EUROPEAN CENTRAL BANK
EUROSYSTEM

Working Paper Series

Antoine Camous, Alejandro Van der Ghote

Financial cycles under diagnostic beliefs

No 2659/ May 2022

Abstract

Swift changes in investors' sentiment, such as the one triggered by COVID-19 global outbreak in March 2020, lead to financial tensions and asset price volatility. We study the interactions of behavioral and financial frictions in an environment with endogenous risk-taking and capital accumulation. Agents form diagnostic expectations about future stochastic outcomes: recent realizations of aggregate shocks are expected to persist. This behavioral friction gives rise to sentiment cycles with excessive investment and occasional safety traps. The interactions with financial frictions lead to an endogenous amplification of financial instability. We discuss implications for policy interventions.

JEL classification: E32, E44, E71.
Key words: Financial Cycles, Diagnostic Beliefs, Macro-prudential Policy.

Non-technical Summary

Popular accounts of financial crises depict cyclic waves of irrational exuberance, where excessive optimism rises in tandem with asset prices, always to be followed by deceptive returns, panics and economic meltdown. These views highlight the importance of psychological factors to financial instability, and will certainly contribute to chronicles of upcoming economic fluctuations.

However, a long tradition in macroeconomics has abstracted from these behavioral elements and stressed that policymakers should be primarily concerned with financial frictions. Policy prescriptions are designed to mitigate excessive risk taking, stabilize financial cycles and improve the resilience of the economy to adverse shocks.

The purpose of this analysis is to incorporate a psychological process of investors' sentiment into an otherwise standard model of financial cycles, to study the interactions of financial and behavioral frictions on risk taking, growth and economic cycles. We then investigate whether behavioral frictions prescribe adjustments of regulatory and policy recommendations.

Specifically, agents form expectations diagnostically rather than rationally: their forecasts of returns is tied to *sentiment*, a measure that reflects past returns. Under this extrapolative process, agents' neglect downside risk in good times and upside risk in bad times. In our model, the implications of diagnostic beliefs are more salient in good times, where the economy is most exposed to aggregate risk. Accordingly, the behavioral friction amplifies financial instability and strengthens the rationale for prudential balance sheet restrictions.

The analysis rests on an environment in which the allocation of physical capital across different productive technologies and growth are endogenous. In the absence of financial and behavioral frictions, the riskier technology provides higher expected returns. All productive resources in this benchmark economy are allocated to this *superior* technology, which is exposed to aggregate risk but grants reinvestment opportunities and supports economic growth.

In contrast, economies with either financial or behavioral frictions display boom-bust cycles, with fluctuations in capital allocation, risk-taking and growth rates. Each of these economies exhibits distinctive characteristics though.

In the behavioral economy (without financial frictions), the dependence of forecasts on sentiment distorts the *willingness* of agents to invest in the superior technology. Follow-

ing negative shocks, sentiment is low and agents' misperceive the relative return to invest in productive technologies: the economy falls into a *safety trap*, where physical capital is allocated to the *inferior* technology, insulated from aggregate risk, but not offering reinvestment and growth opportunities. In contrast, with elevated sentiment, agents expect higher returns from the superior technology exposed to risk. Waves of elevated and depressed sentiment generate cyclic fluctuations in the allocation of physical capital across technologies, hence effectively in exposure to aggregate risk.

Importantly, the implications of forecasts distortions are not symmetric along the sentiment cycle, since exposure to risk and sentiment are positively correlated. The effects of elevated sentiment on the price of physical capital dominates, so that the incentives to reinvest (whenever possible) are stronger than in the benchmark economy.

In the economy with financial frictions (and rational expectations), occasionally binding constraints might restrict the *capacity* of financiers to leverage and operate the superior technology. Negative shocks are amplified by adverse feedback loops between asset prices and portfolio constraints. The risk of contractionary fire-sales depresses the price of physical capital and the incentives to reinvest all along the economic cycle.

These elements provide the basis for understanding how diagnostic beliefs amplify the adverse effects of financial frictions. As argued, the effects of diagnostic beliefs are more salient during booms, where the economy is most exposed to aggregate shocks. Agents with positive sentiment are willing to bid a higher price to acquire physical capital and operate the superior technology, which exposes them to large systematic forecast errors and sharper endogenous contractions when sentiment turns: a small negative shock triggers a larger drop in asset prices, which amplifies the interactions between fire-sales and tightening collateral constraints.

These interactions are enhanced when diagnostic distortions are relatively more sensitive to positive sentiment, giving rise to a *paradox of optimism*: without financial frictions, diagnostic optimism leads to less misallocation and higher growth; in contrast, in economies with financial frictions, diagnostic optimism generates more financial instability, with a poorer allocation of physical capital and less growth.

1 Introduction

"*There will be manias. The manias will be followed by panics.*"
[Akerlof and Shiller (2010)]

Popular accounts of financial crises depict cyclic waves of irrational exuberance, where excessive optimism rises in tandem with asset prices, always to be followed by deceptive returns, panics and economic meltdown. These views highlight the importance of psychological factors to financial instability, and will certainly contribute to chronicles of upcoming economic fluctuations.

However, a long tradition in macroeconomics has abstracted from these behavioral elements and stressed that policymakers should be primarily concerned with financial frictions.[1] Policy prescriptions are designed to mitigate excessive risk taking, stabilize financial cycles and improve the resilience of the economy to adverse shocks.

The purpose of this analysis is to incorporate a psychological process of investors' sentiment into an otherwise standard model of financial cycles, to study the interactions of financial and behavioral frictions on risk taking, growth and economic cycles. We then investigate whether behavioral frictions prescribe adjustments of regulatory and policy recommendations.

Specifically, agents form expectations diagnostically rather than rationally: their forecasts of returns is tied to *sentiment*, a measure that reflects past returns. Under this extrapolative process, agents' neglect downside risk in good times and upside risk in bad times. In our model, the implications of diagnostic beliefs are more salient in good times, where the economy is most exposed to aggregate risk. Accordingly, the behavioral friction amplifies financial instability and strengthens the rationale for prudential balance sheet restrictions.

The analysis rests on an environment in which the allocation of physical capital across different productive technologies and growth are endogenous. In the absence of financial and behavioral frictions, the riskier technology provides higher expected returns. All productive resources in this benchmark economy are allocated to this *superior* technology, which is exposed to aggregate risk but grants reinvestment opportunities and supports economic growth.

[1] The literature review discusses in detail these contributions along with papers that are directly linked to the main themes of our analysis.

In contrast, economies with either financial or behavioral frictions display boom-bust cycles, with fluctuations in capital allocation, risk-taking and growth rates. Each of these economies exhibits distinctive characteristics though.

In the behavioral economy (without financial frictions), the dependence of forecasts on sentiment distorts the *willingness* of agents to invest in the superior technology. Following negative shocks, sentiment is low and agents' misperceive the relative return to invest in productive technologies: the economy falls into a *safety trap*, where physical capital is allocated to the *inferior* technology, insulated from aggregate risk, but not offering reinvestment and growth opportunities. In contrast, with elevated sentiment, agents expect higher returns from the superior technology exposed to risk. Waves of elevated and depressed sentiment generate cyclic fluctuations in the allocation of physical capital across technologies, hence effectively in exposure to aggregate risk.

Importantly, the implications of forecasts distortions are not symmetric along the sentiment cycle, since exposure to risk and sentiment are positively correlated. The effects of elevated sentiment on the price of physical capital dominates, so that the incentives to reinvest (whenever possible) are stronger than in the benchmark economy.

In the economy with financial frictions (and rational expectations), occasionally binding constraints might restrict the *capacity* of financiers to leverage and operate the superior technology. Negative shocks are amplified by adverse feedback loops between asset prices and portfolio constraints. The risk of contractionary fire-sales depresses the price of physical capital and the incentives to reinvest all along the economic cycle.

These elements provide the basis for understanding how diagnostic beliefs amplify the adverse effects of financial frictions. As argued, the effects of diagnostic beliefs are more salient during booms, where the economy is most exposed to aggregate shocks. Agents with positive sentiment are willing to bid a higher price to acquire physical capital and operate the superior technology, which exposes them to large systematic forecast errors and sharper endogenous contractions when sentiment turns: a small negative shock triggers a larger drop in asset prices, which amplifies the interactions between fire-sales and tightening collateral constraints.[2]

These interactions are enhanced when diagnostic distortions are relatively more sensitive to positive sentiment, giving rise to a *paradox of optimism*: without financial fric-

[2]In contrast, when sentiment and the share of physical capital allocated to the superior technology are low, the economy is relatively less exposed to the dynamic implications of forecast errors induced by diagnostic distortions.

tions, diagnostic optimism leads to less misallocation and higher growth; in contrast, in economies with financial frictions, diagnostic optimism generates more financial instability, with a poorer allocation of physical capital and less growth.

We explore the normative implications of diagnostic beliefs for financial regulation.[3] First, sentiment-based fluctuations in economic activity are not necessarily inefficient. A benevolent planner that evaluates economic outcomes under the diagnostic belief system would not curb fluctuations driven solely by waves of sentiment.

In contrast, financial-based inefficiencies are grounded in pecuniary externalities that motivate policy interventions. For instance, unconditional prudential restrictions on individual portfolio choices contribute to stabilize financial cycles, improve the overall performance of the economy and its resilience to adverse shocks: by restricting individual risk taking, policymakers can improve the average allocation of physical capital and support the average growth rate of the economy.

Diagnostic beliefs reinforce the rationale for these interventions, but the adjustments depend on the expectations process adopted by policymakers. A paternalistic policymaker that evaluates economic outcomes from a rational perspective would impose stricter curbs on risk taking than a benevolent policymaker that shares the diagnostic belief systems of agents.

Literature review. This paper relates to two strands of the economic literature. The first is the study of the implications of financial frictions on economic activity. The second is the burgeoning literature in behavioral macroeconomy. Papers that relate to model details and specific findings are discussed in the analysis.

The macroeconomic literature has built on seminal contributions by Bernanke Gertler (1989), Kiotaky and Moore (1997) and Bernanke, Gertler and Gilchrist (1999) to investigate the implications of financial frictions in general equilibrium.[4] Our study maintains the focus of Brunnermeier and Sannikov (2014) on capital (mis)allocation and risk. Collateralised borrowing gives rise to an externality that motivates policy interventions, as in Lorenzoni (2008) and Jeanne and Korinek (2019).

Economic studies have adopted insights from psychology to refine our understanding

[3]Arguably, restrictions on unobservable psychological variables such as sentiment are not plausible policy instruments. Hence we study adjustments of prudential policies aimed at curbing the undesirable consequences of financial frictions, under the additional hypothesis of diagnostic expectations.

[4]See Brunnermeier, Eisenbach and Sannikov (2013) for an extensive survey of the literature.

of individual choices and study the implications for economic activity.[5] In particular, foundational works by Daniel Kahneman and Amos Tversky, e.g. Tversky and Kahneman (1983), have popularized the concept of *representativeness heuristic*. This notion describes the tendency to infer the frequency of an event not based on its general prevalence, but according to its relative prevalence across subsets. Bordalo, Gennaioli and Shleifer (2018) and Bordalo et al. (2019*c*) link representativeness heuristic to extrapolative beliefs, where individual forecasts overweight the likelihood of future states that are representative - or *diagnostic* - of recent realizations. Diagnostic beliefs are systematically associated with forecasts errors, but they contain a kernel of truth, since they rely only on a distorted perception of the true features of data. The prevalence of diagnostic beliefs has been documented with investors' surveys, for instance Bacchetta, Mertens and van Wincoop (2009) and Greenwood and Shleifer (2014): the average belief about future stock market returns is a positive function of recent past returns. Multiple studies have investigated the implications of *diagnostic expectations* for credit cycles - Bordalo, Gennaioli and Shleifer (2018), for asset bubbles - Bordalo et al. (2020*a*) or for stock markets - Adam, Marcet and Beutel (2017) and Bordalo et al. (2019*b*).

Maxted (2019) incorporates this expectations process in an environment with financial frictions to study the implications on the frequency of financial crisis and the behavior of risk-premia around crisis events. Our analysis is related but differs along key dimensions. First, our environment with multiple productive technologies focuses on the implications of capital (mis)allocation for endogenous risk and growth.[6] Our results emphasize how the effects of diagnostic expectations are more salient in booms, where the economy is most exposed to risk. Finally, our analysis discusses the associated normative implications and policy interventions.[7]

Plan. The rest of the document is organized as follows. Section 2 presents the economic environment. Section 3 characterizes the contributions of financial frictions and diagnostic expectations to economic fluctuations. Section **??** discusses the normative implications.

[5] Barberis (2018) provides an overview of behavioral approaches in finance and macro-finance.

[6] Maxted (2019) builds on He and Krishnamurthy (2019), which features a single production technology. Hence, it does not capture the procyclical nature of risk taking that emerges out of capital allocation, which is at the core of our analysis. Also, our central friction is an endogenous collateral constraint, while these papers consider equity issuance constraints.

[7] A growing literature study policy interventions or optimal policy in economies with behavioral frictions, as for instance Benigno and Paciello (2014), Angeletos and Sastry (2018), Farhi and Werning (2019), Farhi and Gabaix (2020).

An Appendix provides a formal treatment of derivations presented in the text.

2 The Model

The environment builds upon Brunnermeier and Sannikov (2014). Time is continuous and the economy is populated by a continuum of risk-neutral households and financiers. These agents differ in access to investment opportunities and financial constraints. The environment presents two additional features. First, agents form expectations diagnostically rather than rationally. That is, individual forecasts overweight the likelihood of future states that are representative of recent realizations.[8] Second, financial constraints are endogenous and forward-looking, and as such depend on the perception of future economic outcomes.

2.1 Technologies

There are two productive technologies, indexed by $j \in \{I, S\}$. Both rely on physical capital $k_{j,t}$ to produce output flows $y_{j,t} = ak_{j,t}$ per unit of time, where $a > 0$ is a constant and common productivity factor. The *Superior* technology $j = S$ allows for internal reinvestment of output to produce physical capital whereas the *Inferior* technology $j = I$ does not. In addition, only the superior technology is exposed to aggregate capital quality shocks.

The stock of physical capital allocated to the superior technology evolves over time according to

$$\frac{dk_{S,t}}{k_{S,t}} = \mathcal{I}(\iota_t)dt + \sigma dZ_t \ , \tag{1}$$

where $\iota_t \geq 0$ is the reinvestment rate per unit of capital (i.e., $\iota_t k_{S,t}$ is total reinvestment), $dZ_t \sim \mathcal{N}(0, dt)$ are exogenous aggregate Brownian shocks and $\sigma > 0$ a volatility parameter. $\mathcal{I}(\cdot) \geq 0$ represents a standard reinvestment technology with adjustment costs and as such it satisfies $\mathcal{I}(0) = 0$, $\mathcal{I}'(0) = +\infty$, $\mathcal{I}'(\cdot) > 0$ and $\mathcal{I}''(\cdot) < 0$. In contrast, the stock of physical capital allocated to the inferior technology remains constant over the immediate period of time, $(t, t + dt)$.[9]

[8] This extrapolative expectation formation process reflects the representativeness heuristic bias initially documented by Tversky and Kahneman (1983) and subsequently incorporated in macro-finance by Bordalo, Gennaioli and Shleifer (2018), Bordalo et al. (2019*c*), Bordalo et al. (2020*b*) and Maxted (2019).

[9] To be clear, aggregate capital quality shocks dZ_t apply equally to all units of physical capital allocated to the superior technology, but do not affect the evolution of the units allocated to the inferior technology.

Because of reinvestment possibilities, the superior technology is relatively more attractive to risk neutral agents with rational beliefs. Under diagnostic beliefs, the inferior technology might be more attractive though, because estimates of relative returns are sensitive to forecasts about future disturbances, as explained next.

2.2 Diagnostic Beliefs

When forming beliefs diagnostically rather than rationally, agents overweight the likelihood of future disturbances that are reminiscent of recent realizations of aggregate shocks. Following Maxted (2019), agents synthesize *information* about past disturbances as

$$\omega_t \equiv \int_0^t e^{-\delta(t-s)} dZ_s \ , \tag{2}$$

where memory decay parameter $\delta > 0$ indicates the rate at which past shocks are discounted. Agents then use synthetized *information* $\omega_t \in \mathbb{R}$ to forecast future disturbances according to

$$d\hat{Z}_t \equiv \hat{\mu}_t \omega_t dt + dZ_t, \tag{3}$$

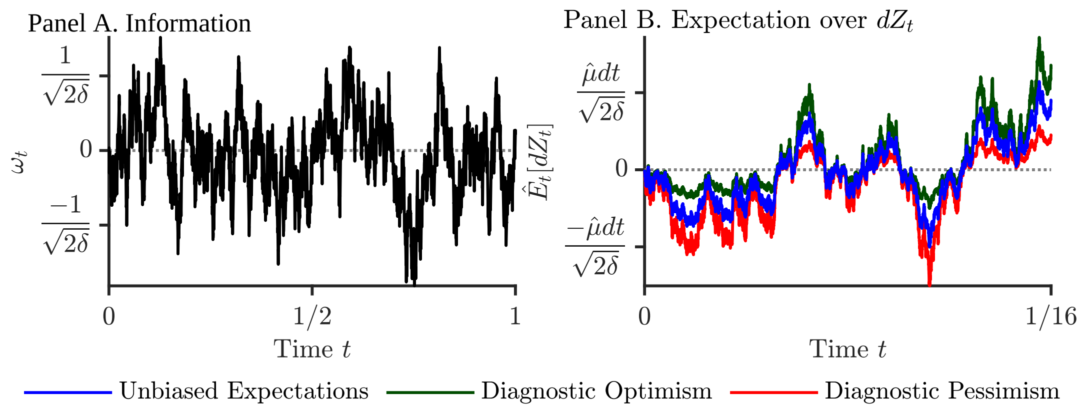
where the *expectation factor* $\hat{\mu}_t \geq 0$ reflects the diagnostic weight of information on expectation formation. Throughout the exposition, *hat variables* indicate diagnostic estimates of future disturbances or related variables.

Agents correctly estimate the distribution and variance of aggregate shocks $d\hat{Z}_t \sim \mathcal{N}(\cdot, dt)$. However, agents systematically misestimate the mean if $\hat{\mu}_t > 0$, in which case past shocks are perceived as informative of the average realization of future disturbances, $\hat{E}_t(dZ_t) = E_t(d\hat{Z}_t) = \hat{\mu}_t \omega_t dt$.

In the analysis, we contrast the baseline case of constant diagnostic beliefs $\hat{\mu}_t = \hat{\mu} > 0$, with state-contingent diagnostic distortions: $\hat{\mu}_t = \left[\Delta \mathbf{1}_{\omega_t<0} + (1-\Delta)\mathbf{1}_{\omega_t>0}\right] 2\hat{\mu}$, with $\Delta \in [0,1]$. Under *diagnostic optimism*, $\Delta \in [0, 1/2)$ and forecasts of future disturbances are relatively more distorted when information $\omega_t > 0$ reflects recent realizations of positive shocks. In contrast, *diagnostic pessimism* arises if $\Delta \in (1/2, 1]$, where forecasts are relatively more sensitive to negative information $\omega_t < 0$.[10]

[10]Da, Huang and Jin (2020) document empirically the presence of asymmetric extrapolative forecasts for stock investors: the dependence of forecasts on past returns is stronger and longer for negative rather than positive recent returns. In the context of our study, this distinction is useful to provide intuitions for our key results, as discussed in Section 3.

Figure 1: Diagnostic Beliefs and Forecasts



Notes: Panel A plots the evolution of information ω_t induced by a generic realizations of shocks dZ_t. Panel B reports the associated expectations over immediate-future disturbances under rational beliefs (dashed line), baseline diagnostic beliefs (blue), diagnostic optimism (green) and diagnostic pessimism (red). Over a sufficiently long time horizon $\omega_t \sim \mathcal{N}(0, 1/2\delta)$ because ω_t evolves according to $d\omega_t = -\delta\omega_t + dZ_t$.

These elements are illustrated in Figure 1, which reports the evolution of information (Panel A) and the expected realization of the disturbance in the immediate future (Panel B). Under rational expectations (dashed line), information has no influence on agents' forecasts and the expected average disturbance is $E_t(dZ_t) = 0$, in contrast to diagnostic beliefs (yellow line), where forecasts reflect the state of information ω_t. The case of diagnostic optimism (pessimism) is reported in red (green) and highlights the increased sensitivity of forecasts to positive (negative) information.

2.3 Rates of Return under Diagnostic Beliefs

Agents rely on diagnostic beliefs to forecast future prices when forming portfolio choices. Critical to these decisions are the returns earned from allocating physical capital to each technology.

Physical capital is traded continuously in liquid markets at spot price $q_t > 0$. Thus, the total return to allocate physical capital in either technology is composed of a dividend yield and the change in the market value of physical capital. The rate of return to allocate physical capital to the superior technology is

$$dR_{S,t} = \frac{a-\iota_t}{q_t}dt + \frac{d\left(q_t k_{S,t}\right)}{q_t k_{S,t}} \ , \tag{4}$$

and in the case of the inferior technology, it is

$$dR_{I,t} = \frac{a}{q_t}dt + \frac{dq_t}{q_t} \ . \tag{5}$$

Agents forecast correctly dividend payouts because output flows during time interval $(t, t+dt)$ are locally deterministic. However, agents may misestimate capital gains or losses because those rates depend on future disturbances. To characterize forecasted returns, we postulate that the equilibrium price of physical capital evolves over time according to an Ito process subject to capital quality shocks dZ_t:

$$\frac{dq_t}{q_t} = \mu_{q,t}dt + \sigma_{q,t}dZ_t \ , \ \text{with} \ \ \sigma_{q,t} \geq 0 \ , \tag{6}$$

where $\mu_{q,t} \in \mathbb{R}$ and $\sigma_{q,t} \geq 0$ are endogenous drift and diffusion processes. The forecasted rates of return to each technology are then given by:[11]

$$d\hat{R}_{S,t} = \left[\frac{a-\iota_t}{q_t} + \mu_{q,t} + \mathcal{I}\left(\iota_t\right) + \left(\sigma_{q,t}+\sigma\right)\hat{\mu}_t\omega_t + \sigma_{q,t}\sigma\right]dt + \left(\sigma_{q,t}+\sigma\right)dZ_t, \tag{7}$$

$$d\hat{R}_{I,t} = \left[\frac{a}{q_t} + \mu_{q,t} + \sigma_{q,t}\hat{\mu}_t\omega_t\right]dt + \sigma_{q,t}dZ_t. \tag{8}$$

Both returns are exposed to *price risk* $\sigma_{q,t}dZ_t$ but only the return to the superior technology is also exposed to *quality risk* σdZ_t. Moreover reinvestment only influences return $d\hat{R}_{S,t}$. Because these reinvestment opportunities are valuable, the expected rate of return of the superior technology is higher if beliefs are rational. However, under diagnostic beliefs, the expected return differential is

$$\hat{E}_t\left[dR_{S,t} - dR_{I,t}\right] = \left[\mathcal{I}\left(\iota_t\right) - \frac{\iota_t}{q_t} + \sigma\hat{\mu}_t\omega_t + \sigma_{q,t}\sigma\right]dt \ , \tag{9}$$

which may be positive or negative depending on the state of information ω_t and its predic-

[11] To derive the formula for $d\hat{R}_{S,t}$, first, apply Ito's product rule to compute $d\left(q_t k_{S,t}\right)/q_t k_{S,t}$ and then, substitute dZ_t with $d\hat{Z}_t$ in the resulting expression. For $d\hat{R}_{I,t}$, first substitute (6) into (5), and then proceed in the same manner as for $d\hat{R}_{S,t}$.

tion about the average realization of physical capital allocated to the superior technology $\hat{E}_t[dk_{S,t}/k_{S,t}] = \sigma\hat{\mu}_t\omega_t dt$.[12]

Information ω_t contributes to economic activity only through its effect on expectations and the *willingness* to take risks and reinvest. Accordingly, and because ω_t has no actual predictive power about dZ_t, we refer to information ω_t as *sentiment*. The other central contribution to economic outcomes is the distribution of aggregate wealth between households and financiers, which influences the *ability* of financiers to hold physical capital and operate the superior technology.

2.4 Rights over Technologies and Financial Frictions

The relative distribution of wealth matters as well because only financiers can operate the superior technology and because these agents are subject to financing constraints. Specifically, first, the net worth of financiers $n_t \geq 0$ cannot be negative, because of limited liability protection. And second, following Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), debt issuance by financiers $b_t \geq 0$ is limited by a *collateral constraint*:

$$q_t k_{f,t} = n_t + b_t \leq \lambda \hat{V}_t\left(n_t\right), \tag{10}$$

where $k_{f,t} \geq 0$ is their holdings of physical capital and $\hat{V}_t\left(n_t\right) \geq n_t$ is the franchise value of a financier with net worth $n_t \geq 0$. This constraint reflects a friction in debt enforcement that allows financiers to walk away with a fraction $1/\lambda \in (0,1)$ of their assets immediately after raising debt, at the cost of losing forever access to the superior technology.[13]

The collateral constraint highlights the interplay of behavioral and financial frictions. Indeed, $\hat{V}_t(\cdot)$ is a present discounted value of future returns and as such it is sensitive to sentiment and economic forecasts. This constraint affects portfolio decisions, as discussed next.

[12] The properties of $\mathcal{I}(\cdot)$ ensure that $\mathcal{I}(\iota_t) - \iota_t/q_t > 0$ holds at the optimal reinvestment rate, see equation (17) below. Information ω_t also predicts an average realization of the price of physical capital of $\hat{E}_t[dq_t/q_t] = \sigma_{q,t}\hat{\mu}_t\omega_t dt$. This other term, however, does not enter the expected return differential, because both returns are equally exposed to price risk.

[13] As verified in the developments, the value of a financier who cannot operate the technology is simply her net worth. Importantly, external financing is restricted to short term debt $b_t \geq 0$, meaning that financial contracts mature at time $t + dt$ and promise a fixed interest rate regardless of disturbance dZ_t. This other friction however does not affect portfolio decisions or equilibrium allocations in the absence of the financial constraints. (See Section 3.1 for details.)

2.5 Portfolio Investment Decisions

Agents adjust their investment portfolio continuously, taking prices and rates of return as given. Financiers maximize the expected dividend payouts to households. Households in turn maximize the expected utility derived from the consumption of output good. This arrangement is interpreted as a delegation of the operation of the superior technology by a household-owner to a financier, and the dividend flow is the residual claim on the delegation.[14]

Households' Decisions. Households discount future consumption flows at a constant subjective time rate, $r > 0$. The lack of financing constraints at the household level has two equilibrium implications.[15] First, households are indifferent between consumption and savings—otherwise, their demand for debt securities would be unbounded. Thus, the interest rate on debt equals the subjective time discount rate. Second, and for the same reason, households weakly prefer debt securities to operating the inferior technology. The forecasted excess return to acquire physical capital and operate the inferior technology over debt, therefore, is non positive,

$$\hat{\alpha}_{I,t} \equiv \frac{1}{dt}\hat{E}_t\left[dR_{I,t}\right] - r = \frac{a}{q_t} + \mu_{q,t} + \sigma_{q,t}\hat{\mu}_t\omega_t - r \leq 0 \; . \tag{11}$$

When $\hat{\alpha}_{I,t} = 0$, households are indifferent between debt and acquiring physical capital to operate the inferior technology. In contrast, when $\hat{\alpha}_{I,t} < 0$, households strictly prefer debt securities over physical capital.

Financiers' Decisions. Financiers pay out dividends once according to an idiosyncratic Poisson process with common and exogenous arrival rate $\theta > 0$. When they do so, each financier transfers all her accumulated net worth to her household-owner, and immediately afterward, she is replaced by a newborn financier that receives a share $\gamma/\theta > 0$ of the market value of the aggregate capital stock as initial endowment, where $\gamma \in (0, 1)$ is a parameter.[16]

[14] As in Maggiori (2017), financiers cannot issue debt to their household-owner. This restriction preserves the nature of the agency problem underlying the collateral constraint, (10).

[15] Appendix **??** details the portfolio problem of households. The equilibrium concept is a competitive equilibrium, as detailed next subsection. The collateral constraint (10) rules out debt default in equilibrium which ensures that the interest rate on debt is de facto locally risk-free.

[16] This payout scheme precludes financiers from accumulating enough wealth that would make financing constraints irrelevant in equilibrium. The initial endowment is required to allow newborn financiers to

The problem of financiers is then to maximize their franchise value $\hat{V}_t(n_t)$, with

$$\hat{V}_t\left(n_t\right) \equiv \max_{k_{f,s},\iota_s\geq 0} \hat{E}_t \int_t^{\infty} \theta e^{-(r+\theta)(s-t)} n_s ds \ , \tag{12}$$

subject to the forecasted law of motion of net worth,

$$d\hat{n}_s = d\hat{R}_{S,s} q_s k_{f,s} - (q_s k_{f,s} - n_s) r ds \ , \tag{13}$$

the solvency constraint $n_s \geq 0$, and the collateral constraint (10). This program presumes that financiers never allocate physical capital to the inferior technology, as verified in Section 2.6.

To solve the program, we make two additional postulates. First, the franchise value $\hat{V}_t\left(n_t\right) \equiv \hat{v}_t n_t$ is linear in net worth n_t. The marginal value of net worth, $\hat{v}_t \geq 1$, is endogenous—but independent of individual choices, and as such, it is common to all financiers. Second, the marginal value $\hat{v}_t$ evolves according to an Ito process with disturbance dZ_t, where the associated drift $\mu_{\hat{v},t} \in \mathbb{R}$ and diffusion $\sigma_{\hat{v},t} \leq 0$ are endogenous.[17]

Under these conditions, the problem of financiers is scale invariant to individual net worth. Therefore, all financiers choose the same portfolio weight on physical capital, $\phi_t \equiv q_t k_{f,t}/n_t \geq 0$, and the marginal value of net worth $\hat{v}_t$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation,

$$\begin{aligned} \left(r+\theta\right)\hat{v}_t \quad &= \quad \max_{\iota_t,\phi_t\geq 0} \left\{\theta + \left[\hat{\mu}_{\hat{v},t} + \hat{\mu}_{n,t} + \hat{\sigma}_{\hat{v},t}\hat{\sigma}_{n,t}\right]\hat{v}_t\right\} \ , \\ &\qquad \text{subject to} : \phi_t \leq \lambda \hat{v}_t \ , \end{aligned} \tag{14}$$

where the drift and diffusion of net worth satisfy

$$\hat{\mu}_{n,t} = \left[\frac{a-\iota_t}{q_t} + \mu_{q,t} + \mathcal{I}\left(\iota_t\right) + \left(\sigma_{q,t}+\sigma\right)\hat{\mu}_t\omega_t + \sigma_{q,t}\sigma - r\right]\phi_t + r \ , \tag{15}$$

$$\hat{\sigma}_{n,t} = \left(\sigma_{q,t}+\sigma\right)\phi_t \ , \tag{16}$$

with $\hat{\mu}_{\hat{v},t} = \mu_{\hat{v},t} + \hat{\mu}_t\omega_t\sigma_{\hat{v},t}$ and $\hat{\sigma}_{\hat{v},t} = \sigma_{\hat{v},t}$.[18]

operate. The overall specification follows from Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).

[17] Non positive diffusion means that the marginal value of net worth is decreasing under positive shocks, a guess verified in the characterization of equilibrium.

[18] The HJB equation is derived in three steps. First, substitute $\hat{V}_t\left(n_t\right) = \hat{v}_t n_t$ into (12) and rewrite the identity as $e^{-(r+\theta)t}\hat{v}_t n_t + \int_0^t \theta e^{-(r+\theta)s} n_s ds = \hat{E}_t \int_0^{\infty} \theta e^{-(r+\theta)s} n_s ds$. Second, note that the RHS of the

The first order condition (FOC) of (14) with respect to the reinvestment rate ι_t is:[19]

$$\mathcal{I}'(\iota_t) = \frac{1}{q_t} \ . \tag{17}$$

This expression highlights that incentives to reinvest are proportional to the price of physical capital q_t.

The FOC with respect to the capital portfolio weight ϕ_t yields

$$\phi_t \begin{bmatrix} = \lambda \hat{v}_t, & \text{if } \hat{\alpha}_{S,t} > 0 \\ \in [0, \lambda \hat{v}_t] & \text{if } \hat{\alpha}_{S,t} = 0 \\ = 0 & \text{if } \hat{\alpha}_{S,t} < 0 \end{bmatrix} , \tag{18}$$

where $\hat{\alpha}_{S,t}$ is the forecasted risk-adjusted excess return to acquire physical capital and operate the superior technology over debt. That is,

$$\begin{aligned} \hat{\alpha}_{S,t} &\equiv \frac{1}{dt}\hat{E}_t \left[dR_{S,t} \right] - r + \left(\sigma_{q,t} + \sigma \right) \sigma_{\hat{v},t} \ , \\ &= \frac{a - \iota_t}{q_t} + \mu_{q,t} + \mathcal{I}\left(\iota_t \right) + \left(\sigma_{q,t} + \sigma \right) \hat{\mu}_t \omega_t + \sigma_{q,t}\sigma - r + \left(\sigma_{q,t} + \sigma \right) \sigma_{\hat{v},t} \ . \end{aligned} \tag{19}$$

When $\hat{\alpha}_{S,t} > 0$, financiers expect a positive excess return from the superior technology over debt. Hence, they issue debt until they reach the limit imposed by the collateral constraint. When $\hat{\alpha}_{S,t} = 0$, financiers are indifferent between portfolio options. Finally, when $\hat{\alpha}_{S,t} < 0$, financiers strictly prefer debt securities and thus do not acquire physical capital. The last term in $\hat{\alpha}_{S,t}$ reflects a compensation that financiers require for holding aggregate risk if they are subject to financing constraints. This *risk premium* term measures the co-movement between the (forecasted) rate of return $d\hat{R}_{S,t}$ and the rate of change of the marginal value of net worth $\hat{v}_t$.

Lastly, replacing optimality conditions into the HJB (14), one gets the following expression that characterizes the marginal value of net worth $\hat{v}_t$:

$$\hat{\alpha}_{S,t}\phi_t + \mu_{\hat{v},t} + \hat{\mu}_t \omega_t \sigma_{\hat{v},t} + \frac{\theta}{\hat{v}_t} - \theta = 0 \ . \tag{20}$$

---

resulting expression is the conditional expectation of a random variable. This implies that the drift process of the RHS is null. Third, apply Ito's Lemma to derive the drift process of the LHS, which is equal to 0, and get the expression reported in the text. See the Appendix for detailed derivations.

[19] In effect, the optimal reinvestment rate is the solution to the problem of maximizing the forecasted expected return to operate the superior technology.

This condition expresses $\hat{v}_t$ as a present discounted value of rents $\hat{\alpha}_{S,t}\phi_t \geq 0$ earned from operating the superior technology. If, for instance, financiers never earn any rent, then $\hat{\alpha}_{S,t}\phi_t = 0$, and the marginal value satisfies $\hat{v}_t = 1$. In contrast, if $\hat{\alpha}_{S,t}\phi_t > 0$ at least occasionally, then $\hat{v}_t \geq 1$.[20]

2.6 Competitive Equilibrium

In equilibrium, households and financiers optimize taking prices and rates of return as given, and the markets for consumption good, physical capital and debt clear. We derive the equilibrium conditions for the allocation and price of physical capital next, and then, we define a Markov competitive equilibrium.

Equilibrium Regimes. Either households or financiers are indifferent between physical capital and debt, that is, either $\hat{\alpha}_{I,t} = 0$ or $\hat{\alpha}_{S,t} = 0$—otherwise, the market for physical capital would not clear. Because $\hat{\alpha}_{I,t} \leq 0$ cannot be positive, at most three regimes can occur in equilibrium, as presented in Table 1.[21] Each regime can be identified by the share of physical capital κ_t allocated to the superior technology. Formally, the share is $\kappa_t \equiv \phi_t\eta_t \in [0,1]$ where $\eta_t \equiv n_t/q_tk_t \in [0,1]$ is the aggregate net worth of financiers as a share of total wealth q_tk_t.[22]

Table 1: Equilibrium Regimes

Regime	Allocation (Physical capital)	Excess returns Households	Excess returns Financiers
1. Financially unconstrained	$\kappa_t = 1$	$\hat{\alpha}_{I,t} < 0$	$\hat{\alpha}_{S,t} = 0$
2. Financially constrained	$\kappa_t = \lambda\hat{v}_t\eta_t \in (0,1)$	$\hat{\alpha}_{I,t} = 0$	$\hat{\alpha}_{S,t} > 0$
3. Safety trap	$\kappa_t = 0$	$\hat{\alpha}_{I,t} = 0$	$\hat{\alpha}_{S,t} < 0$

In the *financially unconstrained* regime, the collateral constraint is slack. Financiers are not only able, but also willing to issue debt and acquire the entire capital stock to operate

[20] The exposition restricts attention to processes $\hat{v}_t$ that are constants if $\hat{\alpha}_{S,t}\phi_t$ is. Note that if $\hat{v}_t$ is constant, then $\mu_{\hat{v},t} = \sigma_{\hat{v},t} = 0$. These elements support the initial guess that the value of a financier who cannot operate the superior technology is simply her net worth.

[21] The characterization presupposes $\hat{\alpha}_{I,t} = 0$ and $\hat{\alpha}_{S,t} = 0$ do not hold simultaneously outside a region with zero probability mass. This is verified in the solution of the model.

[22] Total wealth equals q_tk_t because physical capital is the single real asset. The notation makes no distinction between individual and aggregate variables because a representative financier and a representative household exist in equilibrium.

the superior technology. Households in turn hold financiers' debt. The aggregate capital stock is exposed to quality risk σdZ_t, but it is expected to grow at rate $\mathcal{I}(\iota_t) + \sigma\hat{\mu}_t\omega_t$.

If instead financiers are *financially constrained*, the collateral constraint is binding and restricts their capacity to acquire physical capital. Households hold the remaining stock of physical capital and both technologies are active. Because households are the marginal buyers despite lower valuation, financiers earn a rent per unit of physical capital owned

$$\hat{\alpha}_{S,t} = \mathcal{I}(\iota_t) - \frac{\iota_t}{q_t} + \sigma\hat{\mu}_t\omega_t + \sigma_{q,t}\sigma + (\sigma_{q,t} + \sigma)\,\sigma_{\hat{v},t} > 0 \; . \tag{21}$$

This rent reflects the premium earned from reinvestment opportunities, the exposure to quality risk, and the compensation required for holding aggregate risk. As the collateral constraint tightens, the share of physical capital allocated to the superior technology $\kappa_t = \lambda\hat{v}_t\eta_t$ decreases, as so does the exposure of the aggregate capital stock to quality risk—i.e., $\kappa_t\sigma dZ_t$.

Finally, in a *safety trap*, excess returns are such that financiers are not willing to acquire physical capital. Households issue debt to financiers and allocate the aggregate capital stock to the inferior technology. In this regime, the capital stock is no longer exposed to quality risk, but in the absence of reinvestment, there is no growth: a safety trap.[23]

Markov Equilibrium. In a Markov equilibrium, all endogenous variables are characterized as a mapping over the states. Further, these mappings and the endogenous law of motions of the states are consistent with the equilibrium conditions listed below. As is common, from now on we omit time subscript.

The economy is proportional to the aggregate capital stock. Thus, we interpret the evolution of k_t as the *economic trend* and all other flucuations as the *economic cycle*. The relevant state variables are the wealth share of financiers η_t and sentiment ω_t.

The equilibrium conditions are the following: the reinvestment rate ι satisfies (17); capital portfolio weight ϕ satisfies (18); the fraction of the aggregate capital stock allocated to the superior technology $\kappa = \phi\eta$, the price of physical capital q and investors excess returns satisfy the conditions presented in Table 1 and equations (11) and (19); output per unit of physical capital net of reinvestment is $y/k = a - \iota\kappa$; the marginal value of

[23] In accord with the postulate made to solve (12), financiers are never willing to operate the inferior technology. In regimes 1 and 2, the superior technology yields higher expected excess returns. In case of safety trap, only households are indifferent between allocating physical capital to the inferior technology and debt, because households are risk-neutral whereas financiers are effectively risk-averse.

financiers net worth $\hat{v}$ satisfies (20); the law of motion of the aggregate capital stock is $dk/k = \mathcal{I}(\iota)\,\kappa dt + \sigma\kappa dZ$; the law of motion of sentiment is given by $d\omega = -\delta\omega dt + dZ$; finally the law of motion of the wealth share of financiers is $d\eta/\eta = \mu_\eta dt + \sigma_\eta dZ$, with[24]

$$\begin{aligned}\mu_\eta &= \frac{a-\iota}{q}\phi + \left[\mu_q + \mathcal{I}(\iota) + \sigma_q\sigma - r - (\sigma_q+\sigma)^2\right](\phi-1) - \left(\theta - \frac{\gamma}{\eta}\right), \\ \sigma_\eta &= (\phi-1)(\sigma_q+\sigma).\end{aligned} \tag{22}$$

Solution Method. A characterization of Markov equilibrium requires to solve for the price of physical capital q and the perceived marginal value $\hat{v}$ of financiers' net worth. Any other variables can be expressed as a function of these variables, their derivatives or the states (η, ω). Mappings q and $\hat{v}$ are the solution of a system of second-order partial differential equations (PDEs)—which is solved numerically using spectral methods.[25] Table 2 reports the parameter values. The functional form for the reinvestment function is $\mathcal{I}(\iota) = \chi\iota^{1-\psi}$ where $\chi > 0$ and $\psi \in (0,1)$ are parameters.

Table 2: Parameter Values

	Parameter	Value
1. Technology		
Productivity of physical capital	a	1
Capital quality risk	σ	6%
Productivity of reinvestment	χ	1.9%
Elasticity of reinvestment		0.5
2. Diagnostic beliefs		
Memory decay rate	δ	1
Expectation factor	$\hat{\mu}$	10%
Bias in expectation formation	Δ	0.5
3. Households / Financiers		
Subjective time discount rate	r	2%
Share of divertable assets (inverse)	λ	2
Replacement rate of financiers	θ	5%
Initial endowment of financiers	γ/θ	2.2%

[24] This law of motion follows from applying Ito's quotient rule to $\eta = n_f/qk$, and then subtracting from the resulting expression the net transfers from financiers to households, $\theta - \gamma/\eta$.

[25] In particular, Ito's Lemma allows to express the drift and diffusion processes of endogenous variables as a function of first or second order derivatives of the underlying variables with respect to the state. See the Appendix for details.

3 Sentiment and Financial Cycles

This section examines the equilibrium implications of diagnostic beliefs and financial constraints. Four different economies are being compared to isolate the effects of each friction and their interactions. The characterization of equilibria focuses on the allocation of physical capital, the incentives to reinvest, and the properties of economic cycles. The economy under rational beliefs and without financial constraints serves as a benchmark, characterized by the absence of cyclical fluctuations. Diagnostic beliefs or financial constraints on their own generate recurrent boom-bust cycles. The cycles from diagnostic beliefs feature low sentiment and safety traps during busts and exuberance and excessive reinvestment during booms. Those from financial constraints instead exhibit "fire sales", with two-way feedback loops between asset prices and financial conditions. Finally, the economy with both frictions reveals how diagnostic expectations intensifies instability in financial markets. This economy also highlights a *paradox of optimism* in which diagnostic optimism further intensifies financial instability.

3.1 Benchmark: Rational Economy without Financial Constraints

In the absence of financial constraints, the portfolio decisions of financiers are not restricted by their net worth.[26] If beliefs are rational, moreover, the superior technology is correctly perceived as more profitable. Accordingly, because financiers are both willing and able to take risks and reinvest, the entire capital stock is allocated to the superior technology, and only the *financially unconstrained* regime takes place. The price of physical capital is the present discounted value of the resulting output flows net of reinvestment,

$$q = q_* \equiv \frac{a - \iota_*}{r - \mathcal{I}(\iota_*)} \ , \ \text{with } \iota_* \equiv \arg\max_{\iota \geq 0} \frac{a - \iota}{r - \mathcal{I}(\iota)} \ , \tag{23}$$

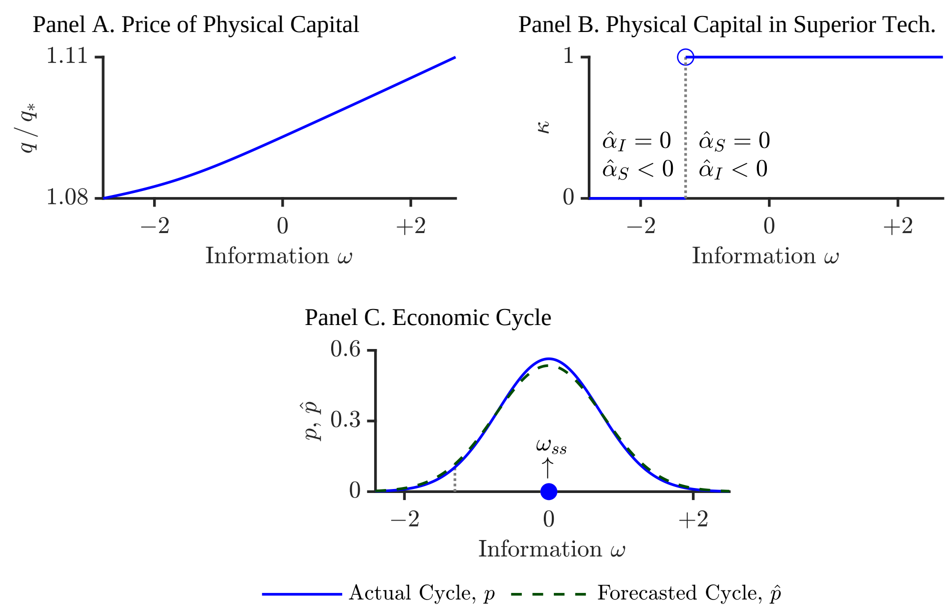
All other detrended variables are stationary as well, hence the absence of cyclical fluctuations. The aggregate capital stock grows at rate $\mathcal{I}(\iota_*) > 0$ in expectation and it is fully exposed to quality risk σdZ.

[26]Formally, there is no financial constraint if negative wealth $n < 0$ is admissible and $\lambda = +\infty$. In this case, regardless of the beliefs system, financiers demand no risk premium (i.e., $(\sigma + \sigma_q)\sigma_{\hat{v}} = 0$) and hence $\hat{\alpha}_S = (a - \iota)/q + \mu_q + \mathcal{I}(\iota) + (\sigma_q + \sigma)\hat{\mu}\omega + \sigma_q\sigma - r$. Also, $\hat{v} = 1$ because $\hat{\alpha}_S\phi = 0$.

3.2 Behavioral Economy without Financial Constraints

If beliefs instead are diagnostic, sentiment do influence the willingness to take risks and reinvest. In the absence of financial constraints, the financially unconstrained regime occurs only when sentiment is relatively high. Safety traps—characterized by no exposure to capital risk but no reinvestment—takes place otherwise. Exposure to capital risk and reinvestment are therefore positively related to sentiment.

Figure 2: Behavioral Economy without Financial Constraints



Notes: Panel A plots the price of physical capital, q, and Panel B plots the share of the aggregate capital stock allocated to the superior technology, κ, as a function of state ω. Reinvestment rate ι is increasing with the price whenever the share is positive. Cut-off $\bar{\omega} < 0$ delimits the two equilibrium regimes: safety traps occur when $\omega < \bar{\omega}$ and the unconstrained regime occurs otherwise. Panel C plots the actual ergodic distribution of sentiment (solid blue line) and the one perceived by agents (dashed green line). Under both distributions the stochastic steady state is $\omega_{ss} = 0$.

Figure 2 displays the equilibrium outcome as a function of the state with $\bar{\omega} \equiv \{\omega \in \mathbb{R} : \hat{\alpha}_I(\omega) = \hat{\alpha}_S(\omega) \text{ with } \hat{v} = 1\} < 0$ being the cut-off that delimits the two regimes.[27] Compared to the benchmark economy, the price of physical capital $q > q_*$ is systematically

[27] The cut-off is negative because reinvestment opportunities are valuable and because the price is positively related to sentiment.

higher (Panel A) and so is reinvestment during the financially unconstrained regime. The reason is that diagnostic beliefs increase the perceived persistence of disturbances, but the effective exposure to capital risk is asymmetric along the economic cycle. For relatively high value of sentiment, the prospect of acquiring physical capital is enhanced by expected positive quality shocks, which support the price q. When sentiment is depressed though, capital is allocated to the inferior technology, and is thus no longer exposed to quality risk.[28]

The economic cycle fluctuates continuously between the two regimes (Panel C), according to an exogenous law of motion: $d\omega = -\delta\omega dt + dZ$. Over a sufficiently long time horizon, sentiment is distributed according to $\omega \sim \mathcal{N}(0, 1/2\delta)$ and safety traps occur at frequency $\Phi(\sqrt{2\delta}\bar{\omega}) \in (0,1)$. However, agents believe the cycle follows $d\hat{\omega} = (-\delta + \hat{\mu})\omega dt + dZ$. The persistence of the cycle $-\delta + \hat{\mu} \geq \delta$ is thus overestimated, as are the variance of information and the frequency of safety traps. Agents then believe that fluctuations in asset prices and real variables are more volatile than what they actually are.

Table 3: State-contingent Diagnostic Beliefs

	Price of physical capital			Frequency of safety traps	
	Min. q	*Av. q*	*Max. q*	Realized	Estimated
Beliefs					
Unbiased expectations ($\Delta = 0.5$)	1.08	1.10	1.13	3.3%	4.1%
Diagnostic optimism ($\Delta = 0$)	1.41	1.42	1.45	0.0%	0.0%
Diagnostic pessimism ($\Delta = 1$)	0.84	0.85	0.86	7.7%	8.8%

State-contingent diagnostic distortions further highlight the properties of the cycle, as reported in Table 3. Under diagnostic optimism—where beliefs are more influenced by positive than negative information—the price of physical capital is even higher because of the larger weight on positive expected returns. Safety traps occur less often because negative information depresses relatively less the attractiveness of the superior technology. In contrast, under diagnostic pessimism of sufficiently high degree, negative expected returns dominate to the extent that the price of physical capital is systematically below the benchmark. Safety traps occur more often.

Overall, the behavioral economy without financing constraints exhibits recurrent boom-bust cycles driven by exuberance during booms and depressed sentiment during busts. Relative to the benchmark, the economy features under reinvestment during safety traps

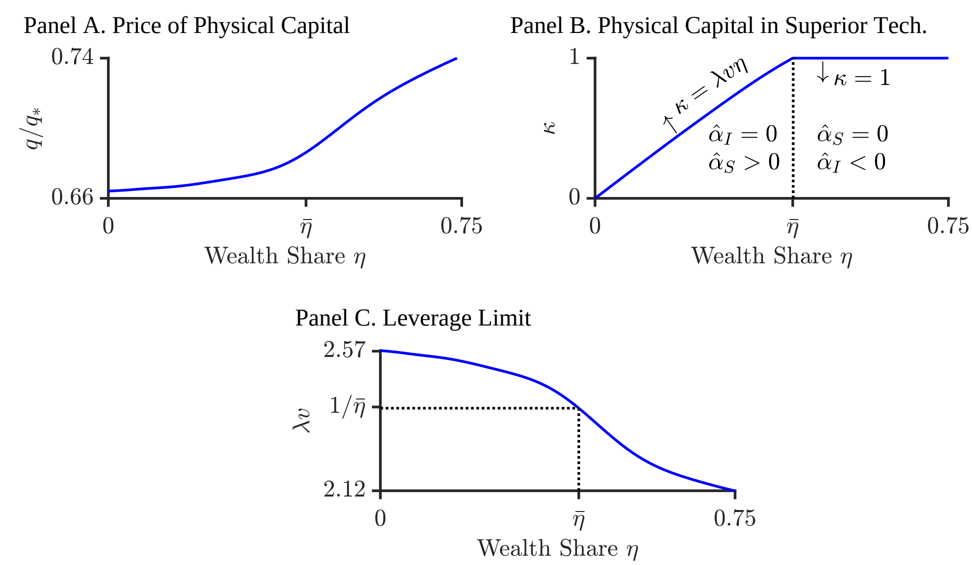
[28] Appendix **??** provides a formal proof of this result which is based on a perturbation argument around $\omega = 0$ in an economy with sufficiently small fluctuations in sentiment.

and over reinvestment outside the traps. The aggregate exposure to capital quality risk is also pro-cyclical.

3.3 Rational Economy with Financial Constraints

If financial constraints instead exists, the relative wealth share η determines the ability of financiers to take risks and reinvest. In particular, when their wealth share is low, financiers lack sufficient borrowing capacity to acquire the entire capital stock. Thus, with rational beliefs, the collateral constraint is binding when η is low—and the financially constrained regime takes place—while the unconstrained regime occurs otherwise. Reinvestment of output and the exposure of physical capital to quality risk are increasing in the wealth share.

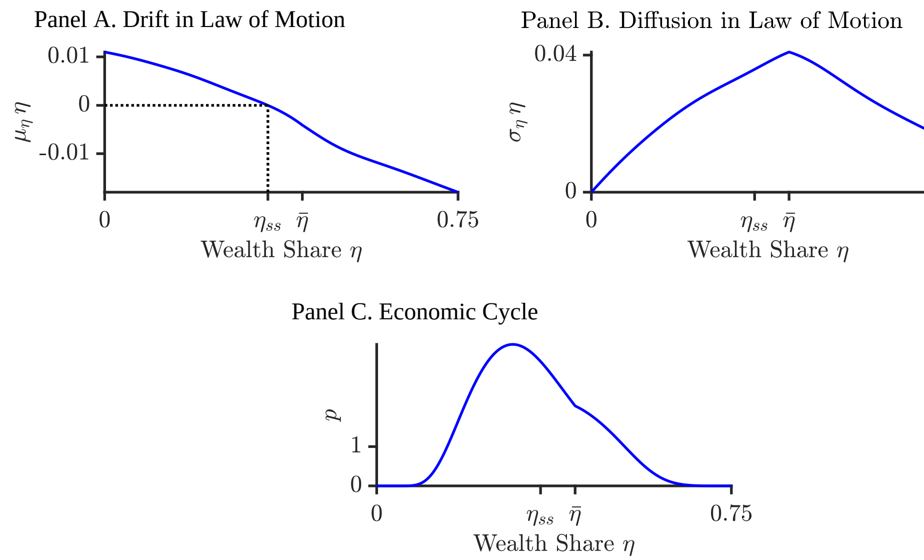
Figure 3: Rational Economy with Financial Constraints (1/2)



Notes: Panel A plots the price of physical capital, q, and panel B plots the share of the aggregate capital stock allocated to the superior technology, κ, as a function of state η. Cut-off $\bar{\eta}$ delimits the two equilibrium regimes: the financially constrained regime occurs when $\eta < \bar{\eta}$ and the financially unconstrained one does so otherwise. Panel C reports the leverage limit, $\phi \leq \lambda v$, which is binding only when $\eta < \bar{\eta}$.

Figure 3 displays the equilibrium outcome as a function of the state with $\bar{\eta} \equiv \{\eta \in \mathbb{R} : \lambda v(\eta)\eta = 1\}$ being the cut-off that delimits the two regimes. Relative to the benchmark economy, the price of physical capital $q < q_*$ is systematically lower (Panel A) and so is the reinvestment rate $\iota < \iota_*$. The price is lower in the constrained regime because households are the marginal buyers. The lower valuation of households contributes to depress the price in the other regime as well, because when financiers are marginal buyers on physical capital, they correctly anticipate a probabilistic transition to the other regime. The risk-premium term in excess return α_S indeed reflects this risk. For a similar reason, the marginal value of net worth $v > v_* \equiv 1$ is systematically enhanced as well (Panel C), despite rents $\alpha_S > 0$ being positive during the constrained regime only.

Figure 4: Rational Economy with Financial Constraints (2/2)



Notes: Panel A plots the drift and panel B reports the diffusion of law of motion of the state $d\eta = \mu_\eta \eta dt + \sigma_\eta \eta dZ$. Cut-off $\bar{\eta}$ delimits the two equilibrium regimes: financially constrained regime occurs when $\eta < \bar{\eta}$ and the financially unconstrained one does so otherwise. Panel C plots the ergodic distribution of wealth share η. The stochastic steady state of the wealth share is $\eta_{ss} < \bar{\eta}$.

The economic cycle fluctuates continuously between the two regimes (Figure 4), according to the following law of motion: $d\eta = \mu_\eta \eta dt + \sigma_\eta \eta dZ$ where drift $\mu_\eta \eta$ and diffusion $\sigma_\eta \eta$ are represented in panels A and B, respectively. Fluctuations in the wealth share are endogenous because portfolio decisions and returns on wealth are jointly determined. The fluctuations exhibit non linear volatility (Panel B) because of the possibility of "fire sales". Following adverse shocks $dZ < 0$, when collateral constraints are binding, financiers are compelled to sell physical capital at discount prices to households, since the shocks erode their net worth. The fall in the price further erodes the net worth of financiers, which tightens the constraints further, and fosters additional "fire-sells". The volatility of the wealth share indeed peaks around $\eta \approx \bar{\eta}$, where financiers are sufficiently well capitalized to hold the entire capital stock, but not capitalized enough to tolerate adverse shocks without liquidating physical capital.[29]

Overall, the rational economy with financial constraints also exhibits recurrent boom-bust cycles. These cycles, however, feature fire sales and non linear volatility in asset prices and financial conditions. Reinvestment is depressed throughout the cycle relative to the benchmark economy. The exposure to quality risk increases gradually with the wealth share of financiers until the unconstrained regime where the exposure is maximum.

3.4 Behavioral Economy with Financial Constraints

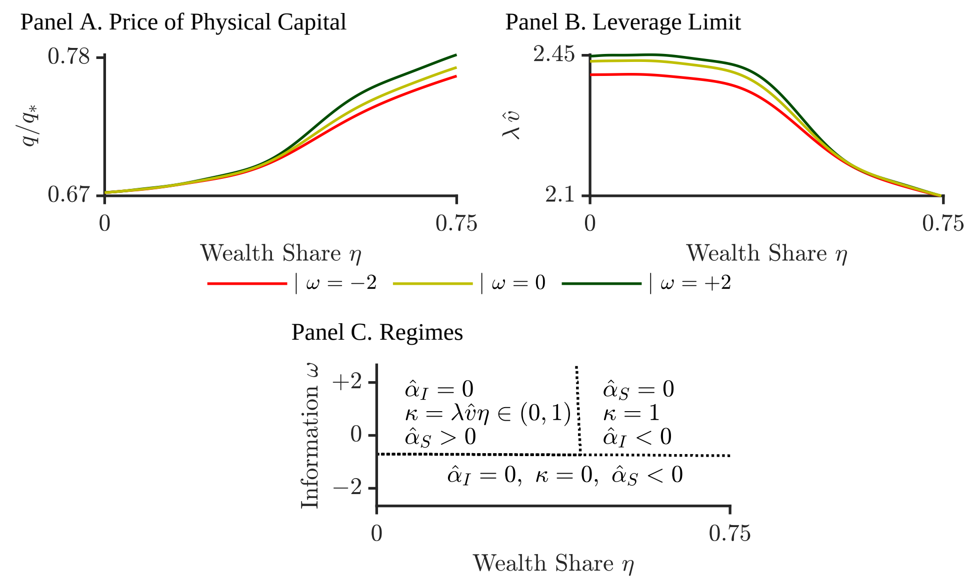
Lastly, with financial constraints and diagnostic beliefs, the relative wealth share η and sentiment ω jointly influence the ability and willingness of financiers to take risks and reinvest. The three regimes occur in equilibrium, along the partitions of the state space described in Sections 3.2 and 3.3.[30]

Figure 5 reports the equilibrium outcome as a function of wealth share η for three levels of sentiment ω. For any given state of the wealth share, improvements in sentiment boost the price of physical capital (Panel A) and relax collateral constraints (Panel B). The effect on the price is stronger for high $\eta > \bar{\eta}$ while the effect on constraints is stronger for low $\eta < \bar{\eta}$. The price is more responsive during the unconstrained regime because the exposure of the aggregate capital stock to quality risk is larger. The leverage limit is more sensitive to sentiment during the constrained regimes because only during that regime financiers earn positive rents on average—which directly enhance the marginal value of net worth $\hat{v}$. A relaxed constraint increases the share of the aggregate capital stock allocated to the superior technology during the constrained regime. It also reduces the cut-off that delimits the two financial regimes.

[29]Formally, fire sales are captured by the interaction between $\sigma_\eta = (\phi - 1)(\sigma_q + \sigma)$ and $\sigma_q = \varepsilon_q \sigma_\eta$, where $\varepsilon_q \equiv (\partial q/\partial \eta)(\eta/q)$ is the elasticity of the price of physical capital with respect to the wealth share of financiers and $\sigma_q = \varepsilon_q \sigma_\eta$ follows from Ito's Lemma. The first relationship has two implications. First, adverse quality shocks to physical capital depresses the wealth share of financiers by $(\phi - 1)\sigma$. Second, the response of the price of physical capital to shocks further depresses the wealth share by $(\phi - 1)\sigma_q$. The second relationship implies that the response of the price depends positively on the response of the wealth share.

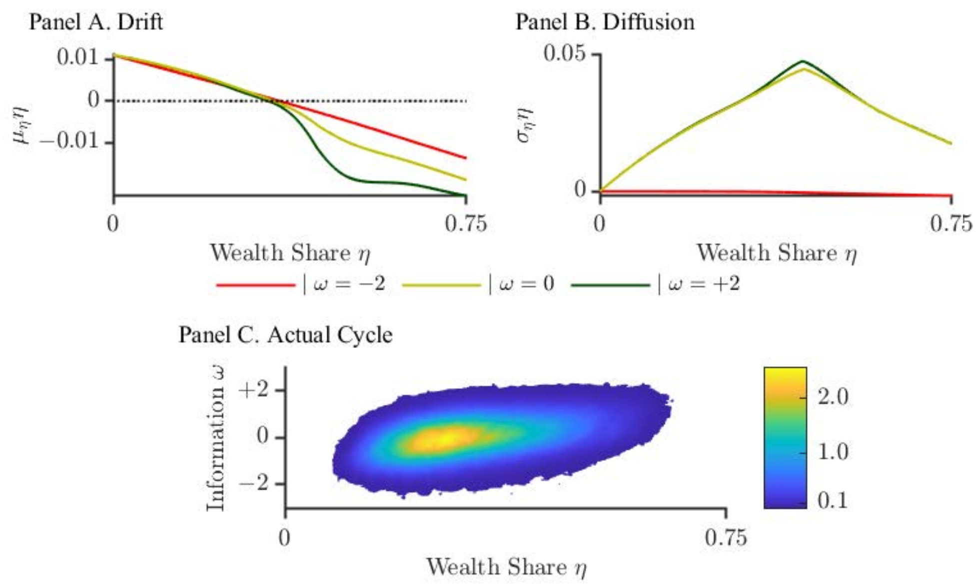
[30]Formally, regimes are delimited by cut-off functions $\bar{\omega}(\eta) \equiv \{\omega \in \mathbb{R} : \hat{\alpha}_I(\omega, \eta) = \hat{\alpha}_S(\omega, \eta)\}$ and $\bar{\eta}(\omega) \equiv \{\eta \in [0, 1] : \lambda \hat{v}(\omega, \eta)\eta = 1\}$, as represented in Figure 5.

Figure 5: Behavioral Economy with Financial Constraints (1/2)



Notes: Panel A plots the price of physical capital q and Panel B reports the leverage limit $\lambda\hat{v}$. Panel C presents the equilibrium regimes as a function of states (ω, η): cut-off $\bar{\omega}(\eta)$ is depicted with the vertical dotted line, while $\bar{\eta}(\omega)$ is reported with the horizontal line. Safety traps occur if $\omega < \bar{\omega}(\eta)$. Otherwise, the constrained regime takes place if $\eta < \bar{\eta}(\omega)$. Reinvestment rate ι is positive and increasing with q only outside the traps.

Figure 6: Behavioral Economy with Financial Constraints (2/2)



Notes: Panel A plots the drift and panel B reports the diffusion of law of motion $d\eta = \mu_\eta \eta dt + \sigma_\eta \eta dZ$, for different level of sentiment ω. Panel C plots the actual joint ergodic distribution of sentiment ω and wealth share η—i.e. not the one perceived by agents.

The economic cycle fluctuates continuously between the three regimes (Figure 6) in accord with the cycles described in Sections 3.2 and 3.3. Financial conditions and sentiment positively co-move because both wealth share η and information ω respond positively to disturbances dZ. The positive co-movement intensifies fire sales and further amplifies the impact of aggregate shocks. Indeed, an adverse shock $dZ < 0$ also contributes to depress sentiment, which further lower households' valuation of physical capital when financiers fire-sale their physical capital.

Diagnostic expectations generate either inflated or depressed forecasts of returns. These forecast errors influence directly the expected profitability of financiers (Panel A). For instance, when sentiment is high, forecasts of positive quality shocks increase the price of physical capital, but financiers with leveraged positions actually earn (on average) negative returns, which erodes their net worth. The opposite happens when sentiment is low: positive forecasts errors contribute to recapitalize financiers. These effects are asymmetric, however, and in particular, they are relatively stronger when the wealth share is high. This is because the exposure of financiers to forecast errors is larger the higher is the share of physical capital allocated to the superior technology.

The combination of intensified fire-sales with procyclical forecast errors share amplify financial instability. Both elements contribute to shift leftward the marginal ergodic distribution of the wealth share relative to the economy with rational beliefs. And they also contribute to depress the average reinvestment rate and the share of physical capital allocated to the superior technology.

Table 4 reports the sensitivity of unconditional averages of reinvestment rate ι and allocative share κ to different parameter values. Relative to the economy with rational beliefs, larger expectation factor $\hat{\mu}$ or higher exogenous risk σ increase instability in financial markets and reduce the unconditional averages (Panel A). Marginal increases in leverage limits (i.e., marginal falls in fraction $1/\lambda$) have similar effects.

State-contingent diagnostic beliefs further highlight the interactions of behavioral and financial frictions and uncover a *paradox of optimism* (Panel B). Specifically, diagnotic expectations biased toward optimism strengthen the negative effects of forecast errors during booms and weaken the positive ones during busts. Thus, diagnostic optimism increases financial instability and deteriorates average wealth share η, reinvestment rate ι and allocative share κ. Pessimistic diagnostic expectations have the opposite effects. These elements indicate that diagnostic optimism combined with financial frictions leads to grimmer realized economic performances, while the opposite is true in economies without financial

TABLE 4: SENSITIVITY ANALYSIS

	Realized averages			Realized frequencies		
	$E[\iota]$	$E[\kappa]$	$E[\eta]$	$\kappa = 0$	$\kappa = \lambda\hat{v}\eta$	$\kappa = 1$
1. Baseline parameter values						
Rational beliefs ($\hat{\mu} = 0\%$)	46%	77%	33%	0%	79%	21%
Diagnostic beliefs ($\hat{\mu} = 10\%$)	49%	67%	31%	11%	74%	15%
2. Baseline with $\hat{\mu} = 10\%$ but with						
Better memory ($\delta = 0.80$)	49%	65%	31%	13%	70%	17%
Higher capital risk ($\sigma = 8\%$)	50%	57%	28%	16%	71%	13%
Higher leverage limits ($\lambda = 2.25$)	54%	66%	28%	10%	72%	18%
3. State-contingent beliefs						
Diagnostic optimism ($\Delta = 0$)	50%	75%	30%	0%	78%	22%
Diagnostic pessimism ($\Delta = 1$)	45%	68%	34%	11%	71%	18%
4. Less intense beliefs ($\hat{\mu} = 3\%$)						
Unbiased expectations ($\Delta = 0.5$)	47%	76%	32%	0%	79%	21%
Diagnostic optimism ($\Delta = 0$)	48%	76%	32%	0%	77%	23%
Diagnostic pessimism ($\Delta = 1$)	47%	78%	33%	0%	76%	24%

frictions, as discussed in Section 3.2).

Overall, diagnostic beliefs and financial constraints interact in a way that generates more volatile economic cycles with stronger financial instability, relative to an economy with rational beliefs. Diagnostic expectations biased toward optimism further intensifies this instability.

4 Conclusion

This paper proposed a tractable macro-finance model with both behavioral and financial frictions to examine the implications of those frictions and their interactions on risk-taking, reinvestment, and economic cycles. Both frictions generate recurrent boom-bust cycles. Behavioral frictions on their own generate safety traps—i.e., episodes without risk but no growth. Financial frictions give rise to fire sales and two-way loops between assets prices and financial conditions that shape pecuniary externalities and exacerbate instability in financial markets. Relative to a rational economy, the behavioral frictions intensify the externalities and instability by inducing more risk-taking during booms. Macro-prudential interventions that demand more balanced investment portfolios thus further help stabilize financial markets and the economic cycle.

References

Adam, Klaus, Albert Marcet, and Johannes Beutel. 2017. "Stock Price Booms and Expected Capital Gains." *American Economic Review*, 107(8): 2352–2408.

Adam, Klaus, Dmitry Matveev, and Stefan Nagel. 2020. "Do survey expectations of stock returns reflect risk adjustments?" *Journal of Monetary Economics.*

Adam, Klaus, Oliver Pfäuti, and Timo Reinelt. 2020. "Falling Natural Rates, Rising Housing Volatility and the Optimal Inflation Target." University of Bonn and University of Mannheim, Germany CRC TR 224 Discussion Paper Series crctr224_2020_235.

Adrian, Tobias, and Markus K. Brunnermeier. 2016. "CoVaR." *American Economic Review*, 106(7): 1705–41.

Akerlof, George A, and Robert J Shiller. 2010. *Animal spirits: How human psychology drives the economy, and why it matters for global capitalism.* Princeton university press.

Angeletos, George-Marios, and Karthik A Sastry. 2018. "Managing Expectations: Instruments vs. Targets." National Bureau of Economic Research Working Paper 25404.

Bacchetta, Philippe, Elmar Mertens, and Eric van Wincoop. 2009. "Predictability in financial markets: What do survey expectations tell us?" *Journal of International Money and Finance*, 28(3): 406–426.

Barberis, Nicholas. 2018. "Psychology-Based Models of Asset Prices and Trading Volume." In *Handbook of Behavioral Economics - Foundations and Applications 1*. Vol. 1 of *Handbook of Behavioral Economics: Applications and Foundations 1*, , ed. B. Douglas Bernheim, Stefano DellaVigna and David Laibson, 79–175. North-Holland.

Benigno, Pierpaolo, and Luigi Paciello. 2014. "Monetary policy, doubts and asset prices." *Journal of Monetary Economics*, 64: 85–98.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer. 2018. "Diagnostic expectations and credit cycles." *The Journal of Finance*, 73(1): 199–227.

Bordalo, Pedro, Nicola Gennaioli, Andrei Shleifer, and Stephen J Terry. 2019*a*. "Real credit cycles." *Harvard University.*

Bordalo, Pedro, Nicola Gennaioli, Rafael La Porta, and Andrei Schliefer. 2019*b*. "Diagnostic Expectations and Stock Returns." *The Journal of Finance*, 74(6): 2839–2874.

Bordalo, Pedro, Nicola Gennaioli, Rafael La Porta, and Andrei Shleifer. 2019*c*. "Diagnostic expectations and stock returns." *The Journal of Finance*, 74(6): 2839–2874.

Bordalo, Pedro, Nicola Gennaioli, Spencer Yongwook Kwon, and Andrei Shleifer. 2020*a*. "Diagnostic bubbles." *Journal of Financial Economics.*

Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer. 2020*b*. "Overreaction in macroeconomic expectations." *American Economic Review*, 110(9): 2748–82.

Brunnermeier, Markus K., and Yuliy Sannikov. 2014. "A macroeconomic model with a financial sector." *American Economic Review*, 104(2): 379–421.

Brunnermeier, Markus K., Thomas Eisenbach, and Yuliy Sannikov. 2013. "Macroeconomics with Financial Frictions: A Survey." *Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society.* New York:Cambridge University Press.

Da, Zhi, Xing Huang, and Lawrence J. Jin. 2020. "Extrapolative beliefs in the cross-section: What can we learn from the crowds?" *Journal of Financial Economics.*

Farhi, Emmanuel, and Iván Werning. 2019. "Monetary Policy, Bounded Rationality, and Incomplete Markets." *American Economic Review*, 109(11): 3887–3928.

Farhi, Emmanuel, and Xavier Gabaix. 2020. "Optimal Taxation with Behavioral Agents." *American Economic Review*, 110(1): 298–336.

Gennaioli, Nicola, and Andrei Shleifer. 2010. "What Comes to Mind*." *The Quarterly Journal of Economics*, 125(4): 1399–1433.

Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial intermediation and credit policy in business cycle analysis." In *Handbook of Monetary Economics.* Vol. 3, 547–599. Elsevier.

Gertler, Mark, and Peter Karadi. 2011. "A model of unconventional monetary policy." *Journal of Monetary Economics*, 58(1): 17–34.

Greenwood, Robin, and Andrei Shleifer. 2014. "Expectations of Returns and Expected Returns." *Review of Financial Studies*, 27(3): 714–746.

He, Zhiguo, and Arvind Krishnamurthy. 2013. "Intermediary asset pricing." *American Economic Review*, 103(2): 732–70.

He, Zhiguo, and Arvind Krishnamurthy. 2019. "A Macroeconomic Framework for Quantifying Systemic Risk." *American Economic Journal: Macroeconomics*, 11(4): 1–37.

Jeanne, Olivier, and Anton Korinek. 2010. "Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach." *American Economic Review*, 100(2): 403–07.

Jeanne, Olivier, and Anton Korinek. 2019. "Managing credit booms and busts: A Pigouvian taxation approach." *Journal of Monetary Economics*, 107(C): 2–17.

Krishnamurthy, Arvind, and Wenhao Li. 2020. "Dissecting mechanisms of financial crises: Intermediation and sentiment." National Bureau of Economic Research.

Lorenzoni, Guido. 2008. "Inefficient Credit Booms." *Review of Economic Studies*, 75(3): 809–833.

Maggiori, Matteo. 2017. "Financial intermediation, international risk sharing, and reserve currencies." *American Economic Review*, 107(10): 3038–71.

Maxted, Peter. 2019. "A macro-finance model with sentiment." Working Paper.

Tversky, Amos, and Daniel Kahneman. 1983. "Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment." *Psychological review*, 90(4): 293.

Van der Ghote, Alejandro. 2020*a*. "Benefits of macro-prudential policy in low interest rate environments." Working Paper.

Van der Ghote, Alejandro. 2020*b*. "Interactions and coordination between monetary and macro-prudential policies." *American Economic Journal: Macroeconomics (forthcoming)*.

Acknowledgements

We thank seminar participants at University of Mannheim, European Central Bank, CESifo Area Conference on Macro, Money, and International Finance 2021, and European Economic Association Congress 2021.
The views expressed in this article are those of the authors and do not necessarily reflect the views of the European Central Bank or the Eurosystem.

Antoine Camous
University of Mannheim, Mannheim, Germany; email: camous@uni-mannheim.de

Alejandro Van der Ghote
European Central Bank, Frankfurt am Main, Germany; email: Alejandro.Van_der_Ghote@ecb.europa.eu

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

PDF ISBN 978-92-899-5108-1 ISSN 1725-2806 doi: 10.2866/335 QB-AR-22-024-EN-N