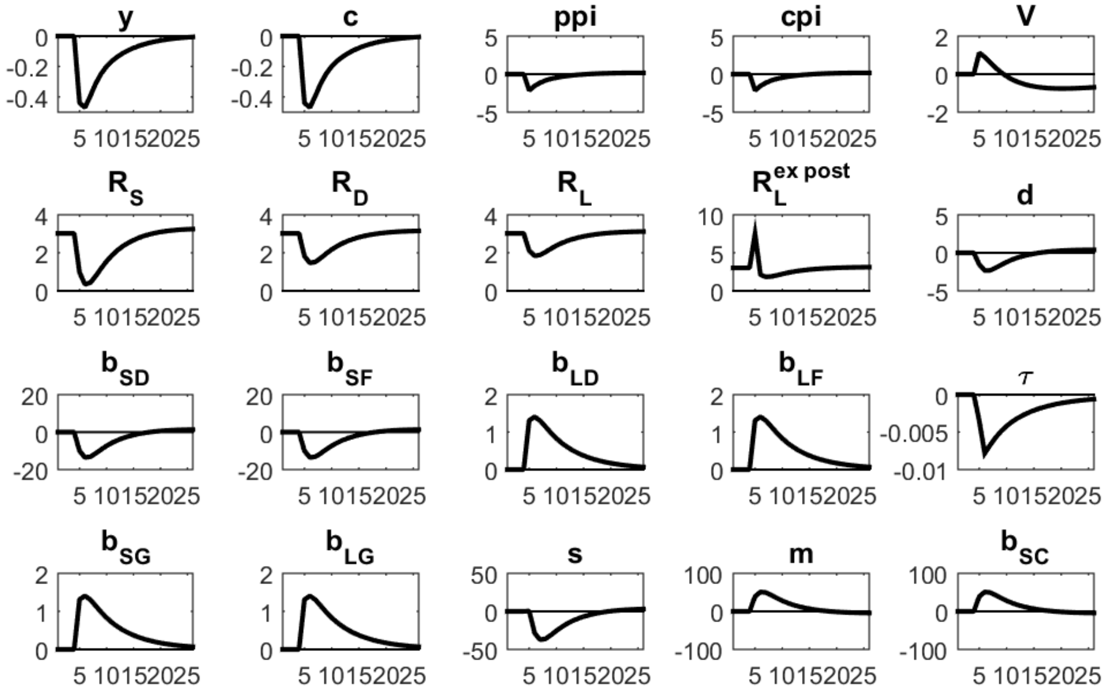


Figure 1: Impulse responses to a *symmetric* negative demand shock to the natural rate in a *structurally symmetric* monetary union. All variables are shown as percentage deviations from their steady-state values, with the exception of inflation and interest rates which are transformed into annualised net nominal levels in percent.

responses of the deposit rate and inflation).

It is worth pointing out that households, on impact, have two options to reduce consumption out of current income. First, they increase their holdings of real money balances, consistent with the decline of the deposit rate (which captures the opportunity cost of holding money balances). The increase in real money balances is accompanied by a corresponding increase in central bank purchases of short-term government debt. Second, households will attempt to hold more deposits. Yet, since the set-up is isomorphic to a single closed economy there is no channel, in the absence of investment, how the additional demand for deposits could at least partly stabilise aggregate demand. As a result of this feature, the decline in consumption is bound to trigger an equiproportionate decline in output. Deposits may fall as well (as is the case in Figure 1).

Finally, as concerns the fiscal side of the model, in response to the shock the government issues more short- and long-term debt (with the reaction bound to be equiproportionate because of (20)). This increase in government debt results from two reinforcing developments, to be inferred from the government budget constraint (19). On the one hand, when the shock gets realised (i.e. in period $t = 5$), the ex post long-term rate, which is relevant for interest payments on outstanding long-term government debt, increases since the price of consols will be bidden up in the rebalancing of bank portfolios (see equation (18)). This one-off effect needs to be funded. On the other hand, seigniorage revenue declines since the central bank receives lower interest income on its assets. Over time, however, government debt levels return to their steady-state levels, in view of stabilising transfers (which react with a lag of one period) as given by the passive feedback rule (21).

### 3.2.2 Replicating unconstrained outcomes if the lower bound is binding: the case of a small shock

Whenever the short-term interest rate is constrained at the lower bound the central bank can still ease monetary policy by exploiting the existence of spreads between long- and short-term interest rates. In particular, the central bank can reduce long-term rates via appropriate purchases of long-term debt, provided that the yield curve is not entirely flat. In such a constellation purchases of long-term debt have the potential to stimulate the economy because they lower the deposit rate. Hence, as long as the deposit rate remains positive, the central bank can stimulate the economy despite being constrained at the short end of the yield curve.

The degree to which monetary policy can be effective in such circumstances depends on the severity of the recessionary shock that initiates the crisis and drives the economy to the lower bound constraint. The relevant measure for the severity of the shock is given by the strength of the downward shift of the yield curve that would have prevailed under the unconstrained interest rate rule, at both the short and the long end. As long as the unconstrained deposit rate stays non-negative, it will be possible for the central bank to replicate the hypothetical outcomes of welfare relevant variables that would have prevailed in the absence of the lower bound constraint. In such circumstances the central bank can make full use of the portfolio balance effect and credibly commit to a rule which substitutes for unavailable short-term interest rate cuts with appropriate cuts of long-term rates, induced by purchases of long-term debt and designed to exactly replicate the deposit rate that would have prevailed under the unconstrained interest rate rule. Intuitively, such design exists since central bank purchases of long-term debt reduce the supply of bonds to be absorbed by private bond holdings and thereby lower the long-term interest rate. If this supply effect, to be activated at the lower bound, overturns the increase in private demand for long-term bonds that would have prevailed in the unconstrained environment (relative to the environment where the lower bound is binding), it is possible to create a constellation of demand and supply in the markets for short-term and long-term bonds which replicates the unconstrained deposit rate. This leads to our first proposition (in which starred variables refer to the hypothetically unconstrained economy and variables without an asterisk to the actual economy).

**Proposition 1:** *Consider the equilibrium allocation* $A^{N*} = \left\{ \hat{c}_t^{N*}, \hat{h}_t^{N*}, \hat{m}_t^{N*} \right\}_{t=0}^{\infty}$ *of welfare relevant variables in a symmetric monetary union that results from an unconstrained interest rate rule consistent with* $R_{D,t}^{N*} \geq 1$*, leading to a welfare level* $W^{N*}$*. If the lower bound constraint on short-term interest rates makes it not feasible to implement this allocation with a conventional policy rule, then there exists a* QE-augmented policy rule *which respects the lower bound and replicates* $A^{N*}$ *and, thus,* $W^{N*}$*.*

**Proof:** See Appendix A.4. ■

The proposition captures a constellation in which the welfare reducing effects from the lower bound constraint on short-term interest rates can exactly be offset by appropriately designed QE-type purchases of long-term debt. The design which is necessary in order to overcome the constraint combines restrictions on the conventional interest rate rule and a purchase rule for long-term government debt.

**Corollary I:** *The* QE-augmented policy rule *is a set consisting of a short-term interest rate rule and a purchase rule for long-term debt, to be activated only if the lower bound constraint on the short-term interest rate becomes binding. For exposition, let us assume that the constraint becomes binding at date* $t_1$ *and that this lasts until date* $t_2$*, leading to the pattern* $R^*_{S,t} < 1$ *if* $t_1 \leq t \leq t_2$*, while* $R^*_{S,t} \geq 1$ *otherwise. Then,* $A^{N*}$ *and, thus,* $W^{N*}$ *can be replicated if the* QE-augmented policy rule *takes the form:*

i. *If* $R^*_{S,t} \geq 1$*, set* $R_{S,t} = R^*_{S,t}$ *and if* $R^*_{S,t} < 1$*, set* $R_{S,t} = 1$

ii. *For* $t < t_1$ *set* $q^N_t = 0$*, while for* $t \geq t_1$ *set* $q^N_t \geq 0$

$R_S$ *denotes the implementable gross interest rate in levels,* $R^*_S$ *the corresponding unconstrained rate, which is suggested by the conventional interest rate rule and which would have prevailed in the absence of the lower bound constraint, and* $q^N$ *the purchases of long-term government debt issued in* $N$*, expressed in real per capita terms, that replicate the values of the deposit rate* $R^{N*}_{D,t}$*, as detailed in the Proof of Proposition 1.*

Four comments are worth to make. First, like the New Keynesian reference model discussed above, our analysis is based on a first-order linearised system of equations. Second, the symmetry assumption implies that under the QE-augmented policy rule the central bank will be able to replicate $A^{S*}$ and thus $W^{S*}$ by adopting $q^S_t = q^N_t$. Third, Proposition 1 covers shocks which by assumption satisfy $R^{N*}_{D,t} \geq 1$. This ensures that the unconstrained deposit rate can be replicated without violating the constraint on implementable deposit rates $R^N_{D,t} \geq 1$, as given by (14). Forth, for Proposition 1 to hold it is crucial that the central bank can credibly commit ex ante to implement the QE-augmented policy rule. This feature is needed in order to replicate hypothetical outcomes resulting from a rule which forward-looking agents would have perceived as being credible. In view of this feature it is easy to see that the QE-augmented policy rule, in fact, embeds the conventional interest rate rule as a special case. In other words, the QE-augmented policy rule coincides with the conventional interest rate rule as long as the lower bound constraint never binds and it activates a purchase rule for long-term debt only if the constraint becomes binding.

Figure 2 illustrates Proposition 1, focusing on a selection of responses already introduced in Figure 1 and assuming, again, that the negative shock to the natural rate gets realised in period $t = 5$. The last chart depicts the stock of long-term government debt issued in $N$ and held by the central bank ($q^N$) in real per capita terms, shown as level deviation from its steady-state value (which is zero) or, equivalently, as a percentage point ratio of steady-state output (which is unity). The light grey full lines ("no ZLB") denote the impulse responses which result from a hypothetical scenario which ignores the binding nature of the lower bound constraint. The negative demand shock triggers a reduction of the short-term interest rate, while by assumption the central bank does not engage in purchases of long-term debt. The size of the shock is such that the central bank would hypothetically reduce the short-term interest rate to about minus two percent. Since short- and long-term debt are imperfect substitutes, the long-term rate falls too, yet not as much as the short-term rate, and, importantly, the deposit rate remains positive. The negative welfare effects of the lower bound constraint are illustrated by means of the dark grey dotted lines ("ZLB no QE"). In this scenario the central bank respects the lower bound constraint (which is binding for five periods), but does not yet conduct quantitative easing. This leads to a significant decline of both output and inflation which drop by about 50 percent more than in the previous scenario of an unconstrained monetary policy. In the final scenario ("ZLB

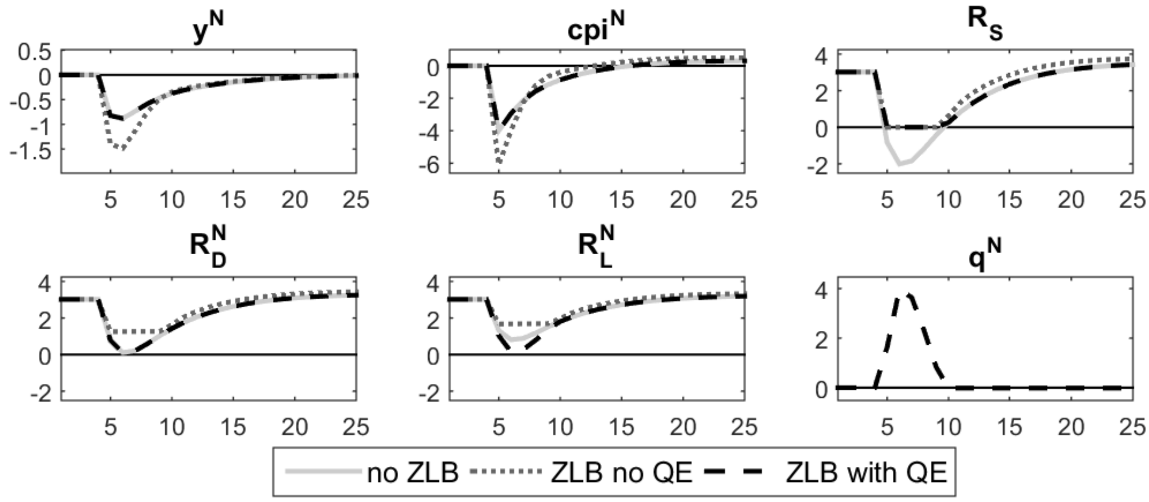


Figure 2: Impulse responses to a *small symmetric* negative demand shock to the natural rate in a *structurally symmetric* monetary union with a binding lower bound constraint. All variables are shown as percentage deviations from their steady-state values, with the exception of inflation and interest rates, which are transformed into annualised net nominal levels in percent, and QE purchases, which are shown as level deviations from the steady state of zero.

with QE", black dashed lines) the central bank exploits the portfolio balance channel to offset the lower bound constraint. Despite being constrained at the short end of the yield curve, the central bank can still stimulate the economy by purchasing long-term debt and thereby reduce long-term rates. Consistent with Proposition 1, the central bank can exactly reproduce the paths of output and inflation (and, hence, welfare) of the unconstrained scenario since the QE-augmented policy rule implements purchase volumes of long-term debt which exactly replicate the deposit rate of the unconstrained scenario. With this criterion being satisfied the decisions of households are not affected by the lower bound constraint. For the particular shock displayed in Figure 2 central bank holdings of long-term debt reach four percent of steady-state output in the first period after the lower bound constraint begins to bind and afterwards gradually decline back to the steady-state value of zero. It is worth pointing out that Figure 2 describes a constellation in which the central bank holdings of long-term debt will have returned to zero at about the same time when the lower bound constraint ceases to be binding.

### 3.2.3 Approximating unconstrained outcomes if the lower bound is binding: the case of a large shock

For a very severe recessionary shock the downward shift of the unconstrained yield curve may well be associated with negative values of the unconstrained deposit rate, i.e. $R_{D,t}^{N*} < 1$. Such values cannot be replicated without violating the constraint on implementable deposit rates ($R_{D,t}^{N} \geq 1$), implying that the hypothetical outcomes of welfare relevant variables that would have prevailed in the absence of the lower bound cannot be reproduced by QE-type purchases of long-term debt. However, in such a constellation the central bank can conduct a combination of QE-type purchases of long-term debt and forward guidance and thereby approximate the unconstrained outcomes to a high degree. The logic of such combined response can be decomposed into two stylised steps. First, as the lower bound constraint becomes binding, the central bank can engage in purchases of long-term debt in order to exploit the portfolio balance channel as far as possible,

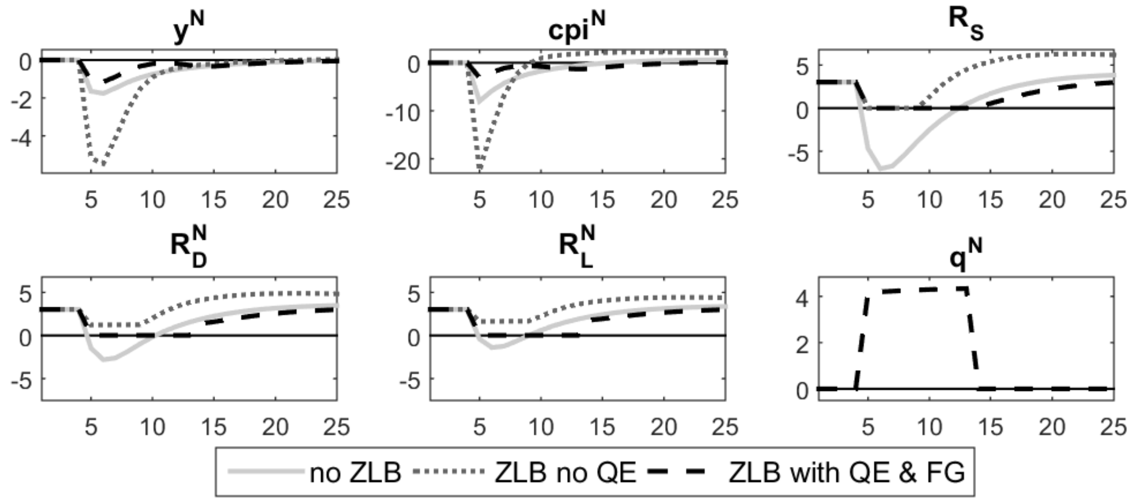


Figure 3: Impulse responses to a *large symmetric* negative demand shock to the natural rate in a *structurally symmetric* monetary union with a binding lower bound constraint. All variables are shown as percentage deviations from their steady-state values, with the exception of inflation and interest rates, which are transformed into annualised net nominal levels in percent, and QE purchases, which are shown as level deviations from the steady state of zero.

pushing thereby the long-term rate to zero. This leads to a flat yield curve and, hence, a zero net deposit rate, making any additional purchases of long-term debt ineffective. Second, in line with the reasoning of Eggertsson and Woodford (2003), albeit now adopted to the crucial role of the deposit rate, the central bank can exercise additional stimulus via forward guidance (i.e. it can promise to keep the short-term interest rate lower for longer as would be indicated by the conventional policy rule). This additional channel can be used to close, at least approximately, remaining gaps with respect to the desirable unconstrained outcomes.

In practice, these two complementary channels of a flattening of the yield curve via purchases of long-term debt and the adoption of forward guidance can, of course, be combined in various ways, as illustrated in Figure 3 by means of example.[25] Compared to Figure 2 the crisis is more severe, leading to stronger reductions of output, inflation and the short- and long-term interest rate in the unconstrained scenario ("no ZLB"). In particular, the shock is severe enough to induce in this hypothetical scenario a negative deposit rate. Hence, there exists no QE-augmented policy rule which could perfectly replicate the unconstrained outcomes. However, Figure 3 shows that these outcomes can approximately be achieved by a combination of a considerable amount of QE-type purchases of long-term debt which pushes the long-term rate to zero and a commitment to keep the short-term rate at the level of zero for one additional period. This combination approximates the unconstrained outcomes very closely ("ZLB with QE & FG"). The fact that both output and inflation drop even less than in the unconstrained scenario can be related to the forward guidance puzzle. As identified by Del Negro et al. (2012), New Keynesian models typically exhibit a strong and front-loaded reaction to forward guidance due to the forward-looking nature of the model agents.

[25] The simulation in Figure 3 is obtained by exogenously fixing the policy rate at the lower bound constraint for one period longer than implied by the unconstrained scenario. In addition, the level of QE is chosen such as to lower the deposit rate as far as possible (i.e. $R_{D,t}^N = 1$), thus still satisfying its non-negativity constraint, for as long as the ZLB is binding.

# 4 Asymmetric monetary Union

This section extends the findings to an asymmetric monetary union in which the two countries cease to be identical because they receive shocks to the natural rate of different magnitude (*"asymmetric shocks"*) or, alternatively, they exhibit structural differences in the transmission of monetary policy (*"asymmetric structures"*). In either case, the complete set of equations (34)-(40) describing the analytical core becomes relevant. As a general feature, asymmetries between countries give rise to equilibrium dynamics characterised by current account imbalances which act as a built-in device to absorb asymmetric adjustment needs of the two countries. We illustrate this general result for asymmetric, yet sufficiently small shocks which ensure that the lower bound will not be reached. Moreover, we show that the scope for current account reactions depends on the degree of financial market integration.

Finally, this section addresses the effects of sufficiently large shocks, which make the lower bound constraint binding (but do not challenge the non-negativity of the unconstrained deposit rates in both countries). We generalise Proposition 1 to an asymmetric monetary union and prove that there exists a *QE-augmented policy rule* which replicates the equilibrium allocations and the welfare levels of the unconstrained conventional policy rule in both countries. We illustrate this result through various simulations and discuss whether *symmetric* or *asymmetric QE purchases* are needed to replicate the outcomes of the unconstrained policy rule, depending on the type of asymmetry.

## 4.1 Outcomes above the lower bound

For simple exposition of how the model works under conventional monetary policy (i.e. above the lower bound), let us consider a constellation of sufficiently small asymmetric shocks in which only country $N$ experiences a negative shock to the natural rate of interest. For comparability with Figure 1, we assume that the shock hitting $N$ is twice as large as in Section 3.2, while no such shock occurs in $S$ such that the aggregate shock, which hits the two equally sized countries, is the same.[26] The key difference to the scenario of a symmetric shock discussed in Figure 1 is that the country which receives the shock is no longer forced to absorb the reduced demand for consumption via an equiproportionate decline in output. In view of the asymmetric nature of the shock, equilibrium dynamics will rather be characterised by current account imbalances which act as a built-in device to absorb asymmetric adjustment needs of the two countries.

Like in Figure 1 the demand-driven recession in $N$ induces the central bank to lower the short-term rate. Via the rebalancing of portfolios this leads in both countries to a decline of newly contracted long-term rates. Since long-term rates fall by less than short-term rates, the composition of portfolios of banks shift in both countries in favour of long-term bonds. Moreover, deposit rates fall. Unlike in Figure 1, however, the decline of the deposit rate in $S$ (i.e. the country which has not received a negative shock) is lower than the decline in $N$ and thus favours, on impact, consumption relative to savings. The additional consumption demand in $S$ falls partly on goods produced in $N$, supported by a change in the terms-of-trade which favours expenditure switching towards $N$. Moreover, $N$ gains the ability to run a current account surplus and thereby to export savings from $N$ to $S$. As discussed in Section 2.6.1, the current account surplus of $N$ can be funded via the central bank balance sheet channel and private capital exports from

[26] In view of the linearised environment, the output response shown in Figure 1 is identical to the aggregate output response in Figure 4b (which features the same economic structure as Figure 1).

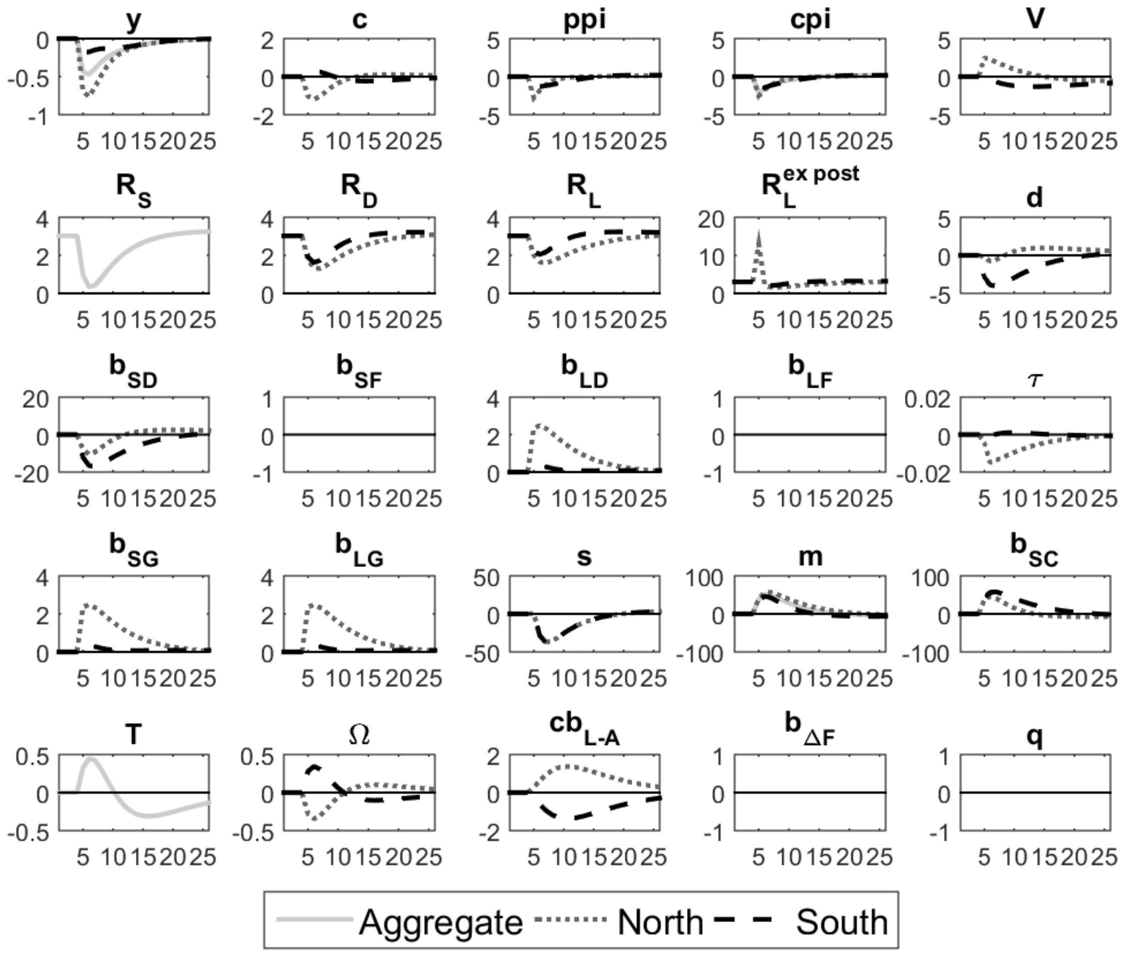


Figure 4a: Impulse responses to an *asymmetric* negative demand shock to the natural rate in a *structurally symmetric* monetary union *without financial market integration*. All variables are shown as percentage deviations from their steady-state values, with the exception of inflation and interest rates, which are transformed into annualised net nominal levels in percent, and the four right variables in the last row, which are shown as level deviations from their steady-state values of zero.

$N$ to $S$.[27] To highlight the differential impact of these two channels we proceed in two steps, summarised in Figures 4a and 4b. All variables are defined as in Figure 1, with the exception of the final row which includes new variables which matter in asymmetric constellations, including the terms of trade ($T$, shown as percentage deviation from the steady-state value of unity and defined from the perspective of $N$). Moreover, the final row shows the evolution of current account imbalances (with a negative $\Omega$ denoting a surplus) as well as the period-$t$ contribution to the funding of current account surpluses from the balance sheet channel ($cb_{L-A}$) and private capital exports ($b_{\Delta F}$), all expressed as level deviations from the steady-state values of zero.[28]

[27] The third channel identified in Section 2.6.1 facilitating the funding of current account imbalances, namely the seigniorage channel, plays in our calibration no role as long as the economy operates above the lower bound, since we assume that the central bank income on short-term bonds is shared, i.e. $\hat{s}_t^N = \hat{s}_t^S$.

[28] In other words, in terms of the decomposition of $\Omega$ via equation (33) offered in Section 2.6.1, the real terms $cb_{L-A}$ and $b_{\Delta F}$ are computed from these definitions in nominal terms:

$$CB_{L-A}^N \equiv M_t^N - B_{SC,t}^N - Q_t^N$$

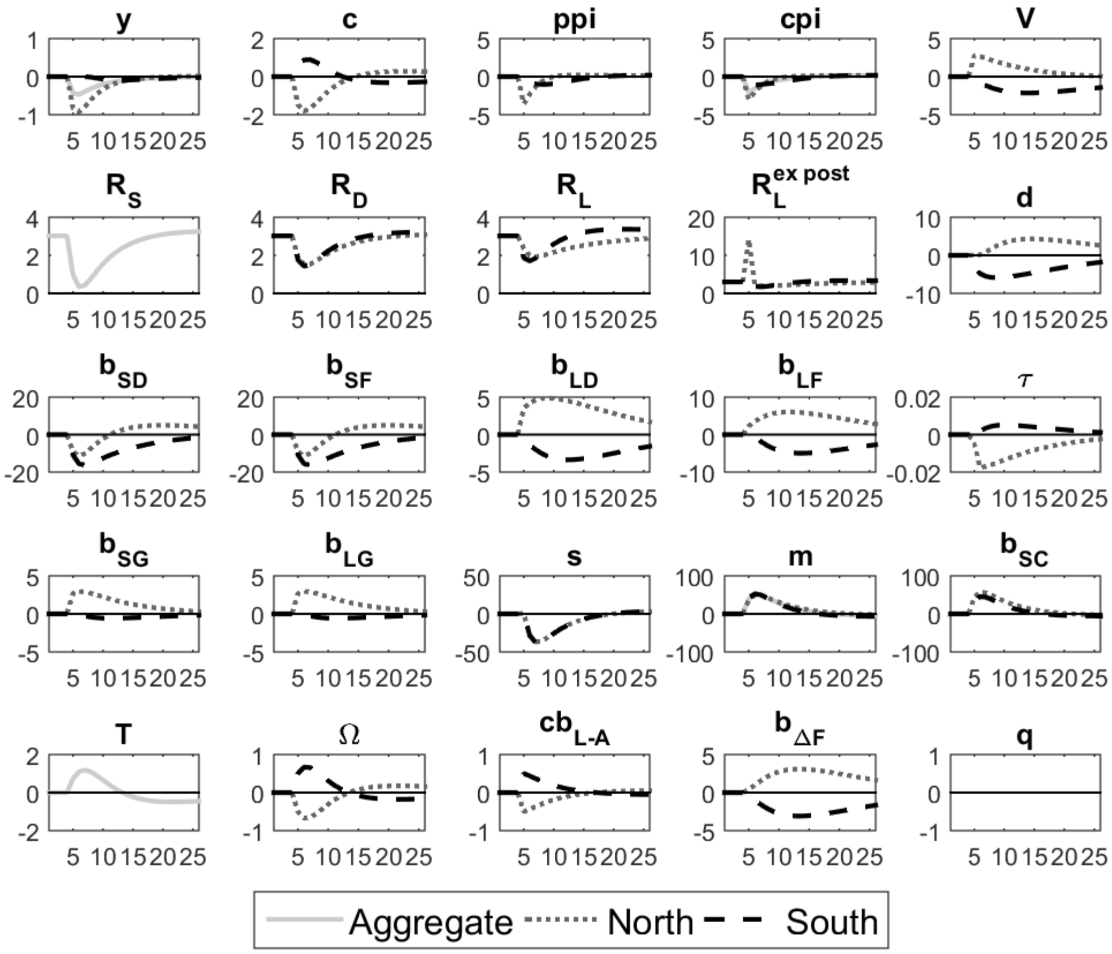


Figure 4b: Impulse responses to an *asymmetric* negative demand shock to the natural rate in a *structurally symmetric* monetary union *with financial market integration*. All variables are shown as percentage deviations from their steady-state values, with the exception of inflation and interest rates, which are transformed into annualised net nominal levels in percent, and the four right variables in the last row, which are shown as level deviations from their steady-state values of zero.

Notice that the variables $\Omega$, $cb_{L-A}$ and $b_{\Delta F}$ can equivalently be interpreted as percentage point ratios of steady-state output.

In a first step, in order to isolate the central bank balance sheet channel, Figure 4a considers the extreme case of a monetary union in which there is no financial integration. In other words, we assume that the banking systems in the two countries operate under autarky such that their deposits can only be invested in domestically issued government bonds (both short-term and long-term), implying $b_{SF} = b_{LF} = 0$ in both countries and, hence, $b_{\Delta F} = 0$. Nevertheless, $N$ can run a current account surplus. In particular, households in $S$ can increase consumption (and households in $N$ decrease consumption) since households in $N$ want to increase real money

$$\alpha B^N_{\Delta F} \equiv \alpha \left[B^N_{SF,t} + B^N_{LF,t}\right] - (1-\alpha)\left[B^S_{SF,t} + B^S_{LF,t}\right]$$

As illustrated by equation (33), for given asset positions inherited from period $t-1$, increases in $CB^N_{L-A}$ and $B^N_{\Delta F}$ contribute to a current account deficit in $S$.

balances by more than households in $S$.[29] Hence, the central bank offers an equilibrium channel which shifts consumption from $N$ to $S$. Figure 4a confirms that this shift in consumption between the two countries (leading to a current account surplus in $N$ and a current account deficit in $S$) corresponds to a central bank balance sheet composition where some of the money held in $N$ is backed by central bank holdings of bonds issued in $S$, as indicated by $cb_{L-A}$.

In a second step, in order to also allow for private capital exports and imports, Figure 4b considers our benchmark monetary union in which financial markets are integrated. In other words, the banking systems in the two countries, in line with the description of the full model in Section 2, manage portfolios consisting of domestic and foreign bonds (both short-term and long-term). Under this additional assumption, households in $S$ can increase consumption by reducing their deposits (which are partly invested in foreign bonds). This matches the desire of households in $N$ to reduce consumption and to save more via holding more deposits (which are partly invested in foreign bonds). Hence, there is scope for private capital exports to absorb some of the savings in $N$, as indicated by $b_{\Delta F}$. This reflects that financial integration offers a second equilibrium channel which facilitates a shift of consumption from $N$ to $S$.

In sum, Figure 4b shows that in the presence of both adjustment channels $N$ can run a larger current account surplus than in the financial autarky case depicted in Figure 4a. Moreover, Figure 4b reveals that the current account surplus is largely driven by private capital exports, while the central bank balance sheet channel loses importance and, in fact, switches sign. In other words, portfolio adjustments in financially integrated markets are a strong substitute for central bank intermediated funding of current account imbalances which arise in response to asymmetric shocks. Finally, it is worth emphasising that it is a priori open whether the shift towards financial market integration favours the stabilisation of the output level in $N$. For the particular calibration underlying Figures 4a and 4b, the decline in output in $N$ will be amplified under financial market integration, but this finding is not robust to alternative numerical specifications.[30]

### 4.2 The model with an occasionally binding lower bound constraint

This section extends the reasoning to specifications of an asymmetric monetary union which occasionally reaches the lower bound on short-term interest rates. For simplicity, we only consider negative shocks to the natural rate which are large enough to make the lower bound constraint binding, but, at the same time, do not challenge the non-negativity of the unconstrained deposit rates in both countries. In view of Section 3.2.2, it is clear that only for such configurations there will be scope for a *QE-augmented policy rule* to overcome the restriction arising from the lower bound constraint.[31] Asymmetric reactions of countries can emerge if the two countries receive shocks to the natural rate of different magnitude (*"asymmetric shocks"*). Alternatively, countries can exhibit structural differences in the transmission of monetary policy (*"asymmetric structures"*). To make this operational we assume below, in line with the discussion in Subsec-

[29]Notice that in both countries the demand for real money balances increases since deposit rates (i.e. the opportunity cost of holding real money balances) decline in $N$ and $S$. However, only country $N$ has received the negative shock to the natural rate which favours savings relative to consumption.

[30]For discussions of the international transmission of shocks in open economy models and the role of financial market integration, see e.g. Sutherland (1996), Corsetti and Pesenti (2001) and Tille (2001).

[31]For larger shocks, similar to Section 3.2.3., forward guidance offers an additional tool to achieve approximately acceptable outcomes.

tion 2.3.2, that the banking systems in the two countries may differ in the degree of home bias of long-term bond holdings. In other words, we relax assumption (17), implying

$$\alpha(1-\omega_N) \neq (1-\alpha)(1-\omega_S).$$

This asymmetry makes the steady-state values of privately held long-term bonds country-specific ($\bar{b}^N_{LP} \neq \bar{b}^S_{LP}$). Consequently, it affects the ratios between long-term and short-term privately held debt ($\delta_N \neq \delta_S$). Moreover, it also changes how banks face portfolio choices between domestic and foreign long-term bonds in the equilibrium dynamics outside the steady state as captured by $\tilde{\nu}_2$, implying $\tilde{\nu}_{2N} \neq \tilde{\nu}_{2S}$. Hence, asymmetric structures generate asymmetric spread dynamics between the long-term rates in the two countries via equation (16) for $N$ and via the corresponding equation for $S$.[32] To simplify the algebra, we assume from now onwards, without loss of generality, $\alpha = 0.5$ (see Appendix A.5 for details).

In general, when conducting purchases of long-term debt at the lower bound, the central bank has a certain flexibility in its response since it can freely choose the portfolio mix of long-term bonds bought in $N$ and $S$. Because of this flexibility, it can be proven that in either case of *"asymmetric shocks"* or *"asymmetric structures"* there exists a *QE-augmented policy rule* which replicates in both countries the equilibrium allocations and the corresponding welfare levels of the unconstrained conventional policy rule.

**Proposition 2:** *Consider the equilibrium allocation of welfare relevant variables, consisting of the pair* $A^{N*} = \left\{\hat{c}^{N*}_t, \hat{h}^{N*}_t, \hat{m}^{N*}_t\right\}_{t=0}^{\infty}$ *and* $A^{S*} = \left\{\hat{c}^{S*}_t, \hat{h}^{S*}_t, \hat{m}^{S*}_t\right\}_{t=0}^{\infty}$, *that results from an unconstrained interest rate rule consistent with* $R^{N*}_{D,t} \geq 1$ *and* $R^{S*}_{D,t} \geq 1$, *leading to welfare levels* $W^{N*}$ *and* $W^{S*}$. *If the lower bound constraint on short-term interest rates makes it not feasible to implement this allocation with a conventional policy rule, then there exists a* QE-augmented policy rule *which respects the lower bound and replicates* $A^{N*}$ *and* $A^{S*}$ *and, thus,* $W^{N*}$ *and* $W^{S*}$.

**Proof:** See Appendix A.5. ■

Proposition 2 summarises a broad range of constellations in which the central bank is able to offset the lower bound restriction even if the countries display asymmetric developments. Essentially, this finding reflects that QE-type purchases of long-term debt can be designed to offer country-specific stimulus via asymmetric purchases of debt issued in the two countries. Hence, extending Proposition 1, the design of QE which is needed to overcome the lower bound constraint in an asymmetric monetary union is a combination of the conventional interest rate rule and possibly country-specific purchasing rules for long-term government bonds issued in the two countries.

**Corollary II:** *The* QE-augmented policy rule *is a set consisting of a short-term interest rate rule and possibly country-specific purchase rules for long-term debt, to be activated only if the lower bound constraint on the short-term interest rate becomes binding. For exposition, let us assume that the constraint becomes binding at date* $t_1$ *and that this lasts until date* $t_2$, *leading*

[32] The non-equality of privately held long-term bonds, when equation (17) does not hold, can be deduced from Footnote 19. Concerning $\tilde{\nu}_{2N} = \frac{\nu_2\beta}{\omega_N(1-\omega_N)\bar{b}^N_{LP}}$ and $\tilde{\nu}_{2S} = \frac{\nu_2\beta}{\omega_S(1-\omega_S)\bar{b}^S_{LP}}$, note that our choice of $\alpha = 0.5$ still ensures $(1-\omega_N)\bar{b}^N_{LP} = (1-\omega_S)\bar{b}^S_{LP}$. Thus, $\tilde{\nu}_{2N} \neq \tilde{\nu}_{2S}$ whenever $\omega_N \neq \omega_S$.

*to the pattern* $R^*_{S,t} < 1$ *if* $t_1 \leq t \leq t_2$*, while* $R^*_{S,t} \geq 1$ *otherwise. Then,* $A^{N*}$ *and* $A^{S*}$ *and, thus,* $W^{N*}$ *and* $W^{S*}$ *can be replicated if the* QE-augmented policy rule t*akes the form:*

i. *If* $R^*_{S,t} \geq 1$*, set* $R_{S,t} = R^*_{S,t}$ *and if* $R^*_{S,t} < 1$*, set* $R_{S,t} = 1$

ii. *For* $t < t_1$ *set* $q^N_t = q^S_t = 0$*, while for* $t \geq t_1$ *set* $q^N_t \geq 0$ *and* $q^S_t \geq 0$

$R_S$ *denotes the implementable gross interest rate in levels,* $R^*_S$ *the corresponding unconstrained rate, which is suggested by the conventional interest rate rule and which would have prevailed in the absence of the lower bound constraint, and* $q^N$ *and* $q^S$ *the purchases of long-term government debt, expressed in real per capita terms, that replicate the values of the deposit rates* $R^{N*}_{D,t}$ *and* $R^{S*}_{D,t}$*, as detailed in the Proof of Proposition 2.*

How does the central bank portfolio of long-term bonds, as prescribed by Proposition 2, look like? To answer this question it is worth repeating that in this paper we look at a design of QE which is able to replicate the outcomes of a hypothetically unconstrained *uniform* monetary policy (i.e. outcomes which would have prevailed in the absence of the lower bound constraint on the *uniform* short-term policy rate). In view of this, the QE-augmented policy rule may well be consistent with a symmetric portfolio design. This will be addressed in the next subsections which illustrate Proposition 2 through simulations.

### 4.2.1 Symmetric structures and asymmetric shocks

Figure 5 reconsiders the constellation of *symmetric structures* and *asymmetric shocks* in which only country $N$ experiences a negative shock to the natural rate of interest, as discussed in the context of Figure 4b. Differently from Figure 4b, however, the shock is large enough to make the lower bound constraint binding (while respecting the non-negativity of the unconstrained deposit rates in both countries). Figure 5 indicates that the *QE-augmented policy rule* can be characterised by a symmetric purchase rule for debt issued in $N$ and $S$, i.e. $q^N = q^S$. Intuitively, this finding reflects that the lower bound imposes a constraint on the uniform instrument of the short-term policy rate. Moreover, irrespective of potential asymmetries in the magnitude of the originating shocks, in our linear framework this constraint creates for both countries a symmetric restriction for the portfolio adjustments induced by the lower bound constraint, since the two countries are assumed to have identical structures for the transmission of monetary policy. In particular, equations (37)-(39) indicate that the rates of return in the two countries will be affected in the same way. This explains why the lower bound constraint can be overcome if the central bank follows a symmetric purchase rule, consisting of purchases of identical per capita amounts of long-term debt issued by the two governments.

It is worth stressing that our model features a conventional interest rate rule with symmetric weights as shown in equation (22). Yet, the desirability of a symmetric purchase rule in response to asymmetric shocks remains unchanged if the conventional interest rate rule attaches asymmetric weights to the two countries. Intuitively, any uniformly applied policy rate creates at the lower bound a symmetric restriction for both countries, irrespective of the origin of this rate in terms of country-specific weighting schemes. This finding can be related to the analysis of Benigno (2004) who showed that optimal conventional monetary policy in an asymmetric monetary union may well be characterised by an interest rate rule which assigns a larger weight to the more rigid economy. Our findings suggest that such a weighting scheme does not necessarily translate into a corresponding weighting scheme for QE-type purchases of long-term bonds at the lower bound constraint, provided that the monetary transmissions channels are identical.

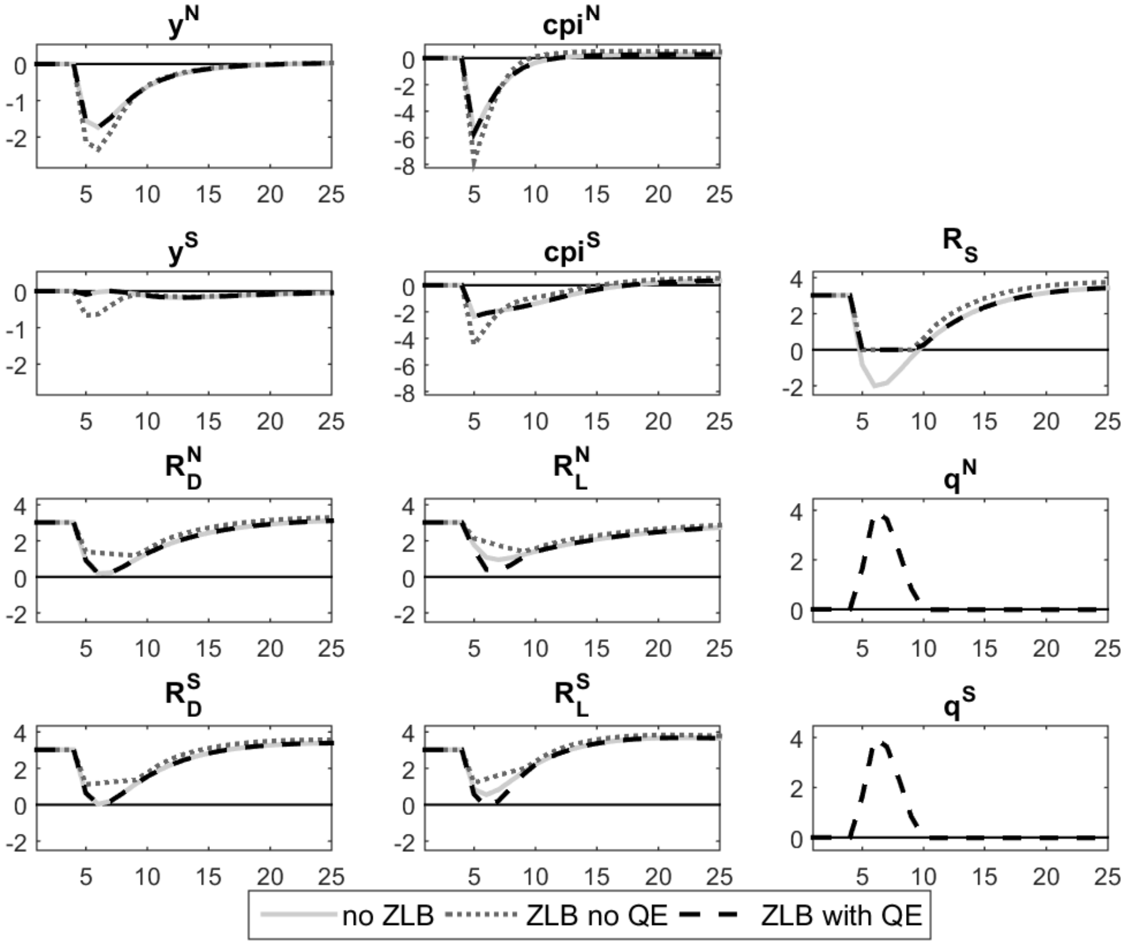


Figure 5: Impulse responses to a *small asymmetric* negative demand shock to the natural rate in a *structurally symmetric* monetary union with a binding lower bound constraint. All variables are shown as percentage deviations from their steady-state values, with the exception of inflation and interest rates, which are transformed into annualised net nominal levels in percent, and QE purchases, which are shown as level deviations from the steady state of zero.

#### 4.2.2 Asymmetric structures and symmetric shocks

Figure 6 describes a constellation in which the lower bound is reached in an environment characterised by *symmetric shocks* and *asymmetric structures* in the transmission of monetary policy. To this end, we assume, ceteris paribus, that the banking system in $S$ has a larger home bias than in $N$. In particular, following our baseline calibration of equally sized countries (i.e. $\alpha = 0.5$), we assume $\omega_S > \omega_N$ which implies $\tilde{\nu}_{2S} < \tilde{\nu}_{2N}$.[33] Figure 6 shows that this structural difference does not lead to strong differences in the impulse responses between the two countries, as long as one ignores the lower bound constraint ("no ZLB"). Yet, with the constraint assumed to be binding ("ZLB no QE"), this will not only deepen the recession in both countries (for the same reasons as discussed above), but it can also be observed that the long-term rates between the two countries differ. This is explained by the different demand pattern for long-term bonds issued in $S$ due to the stronger home bias of bond holdings in that country. Figure 6 indicates that the *QE-augmented policy rule* which overcomes the lower bound constraint("ZLB with QE")

[33]This pattern follows from the exposition in Subsection 2.3.2 and Footnote 32.

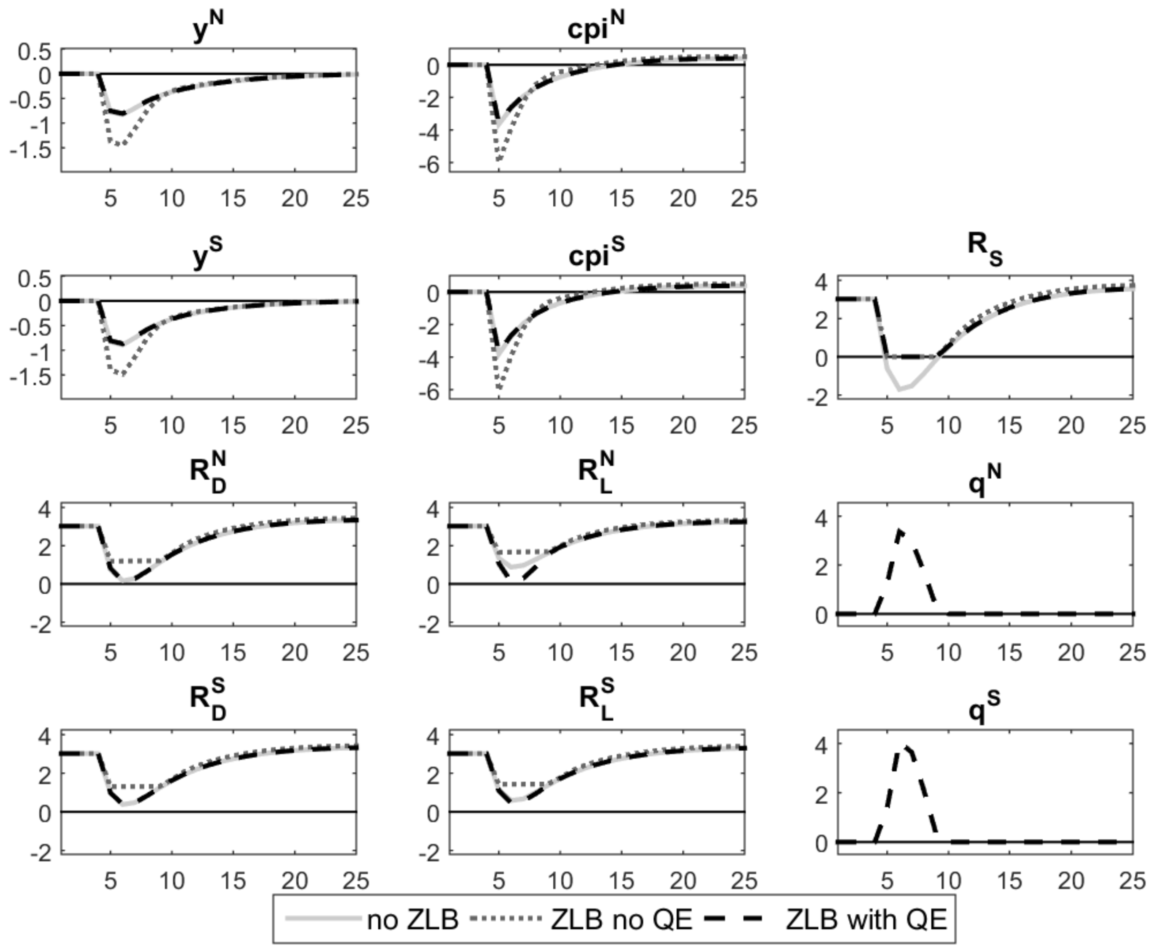


Figure 6: Impulse responses to a *small symmetric* negative demand shock to the natural rate in a *structurally asymmetric* monetary union with a binding lower bound constraint. All variables are shown as percentage deviations from their steady-state values, with the exception of inflation and interest rates, which are transformed into annualised net nominal levels in percent, and QE purchases, which are shown as level deviations from the steady state of zero.

leads to asymmetric purchases of long-term debt by the central bank, favouring the country with the stronger home bias (i.e. $q^S > q^N$). For the particular calibration shown in Figure 6 (with $\omega_S = 0.9 > \omega_N = 0.7$), the central bank needs in the peak to buy 22 percent more of bonds issued in $S$ than in $N$.

Qualitatively, this finding can be rationalised as follows. The unconstrained interest rate rule would trigger a large decline in the uniform short-term rate, inducing a reallocation of portfolios which shifts private demand towards long-term bonds. Because of the asymmetric home bias the private demand for long-term bonds will be asymmetric, biased towards bonds issued in $S$ and leading to different long-term interest rates of the two countries. If the lower bound constraint binds, the decline in the short-term rate is less strong, inducing therefore a smaller asymmetric portfolio shift towards long-term bonds. For the central bank to be able to replicate the unconstrained outcomes it has to compensate for the effects of this shortfall in asymmetric private demand. This can be achieved via asymmetric central bank purchases of long-term debt, to be activated at the lower bound and to be biased towards $S$, with the intention to achieve an asymmetric reduction of the supply of bonds to be absorbed by private bond holdings. Under the QE-augmented policy rule the asymmetric reduction of the supply of privately held long-term bonds overturns the asymmetric private demand for these bonds that would have prevailed in the unconstrained environment (relative to the constrained environment). The design of the asymmetric intervention needs to be such that a constellation of demand and supply in the markets for short-term and long-term bonds emerges which replicates the unconstrained deposit rates in both countries (and via this channel all welfare relevant variables). This reasoning explains why a stronger QE-type intervention is needed in the country with a stronger home bias, i.e. $q^S > q^N$.

In sum, these simulations suggest that, assuming *asymmetric structures* in the transmission of monetary policy, *asymmetric QE purchases* are needed to replicate the outcomes of the unconstrained policy rule. The assumption of a different degree of home bias is consistent with different exposures of banks in the two countries to their own sovereign. When viewed from this perspective, the asymmetric structure would translate into asymmetric central bank purchase volumes of long-term debt at the lower bound, favouring the country where banks are more strongly exposed to their own sovereign. The central bank portfolio bias of QE, in a sense, can fix asymmetric structures. This feature disappears if the home bias of banks in both countries converges, e.g. as a result of uniform regulation. Moreover, in our analysis any asymmetric central bank reaction has no strategic implications since the assumption of a sound fiscal governance structure rules out that asymmetric QE purchases could come together with adverse incentive effects for governments. Incorporating strategic design issues of QE in the presence of a weak fiscal governance structure and excessive exposure of banks to their own sovereign is beyond the scope of this paper.

# 5 Conclusion

This paper is motivated by the idea to develop a tractable model of a monetary union in which the design of monetary policy above and at the lower bound constraint on short-term interest rates can be linked to findings from the literature dealing with single closed economies. As a clear reference point for the analysis of monetary policy in a closed economy, we take the canonical linearised New Keynesian model, with well-understood properties of a conventional (Taylor-type) interest rate rule at positive levels of the interest rate. Moreover, in this setting, once the lower bound constraint becomes binding, central bank purchases of long-term debt (which we label as QE) are ineffective, while forward guidance (i.e. the commitment of the central bank to keep future short-term rates lower for longer when the constraint ceases to be binding) is effective, as shown by Eggertsson and Woodford (2003). Relative to this benchmark, we introduce two extensions. First, in the spirit of Tobin and Brainard (1963), we add a portfolio balance channel which ensures that short-term and long-term bonds become imperfect substitutes. Second, following Benigno (2004), we consider two countries, belonging to a monetary union. The countries share a common monetary policy, while fiscal policies, in the absence of a fiscal union, are decided at the national level, subject to a sound governance structure. For the special case of a symmetric monetary union and abstracting from portfolio adjustment costs the setting becomes isomorphic to the reference model. In general, however, the first extension makes purchases of long-term debt effective, while the second extension leads to the question how the central bank should split its purchases between debt issued in the two countries. We prove that under certain conditions there exists, as the lower bound constraint becomes binding, an interest rate rule augmented by QE which replicates the equilibrium allocations and the corresponding welfare levels that would have prevailed in the two countries under a hypothetically unconstrained and uniformly applied interest rate rule. Moreover, our numerical illustrations for asymmetric monetary unions suggest that the central bank's QE portfolio under the augmented interest rate rule can be symmetric or asymmetric, depending on whether the countries have received shocks of a different magnitude (*"asymmetric shocks"*) or, alternatively, they exhibit structural differences in the transmission of monetary policy (*"asymmetric structures"*).

For tractability, the framework is deliberately simple and many important extensions come to mind. In particular, we leave it for future work to extend the analysis to specifications of optimal monetary policy. Moreover, as evidenced by recent euro area developments, monetary unions without a fiscal union tend to suffer from a weak fiscal governance structure. This leads to sub-optimal outcomes, in line with the analysis of Chari and Kehoe (2008), and makes it desirable in the context of our paper to extend the design of QE to incentive issues. More generally speaking, richer frameworks should be able to distinguish between complete and incomplete monetary unions. Finally, in view of the lower bound constraint addressed in this paper it is worth to extend the analysis to a non-linear environment.

# A Appendix

This appendix presents selected optimisation problems in more detail (see A.1) and lists the full economic system both in non-linear and log-linear terms (see A.2 and A.3). Most of the equations shown refer to country $N$, but there exists by definition a corresponding set of equations belonging to $S$. For a detailed motivation and explanation of the variables refer to the main text. In addition, the appendix contains the proofs of Propositions 1 and 2 (see A.4 and A.5).

## A.1 Two-stage Dixit-Stiglitz aggregation

The overall consumption bundle $c^N$ consumed by the household results from a two-stage Dixit-Stiglitz aggregation. First, the bundle is defined as a combination of domestic and foreign (imported) consumption bundles as

$$c^N \equiv \left[\lambda_N^{\frac{1}{\eta}}(c_D^N)^{\frac{\eta-1}{\eta}} + (1-\lambda_N)^{\frac{1}{\eta}}(c_F^N)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

where $\eta > 0$ is the elasticity of substitution between the $N$ and $S$ goods and $\lambda_N \in [0,1]$ measures the home bias in consumption. The Euler equation first determines how much each household wants to consume overall. Then, the costs of that overall consumption bundle are minimised taking the prices of $N$ and $S$ consumption bundles, $P_p^N$ and $P_p^S$, as given. The resulting price aggregator is the consumer price index:

$$P_c^N \equiv \left[\lambda_N \left(P_p^N\right)^{1-\eta} + (1-\lambda_N)\left(P_p^S\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

The demand functions for $c_D^N$ and $c_F^N$ resulting from the minimisation problems are:

$$c_D^N = \left(\frac{P_p^N}{P_c^N}\right)^{-\eta} \lambda_N c^N \quad \text{and} \quad c_F^N = \left(\frac{P_p^S}{P_c^N}\right)^{-\eta}(1-\lambda_N)c^N$$

The second stage of the aggregation defines domestic and foreign consumption bundles, each made up of differentiated goods produced in the respective country, as

$$c_D^N \equiv \left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\varepsilon}} \int_0^\alpha c^N(n)^{\frac{\varepsilon-1}{\varepsilon}} dn\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad c_F^N \equiv \left[\left(\frac{1}{1-\alpha}\right)^{\frac{1}{\varepsilon}} \int_\alpha^{1-\alpha} c^N(s)^{\frac{\varepsilon-1}{\varepsilon}} ds\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 0$ determines the elasticity of substitution across the differentiated goods within a country. In the same fashion as above, the respective price aggregators are obtained by minimising the costs of each bundle, taking the prices of the differentiated goods as given. The resulting price aggregators, $P_p^N$ and $P_p^S$, and demand functions are:

$$P_p^N = \left[\frac{1}{\alpha}\int_0^\alpha P(n)^{1-\varepsilon} dn\right]^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad P_p^S = \left[\frac{1}{1-\alpha}\int_\alpha^{1-\alpha} P(s)^{1-\varepsilon} ds\right]^{\frac{1}{1-\varepsilon}}$$

$$c^N(n) = \left(\frac{P(n)}{P_p^N}\right)^{-\varepsilon} \frac{c_D^N}{\alpha} \quad \text{and} \quad c^N(s) = \left(\frac{P(s)}{P_p^S}\right)^{-\varepsilon} \frac{c_F^N}{1-\alpha}$$

Given that exports and imports are exchanged at prices $P_p^N$ and $P_p^S$, the terms of trade are defined from $N$'s perspective as $T \equiv \frac{P_p^S}{P_p^N}$. Combining the demand equations with the price definitions yields the demand equations for the differentiated $N$ and $S$ goods:

$$c^N(n) = \left(\frac{P(n)}{P_p^N}\right)^{-\varepsilon} \left(\frac{P_p^N}{P_c^N}\right)^{-\eta} c^N \frac{\lambda_N}{\alpha}$$

$$c^N(s) = \left(\frac{P(s)}{P_p^S}\right)^{-\varepsilon} \left(\frac{P_p^S}{P_c^N}\right)^{-\eta} c^N \frac{1-\lambda_N}{1-\alpha}$$

**Aggregate demand**

The aggregate demand for good $n$ consists, besides the demand by consumers in $N$ and $S$, of adjustment costs faced by firms and banks. When adjusting prices and the portfolio composition, respectively, firms and banks consume resources in form of domestic goods which they buy from domestic firms. The real adjustment costs in $N$ are denoted as $\Xi^N$ and the corresponding resources are bought from firms according to the demand function:

$$\Xi^N(n) = \left(\frac{P(n)}{P_p^N}\right)^{-\varepsilon} \frac{\Xi^N}{\alpha}$$

Integrating the demand equations over the $N$ and $S$ populations, respectively, and adding the adjustment costs results in the aggregate demand functions for firms located in $N$.

$$y(n) = \int_0^\alpha c^N(n)dn + \int_\alpha^1 c^S(n)dn + \int_0^\alpha \Xi^N(n)dn$$

$$y(n) = \left(\frac{P(n)}{P_p^N}\right)^{-\varepsilon} \left(\lambda_N \left(\frac{P_p^N}{P_c^N}\right)^{-\eta} c^N + (1-\lambda_S)\left(\frac{P_p^N}{P_c^S}\right)^{-\eta} \frac{1-\alpha}{\alpha} c^S + \Xi^N\right)$$

The overall aggregate demand (population times output per capita) in country $N$ is obtained by using the appropriate Dixit-Stiglitz aggregator

$$\alpha y^N \equiv \left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\varepsilon}} \int_0^\alpha y(n)^{\frac{\varepsilon-1}{\varepsilon}} dn\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

together with the firm-specific demand equations in a symmetric equilibrium:[34]

$$y^N = \lambda_N \left(\frac{P_p^N}{P_c^N}\right)^{-\eta} c^N + (1-\lambda_S)\left(\frac{P_p^N}{P_c^S}\right)^{-\eta} \frac{1-\alpha}{\alpha} c^S + \Xi^N$$

Noting that the adjustment costs $\Xi^N$ are zero both in the steady state and in the first-order log-linearised system of equations and making use of $(1-\lambda_S)\frac{1-\alpha}{\alpha} = 1-\lambda_N$, the log-linear aggregate demand equation is:

$$\hat{y}_t^N = \lambda_N \hat{c}_t^N + (1-\lambda_N)\hat{c}_t^S + \eta(1-\lambda_N)(\lambda_N+\lambda_S)\hat{T}_t \tag{A.1.1}$$

Consumer prices obey:

$$\hat{\pi}_{c,t}^N = \lambda_N \hat{\pi}_{p,t}^N + (1-\lambda_N)\hat{\pi}_{p,t}^S \tag{A.1.2}$$

[34] A symmetric equilibrium implies that all firms within one country face the same optimisation problem and thus set the same price. Hence, $P(n) = P_p^N$ for $n \in [0, \alpha)$ and $P(s) = P_p^S$ for $s \in [\alpha, 1]$, leading to $y(n) = y^N$ and $y(s) = y^S$.

## A.2 Full set of non-linear equilibrium conditions

### A.2.1 Households

$$P_{c,t}^N c_t^N + D_t^N + M_t^N = R_{D,t-1}^N D_{t-1}^N + M_{t-1}^N + W_t^N h_t^N + \Gamma_t^N \tag{A.2.1}$$

$$MUC_t^N = \left(c_t^N - \varsigma c_{t-1}^N\right)^{-\frac{1}{\sigma}} - \varsigma\beta\frac{\phi_{t+1}^N}{\phi_t^N}\left(c_{t+1}^N - \varsigma c_t^N\right)^{-\frac{1}{\sigma}} \tag{A.2.2}$$

$$MUC_t^N = \mathbb{E}_t\beta\frac{R_{D,t}^N}{\Pi_{c,t+1}^N}\frac{\phi_{t+1}^N}{\phi_t^N}MUC_{t+1}^N \tag{A.2.3}$$

$$\left(h_t^N\right)^{\psi} = w_t^N MUC_t^N \tag{A.2.4}$$

$$\chi_m^{-1}\left(m_t^N\right)^{-\frac{1}{\sigma_m}} = MUC_t\frac{R_{D,t}^N - 1}{R_{D,t}^N} \tag{A.2.5}$$

$$\begin{aligned}\Gamma^N =& P_{c,t}^N \tau_t^N + P_{p,t}^N y_t^N - W_t^N h_t^N \\ &- \left(R_{D,t-1}^N D_{t-1}^N - R_{S,t-1}B_{SP,t-1}^N - R_{L,t}^N B_{LD,t-1}^N - R_{L,t}^S B_{LF,t-1}^N\right) - P_{p,t}^N \Xi_t^N\end{aligned} \tag{A.2.6}$$

$$\Xi_t^N = \frac{\chi}{2}\left(\Pi_{p,t}^N - 1\right)^2 y_t^N + \frac{\nu_1}{2}\left(\delta\frac{B_{SP,t}^N}{B_{LP,t}^N} - 1\right)^2 + \frac{\nu_2}{2}\left(\frac{\omega_N}{1-\omega_N}\frac{B_{LF,t}^N}{B_{LD,t}^N} - 1\right)^2 \tag{A.2.7}$$

$$\frac{\phi_{t+1}^N}{\phi_t^N} \equiv r_{n,t}^N = \left(r_{n,t-1}^N\right)^{\rho_n} e^{\varepsilon_{n,t}^N} \tag{A.2.8}$$

### A.2.2 Firms

$$y_t^N = a h_t^N \tag{A.2.9}$$

$$(1-\varepsilon) + \varepsilon\frac{w_t^N}{a_t^N}\frac{P_{c,t}^N}{P_{p,t}^N} - \chi(\Pi_{p,t}^N - 1)\Pi_{p,t}^N = -\mathbb{E}_t\beta\chi\frac{\Pi_{p,t+1}^N - 1}{\Pi_{c,t+1}^N}(\Pi_{p,t+1}^N)^2\frac{y_{t+1}^N\Delta_{t+1}^N}{y_t^N\Delta_t^N} \tag{A.2.10}$$

### A.2.3 Banks

$$D_t^N = B_{SP,t}^N + B_{LP,t}^N \tag{A.2.11}$$

$$B_{SP,t}^N = B_{SD,t}^N + B_{SF,t}^N \tag{A.2.12}$$

$$B_{SD,t}^N = \alpha B_{SP,t}^N \tag{A.2.13}$$

$$B_{LP,t}^N = B_{LD,t}^N + B_{LF,t}^N \tag{A.2.14}$$

$$R_{D,t}^N = R_{S,t} - \nu_1\left(\delta\frac{b_{SP,t}^N}{b_{LP,t}^N} - 1\right)\frac{\delta}{b_{LP,t}^N}\frac{P_{p,t}^N}{P_{c,t}^N} \tag{A.2.15}$$

$$\begin{aligned}R_{L,t+1}^N =& R_{D,t}^N\left(\omega_N\frac{b_{LP,t}^N}{b_{LD,t}^N}\right)^{\frac{1}{\gamma}} - \nu_1\left(\delta\frac{b_{SP,t}^N}{b_{LP,t}^N} - 1\right)\frac{\delta b_{S,t}^N\omega_N^{\frac{1}{\gamma}}}{\left(b_{LP,t}^N\right)^{\frac{2\gamma-1}{\gamma}}\left(b_{LD,t}^N\right)^{\frac{1}{\gamma}}}\frac{P_{p,t}^N}{P_{c,t}^N} \\ &- \nu_2\left(\frac{\omega_N}{1-\omega_N}\frac{b_{LF,t}^N}{b_{LD,t}^N} - 1\right)\frac{\omega_N b_{LF,t}^N}{(1-\omega_N)\left(b_{LD,t}^N\right)^2}\frac{P_{p,t}^N}{P_{c,t}^N}\end{aligned} \tag{A.2.16}$$

$$
\begin{aligned}
R^S_{L,t+1} =& R^N_{D,t}\left((1-\omega_N)\frac{b^N_{LP,t}}{b^N_{LF,t}}\right)^{\frac{1}{\gamma}} - \nu_1\left(\delta\frac{b^N_{SP,t}}{b^N_{LP,t}}-1\right)\frac{\delta b^N_{S,t}(1-\omega_N)^{\frac{1}{\gamma}}}{\left(b^N_{LP,t}\right)^{\frac{2\gamma-1}{\gamma}}\left(b^N_{LF,t}\right)^{\frac{1}{\gamma}}}\frac{P^N_{p,t}}{P^N_{c,t}} \\
& - \nu_2\left(\frac{\omega_N}{1-\omega_N}\frac{b^N_{LF,t}}{b^N_{LD,t}}-1\right)\frac{\omega_N}{(1-\omega_N)b^N_{LD,t}}\frac{P^N_{p,t}}{P^N_{c,t}} \qquad \text{(A.2.17)}
\end{aligned}
$$

Notice that long-term private debt holdings and the role of the steady-state share $\omega_N$ could be micro-founded with the CES-specification: $B^N_{LP,t} = \left[\omega_N^{\frac{1}{\gamma}}(B^N_{LD,t})^{\frac{\gamma-1}{\gamma}} + (1-\omega_N)^{\frac{1}{\gamma}}(B^N_{LF,t})^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$

### A.2.4 Fiscal sector

$$B^N_{SG,t} + V^N_t B^N_{consols,t} + S^N_t = R_{S,t-1}B^N_{SG,t-1} + (1+V^N_t)B^N_{consols,t-1} + P^N_{c,t}\tau^N_t \qquad \text{(A.2.18)}$$

$$B^N_{LG,t} = V^N_t B^N_{consols,t} \qquad \text{(A.2.19)}$$

$$R^N_{L,t} = \frac{1+V^N_t}{V^N_{t-1}} \qquad \text{(A.2.20)}$$

$$\frac{B^N_{LG,t}}{P^N_{c,t}} - \frac{\bar{B}^N_{LG,t}}{\bar{P}^N_c} = \frac{\delta}{1+\bar{m}_b}\left(\frac{B^N_{SG,t}}{P^N_{c,t}} - \frac{\bar{B}^N_{SG,t}}{\bar{P}^N_c}\right) \qquad \text{(A.2.21)}$$

$$\tau^N_t - \bar{\tau}^N = -\theta\frac{\beta\bar{b}^N_{LP}}{\delta\bar{b}^N_{SG}}\left(R_{S,t-1}\frac{B^N_{SG,t-1}}{P^N_{c,t}} - \bar{R}_S\frac{\bar{B}^N_{SG}}{\bar{P}^N_c}\right) \qquad \text{(A.2.22)}$$

leading to

$$B^N_{SG,t} + B^N_{LG,t} + S^N_t = R_{S,t-1}B^N_{SG,t-1} + R^N_{L,t}B^N_{LG,t-1} + P^N_{c,t}\tau^N_t$$

### A.2.5 Monetary sector

$$M_t = \alpha M^N_t + (1-\alpha)M^S_t \qquad \text{(A.2.23)}$$

$$M_t = \alpha\left(B^N_{SC,t} + Q^N_t\right) + (1-\alpha)\left(B^S_{SC,t} + Q^S_t\right) \qquad \text{(A.2.24)}$$

$$
\begin{aligned}
\alpha S^N_t =& (1-(1-\alpha)\mu_1)(R_{S,t-1}-1)\alpha B^N_{SC,t-1} + \alpha\mu_1(R_{S,t-1}-1)(1-\alpha)B^S_{SC,t-1} \\
& + (1-(1-\alpha)\mu_2)(R^N_{Lt}-1)\alpha Q^N_{t-1} + \alpha\mu_2(R^S_{Lt}-1)(1-\alpha)Q^S_{t-1} \qquad \text{(A.2.25)}
\end{aligned}
$$

$$R_{S,t} = (R_{S,t-1})^{\rho_R}\left[\bar{R}_S\left(\frac{\Pi_{c,t}}{\bar{\Pi}_c}\right)^{\phi_\pi}\left(\frac{y_t}{\bar{y}}\right)^{\phi_y}\right]^{1-\rho_R} e^{\varepsilon_{R,t}} \qquad \text{(A.2.26)}$$

$$\frac{Q^N_t}{P^N_{c,t}} = f^N(.) + \varepsilon^N_{q,t} \qquad \text{(A.2.27)}$$

### A.2.6 Market clearing

$$y_t = \alpha y^N_t + (1-\alpha)y^S_t \qquad \text{(A.2.28)}$$

$$y^N_t = \lambda_N\left(\frac{P^N_{p,t}}{P^N_{c,t}}\right)^{-\eta}c^N_t + (1-\lambda_S)\left(\frac{P^N_{p,t}}{P^S_{c,t}}\right)^{-\eta}\frac{1-\alpha}{\alpha}c^S_t + \Xi^N_t \qquad \text{(A.2.29)}$$

$$P_{p,t}^N \Omega_t^N = P_{c,t}^N c_t^N - P_{p,t}^N \left[ y_t^N - \Xi_t^N \right] \tag{A.2.30}$$

$$B_{SG,t}^N = B_{SD,t}^N + \frac{1-\alpha}{\alpha} B_{SF,t}^S + B_{SC,t}^N \tag{A.2.31}$$

$$B_{LG,t}^N = B_{LD,t}^N + \frac{1-\alpha}{\alpha} B_{LF,t}^S + Q_t^N \tag{A.2.32}$$

### A.2.7 Prices

$$T_t = \frac{P_{p,t}^S}{P_{p,t}^N} \tag{A.2.33}$$

$$\Pi_{c,t+1} = \frac{P_{c,t+1}}{P_{c,t}} \tag{A.2.34}$$

$$\Pi_{c,t+1}^N = \frac{P_{c,t+1}^N}{P_{c,t}^N} \tag{A.2.35}$$

$$P_{c,t} = \left(P_{c,t}^N\right)^{\alpha} \left(P_{c,t}^S\right)^{1-\alpha} \tag{A.2.36}$$

$$P_{c,t}^N = \left[ \lambda_N \left(P_{p,t}^N\right)^{1-\eta} + (1-\lambda_N) \left(P_{p,t}^S\right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{A.2.37}$$

### A.2.8 Resulting price relations (of help in deriving the log-linear system)

$$\frac{P_{c,t}^N}{P_{p,t}^N} = \left[ \lambda_N + (1-\lambda_N) T_t^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

$$\frac{P_{c,t}^S}{P_{p,t}^S} = \left[ \lambda_S + (1-\lambda_S) T_t^{\eta-1} \right]^{\frac{1}{1-\eta}}$$

$$\frac{P_{c,t}^N}{P_{p,t}^S} = \left[ \lambda_N T_t^{\eta-1} + (1-\lambda_N) \right]^{\frac{1}{1-\eta}}$$

$$\frac{P_{c,t}^S}{P_{p,t}^N} = \left[ \lambda_S T_t^{1-\eta} + (1-\lambda_S) \right]^{\frac{1}{1-\eta}}$$

$$\frac{P_{c,t}}{P_{c,t}^N} = \left[ \frac{\lambda_S T_t^{1-\eta} + (1-\lambda_S)}{\lambda_N + (1-\lambda_N) T_t^{1-\eta}} \right]^{\frac{1-\alpha}{1-\eta}}$$

$$\frac{P_{c,t}}{P_{c,t}^S} = \left[ \frac{\lambda_N T_t^{\eta-1} + (1-\lambda_N)}{\lambda_S + (1-\lambda_S) T_t^{\eta-1}} \right]^{\frac{\alpha}{1-\eta}}$$

$$\frac{P_{c,t}^S}{P_{c,t}^N} = \left[ \frac{\lambda_S T_t^{1-\eta} + (1-\lambda_S)}{\lambda_N + (1-\lambda_N) T_t^{1-\eta}} \right]^{\frac{1}{1-\eta}}$$

## A.3 Full set of log-linear equilibrium conditions

### A.3.1 Households

$$\frac{\delta}{\bar{b}_{LP}^N} \hat{c}_t^N + (1+\delta) \hat{d}_t^N + \bar{m}_b \hat{m}_t^N = \frac{1+\delta}{\beta} \left[ \hat{R}_{D,t-1}^N - \hat{\pi}_{c,t}^N + \hat{d}_{t-1}^N \right] + \bar{m}_b \left[ \hat{m}_{t-1}^N - \hat{\pi}_{c,t}^N \right]$$

$$+\frac{\bar{w}^N\bar{h}^N\delta}{\bar{b}_{LP}^N}\left[\hat{w}_t^N+\hat{h}_t^N\right]+\frac{\delta}{\bar{b}_{LP}^N}\tilde{\gamma}_t^N \tag{A.3.1}$$

$$(1-\varsigma\beta)M\hat{U}C_t^N=-\frac{1}{\sigma(1-\varsigma)}\left[\hat{c}_t^N-\varsigma\hat{c}_{t-1}^N\right]+\frac{\varsigma\beta}{\sigma(1-\varsigma)}\left[\hat{c}_{t+1}^N-\varsigma\hat{c}_t^N\right]+\varsigma\beta\hat{r}_{n,t+1}^N \tag{A.3.2}$$

$$M\hat{U}C_t^N=M\hat{U}C_{t+1}^N+\left[\hat{R}_{D,t}^N-\hat{\pi}_{c,t+1}^N-\hat{r}_{n,t}^N\right] \tag{A.3.3}$$

$$\psi\hat{h}_t^N=\hat{w}_t^N+M\hat{U}C_t^N \tag{A.3.4}$$

$$\hat{m}_t^N=-\sigma_m M\hat{U}C_t^N-\frac{\sigma_m\beta}{1-\beta}\hat{R}_{D,t}^N \tag{A.3.5}$$

$$\frac{\delta}{\bar{b}_{LP}^N}\tilde{\gamma}_t^N=\frac{\delta}{\bar{b}_L^N}\tilde{\tau}_t^N+\frac{1}{\beta}\left[\hat{R}_{S,t-1}+\delta\omega_N\hat{R}_{L,t}^N+\delta(1-\omega_N)\hat{R}_{L,t}^S-(1+\delta)\hat{R}_{D,t-1}^N\right]$$
$$+\frac{\delta}{\bar{b}_{LP}^N}\left[\hat{y}_t^N-(1-\lambda_N)\hat{T}_t-\bar{w}^N\bar{h}^N(\hat{w}_t^N+\hat{h}_t^N)\right] \tag{A.3.6}$$

$$\tilde{\Xi}_t=0 \tag{A.3.7}$$

$$\hat{r}_{n,t}^N=\rho_n\hat{r}_{n,t-1}^N+\varepsilon_{n,t}^N \tag{A.3.8}$$

### A.3.2 Firms

$$\hat{y}_t^N=\hat{h}_t^N \tag{A.3.9}$$

$$\hat{\pi}_{p,t}^N=\beta\hat{\pi}_{p,t+1}^N+\frac{\varepsilon-1}{\chi}\left[\hat{w}_t^N+(1-\lambda_N)\hat{T}_t\right] \tag{A.3.10}$$

### A.3.3 Banks

$$\hat{d}_t^N=\frac{1}{1+\delta}\hat{b}_{SP,t}^N+\frac{\delta}{1+\delta}\hat{b}_{LP,t}^N \tag{A.3.11}$$

$$\hat{b}_{SP,t}^N=\alpha\hat{b}_{SD,t}^N+(1-\alpha)\hat{b}_{SF,t}^N \tag{A.3.12}$$

$$\hat{b}_{SD,t}^N=\hat{b}_{SP,t}^N \tag{A.3.13}$$

$$\hat{b}_{LP,t}^N=\omega_N\hat{b}_{LD,t}^N+(1-\omega_N)\hat{b}_{LF,t}^N \tag{A.3.14}$$

$$\hat{R}_{D,t}^N=\frac{1}{1+\delta}\hat{R}_{S,t}+\frac{\delta\omega_N}{1+\delta}\hat{R}_{L,t+1}^N+\frac{\delta(1-\omega_N)}{1+\delta}\hat{R}_{L,t+1}^S \tag{A.3.15}$$

$$\hat{R}_{D,t}^N=\hat{R}_{S,t}+\frac{\nu_1\beta\delta}{\bar{b}_{LP}^N}\left[\hat{b}_{LP,t}^N-\hat{b}_{SP,t}^N\right] \tag{A.3.16}$$

$$\hat{R}_{L,t+1}^N=\hat{R}_{L,t+1}^S+\left(\frac{\nu_2\beta}{\omega_N(1-\omega_N)\bar{b}_{LP}^N}\right)\left[\hat{b}_{LD,t}^N-\hat{b}_{LF,t}^N\right] \tag{A.3.17}$$

### A.3.4 Fiscal sector

$$(1+\bar{m}_b)\hat{b}_{SG,t}^N+\delta\hat{b}_{LG,t}^N+\frac{\bar{m}_b}{\beta}(1-\beta)\hat{s}_t^N=\frac{1+\bar{m}_b}{\beta}\left[\hat{R}_{S,t-1}-\hat{\pi}_{c,t}^N+\hat{b}_{SG,t-1}^N\right]$$
$$+\frac{\delta}{\beta}\left[\hat{R}_{L,t}^N-\hat{\pi}_{c,t}^N+\hat{b}_{LG,t-1}^N\right]+\frac{\delta}{\bar{b}_{LP}^N}\tilde{\tau}_t^N \tag{A.3.18}$$

$$\hat{b}^N_{LG,t} = \hat{V}^N_t + \hat{b}^N_{consols,t} \tag{A.3.19}$$
$$\hat{R}^N_{L,t} = \beta\hat{V}^N_t - \hat{V}^N_{t-1} \tag{A.3.20}$$
$$\hat{b}^N_{LG,t} = \hat{b}^N_{SG,t} \tag{A.3.21}$$
$$\frac{\delta}{\bar{b}^N_{LP}}\tilde{\tau}^N_t = -\theta\left[\hat{R}_{S,t-1} - \hat{\pi}^N_{c,t} + \hat{b}^N_{SG,t-1}\right] \tag{A.3.22}$$

### A.3.5 Monetary sector

$$\hat{m}_t = \alpha\hat{m}^N_t + (1-\alpha)\hat{m}^S_t \tag{A.3.23}$$
$$\hat{m}_t = \alpha\left[\hat{b}^N_{SC,t} + \frac{\delta}{\bar{m}_b\bar{b}^N_{LP}}\tilde{q}^N_t\right] + (1-\alpha)\left[\hat{b}^S_{SC,t} + \frac{\delta}{\bar{m}_b\bar{b}^S_{LP}}\tilde{q}^S_t\right] \tag{A.3.24}$$
$$\begin{aligned}\hat{s}^N_t &= \frac{1}{1-\beta}\hat{R}_{S,t-1} + (1-(1-\alpha)\mu_1)\left[\hat{b}^N_{SC,t-1} - \hat{\pi}^N_{c,t}\right] \\ &+ \mu_1(1-\alpha)\left[\hat{b}^S_{SC,t-1} - \hat{\pi}^S_{c,t} + (\lambda_N + \lambda_S - 1)\hat{T}_t\right] \\ &+ (1-(1-\alpha)\mu_2)\frac{\delta}{\bar{m}_b\bar{b}^N_{LP}}\tilde{q}^N_{t-1} + \mu_2(1-\alpha)\frac{\delta}{\bar{m}_b\bar{b}^S_{LP}}\tilde{q}^S_{t-1}\end{aligned} \tag{A.3.25}$$
$$\hat{R}_{S,t} = \rho_R\hat{R}_{S,t-1} + (1-\rho_R)\left[\phi_\pi\hat{\pi}_{c,t} + \phi_y\hat{y}_t\right] + \varepsilon_{R,t} \tag{A.3.26}$$
$$\tilde{q}^N_t = f^N(.) + \varepsilon^N_{q,t} \tag{A.3.27}$$

### A.3.6 Market clearing

$$\hat{y}_t = \alpha\hat{y}^N_t + (1-\alpha)\hat{y}^S_t \tag{A.3.28}$$
$$\hat{y}^N_t = \lambda_N\hat{c}^N_t + (1-\lambda_N)\hat{c}^S_t + \eta(1-\lambda_N)(\lambda_N+\lambda_S)\hat{T}_t \tag{A.3.29}$$
$$\tilde{\Omega}^N_t = \hat{c}^N_t - \hat{y}^N_t + (1-\lambda_N)\hat{T}_t \tag{A.3.30}$$
$$(1+\bar{m}_b)\hat{b}^N_{SG,t} = \alpha\hat{b}^N_{SD,t} + (1-\alpha)\left[\hat{b}^S_{SF,t} + (\lambda_N+\lambda_S-1)\hat{T}_t\right] + \bar{m}_b\hat{b}^N_{SC,t} \tag{A.3.31}$$
$$\hat{b}^N_{LG,t} = \omega_N\hat{b}^N_{LD,t} + (1-\omega_N)\left[\hat{b}^S_{LF,t} + (\lambda_N+\lambda_S-1)\hat{T}_t\right] + \frac{1}{\bar{b}^N_{LP}}\tilde{q}^N_t \tag{A.3.32}$$

### A.3.7 Prices

$$\hat{T}_t = \hat{T}_{t-1} + \hat{\pi}^S_{p,t} - \hat{\pi}^N_{p,t} \tag{A.3.33}$$
$$\hat{\pi}_{c,t} = \alpha\hat{\pi}^N_{c,t} + (1-\alpha)\hat{\pi}^S_{c,t} \tag{A.3.34}$$
$$\hat{\pi}^N_{c,t} = \lambda_N\hat{\pi}^N_{p,t} + (1-\lambda_N)\hat{\pi}^S_{p,t} \tag{A.3.35}$$

## A.4 Proof of Proposition 1

Recall from the main text that in a symmetric monetary union the analytical core of the linearised dynamics reduces to

$$\hat{R}_{S,t} = \rho_R\hat{R}_{S,t-1} + (1-\rho_R)\left[\phi_\pi\hat{\pi}^N_{c,t} + \phi_y\hat{c}^N_t\right] + \varepsilon_{R,t} \tag{A.4.1}$$
$$\hat{c}^N_t = \hat{c}^N_{t+1} - \sigma\left[\hat{R}^N_{D,t} - \hat{\pi}^N_{c,t+1} - \hat{r}^N_{n,t}\right] \tag{A.4.2}$$

$$\hat{\pi}^N_{c,t} = \beta\hat{\pi}^N_{c,t+1} + \frac{\varepsilon - 1}{\chi}(\psi + \frac{1}{\sigma})\hat{c}^N_t \quad \text{(A.4.3)}$$

and

$$\hat{R}^N_{D,t} = \hat{R}_{S,t} + \tilde{\nu}_1 \left[\hat{b}^N_{LP,t} - \hat{b}^N_{SP,t}\right] \quad \text{(A.4.4)}$$

$$\hat{R}^N_{D,t} = \frac{1}{1+\delta}\hat{R}_{S,t} + \frac{\delta}{1+\delta}\hat{R}^N_{L,t+1} \quad \text{(A.4.5)}$$

Let the hypothetical outcomes of this system induced by the unconstrained interest rate rule be denoted by starred variables. We consider a small shock such that $R^{N*}_{D,t} \geq 1$ will be satisfied by assumption. The linearised QE-augmented policy rule implies that $\hat{R}_{S,t} = \hat{R}^*_{S,t}$ will always be satisfied, unless the lower bound constraint is binding. In line with (25), there exists a uniquely defined sequence $\tilde{q}^N_t$ (derived from $q^N_t = \frac{Q^N_t}{P^N_{c,t}}$ with $\bar{q}^N = 0$) which ensures that, whenever the lower bound constraint becomes binding, $\hat{R}^N_{D,t} = \hat{R}^{N*}_{D,t}$ remains being satisfied in all subsequent periods. The replicability of the sequence $\hat{R}^{N*}_{D,t}$ ensures that the two-dimensional dynamic sub-system (A.4.2) and (A.4.3) will in all periods be solved by $\hat{c}^N_t = \hat{c}^{N*}_t$ and $\hat{\pi}^N_{c,t} = \hat{\pi}^{N*}_{c,t}$. Since

$$\hat{h}^{N*}_t = \hat{y}^{N*}_t = \hat{c}^{N*}_t$$

$$\hat{m}^{N*}_t = \frac{\sigma_m}{\sigma}\hat{c}^{N*}_t - \frac{\sigma_m \beta}{1-\beta}\hat{R}^{N*}_{D,t}$$

this feature ensures that the QE-augmented policy rule replicates $A^{N*} = \left\{\hat{c}^{N*}_t, \hat{h}^{N*}_t, \hat{m}^{N*}_t\right\}^{\infty}_{t=0}$ and thus $W^{N*}$. For the remainder of the proof it is at times convenient to rewrite $\tilde{q}^N_t$ via the term $\hat{q}^N_t$ which is defined as

$$\hat{q}^N_t \equiv \frac{\tilde{q}^N_t}{\bar{b}^N_{LP}}.$$

The proof proceeds in three steps: First, we rearrange equation (A.4.4) in order to establish a relationship between $\hat{R}^N_{D,t}$ and $\tilde{q}^N_t$. Second, as an interim step, we compactly re-write a number of terms to prepare the replication result. Third, we solve for the unique sequence of $\tilde{q}^N_t$, ensuring $\hat{R}^N_{D,t} = \hat{R}^{N*}_{D,t}$ $\forall t$, which can be expressed as a function of i) predetermined variables, ii) starred contemporaneous variables $\hat{c}^{N*}_t$, $\hat{\pi}^{N*}_{c,t}$, and iii) the sequences $\{\hat{R}^{N*}_{D,t+j}\}^{\infty}_{j=0}$, $\{\hat{R}_{S,t+j}\}^{\infty}_{j=0}$.

### A.4.1 Step 1: Establishing a link between $\hat{R}^N_{D,t}$ and $\tilde{q}^N_t$

To rewrite equation (A.4.4) recall that **long-term bonds** satisfy the relationships

$$B^N_{LD,t} = B^N_{LP,t} - B^N_{LF,t}$$

and

$$B^N_{LG,t} = B^N_{LD,t} + \frac{1-\alpha}{\alpha}B^S_{LF,t} + Q^N_t.$$

In a symmetric monetary union cross-holdings of bonds satisfy

$$B^N_{LF,t} = \frac{1-\alpha}{\alpha}B^S_{LF,t},$$

leading to

$$B^N_{LP,t} = B^N_{LG,t} - Q^N_t. \quad \text{(A.4.6)}$$

Similarly, for **short-term bonds** use the relationships

$$B^N_{SD,t} = B^N_{SP,t} - B^N_{SF,t}$$

and

$$B^N_{SG,t} = B^N_{SD,t} + \frac{1-\alpha}{\alpha} B^S_{SF,t} + B^N_{SC,t}$$

In a symmetric monetary union cross-holdings of bonds satisfy

$$B^N_{SF,t} = \frac{1-\alpha}{\alpha} B^S_{SF,t}$$

leading to

$$B^N_{SP,t} = B^N_{SG,t} - B^N_{SC,t}. \tag{A.4.7}$$

The log-linearised versions of (A.4.6) and (A.4.7), after deflating by $P^N_{c,t}$, are given by

$$\hat{b}^N_{SP,t} = \hat{b}^N_{SG,t} \cdot \frac{\bar{B}^N_{SG}}{\bar{B}^N_{SP}} - \hat{b}^N_{SC,t} \cdot \frac{\bar{B}^N_{SC}}{\bar{B}^N_{SP}} \tag{A.4.8}$$

$$\hat{b}^N_{LP,t} = \hat{b}^N_{LG,t} \cdot \frac{\bar{B}^N_{LG}}{\bar{B}^N_{LP}} - \hat{q}^N_t \tag{A.4.9}$$

Using (A.4.8) and (A.4.9) in (A.4.4) leads to

$$\hat{b}^N_{LP,t} - \hat{b}^N_{SP,t} = \hat{b}^N_{LG,t} \cdot \frac{\bar{B}^N_{LG}}{\bar{B}^N_{LP}} - \left[\hat{b}^N_{SG,t} \cdot \frac{\bar{B}^N_{SG}}{\bar{B}^N_{SP}} - \hat{b}^N_{SC,t} \cdot \frac{\bar{B}^N_{SC}}{\bar{B}^N_{SP}}\right] - \hat{q}^N_t$$

This can be simplified if one uses the proportionality feature (20) of the fiscal rule, i.e.

$$\hat{b}^N_{LG,t} = \hat{b}^N_{SG,t}$$

as well as the steady-state restrictions

$$\frac{\bar{B}^N_{LG}}{\bar{B}^N_{LP}} = 1 \qquad \text{and} \qquad \frac{\bar{B}^N_{SG}}{\bar{B}^N_{SP}} = \frac{\bar{B}^N_{SP} + \bar{B}^N_{SC}}{\bar{B}^N_{SP}} = 1 + \frac{\bar{B}^N_{SC}}{\bar{B}^N_{SP}} \equiv 1 + \bar{m}_b$$

to get

$$\hat{b}^N_{LP,t} - \hat{b}^N_{SP,t} = \bar{m}_b \left[\hat{b}^N_{SC,t} - \hat{b}^N_{SG,t}\right] - \hat{q}^N_t.$$

Hence, we can write (A.4.4) as

$$\hat{R}^N_{D,t} = \hat{R}_{S,t} + \tilde{\nu}_1 \left[\bar{m}_b \left[\hat{b}^N_{SC,t} - \hat{b}^N_{SG,t}\right] - \hat{q}^N_t\right]$$

or, equivalently,

$$\frac{\tilde{q}^N_t}{\bar{b}^N_{LP}} = \frac{1}{\tilde{\nu}_1} \left[\hat{R}_{S,t} - \hat{R}^N_{D,t}\right] + \bar{m}_b \left[\hat{b}^N_{SC,t} - \hat{b}^N_{SG,t}\right] \tag{A.4.10}$$

### A.4.2 Step 2: Preparing the replication result

To replicate $\hat{R}^{N*}_{D,t}$ through appropriate variations in $\tilde{q}^N_t$ via equation (A.4.10) we express, as an interim step, the two terms $\hat{b}^N_{SC,t}$ and $\hat{b}^N_{SG,t}$ as functions of $\tilde{q}^N_t$ as well as of i) predetermined variables, ii) starred contemporaneous variables $\hat{c}^{N*}_t$, $\hat{\pi}^{N*}_{c,t}$, and iii) the sequences $\{\hat{R}^{N*}_{D,t+j}\}^\infty_{j=0}$, $\{\hat{R}_{S,t+j}\}^\infty_{j=0}$.

**i) Rearranging the $\hat{b}^N_{SC,t}$-term**:
Use

$$M^N_t = B^N_{SC,t} + Q^N_t$$

to express values of $\hat{b}^N_{SC,t}$ that are consistent with $\hat{c}^{N*}_t$ and $\hat{R}^{N*}_{D,t}$ as

$$\hat{b}^{Nq}_{SC,t} = \frac{\bar{M}^N}{\bar{B}^N_{SC}}\hat{m}^{N*}_t - \frac{\bar{B}^N_{LP}}{\bar{B}^N_{SC}}\hat{q}^N_t = \frac{\sigma_m}{\sigma}\hat{c}^{N*}_t - \frac{\sigma_m\beta}{1-\beta}\hat{R}^{N*}_{D,t} - \frac{\delta}{\bar{m}_b}\hat{q}^N_t \tag{A.4.11}$$

**ii) Rearranging the $\hat{b}^N_{SG,t}$-term**:
Recall that the budget constraint of the government (19) satisfies

$$B^N_{SG,t} + B^N_{LG,t} = R_{S,t-1}B^N_{SG,t-1} + R^N_{L,t}B^N_{LG,t-1} + P^N_{c,t}\tau^N_t - S^N_t,$$

while the seigniorage expression (27) reduces in a symmetric monetary union to

$$S^N_t = (R_{S,t-1}-1)B^N_{SC,t-1} + (R^N_{Lt}-1)Q^N_{t-1}$$

Hence, after deflating by $P^N_{c,t}$, the government budget constraint in real terms is

$$b^N_{SG,t} + b^N_{LG,t} = \frac{R_{S,t-1}}{\Pi^N_{c,t}}b^N_{SG,t-1} + \frac{R^N_{L,t}}{\Pi^N_{c,t}}b^N_{LG,t-1} + \tau^N_t - s^N_t$$

with

$$\begin{aligned} \tau^N_t &= -\theta\frac{\beta\bar{b}^N_{LP}}{\delta\bar{b}^N_{SG}}\left(\frac{R_{S,t-1}}{\Pi^N_{c,t}}b^N_{SG,t-1} - \bar{R}_S\frac{\bar{B}^N_{SG}}{\bar{P}^N_c}\right) + \bar{\tau}^N \\ s^N_t &= \frac{R_{S,t-1}-1}{\Pi^N_{c,t}}b^N_{SC,t-1} + \frac{R^N_{Lt}-1}{\Pi^N_{c,t}}q^N_{t-1} \end{aligned}$$

Linearising the government budget constraint, using the proportionality feature of the fiscal rule (i.e. $\hat{b}^N_{LG,t} = \hat{b}^N_{SG,t}$), yields

$$\hat{b}^N_{SG,t} = \frac{1}{1+\bar{m}_b+\delta}\left[\hat{z}^N_{G,t} + \frac{\delta}{\beta}\hat{R}^N_{L,t}\right] \tag{A.4.12}$$

with

$$\begin{aligned} \hat{z}^N_{G,t} &= \frac{1+\bar{m}_b}{\beta}\left[\hat{R}_{S,t-1} - \hat{\pi}^N_{c,t} + \hat{b}^N_{SG,t-1}\right] - \frac{\delta}{\beta}\left[\hat{\pi}^N_{c,t} - \hat{b}^N_{SG,t-1}\right] - \frac{\bar{m}_b}{\beta}(1-\beta)\hat{s}^N_t + \frac{1}{\bar{b}^N_{SP}}\tilde{\tau}^N_t \\ \tilde{\tau}^N_t &= -\theta\bar{b}^N_{SP}\left[\hat{R}_{S,t-1} - \hat{\pi}^N_{c,t} + \hat{b}^N_{SG,t-1}\right] \\ \hat{s}^N_t &= \frac{1}{1-\beta}\hat{R}_{S,t-1} + \hat{b}^N_{SC,t-1} - \hat{\pi}^N_{c,t} + \frac{\delta}{\bar{m}_b}\hat{q}^N_{t-1} \end{aligned}$$

Notice that the term $\hat{z}^N_{G,t}$ depends on i) predetermined variables and ii) the contemporaneous variable $\hat{\pi}^N_{c,t}$. Let $\hat{z}^{Nq}_{G,t}$ denote the particular value of $\hat{z}^N_{G,t}$ evaluated at $\hat{\pi}^{N*}_{c,t}$ and used below to back out $\hat{q}^N_t$. Next, the term $\hat{R}^N_{L,t}$ can be recursively rewritten, by combining

$$\hat{R}^N_{L,t} = \beta \hat{V}^N_t - \hat{V}^N_{t-1}$$

and (A.4.5), i.e.

$$\hat{R}^N_{D,t} = \frac{1}{1+\delta}\hat{R}_{S,t} + \frac{\delta}{1+\delta}\hat{R}^N_{L,t+1},$$

to yield

$$\begin{aligned} \hat{R}^N_{L,t} &= -\hat{V}^N_{t-1} - \beta \hat{R}^N_{L,t+1} + \beta^2 \hat{V}^N_{t+1} \\ &= -\hat{V}^N_{t-1} + \sum\nolimits_{j=0}^{\infty} \beta^{j+1}\frac{1}{\delta}\left[\hat{R}_{S,t+j} - (1+\delta)\hat{R}^N_{D,t+j}\right] \end{aligned}$$

Let $\hat{R}^{Nq}_{L,t}$ denote the particular value of $\hat{R}^N_{L,t}$ evaluated at $\{\hat{R}^{N*}_{D,t+j}\}^\infty_{j=0}, \{\hat{R}_{S,t+j}\}^\infty_{j=0}$. In sum this implies that there exists a uniquely defined

$$\hat{b}^{Nq}_{SG,t} = \frac{1}{1+\bar{m}_b+\delta}\left[\hat{z}^{Nq}_{G,t} + \frac{\delta}{\beta}\hat{R}^{Nq}_{L,t}\right] \tag{A.4.13}$$

which is a function of the set of predetermined variables $\{\hat{R}_{S,t-1}, \hat{b}^N_{SC,t-1}, \hat{b}^N_{SG,t-1}, \hat{V}^N_{t-1}, \hat{q}^N_{t-1}\}$ as well as of $\hat{\pi}^{N*}_{c,t}$ and $\{\hat{R}^{N*}_{D,t+j}\}^\infty_{j=0}, \{\hat{R}_{S,t+j}\}^\infty_{j=0}$.

### A.4.3 Step 3: Replication result

Finally, use both (A.4.11) and (A.4.13) in (A.4.10), evaluated at $\hat{R}^N_{D,t} = \hat{R}^{N*}_{D,t}$, to establish the uniquely defined sequence

$$\tilde{q}^N_t = \frac{1}{\frac{1}{\bar{b}^N_{LP}} + \frac{1}{\bar{b}^N_{SP}}}\left[\frac{1}{\tilde{\nu}_1}\left(\hat{R}_{S,t} - \hat{R}^{N*}_{D,t}\right) + \bar{m}_b\left(\frac{\sigma_m}{\sigma}\hat{c}^{N*}_t - \frac{\sigma_m\beta}{1-\beta}\hat{R}^{N*}_{D,t} - \hat{b}^{Nq}_{SG,t}\right)\right], \tag{A.4.14}$$

ensuring $\hat{R}^N_{D,t} = \hat{R}^{N*}_{D,t}$ $\forall t$, which can be expressed as a function of i) predetermined variables, ii) starred contemporaneous variables $\hat{c}^{N*}_t$, $\hat{\pi}^{N*}_{c,t}$, and iii) the sequences $\{\hat{R}^{N*}_{D,t+j}\}^\infty_{j=0}$, $\{\hat{R}_{S,t+j}\}^\infty_{j=0}$.

As stated in Corollary $I$, assume that unexpectedly in some period $t_1 > 0$ the lower bound constraint becomes binding, known for sure to last until $t_2$. Then, for $t < t_1$, by construction, $\hat{R}^{N*}_{D,t} = \hat{R}^N_{D,t}$ will be satisfied since $\hat{R}_{S,t} = \hat{R}^*_{S,t}$ and $\tilde{q}^N_t = 0$, while for $t \geqslant t_1$ the unique sequence (A.4.14) ensures that $\hat{R}^{N*}_{D,t}$ will be replicated in all subsequent periods, consistent with $\hat{R}_{S,t} = \hat{R}^*_{S,t}$ for $t \geqslant t_2$. q.e.d.

## A.5 Proof of Proposition 2

The previous proof can be extended to an asymmetric monetary union. Recall from the main text that the analytical core of the linearised dynamics reduces to

$$\hat{R}_{S,t} = \rho_R \hat{R}_{S,t-1} + (1-\rho_R)\left[\phi_\pi \hat{\pi}_{c,t} + \phi_y \hat{c}_t\right] + \varepsilon_{R,t} \tag{A.5.1}$$

as well as a pair of equations (holding in $N$ and $S$, respectively), namely

$$\begin{aligned}\hat{c}_t^N &= \hat{c}_{t+1}^N - \sigma\left[\hat{R}_{D,t}^N - \hat{\pi}_{c,t+1}^N - \hat{r}_{n,t}^N\right] \\ \hat{c}_t^S &= \hat{c}_{t+1}^S - \sigma\left[\hat{R}_{D,t}^S - \hat{\pi}_{c,t+1}^S - \hat{r}_{n,t}^S\right]\end{aligned} \tag{A.5.2}$$

$$\begin{aligned}\hat{\pi}_{pt}^N &= \beta\hat{\pi}_{p,t+1}^N + \frac{\varepsilon-1}{\chi}\left[\psi\hat{y}_t^N + \frac{1}{\sigma}\hat{c}_t^N + (1-\lambda_N)\hat{T}_t\right] \\ \hat{\pi}_{pt}^S &= \beta\hat{\pi}_{p,t+1}^S + \frac{\varepsilon-1}{\chi}\left[\psi\hat{y}_t^S + \frac{1}{\sigma}\hat{c}_t^S - (1-\lambda_S)\hat{T}_t\right]\end{aligned} \tag{A.5.3}$$

and

$$\begin{aligned}\hat{R}_{D,t}^N &= \hat{R}_{S,t} + \tilde{\nu}_1\left[\hat{b}_{LP,t}^N - \hat{b}_{SP,t}^N\right] \\ \hat{R}_{D,t}^S &= \hat{R}_{S,t} + \tilde{\nu}_1\left[\hat{b}_{LP,t}^S - \hat{b}_{SP,t}^S\right]\end{aligned} \tag{A.5.4}$$

$$\begin{aligned}\hat{R}_{L,t+1}^N &= \hat{R}_{L,t+1}^S + \tilde{\nu}_{2N}\left[\hat{b}_{LD,t}^N - \hat{b}_{LF,t}^N\right] \\ \hat{R}_{L,t+1}^S &= \hat{R}_{L,t+1}^N + \tilde{\nu}_{2S}\left[\hat{b}_{LD,t}^S - \hat{b}_{LF,t}^S\right]\end{aligned} \tag{A.5.5}$$

$$\begin{aligned}\hat{R}_{D,t}^N &= \frac{1}{1+\delta_N}\hat{R}_{S,t} + \frac{\delta_N\omega_N}{1+\delta_N}\hat{R}_{L,t+1}^N + \frac{\delta_N(1-\omega_N)}{1+\delta_N}\hat{R}_{L,t+1}^S \\ \hat{R}_{D,t}^S &= \frac{1}{1+\delta_S}\hat{R}_{S,t} + \frac{\delta_S\omega_S}{1+\delta_S}\hat{R}_{L,t+1}^S + \frac{\delta_S(1-\omega_S)}{1+\delta_S}\hat{R}_{L,t+1}^N\end{aligned} \tag{A.5.6}$$

and

$$\tilde{\Omega}_t^N = \hat{c}_t^N - \hat{y}_t^N + (1-\lambda_N)\hat{T}_t, \tag{A.5.7}$$

where, by construction, movements in the current account need to satisfy

$$\tilde{\Omega}_t^S = -\tilde{\Omega}_t^N$$

This representation of the core equations is consistent with Section 4.2 which restricted structural differences between $N$ and $S$, ceteris paribus, to the assumption $\omega_S \neq \omega_N$, while maintaining $\alpha = \frac{1}{2}$. This assumption implies that some values become country-specific, namely

$$\tilde{\nu}_{2N} \neq \tilde{\nu}_{2S},\ \bar{b}_{LP}^N \neq \bar{b}_{LP}^S,\ \delta_N \neq \delta_S,$$

while $\bar{b}_{SP}^N = \bar{b}_{SP}^S = \bar{b}_{SP}$. Notice that in view of (A.5.7), $\hat{c}_t^N = \hat{y}_t^N$ is no longer ensured (as is the case in Proposition 1). Moreover, recall that the two economies are linked by

$$\hat{\pi}_{c,t}^N = \lambda_N\hat{\pi}_{p,t}^N + (1-\lambda_N)\hat{\pi}_{p,t}^S \quad \text{and} \quad \hat{\pi}_{c,t}^S = \lambda_S\hat{\pi}_{p,t}^S + (1-\lambda_S)\hat{\pi}_{p,t}^N$$

$$\begin{aligned}\hat{T}_t &= \hat{T}_{t-1} + \hat{\pi}_{p,t}^S - \hat{\pi}_{p,t}^N \\ \hat{\pi}_{c,t} &= \frac{1}{2}\hat{\pi}_{c,t}^N + \frac{1}{2}\hat{\pi}_{c,t}^S\end{aligned}$$

$$\hat{y}_t \;=\; \frac{1}{2}\hat{y}_t^N + \frac{1}{2}\hat{y}_t^S$$

Let the hypothetical outcomes of this system induced by the unconstrained interest rate rule be denoted by starred variables. We consider a small shock such that $R^{N*}_{D,t} \geq 1$ and $R^{S*}_{D,t} \geq 1$ will be satisfied by assumption. The linearised QE-augmented policy rule implies that $\hat{R}_{S,t} = \hat{R}^*_{S,t}$ will always be satisfied, unless the lower bound constraint is binding. There exists a pair of uniquely defined sequences $\tilde{q}_t^N$ and $\tilde{q}_t^S$ (derived from $q_t^N = \frac{Q_t^N}{P^N_{c,t}}$ and $q_t^S = \frac{Q_t^S}{P^S_{c,t}}$) which ensures that, whenever the lower bound constraint becomes binding, $\hat{R}^N_{D,t} = \hat{R}^{N*}_{D,t}$ and $\hat{R}^S_{D,t} = \hat{R}^{S*}_{D,t}$ as well as $\tilde{\Omega}_t^N = \tilde{\Omega}_t^{N*} = -\tilde{\Omega}_t^{S*}$ remain being satisfied in all subsequent periods. The replicability of the sequences of $\hat{R}^{N*}_{D,t}$, $\hat{R}^{S*}_{D,t}$, $\tilde{\Omega}_t^{N*}$, and $\tilde{\Omega}_t^{S*}$ ensures that the pairs of the equations (A.5.2) and (A.5.3), when combined with (A.5.7) and after inserting the expressions for $\hat{\pi}^N_{c,t}$, $\hat{\pi}^S_{c,t}$ and $\hat{T}_t$, reduce to a 2x2-dynamic sub-system which will be solved in all periods by $\hat{c}_t^N = \hat{c}_t^{N*}$, $\hat{c}_t^S = \hat{c}_t^{S*}$ and $\hat{\pi}^N_{pt} = \hat{\pi}^{N*}_{pt}$, $\hat{\pi}^S_{pt} = \hat{\pi}^{S*}_{pt}$. Next, (A.5.7) can be used to back out $\hat{y}_t^{N*}$ and $\hat{y}_t^{S*}$. In view of the pair of equations

$$\hat{h}_t^{N*} = \hat{y}_t^{N*} \;\text{ and }\; \hat{h}_t^{S*} = \hat{y}_t^{S*}$$

$$\hat{m}_t^{N*} = \frac{\sigma_m}{\sigma}\hat{c}_t^{N*} - \frac{\sigma_m \beta}{1-\beta}\hat{R}^{N*}_{D,t} \;\text{ and }\; \hat{m}_t^{S*} = \frac{\sigma_m}{\sigma}\hat{c}_t^{S*} - \frac{\sigma_m \beta}{1-\beta}\hat{R}^{S*}_{D,t}$$

these features ensure that the QE-augmented policy rule replicates $A^{N*} = \left\{\hat{c}_t^{N*}, \hat{h}_t^{N*}, \hat{m}_t^{N*}\right\}_{t=0}^{\infty}$ and $A^{S*} = \left\{\hat{c}_t^{S*}, \hat{h}_t^{S*}, \hat{m}_t^{S*}\right\}_{t=0}^{\infty}$ and, thus, $W^{N*}$ and $W^{S*}$.

For the remainder of the proof it is at times convenient to rewrite $\tilde{q}_t^N$ and $\tilde{q}_t^S$ via the expressions $\hat{q}_t^N$ and $\hat{q}_t^S$ which are defined as

$$\hat{q}_t^N = \frac{\tilde{q}_t^N}{\bar{b}^N_{LP}} \;\text{ and }\; \hat{q}_t^S = \frac{\tilde{q}_t^S}{\bar{b}^S_{LP}}$$

The proof proceeds in three steps. First, we offer a representation of the pair of equations (A.5.4) as well as of (A.5.7) which expresses $\tilde{q}_t^N$ and $\tilde{q}_t^S$ as functions of $\hat{R}^N_{D,t}, \hat{R}^S_{D,t}$, and $\tilde{\Omega}_t^N = -\tilde{\Omega}_t^S$. Second, as an interim step, we compactly re-write a number of terms to prepare the replication result. Third, we solve for the unique sequences $\tilde{q}_t^N$ and $\tilde{q}_t^S$, ensuring $\hat{R}^N_{D,t} = \hat{R}^{N*}_{D,t}$, $\hat{R}^S_{D,t} = \hat{R}^{S*}_{D,t}$, and $\tilde{\Omega}_t^N = \tilde{\Omega}_t^{N*} = -\tilde{\Omega}_t^{S*}$ $\forall t$, which can be expressed as a function of i) predetermined variables, ii) starred contemporaneous variables $\hat{c}_t^{N*}$, $\hat{c}_t^{S*}$, $\hat{\pi}^{N*}_{c,t}$, $\hat{\pi}^{S*}_{c,t}$ and iii) the sequences $\{\hat{R}^{N*}_{D,t+j}\}_{j=0}^{\infty}$, $\{\hat{R}^{S*}_{D,t+j}\}_{j=0}^{\infty}$, $\{\hat{R}_{S,t+j}\}_{j=0}^{\infty}$.

### A.5.1 Step 1: Establishing links between $\hat{R}^N_{D,t}, \hat{R}^S_{D,t}$, $\tilde{\Omega}_t^N$, and $\tilde{q}_t^N$ and $\tilde{q}_t^S$

**i)** Rearranging equation (A.5.4) for $\hat{R}^N_{D,t}$:
Starting point:

$$\hat{R}^N_{D,t} = \hat{R}_{S,t} + \tilde{\nu}_1\left[\hat{b}^N_{LP,t} - \hat{b}^N_{SP,t}\right]$$

**i1)** Obtain an expression for $\hat{b}^N_{LP,t}$:
Recall that in an asymmetric union there is scope for cross-holdings of bonds.
Use for **long-term bonds**

$$B^N_{LP,t} = B^N_{LD,t} + B^N_{LF,t} \quad \text{and} \quad B^N_{LG,t} = B^N_{LD,t} + B^S_{LF,t} + Q^N_t,$$

leading to

$$B^N_{LP,t} = B^N_{LG,t} - \left(B^S_{LF,t} - B^N_{LF,t}\right) - Q^N_t,$$

which after deflating by $P^N_{c,t}$ can be written as

$$b^N_{LP,t} = b^N_{LG,t} - \Delta_{LT,t} - q^N_t$$

where

$$\Delta_{LT,t} = \frac{P^S_{c,t}}{P^N_{c,t}} b^S_{LF,t} - b^N_{LF,t} \tag{A.5.8}$$

denotes the per capita difference of privately held real foreign long-term bonds between $S$ and $N$, which can be different from zero outside the steady state.
Log-linearising yields, using $\frac{\bar{b}^N_{LG}}{\bar{b}^N_{LP}} = 1$,

$$\hat{b}^N_{LP,t} = \hat{b}^N_{LG,t} - \frac{1}{\bar{b}^N_{LP}} \tilde{q}^N_t - \frac{1}{\bar{b}^N_{LP}} \tilde{\Delta}_{LT,t} \tag{A.5.9}$$

with

$$\tilde{\Delta}_{LT,t} = (1-\omega_S)\bar{b}^S_{LP}\left[\hat{b}^S_{LF,t} + (\lambda_N + \lambda_S - 1)\hat{T}_t\right] - (1-\omega_N)\bar{b}^N_{LP}\hat{b}^N_{LF,t} \tag{A.5.10}$$

**i2)** Obtain an expression for $\hat{b}^N_{SP,t}$:
Similarly, for **short-term bonds** use

$$B^N_{SP,t} = B^N_{SD,t} + B^N_{SF,t} \quad \text{and} \quad B^N_{SG,t} = B^N_{SD,t} + B^S_{SF,t} + B^N_{SC,t}$$

leading to

$$B^N_{SP,t} = B^N_{SG,t} - \left(B^S_{SF,t} - B^N_{SF,t}\right) - B^N_{SC,t},$$

which after deflating by $P^N_{c,t}$ can be written as

$$b^N_{SP,t} = b^N_{SG,t} - \Delta_{ST,t} - b^N_{SC,t}$$

where

$$\Delta_{ST,t} = \frac{P^S_{c,t}}{P^N_{c,t}} b^S_{SF,t} - b^N_{SF,t} \tag{A.5.11}$$

denotes the per capita difference of privately held real foreign short-term bonds between $S$ and $N$, which can be different from zero outside the steady state.
Log-linearising yields, using $\frac{\bar{b}^N_{SG}}{\bar{b}^N_{SP}} = 1 + \bar{m}_b$ and $\bar{m}_b = \frac{\bar{b}^N_{SC}}{\bar{b}^N_{SP}}$,

$$\hat{b}^N_{SP,t} = (1+\bar{m}_b)\hat{b}^N_{SG,t} - \bar{m}_b\hat{b}^N_{SC,t} - \frac{1}{\bar{b}^N_{SP}}\tilde{\Delta}_{ST,t} \tag{A.5.12}$$

with

$$\tilde{\Delta}_{ST,t} = \frac{1}{2}\bar{b}_{SP}\left(\left[\hat{b}^S_{SF,t} + (\lambda_N + \lambda_S - 1)\hat{T}_t\right] - \hat{b}^N_{SF,t}\right)$$

**i3)** Inserting (A.5.9) and (A.5.12) from the previous two steps into (A.5.4) yields

$$\hat{R}^N_{D,t} \;=\; \hat{R}_{S,t} + \tilde{\nu}_1\left[\hat{b}^N_{LG,t} - \hat{q}^N_t - \frac{1}{\bar{b}^N_{LP}}\tilde{\Delta}_{LT,t} - \left((1+\bar{m}_b)\hat{b}^N_{SG,t} - \frac{1}{\bar{b}_{SP}}\left(\bar{m}\hat{b}^N_{SC,t} + \tilde{\Delta}_{ST,t}\right)\right)\right]$$

$$= \quad \hat{R}_{S,t} + \tilde{\nu}_1 \left[ -\bar{m}_b \hat{b}^N_{SG,t} - \frac{1}{\bar{b}^N_{LP}} \tilde{\Delta}_{LT,t} + \frac{1}{\bar{b}_{SP}} \left( \bar{m} \hat{b}^N_{SC,t} + \tilde{\Delta}_{ST,t} \right) - \frac{\tilde{q}^N_t}{\bar{b}^N_{LP}} \right],$$

where $\hat{b}^N_{LG,t} = \hat{b}^N_{SG,t}$ and $\frac{1}{\bar{b}_{SP}} \bar{m} = \bar{m}_b$ have been used.
**i4)** For further reference below, use in the previous equation the money market equilibrium

$$\bar{m} \hat{m}^N_t + \bar{m} \hat{m}^S_t = \bar{m} \hat{b}^N_{SC,t} + \bar{m} \hat{b}^S_{SC,t} + \tilde{q}^N_t + \tilde{q}^S_t$$

to substitute out for $\bar{m} \hat{b}^N_{SC,t}$, leading to

$$\begin{aligned} \hat{R}^N_{D,t} = \hat{R}_{S,t} + \tilde{\nu}_1 \Big[ & - \bar{m}_b \hat{b}^N_{SG,t} - \frac{1}{\bar{b}^N_{LP}} \tilde{\Delta}_{LT,t} + \frac{1}{\bar{b}_{SP}} \left( \tilde{\Delta}_{ST,t} - \bar{m} \hat{b}^S_{SC,t} \right) \\ & + \frac{1}{\bar{b}_{SP}} \left( \bar{m} \hat{m}^N_t + \bar{m} \hat{m}^S_t - \tilde{q}^N_t - \tilde{q}^S_t \right) - \frac{\tilde{q}^N_t}{\bar{b}^N_{LP}} \Big] \end{aligned} \quad \text{(A.5.13)}$$

**ii)** Rearranging equation (A.5.4) for $\hat{R}^S_{D,t}$:
Starting point:

$$\hat{R}^S_{D,t} = \hat{R}_{S,t} + \tilde{\nu}_1 \left[ \hat{b}^S_{LP,t} - \hat{b}^S_{SP,t} \right]$$

**ii1)** Obtain an expression for $\hat{b}^S_{LP,t}$ :
For $S$, the corresponding equation for $b^S_{LP,t}$ is given by

$$b^S_{LP,t} = b^S_{LG,t} + \frac{P^N_{c,t}}{P^S_{c,t}} \Delta_{LT,t} \ - q^S_t$$

Log-linearising yields

$$\hat{b}^S_{LP,t} = \hat{b}^S_{LG,t} - \frac{1}{\bar{b}^S_{LP}} \tilde{q}^S_t + \frac{1}{\bar{b}^S_{LP}} \tilde{\Delta}_{LT,t} \quad \text{(A.5.14)}$$

**ii2)** Obtain an expression for $\hat{b}^S_{SP,t}$ :
For $S$, the corresponding equation for $b^S_{SP,t}$ is given by

$$b^S_{SP,t} = b^S_{SG,t} + \frac{P^N_{c,t}}{P^S_{c,t}} \Delta_{ST,t} - b^S_{SC,t}$$

Log-linearising yields

$$\hat{b}^S_{SP,t} = (1 + \bar{m}_b) \hat{b}^S_{SG,t} - \bar{m}_b \hat{b}^S_{SC,t} + \frac{1}{\bar{b}^S_{SP}} \tilde{\Delta}_{ST,t} \quad \text{(A.5.15)}$$

**ii3)** Inserting (A.5.14) and (A.5.15) from the previous two steps into (A.5.4) for $\hat{R}^S_{D,t}$ yields

$$\begin{aligned} \hat{R}^S_{D,t} &= \hat{R}_{S,t} + \tilde{\nu}_1 \left[ \hat{b}^S_{LG,t} - \hat{q}^S_t + \frac{1}{\bar{b}^S_{LP}} \tilde{\Delta}_{LT,t} - \left( (1 + \bar{m}_b) \hat{b}^S_{SG,t} + \frac{1}{\bar{b}_{SP}} \left[ \tilde{\Delta}_{ST,t} - \bar{m} \hat{b}^S_{SC,t} \right] \right) \right] \\ &= \hat{R}_{S,t} + \tilde{\nu}_1 \left[ -\bar{m}_b \hat{b}^S_{SG,t} + \frac{1}{\bar{b}^S_{LP}} \tilde{\Delta}_{LT,t} - \frac{1}{\bar{b}_{SP}} \left( \tilde{\Delta}_{ST,t} - \bar{m} \hat{b}^S_{SC,t} \right) - \frac{\tilde{q}^S_t}{\bar{b}^S_{LP}} \right] \end{aligned} \quad \text{(A.5.16)}$$

**iii)** Re-expressing equation (A.5.7) for $\tilde{\Omega}^N_t$:

**iii1)** Starting point:

$$\tilde{\Omega}_t^N = \hat{c}_t^N - \hat{y}_t^N + (1-\lambda_N)\hat{T}_t$$

**iii2)** An alternative description for current account movements can be deduced from the funding channels discussed in the main text. Complementing equation (33) for $\Omega_t^S$, i.e.

$$\begin{aligned}
P_{p,t}^S \Omega_t^S &= \frac{\alpha}{1-\alpha}\left[M_t^N - M_{t-1}^N - (B_{SC,t}^N - B_{SC,t-1}^N) - (Q_t^N - Q_{t-1}^N)\right] \\
&+\mu_1\alpha(R_{S,t-1}-1)\left[B_{SC,t-1}^N - B_{SC,t-1}^S\right] \\
&+\mu_2\alpha\left[(R_{L,t}^N-1)Q_{t-1}^N - (R_{L,t}^S-1)Q_{t-1}^S\right] \\
&+\frac{\alpha}{1-\alpha}\left[B_{SF,t}^N - R_{S,t-1}B_{SF,t-1}^N\right] - \left[B_{SF,t}^S - R_{S,t-1}B_{SF,t-1}^S\right] \\
&+\frac{\alpha}{1-\alpha}\left[B_{LF,t}^N - R_{L,t}^S B_{LF,t-1}^N\right] - \left[B_{LF,t}^S - R_{L,t}^N B_{LF,t-1}^S\right]
\end{aligned}$$

the corresponding expression for $\Omega_t^N$ is given by

$$\begin{aligned}
P_{p,t}^N \Omega_t^N &= \frac{1-\alpha}{\alpha}\left[M_t^S - M_{t-1}^S - (B_{SC,t}^S - B_{SCt-1}^S) - (Q_t^S - Q_{t-1}^S)\right] \\
&+\mu_1(1-\alpha)(R_{S,t-1}-1)\left[B_{SC,t-1}^S - B_{SC,t-1}^N\right] \\
&+\mu_2(1-\alpha)\left[(R_{Lt}^S-1)Q_{t-1}^S - (R_{Lt}^N-1)Q_{t-1}^N\right] \\
&+\frac{1-\alpha}{\alpha}\left[B_{SF,t}^S - R_{S,t-1}B_{SF,t-1}^S\right] - \left[B_{SF,t}^N - R_{S,t-1}B_{SF,t-1}^N\right] \\
&+\frac{1-\alpha}{\alpha}\left[B_{LF,t}^S - R_{Lt}^N B_{LF,t-1}^S\right] - \left[B_{LF,t}^N - R_{L,t}^S B_{LF,t-1}^N\right]
\end{aligned}$$

Keeping in mind that the movements in $\Omega_t^S$ and $\Omega_t^N$ are not independent, we focus on the latter equation only.

**iii3)** Expressing the previous equation in real terms (at $\alpha = 1/2$) and using $\Delta_{LT,t}$ and $\Delta_{ST,t}$ as defined in (A.5.8) and (A.5.11), yields

$$\begin{aligned}
\frac{P_{p,t}^N}{P_{c,t}^N}\Omega_t^N &= \Delta_{ST,t} + \Delta_{LT,t} \\
&+\frac{P_{c,t}^S}{P_{c,t}^N}\left[m_t^S - b_{SC,t}^S - q_t^S\right] \\
&+\frac{P_{c,t}^S}{P_{c,t}^N}\left[\frac{1}{\Pi_{c,t}^S}b_{SCt-1}^S + \frac{1}{\Pi_{c,t}^S}q_{t-1}^S - \frac{1}{\Pi_{c,t}^S}m_{t-1}^S)\right] \\
&+\frac{P_{c,t}^S}{P_{c,t}^N}\mu_1\frac{1}{2}(R_{S,t-1}-1)\left[\frac{1}{\Pi_{c,t}^S}b_{SC,t-1}^S - \frac{1}{\Pi_{c,t}^S}\frac{P_{c,t-1}^N}{P_{c,t-1}^S}b_{SC,t-1}^N\right] \\
&+\frac{P_{c,t}^S}{P_{c,t}^N}\mu_2\frac{1}{2}\left[(R_{Lt}^S-1)\frac{1}{\Pi_{c,t}^S}q_{t-1}^S - (R_{Lt}^N-1)\frac{1}{\Pi_{c,t}^S}\frac{P_{c,t-1}^N}{P_{c,t-1}^S}q_{t-1}^N\right] \\
&-\left(R_{S,t-1}\frac{1}{\Pi_{c,t}^N}\Delta_{ST_{t-1}} + R_{Lt}^N\frac{1}{\Pi_{c,t}^N}\Delta_{LT_{t-1}}\right) + \left(R_{L,t}^S - R_{Lt}^N\right)\frac{1}{\Pi_{c,t}^N}b_{LF,t-1}^N
\end{aligned}$$

which can be written more compactly as

$$\frac{P_{p,t}^N}{P_{c,t}^N}\Omega_t^N = \Delta_{ST,t} + \Delta_{LT,t} + \frac{P_{c,t}^S}{P_{c,t}^N}\left[m_t^S - b_{SC,t}^S - q_t^S\right]$$

$$
\begin{aligned}
&+ \left(R^S_{L,t} - R^N_{Lt}\right) \frac{1}{\Pi^N_{c,t}} b^N_{LF,t-1} \\
&+ z^N_{\Omega,t}
\end{aligned}
\tag{A.5.17}
$$

with

$$
\begin{aligned}
z^N_{\Omega,t} &= \frac{P^S_{c,t}}{P^N_{c,t}} \left[ \frac{1}{\Pi^S_{c,t}} b^S_{SCt-1} + \frac{1}{\Pi^S_{c,t}} q^S_{t-1} - \frac{1}{\Pi^S_{c,t}} m^S_{t-1}) \right] \\
&+ \frac{P^S_{c,t}}{P^N_{c,t}} \mu_1 \frac{1}{2} (R_{S,t-1} - 1) \left[ \frac{1}{\Pi^S_{c,t}} b^S_{SC,t-1} - \frac{1}{\Pi^S_{c,t}} \frac{P^N_{c,t-1}}{P^S_{c,t-1}} b^N_{SC,t-1} \right] \\
&+ \frac{P^S_{c,t}}{P^N_{c,t}} \mu_2 \frac{1}{2} \left[ (R^S_{Lt} - 1) \frac{1}{\Pi^S_{c,t}} q^S_{t-1} - (R^N_{Lt} - 1) \frac{1}{\Pi^S_{c,t}} \frac{P^N_{c,t-1}}{P^S_{c,t-1}} q^N_{t-1} \right] \\
&- \left( R_{S,t-1} \frac{1}{\Pi^N_{c,t}} \Delta_{ST_{t-1}} + R^N_{Lt} \frac{1}{\Pi^N_{c,t}} \Delta_{LT_{t-1}} \right)
\end{aligned}
$$

Linearising (A.5.17) yields

$$
\begin{aligned}
\tilde{\Omega}^N_t &= \tilde{\Delta}_{LT,t} + \tilde{\Delta}_{ST,t} - \bar{m} \hat{b}^S_{SC,t} + \bar{m} \hat{m}^S_t - \tilde{q}^S_t \\
&+ \frac{1}{\beta} \bar{b}^N_{LF} \left[ \hat{R}^S_{L,t} - \hat{R}^N_{L,t} \right] \\
&+ \tilde{z}^N_{\Omega,t}
\end{aligned}
\tag{A.5.18}
$$

with

$$
\begin{aligned}
\tilde{z}^N_{\Omega,t} &= \bar{m} \left( \hat{b}^S_{SCt-1} - \hat{m}^S_{t-1} \right) + \tilde{q}^S_{t-1} - \frac{1}{\beta} \left( \tilde{\Delta}_{ST_{t-1}} + \tilde{\Delta}_{LT_{t-1}} \right) \\
&+ \mu_1 \frac{1}{2} \left( \frac{1}{\beta} - 1 \right) \bar{m} \left( \hat{b}^S_{SCt-1} - \hat{b}^N_{SCt-1} + (\lambda_N + \lambda_S - 1) \hat{T}_{t-1} \right) \\
&+ \mu_2 \frac{1}{2} \left( \frac{1}{\beta} - 1 \right) \left( \tilde{q}^S_{t-1} - \tilde{q}^N_{t-1} \right)
\end{aligned}
\tag{A.5.19}
$$

**iv)** Establishing links between $\hat{R}^N_{D,t}$, $\hat{R}^S_{D,t}$, $\tilde{\Omega}^N_t$, and $\tilde{q}^N_t$ and $\tilde{q}^S_t$:
The expression for $\tilde{\Omega}^N_t$,when rewritten as

$$
\tilde{\Delta}_{ST,t} - \bar{m} \hat{b}^S_{SC,t} = \tilde{\Omega}^N_t - \tilde{\Delta}_{LT,t} - \bar{m} \hat{m}^S_t - \frac{1}{\beta} \bar{b}^N_{LF} \left[ \hat{R}^S_{L,t} - \hat{R}^N_{L,t} \right] - \tilde{z}^N_{\Omega t} + \tilde{q}^S_t
$$

can be used to substitute out for the term $\tilde{\Delta}_{ST,t} - \bar{m} \hat{b}^S_{SC,t}$ in (A.5.13) and (A.5.16), leading to the following expression for $\hat{R}^N_{D,t}$:

$$
\begin{aligned}
\hat{R}^N_{D,t} &= \hat{R}_{S,t} + \tilde{\nu}_1 \left[ -\bar{m}_b \hat{b}^N_{SG,t} - \frac{1}{\bar{b}^N_{LP}} \tilde{\Delta}_{LT,t} + \frac{1}{\bar{b}_{SP}} \left( \bar{m} \hat{m}^N_t + \bar{m} \hat{m}^S_t - \tilde{q}^N_t - \tilde{q}^S_t \right) - \frac{\tilde{q}^N_t}{\bar{b}^N_{LP}} \right] \\
&+ \tilde{\nu}_1 \frac{1}{\bar{b}_{SP}} \left[ \tilde{\Omega}^N_t - \tilde{\Delta}_{LT,t} - \bar{m} \hat{m}^S_t - \frac{1}{\beta} \bar{b}^N_{LF} \left[ \hat{R}^S_{L,t} - \hat{R}^N_{L,t} \right] - \tilde{z}^N_{\Omega t} + \tilde{q}^S_t \right] \\
&= \hat{R}_{S,t} + \tilde{\nu}_1 \left[ \frac{1}{\bar{b}_{SP}} \tilde{\Omega}^N_t - \bar{m}_b \hat{b}^N_{SG,t} - \left( \frac{1}{\bar{b}^N_{LP}} + \frac{1}{\bar{b}_{SP}} \right) \tilde{\Delta}_{LT,t} + \frac{1}{\bar{b}_{SP}} \bar{m} \hat{m}^N_t \right]
\end{aligned}
$$

$$
\begin{aligned}
&-\tilde{\nu}_1\left[\frac{1}{\bar{b}_{SP}}\frac{1}{\beta}\bar{b}_{LF}^N\left[\hat{R}_{L,t}^S-\hat{R}_{L,t}^N\right]+\frac{1}{\bar{b}_{SP}}\tilde{z}_{\Omega t}^N\right] \\
&-\tilde{\nu}_1\left(\frac{1}{\bar{b}_{LP}^N}+\frac{1}{\bar{b}_{SP}}\right)\tilde{q}_t^N
\end{aligned} \tag{A.5.20}
$$

Similarly, we get for $\hat{R}_{D,t}^S$ :

$$
\begin{aligned}
\hat{R}_{D,t}^S &= \hat{R}_{S,t}+\tilde{\nu}_1\left[-\bar{m}_b\hat{b}_{SG,t}^S+\frac{1}{\bar{b}_{LP}^S}\tilde{\Delta}_{LT,t}-\frac{\tilde{q}_t^S}{\bar{b}_{LP}^S}\right] \\
&\quad -\tilde{\nu}_1\frac{1}{\bar{b}_{SP}}\left[\tilde{\Omega}_t^N-\tilde{\Delta}_{LT,t}-\bar{m}\hat{m}_t^S-\frac{1}{\beta}\bar{b}_{LF}^N\left[\hat{R}_{L,t}^S-\hat{R}_{L,t}^N\right]-\tilde{z}_{\Omega t}^N+\tilde{q}_t^S\right] \\
&= \hat{R}_{S,t}+\tilde{\nu}_1\left[-\frac{1}{\bar{b}_{SP}}\tilde{\Omega}_t^N-\bar{m}_b\hat{b}_{SG,t}^S+\left[\frac{1}{\bar{b}_{LP}^S}+\frac{1}{\bar{b}_{SP}}\right]\tilde{\Delta}_{LT,t}+\frac{1}{\bar{b}_{SP}}\bar{m}\hat{m}_t^S\right] \\
&\quad +\tilde{\nu}_1\left[\frac{1}{\bar{b}_{SP}}\frac{1}{\beta}\bar{b}_{LF}^N\left[\hat{R}_{L,t}^S-\hat{R}_{L,t}^N\right]+\frac{1}{\bar{b}_{SP}}\tilde{z}_{\Omega t}^N\right] \\
&\quad -\tilde{\nu}_1\left(\frac{1}{\bar{b}_{LP}^S}+\frac{1}{\bar{b}_{SP}}\right)\tilde{q}_t^S
\end{aligned} \tag{A.5.21}
$$

### A.5.2 Step 2: Preparing the replication result

To replicate $\hat{R}_{D,t}^{N*}$, $\hat{R}_{D,t}^{S*}$ and $\tilde{\Omega}_t^{N*}$ through appropriate variations in $\tilde{q}_t^N$ and $\tilde{q}_t^S$ via equations (A.5.20) and (A.5.21) we express, as an interim step, $\hat{R}_{L,t}^N$, $\hat{R}_{L,t}^S$, $\hat{b}_{SG,t}^N$, $\hat{b}_{SG,t}^S$ and $\tilde{\Delta}_{LT,t}$as functions of $\tilde{q}_t^N$ and $\tilde{q}_t^S$, as well as of i) predetermined variables, ii) starred contemporaneous variables $\hat{c}_t^{N*}$, $\hat{c}_t^{S*}$, $\hat{\pi}_{c,t}^{N*}$, $\hat{\pi}_{c,t}^{S*}$ and iii) the sequences $\{\hat{R}_{D,t+j}^{N*}\}_{j=0}^{\infty}$, $\{\hat{R}_{D,t+j}^{S*}\}_{j=0}^{\infty}$, $\{\hat{R}_{S,t+j}\}_{j=0}^{\infty}$. Notice that the term $\tilde{z}_{\Omega t}^N$ as derived in (A.5.19) is already in the required format.

**i) Rearranging the $\hat{R}_{L,t}^N$ and $\hat{R}_{L,t}^S$-terms**:
**i1)** Calculation of $\hat{R}_{L,t+1}^N$ and $\hat{R}_{L,t+1}^S$:
In anticipation of the forward-looking determination of $\hat{R}_{L,t}^N$ and $\hat{R}_{L,t}^S$ we first solve for $\hat{R}_{L,t+1}^N$ and $\hat{R}_{L,t+1}^S$, using the expressions for $N$ and $S$ in (A.5.6), i.e.

$$
\begin{aligned}
\hat{R}_{D,t}^N &= \frac{1}{1+\delta_N}\hat{R}_{S,t}+\frac{\delta_N\omega_N}{1+\delta_N}\hat{R}_{L,t+1}^N+\frac{\delta_N(1-\omega_N)}{1+\delta_N}\hat{R}_{L,t+1}^S \\
\hat{R}_{D,t}^S &= \frac{1}{1+\delta_S}\hat{R}_{S,t}+\frac{\delta_S\omega_S}{1+\delta_S}\hat{R}_{L,t+1}^S+\frac{\delta_S(1-\omega_S)}{1+\delta_S}\hat{R}_{L,t+1}^N
\end{aligned}
$$

Let

$$
H_1\cdot\begin{bmatrix}\hat{R}_{L,t+1}^N\\ \hat{R}_{L,t+1}^S\end{bmatrix}=\begin{bmatrix}\hat{R}_{D,t}^N-\frac{1}{1+\delta_N}\hat{R}_{S,t}\\ \hat{R}_{D,t}^S-\frac{1}{1+\delta_S}\hat{R}_{S,t}\end{bmatrix}
$$

$$
H_1=\begin{bmatrix}\frac{\delta_N\omega_N}{1+\delta_N} & \frac{\delta_N(1-\omega_N)}{1+\delta_N}\\ \frac{\delta_S(1-\omega_S)}{1+\delta_S} & \frac{\delta_S\omega_S}{1+\delta_S}\end{bmatrix}
$$

The determinant of $H_1$ is given by

$$
|H_1|=\frac{\delta_N\omega_N}{1+\delta_N}\frac{\delta_S\omega_S}{1+\delta_S}-\frac{\delta_S(1-\omega_S)}{1+\delta_S}\frac{\delta_N(1-\omega_N)}{1+\delta_N}=\frac{\delta_N\delta_S}{(1+\delta_N)(1+\delta_S)}[\omega_S+\omega_N-1]\neq 0.
$$

Hence,

$$\begin{aligned}
\hat{R}^N_{L,t+1} &= \frac{\left(\hat{R}^N_{D,t} - \frac{1}{1+\delta_N}\hat{R}_{S,t}\right)\frac{\delta_S\omega_S}{1+\delta_S} - \left(\hat{R}^S_{D,t} - \frac{1}{1+\delta_S}\hat{R}_{S,t}\right)\frac{\delta_N(1-\omega_N)}{1+\delta_N}}{|H_1|} \\
&= \frac{\frac{\delta_S\omega_S}{1+\delta_S}\hat{R}^N_{D,t} - \frac{\delta_S\omega_S}{1+\delta_S}\frac{1}{1+\delta_N}\hat{R}_{S,t} - \frac{\delta_N(1-\omega_N)}{1+\delta_N}\hat{R}^S_{D,t} + \frac{\delta_N(1-\omega_N)}{1+\delta_N}\frac{1}{1+\delta_S}\hat{R}_{S,t}}{|H_1|} \\
&= \frac{(1+\delta_N)\,\delta_S\omega_S\hat{R}^N_{D,t} - \delta_S\omega_S\hat{R}_{S,t} - (1+\delta_S)\,\delta_N(1-\omega_N)\hat{R}^S_{D,t} + \delta_N(1-\omega_N)\hat{R}_{S,t}}{\delta_N\delta_S\,(\omega_S+\omega_N-1)} \\
&= \frac{\delta_N(1-\omega_N) - \delta_S\omega_S}{\delta_N\delta_S\,(\omega_S+\omega_N-1)}\hat{R}_{S,t} \\
&\quad + \frac{(1+\delta_N)\,\delta_S\omega_S}{\delta_N\delta_S\,(\omega_S+\omega_N-1)}\hat{R}^N_{D,t} - \frac{(1+\delta_S)\,\delta_N(1-\omega_N)}{\delta_N\delta_S\,(\omega_S+\omega_N-1)}\hat{R}^S_{D,t}
\end{aligned}$$

or, more compactly,

$$\begin{aligned}
\hat{R}^N_{L,t+1} &= \psi_{1N}\hat{R}_{S,t} + \psi_{2N}\hat{R}^N_{D,t} + \psi_{3N}\hat{R}^S_{D,t} \qquad \text{(A.5.22)} \\
\psi_{1N} &= \frac{\delta_N(1-\omega_N) - \delta_S\omega_S}{\delta_N\delta_S\,(\omega_S+\omega_N-1)} \\
\psi_{2N} &= \frac{(1+\delta_N)\,\delta_S\omega_S}{\delta_N\delta_S\,(\omega_S+\omega_N-1)} \\
\psi_{3N} &= -\frac{(1+\delta_S)\,\delta_N(1-\omega_N)}{\delta_N\delta_S\,(\omega_S+\omega_N-1)}
\end{aligned}$$

Similarly

$$\begin{aligned}
\hat{R}^S_{L,t+1} &= \frac{\left(\hat{R}^S_{D,t} - \frac{1}{1+\delta_S}\hat{R}_{S,t}\right)\frac{\delta_N\omega_N}{1+\delta_N} - \left(\hat{R}^N_{D,t} - \frac{1}{1+\delta_N}\hat{R}_{S,t}\right)\frac{\delta_S(1-\omega_S)}{1+\delta_S}}{|H_1|} \\
&= \frac{\frac{\delta_N\omega_N}{1+\delta_N}\hat{R}^S_{D,t} - \frac{\delta_N\omega_N}{1+\delta_N}\frac{1}{1+\delta_S}\hat{R}_{S,t} - \frac{\delta_S(1-\omega_S)}{1+\delta_S}\hat{R}^N_{D,t} + \frac{\delta_S(1-\omega_S)}{1+\delta_S}\frac{1}{1+\delta_N}\hat{R}_{S,t}}{|H_1|} \\
&= \frac{(1+\delta_S)\,\delta_N\omega_N\hat{R}^S_{D,t} - \delta_N\omega_N\hat{R}_{S,t} - (1+\delta_N)\,\delta_S(1-\omega_S)\hat{R}^N_{D,t} + \delta_S(1-\omega_S)\hat{R}_{S,t}}{\delta_N\delta_S\,(\omega_S+\omega_N-1)} \\
&= \frac{\delta_S(1-\omega_S) - \delta_N\omega_N}{\delta_N\delta_S\,(\omega_S+\omega_N-1)}\hat{R}_{S,t} \\
&\quad + \frac{(1+\delta_S)\,\delta_N\omega_N}{\delta_N\delta_S\,(\omega_S+\omega_N-1)}\hat{R}^S_{D,t} - \frac{(1+\delta_N)\,\delta_S(1-\omega_S)}{\delta_N\delta_S\,(\omega_S+\omega_N-1)}\hat{R}^N_{D,t}
\end{aligned}$$

or, more compactly,

$$\begin{aligned}
\hat{R}^S_{L,t+1} &= \psi_{1S}\hat{R}_{S,t} + \psi_{2S}\hat{R}^N_{D,t} + \psi_{3S}\hat{R}^S_{D,t} \qquad \text{(A.5.23)} \\
\psi_{1S} &= \frac{\delta_S(1-\omega_S) - \delta_N\omega_N}{\delta_N\delta_S\,(\omega_S+\omega_N-1)} \\
\psi_{2S} &= -\frac{(1+\delta_N)\,\delta_S(1-\omega_S)}{\delta_N\delta_S\,(\omega_S+\omega_N-1)} \\
\psi_{3S} &= \frac{(1+\delta_S)\,\delta_N\omega_N}{\delta_N\delta_S\,(\omega_S+\omega_N-1)}
\end{aligned}$$

**i2)** The term $\hat{R}^N_{L,t}$ can be recursively rewritten, by combining

$$\hat{R}^N_{L,t} = \beta\hat{V}^N_t - \hat{V}^N_{t-1}$$

and (A.5.22), i.e.

$$\hat{R}^N_{L,t+1} = \psi_{1N}\hat{R}_{S,t} + \psi_{2N}\hat{R}^N_{D,t} + \psi_{3N}\hat{R}^S_{D,t}$$

to yield

$$\begin{aligned}
\hat{R}^N_{L,t} &= -\hat{V}^N_{t-1} - \beta\hat{R}^N_{L,t+1} + \beta^2\hat{V}^N_{t+1} \\
&= -\hat{V}^N_{t-1} + \sum\nolimits_{j=0}^{\infty}\beta^{j+1}\left[\psi_{1N}\hat{R}_{S,t+j} + \psi_{2N}\hat{R}^N_{D,t+j} + \psi_{3N}\hat{R}^S_{D,t+j}\right]
\end{aligned}$$

Let $\hat{R}^{Nq}_{L,t}$ denote the particular value of $\hat{R}^N_{L,t}$ evaluated at $\{\hat{R}^{N*}_{D,t+j}\}_{j=0}^{\infty}$, $\{\hat{R}^{S*}_{D,t+j}\}_{j=0}^{\infty}$, $\{\hat{R}_{S,t+j}\}_{j=0}^{\infty}$.

**i3)** The term $\hat{R}^S_{L,t}$ can be recursively rewritten, by combining

$$\hat{R}^S_{L,t} = \beta\hat{V}^S_t - \hat{V}^S_{t-1}$$

and (A.5.23), i.e.

$$\hat{R}^S_{L,t+1} = \psi_{1S}\hat{R}_{S,t} + \psi_{2S}\hat{R}^N_{D,t} + \psi_{3S}\hat{R}^S_{D,t}$$

to yield

$$\begin{aligned}
\hat{R}^S_{L,t} &= -\hat{V}^S_{t-1} - \beta\hat{R}^S_{L,t+1} + \beta^2\hat{V}^S_{t+1} \\
&= -\hat{V}^S_{t-1} + \sum\nolimits_{j=0}^{\infty}\beta^{j+1}\left[\psi_{1S}\hat{R}_{S,t+j} + \psi_{2S}\hat{R}^N_{D,t+j} + \psi_{3S}\hat{R}^S_{D,t+j}\right]
\end{aligned}$$

Let $\hat{R}^{Sq}_{L,t}$ denote the particular value of $\hat{R}^S_{L,t}$ evaluated at $\{\hat{R}^{N*}_{D,t+j}\}_{j=0}^{\infty}$, $\{\hat{R}^{S*}_{D,t+j}\}_{j=0}^{\infty}$, $\{\hat{R}_{S,t+j}\}_{j=0}^{\infty}$.

**ii) Rearranging the $\hat{b}^N_{SG,t}$ and $\hat{b}^S_{SG,t}$-terms:**

**ii1)** Rearranging $\hat{b}^N_{SG,t}$ :

Linearising the government budget constraint, using the proportionality feature of the fiscal rule (i.e. $\hat{b}^N_{LG,t} = \hat{b}^N_{SG,t}$), yields

$$\hat{b}^N_{SG,t} = \frac{1}{1+\bar{m}_b+\delta_N}\left[\hat{z}^N_{G,t} + \frac{\delta_N}{\beta}\hat{R}^N_{L,t}\right]$$

with

$$\begin{aligned}
\hat{z}^N_{G,t} &= \frac{1+\bar{m}_b}{\beta}\left[\hat{R}_{S,t-1} - \hat{\pi}^N_{c,t} + \hat{b}^N_{SG,t-1}\right] - \frac{\delta_N}{\beta}\left[\hat{\pi}^N_{c,t} - \hat{b}^N_{SG,t-1}\right] - \frac{\bar{m}_b}{\beta}(1-\beta)\hat{s}^N_t + \frac{1}{\bar{b}^N_{SP}}\tilde{\tau}^N_t \\
\tilde{\tau}^N_t &= -\theta\bar{b}^N_{SP}\left[\hat{R}_{S,t-1} - \hat{\pi}^N_{c,t} + \hat{b}^N_{SG,t-1}\right] \\
\hat{s}^N_t &= \frac{1}{1-\beta}\hat{R}_{S,t-1} + \left(1-\frac{1}{2}\mu_1\right)\left[\hat{b}^N_{SC,t-1} - \hat{\pi}^N_{c,t}\right] \\
&\quad +\mu_1\frac{1}{2}\left[\hat{b}^S_{SC,t-1} - \hat{\pi}^S_{c,t} + (\lambda_N+\lambda_S-1)\hat{T}_t\right] \\
&\quad +\left(1-\frac{1}{2}\mu_2\right)\frac{\delta_N}{\bar{m}_b}\hat{q}^N_{t-1} + \mu_2\frac{1}{2}\frac{\delta_S}{\bar{m}_b}\hat{q}^S_{t-1}
\end{aligned}$$

Notice that the term $\hat{z}^N_{G,t}$ depends on i) predetermined variables and ii) the contemporaneous variables $\hat{\pi}^N_{c,t}$ and $\hat{\pi}^S_{c,t}$. Let $\hat{z}^{Nq}_{G,t}$ denote the particular value of $\hat{z}^N_{G,t}$ evaluated at $\hat{\pi}^{N*}_{c,t}$ and $\hat{\pi}^{S*}_{c,t}$ used below to back out $\tilde{q}_t^N$ and $\tilde{q}_t^S$. Combine with $\hat{R}_{L,t}^{Nq}$ as established above to obtain the uniquely defined

$$\hat{b}_{SG,t}^{Nq} = \frac{1}{1+\bar{m}_b+\delta_N}\left[\hat{z}_{G,t}^{Nq} + \frac{\delta_N}{\beta}\hat{R}_{L,t}^{Nq}\right] \tag{A.5.24}$$

which is a function of the set of predetermined variables $\{\hat{R}_{S,t-1}, \hat{b}_{SC,t-1}^N, \hat{b}_{SC,t-1}^S, \hat{b}_{SG,t-1}^N, \hat{V}_{t-1}^N, \hat{q}_{t-1}^N, \hat{q}_{t-1}^S\}$ as well as of $\hat{\pi}_{c,t}^{N*}$, $\hat{\pi}_{c,t}^{S*}$ and $\{\hat{R}_{D,t+j}^{N*}\}_{j=0}^{\infty}, \{\hat{R}_{D,t+j}^{S*}\}_{j=0}^{\infty}, \{\hat{R}_{S,t+j}\}_{j=0}^{\infty}$.

**ii2)** Rearranging $\hat{b}_{SG,t}^S$:

Similarly, there exists a uniquely defined sequence

$$\hat{b}_{SG,t}^{Sq} = \frac{1}{1+\bar{m}_b+\delta_S}\left[\hat{z}_{G,t}^{Sq} + \frac{\delta_S}{\beta}\hat{R}_{L,t}^{Sq}\right] \tag{A.5.25}$$

which is a function of the set of predetermined variables $\{\hat{R}_{S,t-1}, \hat{b}_{SC,t-1}^N, \hat{b}_{SC,t-1}^S, \hat{b}_{SG,t-1}^S, \hat{V}_{t-1}^S, \hat{q}_{t-1}^N, \hat{q}_{t-1}^S\}$ as well as of $\hat{\pi}_{c,t}^{N*}$, $\hat{\pi}_{c,t}^{S*}$ and $\{\hat{R}_{D,t+j}^{N*}\}_{j=0}^{\infty}, \{\hat{R}_{D,t+j}^{S*}\}_{j=0}^{\infty}, \{\hat{R}_{S,t+j}\}_{j=0}^{\infty}$.

**iii) Rearranging the $\tilde{\Delta}_{LT,t}$-term**:

**iii1)** As derived in (A.5.10) we start out from

$$\tilde{\Delta}_{LT,t} = (1-\omega_S)\bar{b}_{LP}^S\left[\hat{b}_{LF,t}^S + (\lambda_N+\lambda_S-1)\hat{T}_t\right] - (1-\omega_N)\bar{b}_{LP}^N\hat{b}_{LF,t}^N,$$

implying that $\tilde{\Delta}_{LT,t}$ is a function of $\hat{b}_{LF,t}^N$ and $\hat{b}_{LF,t}^S$.

**iii2)** In order to obtain expressions for $\hat{b}_{LF,t}^N$ and $\hat{b}_{LF,t}^S$, we start out from the expressions for $N$ and $S$ in (A.5.5), i.e.

$$\begin{aligned}
\hat{R}_{L,t+1}^N &= \hat{R}_{L,t+1}^S + \tilde{\nu}_{2N}\left[\hat{b}_{LD,t}^N - \hat{b}_{LF,t}^N\right]\\
\hat{R}_{L,t+1}^S &= \hat{R}_{L,t+1}^N + \tilde{\nu}_{2S}\left[\hat{b}_{LD,t}^S - \hat{b}_{LF,t}^S\right]
\end{aligned}$$

**iii3)** Obtain $\hat{b}_{LD,t}^N - \hat{b}_{LF,t}^N$ **:**

Linearisation of

$$b_{LD,t}^N = b_{LP,t}^N - b_{LF,t}^N$$

yields

$$\omega_N\hat{b}_{LD,t}^N = \hat{b}_{LP,t}^N - (1-\omega_N)\hat{b}_{LF,t}^N$$

or, alternatively,

$$\hat{b}_{LD,t}^N = \frac{1}{\omega_N}\hat{b}_{LP,t}^N - \frac{1-\omega_N}{\omega_N}\hat{b}_{LF,t}^N$$

Subtracting $\hat{b}_{LF,t}^N$ gives

$$\hat{b}_{LD,t}^N - \hat{b}_{LF,t}^N = \frac{1}{\omega_N}\left(\hat{b}_{LP,t}^N - \hat{b}_{LF,t}^N\right)$$

Next, combine this with (A.5.9), i.e.

$$\hat{b}_{LP,t}^N = \hat{b}_{LG,t}^N - \frac{\tilde{q}_t^N}{\bar{b}_{LP}^N} - \frac{1}{\bar{b}_{LP}^N}\tilde{\Delta}_{LT,t}$$

to get

$$\hat{b}_{LD,t}^N - \hat{b}_{LF,t}^N = \frac{1}{\omega_N}\left(\hat{b}_{LG,t}^N - \frac{\tilde{q}_t^N}{\bar{b}_{LP}^N} - \frac{1}{\bar{b}_{LP}^N}\tilde{\Delta}_{LT,t} - \hat{b}_{LF,t}^N\right) \tag{A.5.26}$$

**iii4)** Obtain $\hat{b}^S_{LD,t} - \hat{b}^S_{LF,t}$ **:**
Similarly, for $S$ we get

$$\hat{b}^S_{LD,t} - \hat{b}^S_{LF,t} = \frac{1}{\omega_S}\left(\hat{b}^S_{LP,t} - \hat{b}^S_{LF,t}\right)$$

and combine it with (A.5.14), i.e.

$$\hat{b}^S_{LP,t} = \hat{b}^S_{LG,t} - \frac{\tilde{q}^S_t}{\bar{b}^S_{LP}} + \frac{1}{\bar{b}^S_{LP}}\tilde{\Delta}_{LT,t}$$

to get

$$\hat{b}^S_{LD,t} - \hat{b}^S_{LF,t} = \frac{1}{\omega_S}\left(\hat{b}^S_{LG,t} - \frac{\tilde{q}^S_t}{\bar{b}^S_{LP}} + \frac{1}{\bar{b}^S_{LP}}\tilde{\Delta}_{LT,t} - \hat{b}^S_{LF,t}\right) \tag{A.5.27}$$

**iii5)** Obtain $\hat{b}^N_{LF,t}$ and $\hat{b}^S_{LF,t}$:
To obtain $\hat{b}^N_{LF,t}$ and $\hat{b}^S_{LF,t}$ insert (A.5.26) and (A.5.27) as well as (A.5.10) into (A.5.5) for $N$ and $S$, leading to

$$\hat{R}^N_{L,t+1} - \hat{R}^S_{L,t+1} = \tilde{\nu}_{2N}\frac{1}{\omega_N}\left[\hat{b}^N_{LG,t} - \frac{\tilde{q}^N_t}{\bar{b}^N_{LP}} - \frac{1}{\bar{b}^N_{LP}}\tilde{\Delta}_{LT,t} - \hat{b}^N_{LF,t}\right]$$

$$\begin{aligned}\hat{R}^N_{L,t+1} - \hat{R}^S_{L,t+1} = \tilde{\nu}_{2N}\frac{1}{\omega_N}\Big[\hat{b}^N_{LG,t} - \frac{\tilde{q}^N_t}{\bar{b}^N_{LP}} - (1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}\hat{b}^S_{LF,t} \\ - \omega_N\hat{b}^N_{LF,t} - (1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}(\lambda_N + \lambda_S - 1)\hat{T}_t\Big]\end{aligned}$$

Similarly,

$$\hat{R}^S_{L,t+1} - \hat{R}^N_{L,t+1} = \tilde{\nu}_{2S}\frac{1}{\omega_S}\left[\hat{b}^S_{LG,t} - \frac{\tilde{q}^S_t}{\bar{b}^S_{LP}} + \frac{1}{\bar{b}^S_{LP}}\tilde{\Delta}_{LT,t} - \hat{b}^S_{LF,t}\right]$$

$$\begin{aligned}\hat{R}^S_{L,t+1} - \hat{R}^N_{L,t+1} = \tilde{\nu}_{2S}\frac{1}{\omega_S}\Big[\hat{b}^S_{LG,t} - \frac{\tilde{q}^S_t}{\bar{b}^S_{LP}} - \omega_S\hat{b}^S_{LF,t} \\ - (1-\omega_N)\frac{\bar{b}^N_{LP}}{\bar{b}^S_{LP}}\hat{b}^N_{LF,t} + (1-\omega_S)(\lambda_N + \lambda_S - 1)\hat{T}_t\Big]\end{aligned}$$

This pair of equations can be rewritten as

$$H_2 \cdot \begin{bmatrix} \hat{b}^N_{LF,t} \\ \hat{b}^S_{LF,t} \end{bmatrix} = \begin{bmatrix} \omega_N\frac{\hat{R}^S_{L,t+1} - \hat{R}^N_{L,t+1}}{\tilde{\nu}_{2N}} + \left[\hat{b}^N_{LG,t} - \frac{\tilde{q}^N_t}{\bar{b}^N_{LP}} - (1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}(\lambda_N + \lambda_S - 1)\hat{T}_t\right] \\ \omega_S\frac{\hat{R}^N_{L,t+1} - \hat{R}^S_{L,t+1}}{\tilde{\nu}_{2S}} + \left[\hat{b}^S_{LG,t} - \frac{\tilde{q}^S_t}{\bar{b}^S_{LP}} + (1-\omega_S)\left[(\lambda_N + \lambda_S - 1)\hat{T}_t\right]\right] \end{bmatrix}$$

$$H_2 = \begin{bmatrix} \omega_N & (1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}} \\ (1-\omega_N)\frac{\bar{b}^N_{LP}}{\bar{b}^S_{LP}} & \omega_S \end{bmatrix}$$

The determinant of $H_2$ is given by

$$Det(H_2) = \omega_N\omega_S - (1-\omega_N)(1-\omega_S) = \omega_N + \omega_S - 1 \neq 0$$

Solving for $\hat{b}^N_{LF,t}$ yields

$$\begin{aligned}
\hat{b}^N_{LF,t} &= \frac{1}{\omega_N+\omega_S-1}\left[\omega_N\omega_S\frac{\hat{R}^S_{L,t+1}-\hat{R}^N_{L,t+1}}{\tilde{\nu}_{2N}}-\omega_S\left(1-\omega_S\right)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}\frac{\hat{R}^N_{L,t+1}-\hat{R}^S_{L,t+1}}{\tilde{\nu}_{2S}}\right] \\
&+\frac{1}{\omega_N+\omega_S-1}\omega_S\left[\hat{b}^N_{LG,t}-\frac{\tilde{q}^N_t}{\bar{b}^N_{LP}}-(1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}(\lambda_N+\lambda_S-1)\hat{T}_t\right] \\
&-\frac{1}{\omega_N+\omega_S-1}(1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}\left[\hat{b}^S_{LG,t}-\frac{\tilde{q}^S_t}{\bar{b}^S_{LP}}+(1-\omega_S)\left[(\lambda_N+\lambda_S-1)\hat{T}_t\right]\right] \\
&= \frac{1}{\omega_N+\omega_S-1}\left[\frac{\omega_N\omega_S}{\tilde{\nu}_{2N}}+\frac{\omega_S\left(1-\omega_S\right)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}}{\tilde{\nu}_{2S}}\right]\left(\hat{R}^S_{L,t+1}-\hat{R}^N_{L,t+1}\right) \\
&+\frac{1}{\omega_N+\omega_S-1}\omega_S\left[\hat{b}^N_{LG,t}-\frac{\tilde{q}^N_t}{\bar{b}^N_{LP}}\right]-\frac{1}{\omega_N+\omega_S-1}(1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}\left[\hat{b}^S_{LG,t}-\frac{\tilde{q}^S_t}{\bar{b}^S_{LP}}\right] \\
&-\frac{1}{\omega_N+\omega_S-1}(1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}(\lambda_N+\lambda_S-1)\hat{T}_t
\end{aligned} \tag{A.5.28}$$

Similarly, solving for $\hat{b}^S_{LF,t}$yields

$$\begin{aligned}
\hat{b}^S_{LF,t} &= \frac{1}{\omega_N+\omega_S-1}\left[\frac{\omega_N\omega_S}{\tilde{\nu}_{2S}}+\frac{(1-\omega_N)\omega_N\frac{\bar{b}^N_{LP}}{\bar{b}^S_{LP}}}{\tilde{\nu}_{2N}}\right]\left(\hat{R}^N_{L,t+1}-\hat{R}^S_{L,t+1}\right) \\
&+\frac{1}{\omega_N+\omega_S-1}\omega_N\left[\hat{b}^S_{LG,t}-\frac{\tilde{q}^S_t}{\bar{b}^S_{LP}}+(1-\omega_S)\left[(\lambda_N+\lambda_S-1)\hat{T}_t\right]\right] \\
&-\frac{1}{\omega_N+\omega_S-1}(1-\omega_N)\frac{\bar{b}^N_{LP}}{\bar{b}^S_{LP}}\left[\hat{b}^N_{LG,t}-\frac{\tilde{q}^N_t}{\bar{b}^N_{LP}}-(1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}(\lambda_N+\lambda_S-1)\hat{T}_t\right] \\
&= \frac{1}{\omega_N+\omega_S-1}\left[\frac{\omega_N\omega_S}{\tilde{\nu}_{2S}}+\frac{(1-\omega_N)\omega_N\frac{\bar{b}^N_{LP}}{\bar{b}^S_{LP}}}{\tilde{\nu}_{2N}}\right]\left(\hat{R}^N_{L,t+1}-\hat{R}^S_{L,t+1}\right) \\
&+\frac{1}{\omega_N+\omega_S-1}\omega_N\left[\hat{b}^S_{LG,t}-\frac{\tilde{q}^S_t}{\bar{b}^S_{LP}}\right]-\frac{1}{\omega_N+\omega_S-1}(1-\omega_N)\frac{\bar{b}^N_{LP}}{\bar{b}^S_{LP}}\left[\hat{b}^N_{LG,t}-\frac{\tilde{q}^N_t}{\bar{b}^N_{LP}}\right] \\
&+\frac{1}{\omega_N+\omega_S-1}(1-\omega_S)(\lambda_N+\lambda_S-1)\hat{T}_t
\end{aligned} \tag{A.5.29}$$

**iii6)** Obtain $\tilde{\Delta}_{LT,t}$ :
Inserting the expressions (A.5.28) and (A.5.29) for $\hat{b}^N_{LF,t}$ and $\hat{b}^S_{LF,t}$ into (A.5.10), i.e.

$$\begin{aligned}
\tilde{\Delta}_{LT,t} &= (1-\omega_S)\bar{b}^S_{LP}\hat{b}^S_{LF,t}-(1-\omega_N)\bar{b}^N_{LP}\hat{b}^N_{LF,t} \\
&+(1-\omega_S)\bar{b}^S_{LP}(\lambda_N+\lambda_S-1)\hat{T}_t
\end{aligned}$$

yields

$$\tilde{\Delta}_{LT,t}=\xi_1\cdot\left[\hat{R}^N_{L,t+1}-\hat{R}^S_{L,t+1}\right]+\xi_2\cdot\left[\hat{b}^N_{LG,t}-\frac{\tilde{q}^N_t}{\bar{b}^N_{LP}}\right]+\xi_3\left[\hat{b}^S_{LG,t}-\frac{\tilde{q}^S_t}{\bar{b}^S_{LP}}\right]+\xi_4(\lambda_N+\lambda_S-1)\hat{T}_t \tag{A.5.30}$$

with

$$
\begin{aligned}
\xi_1 &= (1-\omega_S)\bar{b}^S_{LP}\frac{1}{\omega_N+\omega_S-1}\left[\frac{\omega_N\omega_S}{\tilde{\nu}_{2S}}+\frac{(1-\omega_N)\omega_N\frac{\bar{b}^N_{LP}}{\bar{b}^S_{LP}}}{\tilde{\nu}_{2N}}\right] \\
&\quad +(1-\omega_N)\bar{b}^N_{LP}\frac{1}{\omega_N+\omega_S-1}\left[\frac{\omega_N\omega_S}{\tilde{\nu}_{2N}}+\frac{\omega_S\left(1-\omega_S\right)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}}}{\tilde{\nu}_{2S}}\right] \\
&= \frac{1}{\omega_N+\omega_S-1}\left[\bar{b}^S_{LP}\frac{\omega_S(1-\omega_S)}{\tilde{\nu}_{2S}}+\bar{b}^N_{LP}\frac{\omega_N(1-\omega_N)}{\tilde{\nu}_{2N}}\right]
\end{aligned}
$$

$$
\begin{aligned}
\xi_2 &= -(1-\omega_S)\bar{b}^S_{LP}\frac{1}{\omega_N+\omega_S-1}(1-\omega_N)\frac{\bar{b}^N_{LP}}{\bar{b}^S_{LP}}-(1-\omega_N)\bar{b}^N_{LP}\frac{1}{\omega_N+\omega_S-1}\omega_S \\
&= -\bar{b}^N_{LP}\frac{1-\omega_N}{\omega_N+\omega_S-1}
\end{aligned}
$$

$$
\begin{aligned}
\xi_3 &= (1-\omega_S)\bar{b}^S_{LP}\frac{1}{\omega_N+\omega_S-1}\omega_N+(1-\omega_N)\bar{b}^N_{LP}\frac{1}{\omega_N+\omega_S-1}(1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}} \\
&= \bar{b}^S_{LP}\frac{1-\omega_S}{\omega_N+\omega_S-1}
\end{aligned}
$$

$$
\begin{aligned}
\xi_4 &= (1-\omega_S)\bar{b}^S_{LP}\frac{1}{\omega_N+\omega_S-1}(1-\omega_S)+(1-\omega_N)\bar{b}^N_{LP}\frac{1}{\omega_N+\omega_S-1}(1-\omega_S)\frac{\bar{b}^S_{LP}}{\bar{b}^N_{LP}} \\
&\quad +(1-\omega_S)\bar{b}^S_{LP} \\
&= \bar{b}^S_{LP}(1-\omega_S)\left[\frac{1}{\omega_N+\omega_S-1}(1-\omega_S)+(1-\omega_N)\frac{1}{\omega_N+\omega_S-1}+1\right] \\
&= \bar{b}^S_{LP}\frac{1-\omega_S}{\omega_N+\omega_S-1}
\end{aligned}
$$

**iii7)** Simplify the expression for $\tilde{\Delta}_{LT,t}$:
To further simplify (A.5.30) use (A.5.22) and (A.5.23) for $\hat{R}^N_{L,t+1}$and $\hat{R}^S_{L,t+1}$ to establish

$$
\hat{R}^N_{L,t+1}-\hat{R}^S_{L,t+1}=(\psi_{1N}-\psi_{1S})\hat{R}_{S,t}+(\psi_{2N}-\psi_{2S})\hat{R}^N_{D,t}+(\psi_{3N}-\psi_{3S})\hat{R}^S_{D,t}
$$

or, equivalently,

$$
\begin{aligned}
\hat{R}^N_{L,t+1}-\hat{R}^S_{L,t+1} &= \frac{\delta_N-\delta_S}{\delta_N\delta_S\left(\omega_S+\omega_N-1\right)}\hat{R}_{S,t} \\
&\quad +\frac{\left(1+\delta_N\right)\delta_S}{\delta_N\delta_S\left(\omega_S+\omega_N-1\right)}\hat{R}^N_{D,t} \\
&\quad -\frac{\left(1+\delta_S\right)\delta_N}{\delta_N\delta_S\left(\omega_S+\omega_N-1\right)}\hat{R}^S_{D,t},
\end{aligned}
$$

which can be simplified as

$$
\hat{R}^N_{L,t+1}-\hat{R}^S_{L,t+1}=\frac{\delta_N-\delta_S}{\delta_N\delta_S}\frac{1}{\omega_N-\omega_S-1}\hat{R}_{S,t}+\frac{1+\delta_N}{\delta_N}\frac{1}{\omega_N-\omega_S-1}\hat{R}^N_{D,t}-\frac{1+\delta_S}{\delta_S}\frac{1}{\omega_N-\omega_S-1}\hat{R}^S_{D,t}
$$

Hence, (A.5.30) can be rewritten as

$$\begin{aligned}
\tilde{\Delta}_{LT,t} &= \frac{\bar{b}^S_{LP}\frac{\omega_S(1-\omega_S)}{\tilde{\nu}_{2S}} + \bar{b}^N_{LP}\frac{\omega_N(1-\omega_N)}{\tilde{\nu}_{2N}}}{\omega_N+\omega_S-1} \cdot \left[\frac{\delta_N-\delta_S}{\delta_N\delta_S}\frac{1}{\omega_N-\omega_S-1}\hat{R}_{S,t}\right] \\
&+\frac{\bar{b}^S_{LP}\frac{\omega_S(1-\omega_S)}{\tilde{\nu}_{2S}} + \bar{b}^N_{LP}\frac{\omega_N(1-\omega_N)}{\tilde{\nu}_{2N}}}{\omega_N+\omega_S-1} \cdot \left[\frac{1+\delta_N}{\delta_N}\frac{1}{\omega_N-\omega_S-1}\hat{R}^N_{D,t} - \frac{1+\delta_S}{\delta_S}\frac{1}{\omega_N-\omega_S-1}\hat{R}^S_{D,t}\right] \\
&-\bar{b}^N_{LP}\frac{1-\omega_N}{\omega_N+\omega_S-1} \cdot \left[\hat{b}^N_{LG,t} - \frac{\tilde{q}^N_t}{\bar{b}^N_{LP}}\right] + \bar{b}^S_{LP}\frac{1-\omega_S}{\omega_N+\omega_S-1}\left[\hat{b}^S_{LG,t} - \frac{\tilde{q}^S_t}{\bar{b}^S_{LP}}\right] \\
&+\bar{b}^S_{LP}\frac{1-\omega_S}{\omega_N+\omega_S-1}(\lambda_N+\lambda_S-1)\hat{T}_t
\end{aligned}$$

**iii8)** Let

$$\tilde{\Delta\Delta}_{LT,t} = \tilde{\Delta}_{LT,t} + \frac{1-\omega_S}{\omega_N+\omega_S-1}\tilde{q}^S_t - \frac{1-\omega_N}{\omega_N+\omega_S-1}\tilde{q}^N_t$$

and use $\hat{b}^N_{LG,t} = \hat{b}^N_{SG,t}$, $\hat{b}^S_{LG,t} = \hat{b}^S_{SG,t}$, evaluated at $\hat{b}^{Nq}_{SG,t}$ and $\hat{b}^{Sq}_{SG,t}$, in order to establish the uniquely defined sequence

$$\begin{aligned}
\tilde{\Delta\Delta}^q_{LT,t} &= \frac{\bar{b}^S_{LP}\frac{\omega_S(1-\omega_S)}{\tilde{\nu}_{2S}} + \bar{b}^N_{LP}\frac{\omega_N(1-\omega_N)}{\tilde{\nu}_{2N}}}{\omega_N+\omega_S-1} \cdot \left[\frac{\delta_N-\delta_S}{\delta_N\delta_S}\frac{1}{\omega_N-\omega_S-1}\hat{R}_{S,t}\right] \\
&+\frac{\bar{b}^S_{LP}\frac{\omega_S(1-\omega_S)}{\tilde{\nu}_{2S}} + \bar{b}^N_{LP}\frac{\omega_N(1-\omega_N)}{\tilde{\nu}_{2N}}}{\omega_N+\omega_S-1} \cdot \left[\frac{1+\delta_N}{\delta_N}\frac{1}{\omega_N-\omega_S-1}\hat{R}^{N*}_{D,t} - \frac{1+\delta_S}{\delta_S}\frac{1}{\omega_N-\omega_S-1}\hat{R}^{S*}_{D,t}\right] \\
&-\bar{b}^N_{LP}\frac{1-\omega_N}{\omega_N+\omega_S-1} \cdot \hat{b}^{Nq}_{SG,t} + \bar{b}^S_{LP}\frac{1-\omega_S}{\omega_N+\omega_S-1}\hat{b}^{Sq}_{SG,t} \\
&+\bar{b}^S_{LP}\frac{1-\omega_S}{\omega_N+\omega_S-1}(\lambda_N+\lambda_S-1)\hat{T}^q_t,
\end{aligned}$$

which is a function of the set of predetermined variables $\{\hat{R}_{S,t-1}, \hat{b}^N_{SC,t-1}, \hat{b}^S_{SC,t-1}, \hat{b}^N_{SG,t-1}, \hat{b}^S_{SG,t-1}, \hat{V}^N_{t-1}, \hat{V}^S_{t-1}, \hat{q}^N_{t-1}, \hat{q}^S_{t-1}\}$ as well as of $\hat{\pi}^{N*}_{c,t}$, $\hat{\pi}^{S*}_{c,t}$ and $\{\hat{R}^{N*}_{D,t+j}\}^\infty_{j=0}$, $\{\hat{R}^{S*}_{D,t+j}\}^\infty_{j=0}$, $\{\hat{R}_{S,t+j}\}^\infty_{j=0}$.

### A.5.3 Step 3: Replication result

Finally, use $\tilde{\Delta\Delta}^q_{LT,t}$, $\hat{b}^{Nq}_{SG,t}$, $\hat{b}^{Sq}_{SG,t}$, $\hat{R}^{Sq}_{L,t}$, $\hat{R}^{Nq}_{L,t}$ in the two equations (A.5.20) and (A.5.21) for $\hat{R}^N_{D,t}$ and $\hat{R}^S_{D,t}$, evaluated at $\hat{R}^N_{D,t} = \hat{R}^{N*}_{D,t}$, $\hat{R}^S_{D,t} = \hat{R}^{S*}_{D,t}$, $\tilde{\Omega}^N_t = \tilde{\Omega}^{N*}_t$, to establish

$$\begin{aligned}
\hat{R}^{N*}_{D,t} &= \hat{R}_{S,t} + \tilde{\nu}_1\left[\frac{1}{\bar{b}_{SP}}\tilde{\Omega}^{N*}_t - \bar{m}_b\hat{b}^{Nq}_{SG,t} + \frac{1}{\bar{b}_{SP}}\bar{m}\hat{m}^{N*}_t\right] \\
&-\tilde{\nu}_1\left[\left(\frac{1}{\bar{b}^N_{LP}} + \frac{1}{\bar{b}_{SP}}\right)\left[\tilde{\Delta\Delta}^q_{LT,t} + \frac{1-\omega_N}{\omega_N+\omega_S-1}\tilde{q}^N_t - \frac{1-\omega_S}{\omega_N+\omega_S-1}\tilde{q}^S_t\right]\right] \\
&-\tilde{\nu}_1\left[\frac{1}{\bar{b}_{SP}}\frac{1}{\beta}\bar{b}^N_{LF}\left[\hat{R}^{Sq}_{L,t} - \hat{R}^{Nq}_{L,t}\right] + \frac{1}{\bar{b}_{SP}}\tilde{z}^N_{\Omega t}\right] \\
&-\tilde{\nu}_1\left(\frac{1}{\bar{b}^N_{LP}} + \frac{1}{\bar{b}_{SP}}\right)\tilde{q}^N_t
\end{aligned} \tag{A.5.31}$$

and

$$\hat{R}^{S*}_{D,t} = \hat{R}_{S,t} + \tilde{\nu}_1\left[-\frac{1}{\bar{b}_{SP}}\tilde{\Omega}^{N*}_t - \bar{m}_b\hat{b}^{Sq}_{SG,t} + \frac{1}{\bar{b}_{SP}}\bar{m}\hat{m}^{S*}_t\right]$$

$$+\tilde{\nu}_1 \left[\left[\frac{1}{\bar{b}^S_{LP}} + \frac{1}{\bar{b}_{SP}}\right]\left[\tilde{\Delta\Delta}^q_{LT,t} + \frac{1-\omega_N}{\omega_N+\omega_S-1}\tilde{q}^N_t - \frac{1-\omega_S}{\omega_N+\omega_S-1}\tilde{q}^S_t\right]\right]$$
$$+\tilde{\nu}_1 \left[\frac{1}{\bar{b}_{SP}}\frac{1}{\beta}\bar{b}^N_{LF}\left[\hat{R}^{Sq}_{L,t} - \hat{R}^{Nq}_{L,t}\right] + \frac{1}{\bar{b}_{SP}}\tilde{z}^N_{\Omega t}\right]$$
$$-\tilde{\nu}_1 \left(\frac{1}{\bar{b}^S_{LP}} + \frac{1}{\bar{b}_{SP}}\right)\tilde{q}^S_t \qquad \text{(A.5.32)}$$

This pair of equations can be compactly written as

$$H_3 \cdot \begin{bmatrix} \tilde{q}^N_t \\ \tilde{q}^S_t \end{bmatrix} = \begin{bmatrix} \Theta^q_N \\ \Theta^q_S \end{bmatrix} \qquad \text{(A.5.33)}$$

with

$$H_3 = \begin{bmatrix} \frac{\omega_S}{\omega_N+\omega_S-1} & -\frac{1-\omega_S}{\omega_N+\omega_S-1} \\ -\frac{1-\omega_N}{\omega_N+\omega_S-1} & \frac{\omega_N}{\omega_N+\omega_S-1} \end{bmatrix}$$

$$\begin{aligned}
\Theta^q_N \;=\;& \frac{1}{\frac{1}{\bar{b}^N_{LP}} + \frac{1}{\bar{b}_{SP}}}\frac{\hat{R}_{S,t} - \hat{R}^{N*}_{D,t}}{\tilde{\nu}_1} - \tilde{\Delta\Delta}^q_{LT,t} \\
&+ \frac{1}{\frac{1}{\bar{b}^N_{LP}} + \frac{1}{\bar{b}_{SP}}}\left[\frac{1}{\bar{b}_{SP}}\tilde{\Omega}^{N*}_t - \bar{m}_b\hat{b}^{Nq}_{SG,t} + \frac{1}{\bar{b}_{SP}}\bar{m}\hat{m}^{N*}_t\right] \\
&- \frac{1}{\frac{1}{\bar{b}^N_{LP}} + \frac{1}{\bar{b}_{SP}}}\left[\frac{1}{\bar{b}_{SP}}\frac{1}{\beta}\bar{b}^N_{LF}\left[\hat{R}^{Sq}_{L,t} - \hat{R}^{Nq}_{L,t}\right] + \frac{1}{\bar{b}_{SP}}\tilde{z}^N_{\Omega t}\right]
\end{aligned}$$

$$\begin{aligned}
\Theta^q_S \;=\;& \frac{1}{\frac{1}{\bar{b}^S_{LP}} + \frac{1}{\bar{b}_{SP}}}\frac{\hat{R}_{S,t} - \hat{R}^{S*}_{D,t}}{\tilde{\nu}_1} + \tilde{\Delta\Delta}^q_{LT,t} \\
&+ \frac{1}{\frac{1}{\bar{b}^N_{LP}} + \frac{1}{\bar{b}_{SP}}}\left[-\frac{1}{\bar{b}_{SP}}\tilde{\Omega}^{N*}_t - \bar{m}_b\hat{b}^{Sq}_{SG,t} + \frac{1}{\bar{b}_{SP}}\bar{m}\hat{m}^{S*}_t\right] \\
&+ \frac{1}{\frac{1}{\bar{b}^N_{LP}} + \frac{1}{\bar{b}_{SP}}}\left[\frac{1}{\bar{b}_{SP}}\frac{1}{\beta}\bar{b}^N_{LF}\left[\hat{R}^{Sq}_{L,t} - \hat{R}^{Nq}_{L,t}\right] + \frac{1}{\bar{b}_{SP}}\tilde{z}^N_{\Omega t}\right]
\end{aligned}$$

and with the determinant of $H_3$ given by

$$Det(H_3) = \begin{vmatrix} \frac{\omega_S}{\omega_N+\omega_S-1} & -\frac{1-\omega_S}{\omega_N+\omega_S-1} \\ -\frac{1-\omega_N}{\omega_N+\omega_S-1} & \frac{\omega_N}{\omega_N+\omega_S-1} \end{vmatrix} = \frac{1}{\omega_N+\omega_S-1} \neq 0.$$

Equation (A.5.33) is solved by the pair of uniquely defined sequences $\tilde{q}^N_t$ and $\tilde{q}^S_t$, i.e.

$$\tilde{q}^N_t = (\omega_N+\omega_S-1)\begin{vmatrix} \Theta^q_N & -\frac{1-\omega_S}{\omega_N+\omega_S-1} \\ \Theta^q_S & \frac{\omega_N}{\omega_N+\omega_S-1} \end{vmatrix} = \omega_N \cdot \Theta^q_N + (1-\omega_S)\cdot\Theta^q_S \qquad \text{(A.5.34)}$$

$$\tilde{q}^S_t = (\omega_N+\omega_S-1)\begin{vmatrix} \frac{\omega_S}{\omega_N+\omega_S-1} & \Theta^q_N \\ -\frac{1-\omega_N}{\omega_N+\omega_S-1} & \Theta^q_S \end{vmatrix} = (1-\omega_N)\cdot\Theta^q_N + \omega_S\cdot\Theta^q_S \qquad \text{(A.5.35)}$$

ensuring $\hat{R}^N_{D,t} = \hat{R}^{N*}_{D,t}$, $\hat{R}^S_{D,t} = \hat{R}^{S*}_{D,t}$, and $\tilde{\Omega}^N_t = \tilde{\Omega}^{N*}_t = -\tilde{\Omega}^{S*}_t\ \forall t$, which can be expressed as a function of i) predetermined variables, ii) starred contemporaneous variables $\hat{c}^{N*}_t$, $\hat{c}^{S*}_t$, $\hat{\pi}^{N*}_{c,t}$, $\hat{\pi}^{S*}_{c,t}$

and iii) the sequences $\{\hat{R}^{N*}_{D,t+j}\}^{\infty}_{j=0}, \{\hat{R}^{S*}_{D,t+j}\}^{\infty}_{j=0}, \{\hat{R}_{S,t+j}\}^{\infty}_{j=0}$.

As stated in Corollary $II$, assume that unexpectedly in some period $t_1 > 0$ the lower bound constraint becomes binding, known for sure to last until $t_2$. Then, for $t < t_1$, by construction, $\hat{R}^{N*}_{D,t} = \hat{R}^{N}_{D,t}$, $\hat{R}^{S}_{D,t} = \hat{R}^{S*}_{D,t}$ and $\tilde{\Omega}^N_t = \tilde{\Omega}^{N*}_t = -\tilde{\Omega}^{S*}_t$ will be satisfied since $\hat{R}_{S,t} = \hat{R}^*_{S,t}$ and $\tilde{q}^N_t = \tilde{q}^S_t = 0$, while for $t \geqslant t_1$ the unique sequences (A.5.34) and (A.5.35) ensure that $\hat{R}^{N*}_{D,t}$, $\hat{R}^{S*}_{D,t}$ and $\tilde{\Omega}^{N*}_t = -\tilde{\Omega}^{S*}_t$ will be replicated in all subsequent periods, consistent with $\hat{R}_{S,t} = \hat{R}^*_{S,t}$ for $t \geqslant t_2$. q.e.d.

Notice that Proposition 1 is a special case of Proposition 2. This can be seen by going back to (A.5.31). Imposing $\tilde{\Delta}_{LT,t} = 0$ implies

$$\Delta\tilde{\Delta}_{LT,t} = \frac{1-\omega_S}{\omega_N+\omega_S-1}\tilde{q}^S_t - \frac{1-\omega_N}{\omega_N+\omega_S-1}\tilde{q}^N_t$$

Moreover, Proposition 1 assumes $\tilde{\Omega}^N_t = 0$, $\tilde{\Delta}_{ST,t} = 0$, while $\hat{R}^S_{L,t} = \hat{R}^N_{L,t}$ by construction, and $\tilde{q}^S_t + \bar{m}\hat{b}^S_{SC,t} = \bar{m}\hat{m}^S_t$, implying $\tilde{z}^N_{\Omega t} = 0$ in (A.5.18). Substituting these expressions into (A.5.31) yields equation (A.4.14).

## References


Andrés, J., J. López-Salido, and E. Nelson (2004): "Tobin's imperfect asset substitution in optimizing general equilibrium," *Journal of Money, Credit and Banking*, 36, 665–690.

Benhabib, J., S. Schmitt-Grohé, and M. Uribe (2001): "The perils of Taylor rules," *Journal of Economic Theory*, 96, 40–69.

Benigno, P. (2004): "Optimal monetary policy in a currency area," *Journal of International Economics*, 63, 293–320.

Bernanke, B., V. Reinhart, and B. Sack (2004): "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment," *Brookings Papers on Economic Activity*, 35, 1–100.

Blanchard, O., C. Erceg, and J. Lindé (2017): "Jump-Starting the Euro Area Recovery: Would a Rise in Core Fiscal Spending Help the Periphery?" *NBER Macroeconomics Annual*, 31, 103–182.

Bletzinger, T. (2017): "Optimal QE in a heterogeneous monetary union above the lower bound constraint on interest rates," Mimeo.

Brunnermeier, M. and Y. Koby (2017): "The 'Reversal Interest Rate': An Effective Lower Bound on Monetary Policy," Mimeo.

CEPR (2018): "Reconciling risk sharing with market discipline: A constructive approach to euro area reform," CEPR Policy Insight, Centre for Economic Policy Research.

Chari, V. and P. Kehoe (2008): "Time inconsistency and free-riding in a monetary union," *Journal of Money, Credit and Banking*, 40, 1329–1356.

Christiano, L., R. Motto, and M. Rostagno (2010): "Financial factors in economic fluctuations," Working Paper Series 1192, European Central Bank.

Coenen, G. and V. Wieland (2003): "The zero-interest-rate bound and the role of the exchange rate for monetary policy in Japan," *Journal of Monetary Economics*, 50, 1071–1101.

Cooper, R. and H. Kempf (2004): "Overturning Mundell: Fiscal policy in a monetary union," *Review of Economic Studies*, 71, 371–396.

Corsetti, G. and P. Pesenti (2001): "Welfare and macroeconomic interdependence," *Quarterly Journal of Economics*, 116, 421–445.

Cúrdia, V. and M. Woodford (2011): "The central-bank balance sheet as an instrument of monetary policy," *Journal of Monetary Economics*, 58, 54–79.

Del Negro, M., M. Gianonni, and C. Patterson (2012): "The Forward Guidance Puzzle," Staff Reports 574, Federal Reserve Bank of New York.

den Haan, W. (2016): *Quantitative Easing: Evolution of economic thinking as it happened on Vox*, Centre for Economic Policy Research, CEPR Press.

Eggertsson, G., R. Juelsrud, and E. Wold (2017): "Are Negative Nominal Interest Rates Expansionary?" NBER Working Papers 24039, National Bureau of Economic Research.

Eggertsson, G. and M. Woodford (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 34, 139–235.

European Central Bank (2015): "Account of the ECB's Governing Council meeting on 21-22 January 2015," available at: http://www.ecb.europa.eu/press/accounts/2015/html/mg150219.en.html.

European Commission (2016): "Completing Europe's Economic and Monetary Union," Report by the five presidents.

Farhi, E. and J. Tirole (2016): "Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops," *Review of Economic Studies*, forthcoming.

Farhi, E. and I. Werning (2017): "Fiscal Unions," *American Economic Review*, forthcoming.

Ferrero, A. (2009): "Fiscal and monetary rules for a currency union," *Journal of International Economics*, 77, 1–10.

Gerali, A., S. Neri, L. Sessa, and F. Signoretti (2010): "Credit and Banking in a DSGE Model of the Euro Area," *Journal of Money, Credit and Banking*, 42, 107–141.

Harrison, R. (2011): "Asset purchase policies and portfolio balance effects: a DSGE analysis," in *Interest Rates, Prices and Liquidity: Lessons from the Financial Crisis*, ed. by J. S. Chadja and S. Holly, Cambridge University Press, 117–143.

——— (2012): "Asset purchase policy at the effective lower bound for interest rates," Working Paper 444, Bank of England.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, 233–267.

Leeper, E. (1991): "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, 27, 129–147.

McKay, A., E. Nakamura, and J. Steinsson (2016): "The Power of Forward Guidance Revisited," *American Economic Review*, 106, 3133–3158.

Mertens, K. and M. Ravn (2014): "Fiscal Policy in an Expectations-Driven Liquidity Trap," *Review of Economic Studies*, 81, 1637–1667.

Obstfeld, M. and K. Rogoff (1995): "Exchange Rate Dynamics Redux," *Journal of Political Economy*, 103, 624–660.

——— (2000): "New directions for stochastic open economy models," *Journal of International Economics*, 50, 117–153.

Orphanides, A. (2017): "The fiscal-monetary policy mix in the euro area: challenges at the zero lower bound," CEPR Discussion Paper 12039, Centre for Economic Policy Research.

Orphanides, A. and V. Wieland (2000): "Efficient Monetary Policy Design Near Price Stability," *Journal of the Japanese and International Economies*, 14, 327–365.

Reis, R. (2016): "Can the central bank alleviate fiscal burdens?" NBER Working Papers 23014, National Bureau of Economic Research.

Rotemberg, J. (1982): "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies*, 49, 517–31.

Sinn, H.-W. and T. Wollmershäuser (2012): "Target loans, current account balances and capital flows: the ECB's rescue facility," *International Tax and Public Finance*, 19, 468–508.

Smets, F. and R. Wouters (2003): "An Estimated Dynmaic Stochastic General Equilibrium Model of the Euro Area," *European Economic Association*, 1, 1123–1175.

——— (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.

Sutherland, A. (1996): "Financial Market Integration and Macroeconomic Volatility," *Scandinavian Journal of Economics*, 98, 521–539.

Tille, C. (2001): "The role of consumption substitutability in the international transmission of monetary shocks," *Journal of International Economics*, 53, 421–444.

Tobin, J. (1969): "A general equilibrium approach to monetary theorey," *Journal of Money, Credit and Banking*, 1, 15–29.

Tobin, J. and W. C. Brainard (1963): "Financial intermediaries and the effeeffective of monetary controls," *The American Economic Review*, 53, 383–400.

Vayanos, D. and J.-L. Vila (2009): "A Preferred-Habitat Model of the Term Structure of Interest Rates," NBER Working Paper 15487, National Bureau of Economic Research.

Wallace, N. (1981): "A Modigliani-Miller Theorem for Open-Market Operations," *The American Economic Review*, 71, 267–274.

Woodford, M. (2003): *Interest and Prices*, Princeton University Press.

——— (2012): "Methods of policy accommodation at the interest-rate lower bound," in *The Changing Policy Landscape*, Proceedings of Federal Reserve Bank of Kansas Economic Policy Symposium, 185–288.

Wu, J. and J. Zhang (2017): "A shadow rate New Keynesian model," NBER Working Papers 22856, National Bureau of Economic Research.

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