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Public money as a store of value, heterogeneous beliefs, and banks: implications of CBDC

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Abstract

The bulk of euro-denominated cash is held for store of value purposes, with such holdings sharply increasing in times of high economic uncertainty. We develop a Diamond and Dybvig model with public money as a store of value and heterogeneous beliefs about bank stability that accounts for this evidence. Consumers who are sufficiently pessimistic prefer to hold cash. In our model, the introduction of a central bank digital currency (CBDC) as a store of value that is superior to cash leads to bank disintermediation as some depositors opt for switching to CBDC based on their beliefs. While CBDC partially replaces deposits, long-term lending decreases less than proportionally as remaining depositors are, on average, more optimistic about bank stability and banks re-balance their portfolio accordingly. The appropriate calibration of CBDC design features such as remuneration and quantity limits can mitigate these effects. We study the individual and social welfare implications of introducing CBDC as a store of value.

Keywords: Cash, central bank digital currency, bank disintermediation, bank stability, welfare.

JEL Codes: E41, E58, G11, G21

Non-technical summary

A large proportion of euro-denominated cash is held for store of value purposes. In addition, the evidence shows that such cash holdings sharply increase in times of high economic uncertainty and that only a fraction of the population holds cash outside a bank account as a store of value.

These empirical observations raise questions about the implications of introducing a CBDC that, even if introduced for transaction purposes, may also serve as a store of value. How would such cash holdings and the proportion of the population that hold public money as a safe liquid asset be affected? Would CBDC replace deposits? How would that affect bank lending, productive investment and welfare?

This paper proposes a Diamond and Dybvig model with public money as a store of value and heterogeneous prior beliefs about the probability of a bank run to study these issues. The model captures the above-mentioned evidence on demand for cash as a store of value as well as the empirical fact that prior beliefs about bank stability and the future state of the economy are heterogeneous across agents and crucially affect individual portfolio choices.

In particular, the model shows that only consumers whose subjective probability of a bank run is sufficiently high opt for holding cash as a store of value instead of bank deposits (i.e., those who are sufficiently pessimistic about bank stability). Importantly, the demand for public money as a safe liquid asset and the proportion of consumers who prefer to hold cash rather than bank deposits increase with uncertainty (i.e., dispersion in beliefs). This suggests that, in a dynamic arrangement with shocks to belief dispersion, the model captures the patterns of demand for cash as a safe asset over time.

The issuance of a non-interest bearing central bank digital currency (CBDC) that is a technologically superior store of value compared to cash increases the attractiveness of holding public money. Cash holders and some of the pre-existent depositors switch to CBDC, inducing some bank disintermediation. Although CBDC partially replaces deposits, bank lending and productive investment decrease less than proportionally to deposits as remaining depositors are, on average, more optimistic about bank stability and banks rebalance their portfolio accordingly. That is, maturity transformation increases in relative terms. The appropriate calibration of CBDC design features such as remuneration and quantity limits can mitigate any bank disintermediation and the extent to which CBDC is used as a store of value. We study the individual and social welfare implications of introducing CBDC as a store of value.

1 Introduction

In recent years, the use of digital payment methods for transactions has been increasing at the expense of cash, a pattern that has become more pronounced since the outbreak of the Covid-19 crisis (see, e.g., Auer et al. (2020b); Zamora-Pérez (2021)). In response to this shift, central banks have started to investigate the benefits and implications of issuing central bank digital currencies (CBDCs) (Auer et al., 2020a). The ultimate goal of a CBDC is to ensure that individuals operating in an increasingly digitalized economy continue to have access to public money as a means of payment. However, there are concerns that, if also widely used as a store of value, CBDCs may disintermediate banks (see e.g. ECB (2020); FED (2022)). After all, the cash-to-GDP ratio has continued to steadily increase, which is suggestive of a strong demand for cash as a store of value.

This paper develops a banking model a la Diamond and Dybvig (1983) with public money as a storage technology and studies the effects of CBDC as a store of value. In the model, banks provide insurance for idiosyncratic liquidity shocks, which exposes the bank to the possibility of a bank run. As in Cooper and Ross (1998); Ennis and Keister (2006), we use an equilibrium selection rule that builds on an exogenous probability of a bank run, which is assumed to capture bank stability and the state of the economy. Our results rely on the introduction of heterogeneous beliefs about the probability of a bank run. This allows to model how consumers choose between bank deposits and public money, and how banks adjust their lending in response. Thus, we focus on ex ante portfolio choices, not on actions during a bank run.

Under certain conditions, the introduction of a CBDC that is a more attractive store of value compared to cash leads to bank disintermediation as it increases the demand for public money at the expense of bank deposits. Interestingly, as the demand for public money increases, the average depositor is more optimistic about bank stability. Consequently, the bank optimally rebalances its portfolio towards a larger share of long-term lending. Thus, while in absolute terms the issuance of CBDC as a store of value leads to a decline in bank funding and lending, in relative terms it translates into increased maturity transformation. The appropriate calibration of CBDC design features such as remuneration and quantity limits can mitigate any bank disintermediation and the extent to which CBDC is used as a store of value.

Our analysis is motivated by evidence on a strong demand for cash as a store of value in the euro area. We de-trend the cash-to-GDP ratio, and decompose aggregate cash holdings into an estimated transactions demand and a store of value component. The evidence suggests that: (i) the bulk of cash is held for store of value purposes; (ii) cash holdings have a prominent cyclical component and due to their role as a safe haven markedly increase in times of uncertainty and economic downturn; (iii) only a fraction of the population holds cash as a store of value.

Our baseline model is based on the Ennis and Keister (2006) version of the Diamond and Dybvig set-up, augmented with a private choice on a store of value. The central bank issues cash and reserves that serve as safe storage technologies. To capture their technological difference, we assume that cash holdings imply a storage cost whereas (digital) reserves do not.[1] Only banks have access to central bank reserves. They invest in reserves and long-term loans.

The representative bank offers the contract that maximizes the expected utility of its depositors. We study consumers' choice between cash and deposits.[2] For consumers to prefer cash over deposits, the expected utility derived from holding cash must exceed that obtained from deposits. Given the technological superiority of reserves, the bank never chooses to offer a deposit contract inferior to cash. That is, in the baseline model there is no demand for cash regardless of the probability of a bank run. These findings are sharply at odds with the empirical evidence on cash demand.

We modify the baseline model to introduce heterogeneous beliefs about the probability of a bank run. We refer to this set-up as "The Model". Consumers no longer agree on the probability of the bank run equilibrium and draw their ex ante beliefs from a distribution function. The belief dispersion indirectly captures the level of uncertainty in the economy. Consistent with the literature, the bank offers a single deposit contract that maximizes the expected utility of its depositors, which depends on their average beliefs.[3]

The Model accounts for the main empirical observations on demand for cash as a store of value. First, only some consumers hold cash as a store of value as they are more pessimistic about bank stability. Second, when the dispersion in beliefs (i.e., uncertainty) increases, demand for cash soars at the expense of bank deposits. This suggests that, in a dynamic arrangement with shocks to belief dispersion, the model captures the patterns of demand for cash as a safe

[1] In particular, the net exchange value of reserves is normalized to unity while cash is subject to storage costs and, hence, its net exchange of value is below one.

[2] The baseline model does not explicitly consider mixed portfolios. For an extension of this model that allows for consumers to simultaneously hold cash and bank deposits, see Appendix B.

[3] Heterogeneous beliefs are commonly used in the field of behavioral finance (Hong and Stein, 2007) to capture disagreement among agents. In practice the objective probability of a state of the world is hard to estimate. Thus, individuals have their corresponding subjective beliefs on which they base their investment decisions (Giglio et al., 2021; Meeuwis et al., 2022).

asset over time.

The Model is applied to study the main implications of introducing CBDC as a store of value (that is superior to cash) for banks and consumer welfare. The attractiveness of holding public money increases with the adoption of CBDC as a store of value, which has several consequences. First, cash holders always benefit by fully switching to CBDC. Second, the threshold that defines the set of consumers who prefer to hold public money declines, inducing some depositors to switch to CBDC (bank disintermediation). These consumers also benefit from the introduction of CBDC as a store of value unless individual welfare depends on the true probability of a bank run, and such probability is sufficiently low. Third, the shrinkage in the set of depositors alters the bank deposit contract (which depends on the average depositor's belief). In general, depositors are worse-off with the introduction of CBDC as a store of value, although the most optimistic ones may benefit if individual welfare depends on their own beliefs rather than on actual bank stability.

We assess the desirability of introducing CBDC as a store of value in The Model by solving the problem of an utilitarian social planner under different sets of assumptions, following the welfare criterion proposed by Brunnermeier et al. (2014) where possible.

Related literature

Our paper connects to three main strands of the literature. A first strand models the demand and supply of safe liquid assets (Stein, 2012; Gorton and Ordonez, 2022). Our model contributes by explicitly modelling the lasting implications of demand for cash as a safe store of value on bank intermediation. It is the first to explicitly model cash as a storage technology alternative to bank deposits in a context of heterogeneous beliefs about bank stability. Allen et al. (2014) introduce fiat money in a canonical bank-run model as a nominal means of payment (rather than as a store of value).Ennis and Keister (2003) also explicitly model a storage alternative for bank deposits, while Peck and Setayesh (2022) consider a productive investment technology as the alternative.

This paper also relates to the literature on heterogeneous beliefs and disagreement. Giglio et al. (2021) use survey data to provide robust evidence on: (i) the link between beliefs and portfolio allocations, both across retail investors and over time, and (ii) a persistent heterogeneity in beliefs across individuals. It is well documented that different views on interpreting signals lead to persistent disagreement over economic variables (Harris and Raviv, 1993; Kandel and Pearson, 1995; Meeuwis et al., 2022). Patton and Timmermann (2010) shows that even professional forecasters persistently disagree with a belief dispersion that is counter-cyclical and highest in times of economic recession and uncertainty.

Heterogeneous beliefs are commonly used in behavioral finance and asset pricing to interpret empirical findings on trading and disagreement (Hong and Stein, 2007; Chand et al., 2021). Papers in the macro-finance literature that assume heterogeneous beliefs include Geanakoplos (2010); Scheinkman and Xiong (2003); Martin and Papadimitriou (2021); Caballero and Simsek (2020). We consider heterogeneous beliefs to explain why some consumers prefer cash rather than bank deposits as a store of value.

Finally, our paper contributes to the growing literature on the implications of CBDC for bank intermediation and the real economy. Brunnermeier and Niepelt (2019); Whited et al. (2022) show that, under certain conditions, public and private monies are equivalent and, thus, introducing a CBDC does not have any allocative or macroeconomic consequences.[4] Papers that study the effects of CBDC on the banking sector by making one or various assumptions that impede the equivalence result to hold include Piazzesi and Schneider (2022); Williamson (2022); Bacchetta and Perazzi (2021); Adalid et al. (2022); Ahnert et al. (2022); Keister and Sanches (2022); Abad et al. (2023), among others. In most models, the equivalence result does not hold due to the presence of a market imperfection or a regulatory constraint. Such frictions include imperfect competition in the bank deposit market (see, e.g., Andolfatto (2021); Chiu et al. (2021)), central bank collateral requirements (Assenmacher et al. (2021); Burlon et al. (2022); Williamson (2022)), and liquidity regulation (Meller and Soons, 2023). In our model, it is incomplete information that undermines the equivalence result, through two channels. First, the central bank and consumers face an adverse selection problem which precludes them from investing in long-term loans. Second, consumers do not know the objective probability of a bank run.

From a modelling perspective, our paper connects with the CBDC literature that builds on the Diamond and Dybvig framework. Fernández-Villaverde et al. (2021) find that, under certain assumptions, CBDC leads to a central bank deposit monopoly. In contrast to us, they crucially assume that depositors ex ante do not expect any bank run, and the central bank can indirectly engage in long-term lending by signing contracts with investment banks. Skeie (2020); Tercero (2022) show that the usage of CBDC as a nominal means of payment requires its rate of return

[4] Niepelt (2020) shows that the required conditions are very restrictive and unlikely to hold in practice.

to be higher for bank deposits. Schilling et al. (2020) present a CBDC trilemma according to which the central bank can only achieve two out of the three goals of efficiency, financial stability (i.e., absence of runs), and price stability. Similar to ours, Keister and Monnet (2022) find that the issuance of a CBDC induces a re-balancing effect in the bank asset portfolio towards more long-term lending. In our model, this relative increase in maturity transformation is attributed to the fact that the average bank depositor becomes more optimistic about bank stability.

The rest of the paper is structured as follows. Section 2 documents some empirical facts on the demand for cash as a store of value. Section 3 presents the baseline model. Section 4 develops The Model by introducing heterogeneous beliefs about the probability of a bank run. Section 5 extends The Model by allowing for CBDC as a store of value. Section 6 performs a welfare analysis. Section 7 concludes.

2 Empirical evidence

Some empirical observations help motivate the paper. Figure 1a plots the ratio of aggregate cash holdings defined as the value of euro-denominated banknotes in circulation to GDP for the period 2003 - 2021 at annual frequency. The cash ratio has steadily increased over the last decades, even though (digital) transaction efficiency has risen.

According to recent studies, the use of cash for transactions has decreased, a pattern that has become particularly pronounced in the euro area (and elsewhere) since the onset of the COVID-19 crisis (see, e.g., ECB (2022); Auer et al. (2020b); Zamora-Pérez (2021)). Despite this fact, the upward trend in cash holdings, as also documented by Ashworth and Goodhart (2020), has not been reversed. On the contrary, Figure 1b suggests that cash holdings jumped in response to the COVID-19 shock and have stayed well above their historical trend since then. More generally, Figure 1b shows that cash holdings have only significantly deviated from their trend and remained well above it around the Great Recession (2009) and the COVID-19 crisis (2020-2021). This finding is in line with the empirical studies that show the strong dependence of cash demand as a store of value on uncertainty and the state of the economy (Jobst and Stix, 2017; Rösl and Seitz, 2021). Arguably, as perceived bank stability decreases, a flight-to-safety by depositors from bank deposits to cash takes place (Baubeau et al., 2021). Interestingly, in 2022 37% of euro area survey respondents reported to hold cash at home as a precautionary store of value (ECB, 2022), up from 34% in 2019 and 24% in 2016 (Esselink and Hernández, 2017).

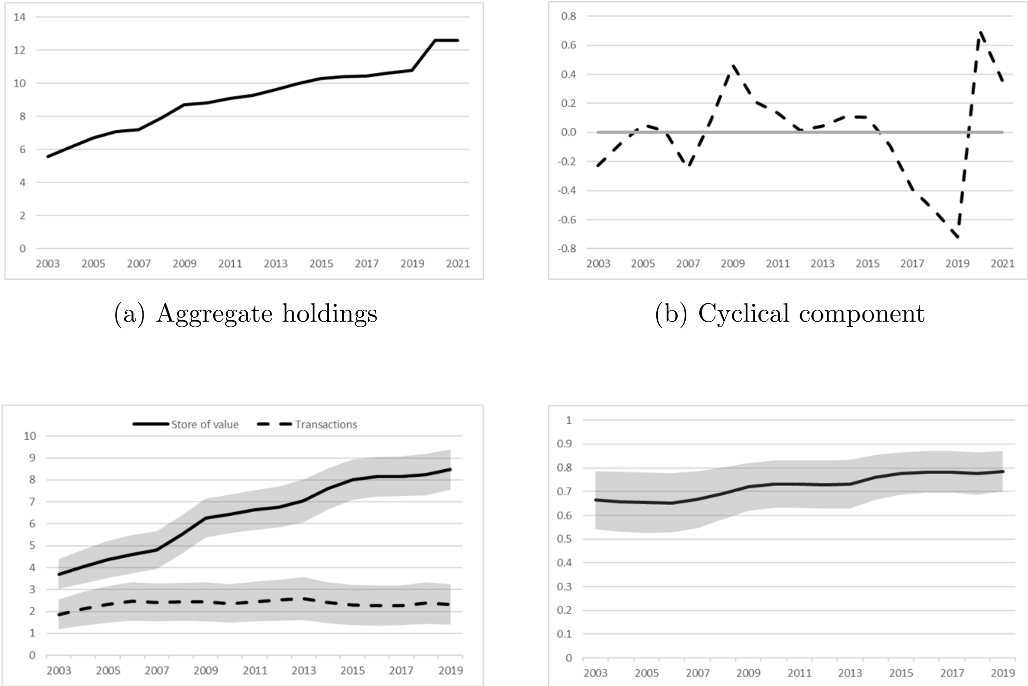
This suggests that only a fraction of the population holds cash as a store of value, arguably as individuals differ in their perceptions about bank stability. According to Zamora-Pérez (2021), in 2019 the amount of cash reserves per-adult in the euro area lied between €1,270 and €2,310.

Similar to the seasonal method applied in Assenmacher et al. (2019); Zamora-Pérez (2021), we decompose the annual series of euro-denominated cash holdings into two estimated components: (i) cash holdings for transaction purposes, and (ii) cash holdings as a store of value. Figure 1c displays the two estimated components of total cash holdings. Decomposition estimates are produced by comparing the seasonality of total banknote circulation with the seasonality of a purely transactional benchmark variable. While the estimated value of banknotes for transactions (dotted line) has not significantly changed over the last two decades, the estimated value of cash holdings as a store of value (solid line) has steadily increased over the same period, suggesting that the upward trend and cyclical deviations in the cash ratio is mostly to be attributed to the demand for cash as a store of value.

Figure 1d confirms the increasing relative importance of cash holdings as a store of value as opposed to that of cash as a means of payment. In particular, the estimated value of banknotes held as a store of value in 2003 already stood at around 65 percent of total cash holdings; a fraction that has been increasing since then until reaching roughly 80 percent of total banknotes in circulation in 2021.

Our paper offers a modification of the canonical bank-run model that accounts for the main empirical observations on demand for cash as a store of value.

Figure 1: Euro denominated cash holdings



(a) Aggregate holdings

(b) Cyclical component

(c) Estimated components

(d) Estimated store of value holdings

Notes: Cash holdings are defined as the value of euro-denominated banknotes in net circulation as a percent of annual GDP. Variables represented in panels (a) and (c) are expressed in percentage points. The one plotted in Figure (b) is expressed in percentage deviations from the HP trend with a standard smoothing parameter of 100. The series displayed in panel (d) is expressed as a proportion of total cash holdings. Data: ECB and own calculations.

3 The baseline model

The baseline model extends the Diamond and Dybvig-type banking model of Cooper and Ross (1998); Ennis and Keister (2006) to allow for cash as a store of value.

3.1 Environment

There are three dates $t = 0, 1, 2$ and a single good per date which works as a numeraire and can be used for investment at $t = 0$ and consumption at $t = 1$ and $t = 2$. A unit continuum of ex ante identical consumers indexed by $i \in [0, 1]$ has an endowment normalized to one at $t = 0$. Consumer preferences are given by

$$U(c_1, c_2, \theta_i) = u(c_1 + \theta_i c_2),$$

where c_t is consumption at date t and the utility function u is strictly increasing, strictly concave, continuously differentiable, and satisfies the Inada conditions. The idiosyncratic liquidity shock $\theta_i \in \{0, 1\}$ is realized at $t = 1$ and privately observed by each consumer. If $\theta_i = 0$, consumer i is impatient and wishes to consume at the interim date only; otherwise, she is patient and values consumption at either the interim or final date. The probability of each consumer becoming impatient is a constant λ.

Consumers can invest in two types of assets at $t = 0$ to transfer wealth to future dates: retail central bank money ("cash") and bank deposits. Without loss of generality and for the sake of simplicity, we do not allow for mixed portfolios in this section. That is, consumers have to place their entire endowment either in deposits or in cash. Appendix B offers a numerical solution to the consumer's problem in a version of The Model in which a consumer can have a mixed portfolio by simultaneously allocating a positive proportion of her endowment in both, deposits and cash.

There is a central bank that exchanges endowment for cash at $t = 0$ and $t = 1$ and repays consumption goods on demand at $t = 1$ and $t = 2$. While the central bank faces no direct storage costs, holding cash comes with a proportional cost $f > 0$ incurred whenever the cash is exchanged for consumption or any other asset, so a unit of cash has a net exchange value of $1 - f$ whenever used.[5]

Second, consumers can pool resources to form a bank that invests their endowments on their behalf. At $t = 0$ the bank invests an amount x of its deposit funding D_0 received from consumers in a long-term investment technology, and $D_0 - x$ in wholesale central bank money ("reserves"). Reserves can only be accessed by the bank and the net exchange value per unit of reserves is normalized to one.

The long-term investment technology (long-term lending) can be of two types, good and bad. The good type yields a return of R units upon maturity at $t = 2$ and has no liquidation value at $t = 1$.[6] This technology offers a higher long-term return than cash or reserves, but it is less liquid. The bad type —a lemon—never generates any return, similar to Dang et al. (2017).

Only a bank can screen potential borrowers and prevent investment in the bad technology,

[5] This cost could correspond to resources spent to prevent theft before its conversion or on other storage and transportation costs.

[6] Jacklin and Bhattacharya (1988); Haubrich and King (1990) also assume that the long-term asset yields a zero payoff when liquidated early. In contrast, Diamond and Dybvig (1983) assume that the liquidation value is equal to the initial investment, while Ennis and Keister (2006); Cooper and Ross (1998) include a liquidation cost $\tau \in [0, 1]$.

as in Holmstrom and Tirole (1997). As in Allen and Gale (1998), we assume that the implied adverse selection problem precludes the consumers and the central bank from investing directly or indirectly (via lending to the bank) in the long-term technology.[7] Thus, the bank has two functions in this economy: (i) it serves as a conduit for investment in good long-term technologies, while screening bad ones, and (ii) it provides insurance against idiosyncratic liquidity risk by offering demand deposits to consumers, as in Diamond and Dybvig (1983). Specifically, at $t = 0$ the bank offers a contract that promises a payment of c_1^B if a consumer withdraws at $t = 1$ and c_2^B if she does not. However, such promises are only fulfilled if the consumers withdrawing at $t = 1$ are the proportion λ of impatient ones. As in Allen and Gale (1998), we assume that if the proportion of early withdrawers exceeds λ, the bank "defaults" and makes a liquidation payment c_R^B to all consumers attempting to withdraw at $t = 1$ (and zero to the rest).[8]

The timing of events in the baseline model is as follows. First, each consumer chooses between holding cash or depositing with the bank. The bank, on behalf of its depositors, invests x in the long-term technology and $D_0 - x$ in reserves. At date $t = 1$, the liquidity shock hits and all impatient consumers attempt to withdraw their bank deposits. The actions of patient consumers depend on: (i) what she expects other patient consumers will do, and (ii) the deposit contract. To simplify the discussion, we will focus on the case in which consumers play symmetric pure strategies. If a patient consumer believes other patient consumers will not withdraw and the deposit contract is incentive compatible, ($c_2^B \geq c_1^B$), she will optimally decide not to withdraw her bank deposits. If all patient consumers follow this behavior, a "good" non-run equilibrium can be sustained. However, if she expects all other patient consumers to withdraw and the bank does not have enough resources to pay c_1^B to all depositors, she will optimally decide to withdraw. If all patient consumers follow this behavior, a "bad" bank run equilibrium emerges. When the bank cannot cover the required repayment in case all patient depositors withdraw at $t = 1$, the deposit contract is said to be run-prone.[9] In contrast, if the bank has enough reserves to meet all of its short-term obligations, waiting to withdraw is a dominant strategy as the payment at $t = 2$ is larger than the payment at $t = 1$. In that case, the deposit contract is said to be run-proof.

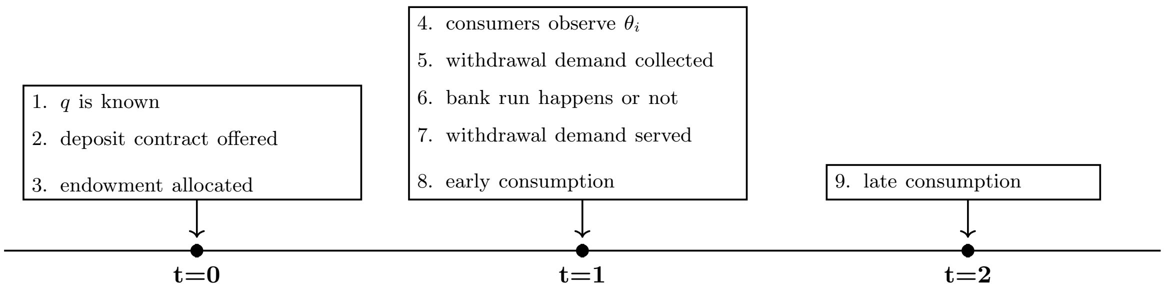
[7] Arguably, in practice, adverse selection explains, among others, why central bank lending and asset purchases are subject to strict risk management frameworks.

[8] Consumers cannot trade at dates $t = 1$ and $t = 2$. Jacklin (1987) and Wallace (1988) consider a credit market at date $t = 1$.

[9] Given the assumption that the long-term investment technology has no liquidation value at $t = 1$, these resources amount to the reserves held by the bank.

In order to describe the ex-ante optimal deposit contract anticipating the possibility of multiple equilibria, we follow Cooper and Ross (1998) and Ennis and Keister (2006) and assume a sunspots-based equilibrium selection rule: if both equilibria exist, a bank run occurs with an exogenous probability q. The probability q is constant and does not depend on actual bank reserves, and we have that $(1-q)R > 1$. Figure 2 summarizes the timeline of the game.

Figure 2: Timeline of the baseline model



3.2 Optimal demand for cash

To determine the demand for cash, we specify the problem of a bank that behaves competitively in the sense that it offers the contract that maximizes the expected utility of its depositors. Let $\bar{\lambda}$ denote the fraction of depositors that can be served at the interim date under the underlying contract. Variable y represents reserves that are needed to repay impatient depositors whereas y^l represents excess liquidity, i.e. reserves in excess of what is required to repay impatient depositors only. The bank's problem solves

$$\max_{c_1^B, c_2^B, c_R^B, x, y, y^l} (1 - q\mathbb{1}_{\bar{\lambda}<1}) \left[\lambda u(c_1^B) + (1-\lambda) u(c_2^B) \right] + q\mathbb{1}_{\bar{\lambda}<1} u(c_R^B) \tag{A}$$

subject to

$$x + y + y^l = D_0, \qquad (1) \qquad\qquad \lambda c_1^B = y, \qquad (2)$$

$$(1-\lambda)c_2^B = Rx + y^l, \qquad (3) \qquad\qquad c_R^B = y + y^l, \qquad (4)$$

$$0 \leq c_1^B \leq c_2^B, \qquad (5) \qquad\qquad c_1^B, c_2^B, x, y, y^l \geq 0, \qquad (6)$$

The indicator function $\mathbb{1}_{\bar{\lambda}<1}$ reflects the equilibrium selection rule. A bank run only occurs with probability q if $\bar{\lambda} < 1$ and otherwise occurs with probability zero. The maximum fraction of depositors that can be served at the interim date without a default is given by

$$\bar{\lambda} = \frac{y + y^l}{c_1^B}. \tag{7}$$

Problem A states that the bank maximizes the expected utility of its depositors subject to the following constraints. Expression 1 stipulates that the bank invest all its deposit funding. The bank is a deposit taker. According to expression 2, the bank must hold enough reserves to cover the promised interim return. Since there is no aggregate uncertainty, the bank knows that a fraction λ of depositors will have liquidity needs. Expression 3 states that the final payment equals the sum of the return on long-term lending and the remaining reserves after having serviced early withdrawals. Expression 4 dictates that the payment in case of a bank run is equal to the liquidation value of the bank. Expression 5 is the incentive compatibility constraint, which ensures that patient consumers have no incentive to withdraw at the interim date in absence of a bank run.

The optimal deposit contract solves Problem A for payments (c_1^B, c_2^B, c_R^B), the bank asset allocation (x, y), subject to the level of deposit funding, D_0. Therefore, it also implicitly includes the demand for cash ($M_0 = 1 - D_0$).

Proposition 1 shows that regardless of the terms of the optimal deposit contract, there is never a positive demand for cash in the baseline model.

Proposition 1: *In the baseline model,* $M^0 = 0$, $\forall\ q \in (0, 1)$.

The reasoning for Proposition 1 is as follows. A bank, whose objective is to maximize depositor utility, can always offer a run-proof contract for any realization of q. The expected utility obtained from the best run-proof is certainly as high as the expected utility obtained from the run-proof contract that includes only reserve holdings. In turn, the utility obtained from cash holdings is strictly lower than that from a run-proof deposit contract with only reserve holdings due to the storage costs (and the lack of liquidity insurance). Thus, the bank never chooses to offer a deposit contract inferior to cash holdings and a consumer never prefers to hold cash instead of bank deposits.

To further characterize the solution to the baseline model, we assume a utility function of the constant-relative-risk-aversion form

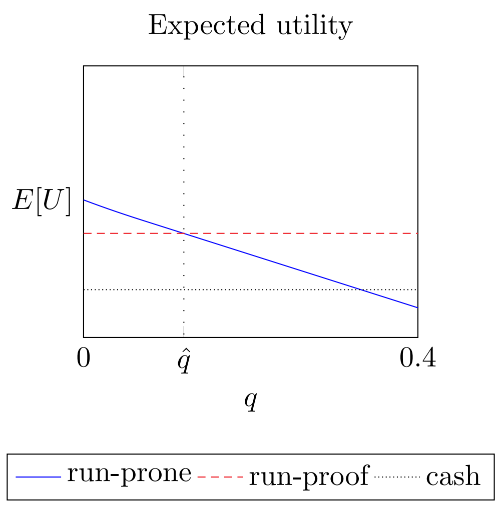
$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad \text{with } \gamma > 1, \tag{8}$$

Corollary 1 states a cut-off value $\hat{q}$ that determines whether the solution to Problem (A) is a run-prone or a run-proof contract, similar to Proposition 5 in Cooper and Ross (1998).

Corollary 1: *There exists a $\hat{q} \in (0, 1)$ such that if $q > \hat{q}$ the optimal deposit contract is run-proof whereas if $q < \hat{q}$ it is run-prone. Regardless of the probability of a bank run, there is no demand for cash in the baseline model.*

The intuition of the proof contained in the Appendix is as follows. When $q = 0$, clearly the optimal contract is the run-prone contract that maximizes expected return by lending long-term. The bank optimally responds to a higher q by substituting long-term loans for additional reserves to increase its liquidation value and thus the payment in case of a bank run. This substitution lowers the expected utility obtained from the run-prone contract. As an alternative, the bank can offer the best run-proof contract, in which case the expected utility is independent from q. When $q > \hat{q}$, the expected utility from the best run-proof contract exceeds that from the best run-prone contract, while when $q < \hat{q}$ a run-prone contract results in higher expected utility. Figure (3) illustrates this intuition by means of a simulation.

Figure 3: Optimal deposit contract vs cash



The simulation uses $R = 1.5$, $\lambda = 0.3$, $f = 0.2$, and $\gamma = 1.5$.

To summarize, the baseline model fails to explain any of the empirical facts on cash holdings as a store of value presented in section 2. Notably, in the baseline model there is no demand for cash regardless of the state of the economy. The next section extends the baseline model to account for the empirical findings on demand for cash as a safe store of value.

4 The Model

This section extends the baseline model to allow for individual heterogeneous beliefs about the probability of a bank run.

4.1 Heterogeneous beliefs

The baseline model assumes that if multiple equilibria exist, a bank run occurs with an exogenous probability q that is known ex ante by all consumers at $t = 0$ and before they decide on how to allocate their endowment. Consider instead that consumers do not have such information but have heterogeneous beliefs (at $t = 0$) about the probability of a bank run.

Formally, a consumer i has belief q_i at $t = 0$ about the probability of a bank run at $t = 1$, if it exists. At $t = 0$ each consumer draws her belief q_i from a cumulative distribution $F(q, \sigma)$ with support $[0, 1]$ and density $f(q, \sigma)$. We assume that a greater σ correlates with greater aggregate belief dispersion in the sense of a mean preserving spread (see Rothschild and Stiglitz (1978); Stiglitz and Weiss (1981)), i.e. for $\sigma_1 > \sigma_2$ it holds that

$$\int_0^1 q_i f(q, \sigma_1) dq = \int_0^1 q_i f(q, \sigma_2) dq,$$

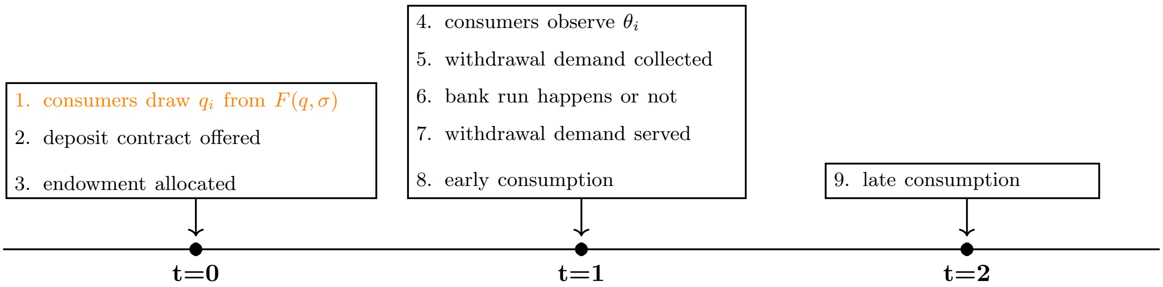
while for any $t > 0$ it holds that

$$\int_0^t F(q, \sigma_1) dq \geq \int_0^t F(q, \sigma_2) dq.$$

Except for their beliefs, consumers remain ex ante identical.

Figure 4 presents the timeline. Importantly, in this set-up consumers make their portfolio choice at $t = 0$ based on their belief q_i about the probability of a bank run at $t = 1$. Note that the baseline model can be interpreted as the case for which $\sigma = 0$ as all consumers agree on the probability q of a bank run.

Figure 4: Timeline of The Model



4.2 Optimal demand for cash

We turn our attention to the banks' problem and the household's portfolio choice, asking under what conditions the optimal demand for cash as a store of value is positive. If the chosen deposit contract is run-proof, individual beliefs q_i are irrelevant and the results presented in section 3 apply.

If the chosen deposit contract is run-prone, a bank run may occur. Consistent with the literature and the baseline model, we assume that the bank offers a single deposit contract that maximizes the expected utility of its depositors.[10] In other words, the bank offers a deposit contract based on the average individual belief of its depositors.

Consumers who are sufficiently pessimistic about bank stability (sufficiently high q_i) believe to be better off with cash than with the run-prone deposit contract. Proposition 2 states that if the deposit contract that solves the bank's problem is run-prone and the depositor is sufficiently pessimistic, or $q_i > \tilde{q}$, consumer i prefers to hold cash rather than bank deposits. The threshold value $\tilde{q}$ that defines the set of consumers who prefer to hold public money is given by

$$\tilde{q} = \frac{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(1-f)}{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})},$$

which is the subjective probability of a bank run for which a consumer is indifferent between placing her endowment in bank deposits and placing it in cash.[11]

[10] In theory, even when beliefs are private information, the bank could offer a continuum of deposit contracts, permitting each depositor to self-select her preferred option based on her subjective beliefs. Allowing for such a technology would translate into a lower but still positive demand for cash as long as the deposit contracts offered by the bank are run-prone. Note that a representative bank cannot simultaneously offer run-prone and run-proof contracts.

[11] Without loss of generality and for the sake of tractability, we have assumed that at $t = 0$ each individual places her endowment either in cash or in deposits. See Appendix B for the general version of the consumer's problem which allows each consumer to simultaneously allocate a positive proportion of her endowment in both

Proposition 2: *Given a certain run-prone deposit contract: (i) consumers with $q_i > \tilde{q}$ place their endowment in cash, (ii) a proportion $(1-\tilde{q})$ of consumers holds cash, and (iii) $M_0 = \int_{\tilde{q}}^{1} f(q,\sigma)dq$.*

Provided that the deposit contract offered by the bank is run-prone, consumers who are sufficiently pessimistic about bank stability hold cash. Aggregate demand for cash is given by the sum of individual cash holdings for all consumers with $q_i > \tilde{q}$.

Despite the fact that the bank cannot observe individual beliefs about the probability of a bank run, the bound $\tilde{q}$ and, thus, the fraction of consumers who optimally place their endowment in deposits is known to the bank and depends on the chosen deposit contract. Consequently, the bank solves

$$\max_{c_1^B, c_2^B, c_R^B, x, y} \left[1 - \int_0^{\tilde{q}} q_i f(q,\sigma_1)dq\right]\left[\lambda u(c_E) + (1-\lambda)u(c_L)\right] + \left[\int_0^{\tilde{q}} q_i f(q,\sigma_1)dq\right]u(c_R), \tag{B}$$

subject to the same constraints as Problem (A). Importantly, the beliefs q_i are assumed to be unaffected by the chosen deposit contract.

Denote the belief of the average depositor as $\int_0^{\tilde{q}} q_i f(q,\sigma_1)dq = \bar{q}$. Problem (B) results in the following optimality condition

$$(1-\bar{q})\left[Ru'(c_B^2) - u'(c_1^B)\right] = \bar{q}u'(c_R^B). \tag{9}$$

Without loss of generality, Figure 5 uses a particular Beta distribution to show that, on average, depositors are relatively optimistic about bank stability or $\bar{q} < E[q_i]$.[12] Consumers with $q_i > \tilde{q}$ are cash holders as they believe they are better off holding cash. Aggregate cash holdings are given by the shaded area. Note that $\bar{q}$ is the average belief of those consumers that deposit with the bank, so it must be that $\bar{q} < \tilde{q}$.

Similar to Corollary 1, Corollary 2 defines a cut-off value $\hat{\hat{q}}$ that determines whether the solution to Problem (B) is a run-prone or a run-proof contract under the assumption that expression (8) applies.[13] The bank offers a run-prone contract when the average beliefs of its depositors $\bar{q}$ is sufficiently low.

deposits and cash (i.e. a mixed portfolio). A simulation shows how consumers with an interior belief choose a mixed portfolio.

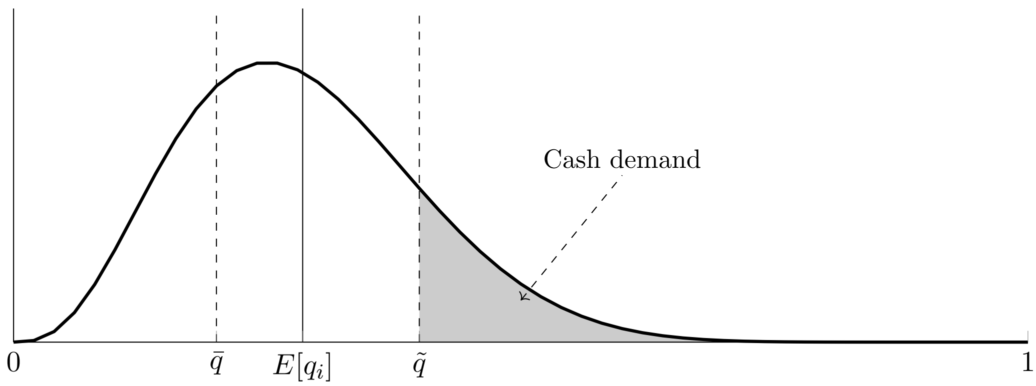
[12]This result applies to all distributions implying a run-prone contract.

[13]The remainder of the analysis continues assuming that this specification of the utility function applies.

Corollary 2: *There exists a $\hat{\hat{q}} \in (0,1)$ such that if $\bar{q} > \hat{\hat{q}}$ the solution to Problem (B) is a run-proof contract and if $\bar{q} < \hat{\hat{q}}$ it is a run-prone contract.*

Corollary 2 follows from the proof of Corollary 1, which applies for all D_0. The difference between $\hat{\hat{q}}$ and $\hat{q}$ is due to the difference between $\bar{q}$ and q in the bank's objective function, which relates to the existence of a demand for cash and, ultimately, to the presence of heterogeneous beliefs about bank stability.

Figure 5: Aggregate demand for cash



Notes: The illustration uses $q_i \sim Beta(4, 10)$.

4.3 Uncertainty and demand for cash

The dispersion in individual beliefs, σ, can be interpreted as a measure of aggregate uncertainty. This section investigates the main implications of an exogenous shift in σ (mean-preserving spread) for the demand for cash and the deposit contract offered by the bank. We assume here that the contract offered by the bank is run-prone (thus $\bar{q} < \hat{\hat{q}}$, see Corollary 2).

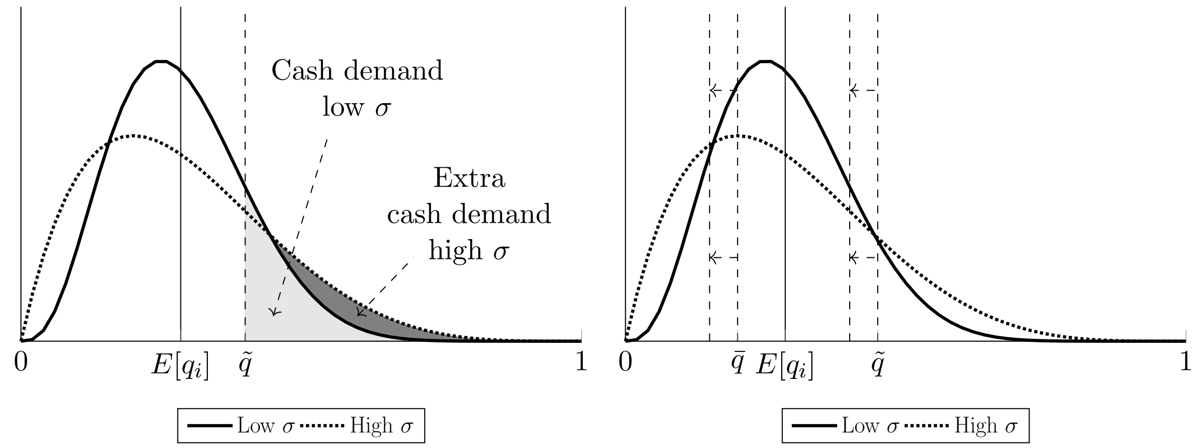
We obtain two results. First, for any given run-prone deposit contract, an increase in beliefs' dispersion (mean-preserving spread) leads to an increase in aggregate demand for cash as the mass of consumers in the tails of the distribution increases (see Figure 6a).[14]

The second effect is particularly interesting. Since the average depositor is now comparatively more optimistic about bank stability ($\bar{q}$ declines, see Figure 6b), the bank adjusts its own liquidity risk profile. As depositors on average perceive a bank run to be less likely, the bank increases

[14]Recall from section 4.1 that a greater value of σ implies a greater dispersion in beliefs but does not affect the mean of the distribution.

long-term lending as a share of its portfolio.[15] As a result, the liquidation value of the bank decreases and so does the bound $\tilde{q}$ since the return on bank deposits increases. Proposition 3 summarizes the main implications of an increase in beliefs' dispersion, σ, for the optimal run-prone contract.

Figure 6: Impact of uncertainty



(a) cash demand (b) q-values

Notes: The illustration uses $q_i \sim Beta(4, 10)$ as the low σ distribution (solid line) and $q_i \sim Beta(2, 5)$ as the high σ distribution (dotted line).

Proposition 3: *Assume $\bar{q} < \hat{\hat{q}}$ so that banks offer a run-prone deposit contract. As σ increases: (i) cash demand increases, (ii) bank deposits and the average belief of depositors $\bar{q}$ decrease, (iii) the bank reduces reserve holdings and long-term lending, (iv) the bank reduces the share of reserves in its portfolio $\frac{y}{D_0}$, (v) the bound $\tilde{q}$ decreases.*

The interpretation of Proposition 3 is as follows. In times of high uncertainty, more consumers prefer cash rather than bank deposits, similar to a flight-to-safety. As a result, the remaining depositors are - on average - more confident about bank stability. The bank optimally responds to this shift in the belief of its average depositors by offering a relatively higher payment in the good equilibrium and a relatively lower payment in case of a bank run. It does so by re-balancing its asset portfolio towards more long-term lending, which increases maturity transformation.[16]

In a nutshell, The Model accounts for key empirical findings on cash holdings as a safe

[15] Optimality condition (9) indicates that the share of bank reserves, y, is strictly decreasing in $\bar{q}$.
[16] Note that we ignore any feedback effects of bank's portfolio choices on the probability of a bank run, which is beyond the scope of this paper.

liquid asset: (i) at the aggregate level there is a demand for cash as a store of value, (ii) only a certain proportion of consumers hold cash (i.e., those who are sufficiently pessimistic about bank stability and the future state of the economy), and (iii) aggregate demand for cash and the proportion of consumers who hold public money for safety reasons increase with uncertainty.

This modification of the Diamond and Dybvig model offers a suitable set-up to study the implications of introducing a CBDC for banks and the demand for public money as a store of value. The next section investigates these issues in the context of the proposed Model.

5 The Model with CBDC

This section extends the Model to allow for the central bank to issue central bank digital currency (CBDC) along with cash and reserves.[17] To simplify the analysis, the introduction of CBDC is assumed to affect neither the probability of a bank run nor individual beliefs about such probability.

5.1 CBDC vs cash

As for the case of cash, the central bank exchanges endowment for CBDC at $t = 0$ and $t = 1$ and repays consumption goods on demand at $t = 1$. When compared to cash, CBDC is characterized by three key distinctive features. From a technological perspective, CBDC is assumed to be a superior store of value compared to cash, captured by lower storage costs, $f^{DC} < f$.[18] The interest rate on CBDC holdings, r^{DC}, can be equal to zero or negative.[19] In addition, the authority could impose a quantity limit on CBDC supply.[20]

For any given run-prone contract offered by the bank, a consumer prefers to hold cash or CBDC depending on the exchange of value of each of the two forms of public money. Proposition 4 summarizes this choice.

[17] In The Model with CBDC, all monetary instruments issued by the central bank, including CBDC, solely serve as a store of value. The study of the implications of CBDC as a means of payment is beyond the scope of this paper.

[18] Note that, in practice, the extent to which CBDC is a technologically superior store of value in advanced economies will crucially depend on design features for which no decisions have been made yet. In practice, and given certain distinctive features of cash (e.g., anonimity, resilience), it should not be ruled out a situation in which - under certain conditions - CBDC is a comparatively less attractive storage technology.

[19] In The Model with CBDC the interest rate on CBDC holdings cannot be strictly positive since the central bank would not have revenues to cover the related expense.

[20] Bindseil (2020); Bindseil and Panetta (2020) include a policy proposal aimed at avoiding excessive CBDC holdings and discouraging the use of CBDC as a form of investment via CBDC quantity limits and dissuasive remuneration.

Proposition 4: *For any given run-prone contract offered by the bank, a consumer strictly prefers to hold CBDC rather than cash if* $(1 - f^{DC} + r^{DC}) > (1 - f)$.

Under a run-prone deposit contract, Proposition 4 has several implications. First, by adequately calibrating r^{DC}, the central bank can determine whether consumers prefer to hold cash or CBDC as a store of value.[21] Second, by introducing a limit on CBDC supply $\bar{M}^{DC} < M_0$, where $\bar{M}^{DC}$ denotes the CBDC quantity limit, the central bank can calibrate the amount of CBDC held as a store of value. Consequently, if the only difference between CBDC and cash is given by $f > f^{DC}$ (i.e., no binding limits on CBDC supply and $r^{DC} = 0$) CBDC fully replaces cash as a safe store of value.

5.2 CBDC vs deposits

The introduction of a CBDC may also affect the run-prone contract offered by the bank and, ultimately, the store of value choice made by consumers. In other words, the issuance of a CBDC in The Model may affect both, the demand for cash as well as bank intermediation.

Consider, again, the reference CBDC case in which $f > f^{DC}$, $r^{DC} = 0$, and there are no binding limits on CBDC supply. Then, the threshold for q that defines the set of consumers who prefer to hold public money is no longer given by $\tilde{q}$ since now it depends on f^{DC} rather than on f (recall expression 13). We find that $\tilde{\tilde{q}} < \tilde{q}$, where $\tilde{\tilde{q}}$ is the threshold with CBDC. Proposition 5 summarizes the main implications of this result.

Proposition 5: *Assume $\bar{q} < \hat{\hat{q}}$, so that banks offer a run-prone deposit contract, and $f > f^{DC}$. Then, the introduction of a CBDC leads to a decline in the threshold that defines the set of consumers who prefer to hold public money ($\tilde{\tilde{q}} < \tilde{q}$). The effect is: (i) an increase in the demand for public money, M_0, (ii) a decline in bank deposits, D_0, and in the average belief of depositors, $\bar{q}$, (iii) a decrease in reserves and long-term lending, and (iv) a reduction in the share of reserves in the bank's portfolio, $\frac{y}{D_0}$.*

Intuitively, the introduction of a superior public storage technology leads to a reduction in the threshold that defines the set of consumers who prefer to hold public money. That is, there is a positive fraction of consumers who switch from bank deposits to CBDC on the basis of their pre-existent beliefs. This results in a decline in bank deposit funding. The corresponding decrease

[21] In practice, all central banks in advanced economies have emphasized that CBDC would complement cash and the use and acceptance of cash would continue to be supported (see, e.g., BIS (2020)

in long-term lending is less than proportional (i.e., increased relative maturity transformation); remaining depositors are - on average - more optimistic about bank stability and, hence, the representative bank optimally increases the share of long-term lending.

6 Welfare

This section studies the welfare implications of introducing CBDC in The Model. Our analysis assumes that the optimal deposit contract is run-prone; the only source of heterogeneity across consumers is their individual beliefs, q_i; and the only difference between CBDC and cash is given by $f > f^{DC}$ (i.e., no binding limits on CBDC supply and $r^{DC} = 0$).

We define social welfare as the total sum of all consumers' individual welfare. The utilitarian social planner has to choose between introducing CBDC as a store of value and not doing so. She makes the choice that maximizes a measure of social welfare. We differentiate between two general cases. In "Case 1" individual welfare of consumer i depends on the true probability of a bank run, q^{true}. In "Case 2" individual welfare of consumer i depends on her individual beliefs, q_i. In each case, we further distinguish between a sub-case "A" in which the social planner has all relevant information to maximize social welfare and a sub-case "B" in which she does not have all relevant information and the measure of social welfare that she maximizes is her own estimate.

Table 1 defines the measure of social welfare maximized by the utilitarian social planner in each of these cases. SW_x refers to the measure of social welfare, where $x \in \{1A, 1B, 2A, 2B\}$. Individual welfare of consumer i is denoted by $W^i(.)$, which may depend on the true probability, $W^i(q^{true})$, or on individual beliefs, $W^i(q_i)$. Similarly, $\hat{W}^i(.)$ refers to the social planner's estimate for individual welfare of consumer i, which may depend on the estimate of the planner for the true probability, $\hat{W}^i(\hat{q}^{true})$, or on the estimate for the corresponding individual belief, $\hat{W}^i(\hat{q}_i)$.

Table 1: Social planner's objective function

	A. Social planner with complete information	B. Social planner with incomplete information
Case 1: Individual welfare depends on q^{true}	$SW_{1A} = \int_0^1 W^i(q^{true})di$	$SW_{1B} = \int_0^1 \hat{W}^i(\hat{q}^{true})di$
Case 2: Individual welfare depends on q_i	$SW_{2A} = \int_0^1 W^i(q_i)di$	$SW_{2B} = \int_0^1 \hat{W}^i(\hat{q}_i)di$

In contrast, the problem of consumer i does not changes across cases since, in this environment, individuals always behave according to their beliefs. Formally, consumer i solves

$$\max_{d_i} \mathrm{E}[U^i] = (1 - q_i)\left[\lambda u(c_E^i) + (1 - \lambda)u(c_L^i)\right] + q_i u(c_R^i), \tag{10}$$

with

$$c_E^i = d_i \epsilon \frac{c_1^B}{D_0} + (1 - d_i)\epsilon(1 - f), \qquad c_L^i = d_i \epsilon \frac{c_2^B}{D_0} + (1 - d_i)\epsilon(1 - f),$$
$$c_R^i = d_i \epsilon \frac{c_R^B}{D_0} + (1 - d_i)\epsilon(1 - f),$$

where ϵ denotes individual consumer's endowment (which is identical across all consumers) and $d_i \in \{0, 1\}$ determines whether i places her endowment in public money or deposits. Importantly, actual welfare of consumer i may differ from her objective function or not depending on whether individual welfare depends on q^{true} or on q_i, respectively.

Next, we study the individual and social welfare implications of introducing CBDC under each of these cases. In order to do so, we numerically solve the problem of individual consumers and of the social planner for all possible values of the relevant probability.

6.1 Welfare depends on actual bank stability

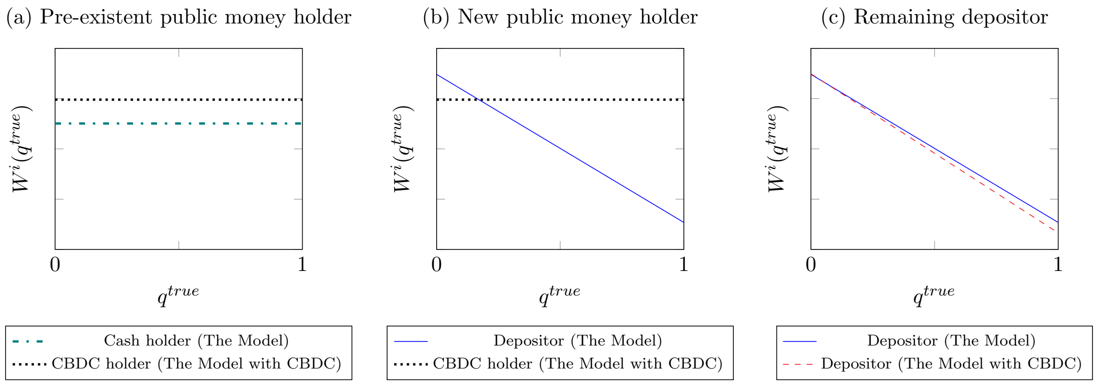
We first consider the case in which individual welfare depends on the true probability of a bank run. Depending on their response to the introduction of CBDC, we differentiate between three types of agents: (i) consumers who remain as public money holders and fully replace cash with CBDC (i.e., pre-existent public money holders); (ii) consumers who switch from deposits to CBDC (i.e., new public money holders); and (iii) consumers who remain as depositors (i.e., remaining depositors).[22]

Figure 7 plots individual welfare for each of the three types of consumers in The Model with and without CBDC, for the entire range of possible q^{true} values. Pre-existent public money holders benefit from the introduction of CBDC by fully switching from cash to the central bank digital currency. The increase in their welfare is proportional to the difference between cash and CBDC storage costs (Figure 7a). Such increase in the individual welfare of holding public money implies that the threshold that defines the set of consumers who prefer to hold public money

[22] Theoretically, there could be a fourth type of agent; public money holders who fully replace cash with deposits. However, we know that - by revealed preference - no cash holder would respond by switching to deposits since they already had that option before the introduction of CBDC.

decreases with CBDC (Proposition 5). Thus, some of the consumers who were holding deposits switch to CBDC based on their individual beliefs. These new public money holders are indeed better-off when the true probability of a bank run is sufficiently high (Figure 7b). As explained in Proposition 5, the change in the depositor base translates into lower maturity transformation and productive investment. The feasibility set of the bank contracts and, consequently, remaining depositors are worse-off with CBDC for virtually the entire range of possible

Figure 7: Simulated individual welfare by consumer type in Case 1

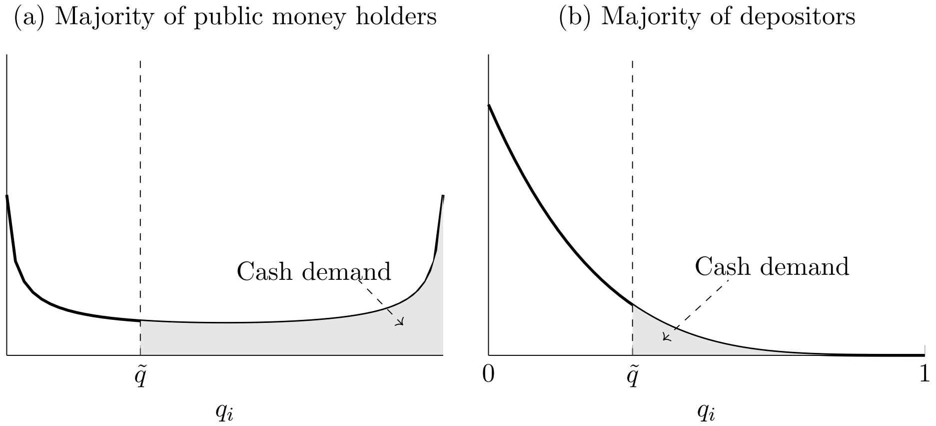


Notes: The figure displays simulated individual welfare under Case 1 in The Model with and without CBDC for each type of consumer and the entire range of q^{true} values. Simulations use $R = 1.5$, $\lambda = 0.3$, $\gamma = 1.5$, $f = 0.2$, $f^{DC} = 0$, and $q_i \sim Beta(1, 5)$. It follows that $D_0 = 0.89$ and $D_0^{DC} = 0.62$.

Case 1A: In this case, the social planner knows the value of q^{true}.[23] Importantly, the impact of CBDC on social welfare depends on the value of q^{true} and on the distribution of individual beliefs. If the distribution of individual beliefs is such that the bulk of consumers are relatively pessimistic and, thus, the majority of society prefers to hold public money (Figure 8a), the introduction of CBDC as a store of value will lead to an increase in social welfare regardless of the value of q^{true} (Figure 9a). In this case, the social planner decides to introduce CBDC as a store of value regardless of q^{true} and The Model with CBDC is said to be associated with a belief-neutral superior allocation, according to the welfare criterion proposed in Brunnermeier et al. (2014).

[23]This case could be interpreted as one in which a paternalist social planner knows what is best for society and does it regardless of consumers' individual behaviour.

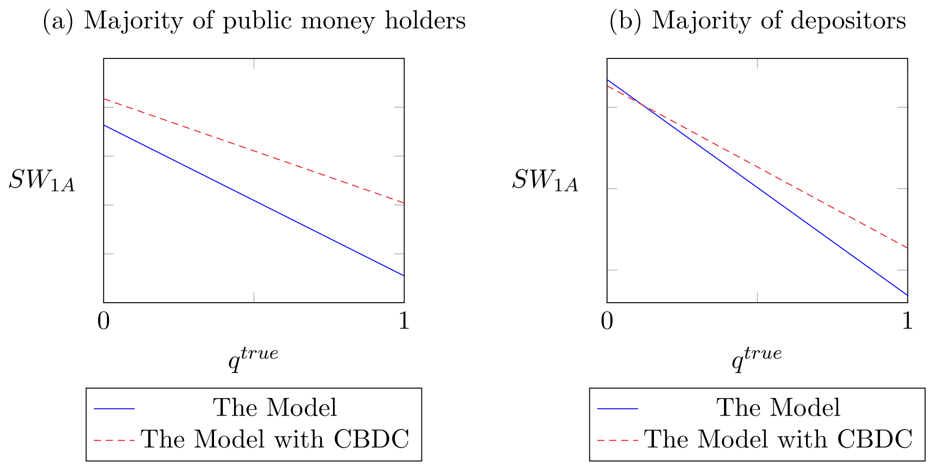
Figure 8: Illustrative distributions of individual beliefs



Notes: Panel (a) plots a distribution such that $q_i \sim Beta(1,5)$ and $\tilde{q} = 0.33$. Panel (b) displays a distribution such that $q_i \sim Beta(0.5, 0.5)$ and $\tilde{q} = 0.31$.

By way of contrast, if the distribution of individual beliefs is such that the majority of consumers are relatively optimistic and, thus, prefer to hold bank deposits (Figure 8b), the choice of the social planner will depend on the value of q^{true} (Figure 9b). In this case, the welfare criterion by Brunnermeier et al. (2014) suggests that the two allocations are "incomparable".

Figure 9: Simulated social welfare for alternative belief distributions in Case 1A

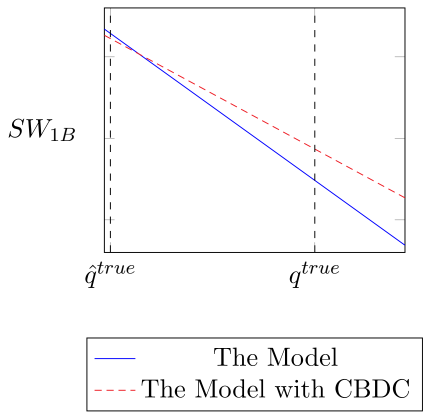


Notes: The figure displays simulated social welfare under Case 1 in The Model with and without CBDC for the entire range of q^{true} values and for two different distributions of individual beliefs. The parameterization of The Model is such that $R = 1.5$, $\lambda = 0.3$, $\gamma = 1.5$, $f = 0.2$, $f^{DC} = 0$. Simulations represented in panel (a) use $q_i \sim Beta(1,5)$, in which case it follows that $D_0 = 0.89$ and $D_0^{DC} = 0.62$. Simulations represented in panel (b) assume $q_i \sim Beta(0.5, 0.5)$, in which case it follows that $D_0 = 0.40$ and $D_0^{DC} = 0.25$.

Case 1B: In this case, the social planner does not know the value of q^{true}. Thus, she maximizes a measure of social welfare based on her estimate of q^{true}. This implies that the solution to the planner's problem may not coincide with that of maximizing actual social welfare. Depending on the distribution of individual beliefs and on the estimation error of the social planner, this could imply that the choice of the social planner differs from the one that would be derived from the maximization of actual social welfare.

If the distribution of individual beliefs is such that the majority of consumers prefer to hold public money (Figure 8a), the choice of the social planner will be identical to the one based on the maximization of actual social welfare regardless of the value of q^{true} and of the planner's estimation error. However, if the majority of consumers are depositors (Figure 8b) and the estimation error is sufficiently large the opposite may hold. Figure 10 illustrates a case in which, due to a significant underestimation of q^{true}, the planner chooses not to introduce CBDC as a store of value whereas the choice based on the maximization of actual social welfare would be to introduce it.

Figure 10: Simulated social welfare and the planner's choice in Case 1B



Notes: The figure displays simulated social welfare under Case 1 in The Model with and without CBDC for the entire range of q^{true} values and a particular distribution of beliefs. The parameterization is such that $R = 1.5$, $\lambda = 0.3$, $\gamma = 1.5$, $f = 0.2$, $f^{DC} = 0$ and $q_i \sim Beta(0.5, 0.5)$, in which case it follows that $D_0 = 0.40$ and $D_0^{DC} = 0.25$.

6.2 Welfare depends on individual perceptions about bank stability

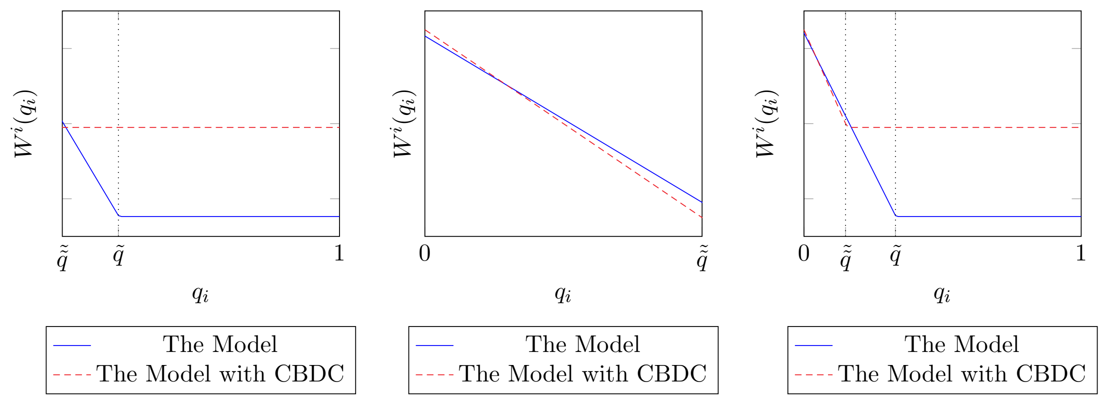
Then, we consider the case in which individual welfare of consumer i depends on her belief q_i. This implies that individual behaviour is unambiguously consistent with the maximization of individual welfare. That is, all consumers who modify their portfolio choice in response to the introduction of CBDC (i.e., those for which $q_i > \tilde{\tilde{q}}$) are better-off. This group of consumers comprises all pre-existent public money holders (who fully switch from cash to CBDC) and new public money holders (who switch from deposits to CBDC). Based on numerical simulations, Figure 11a makes this clear by plotting the individual welfare of consumer i in The Model with and without CBDC, for the range of q_i values that satisfy $q_i > \tilde{\tilde{q}}$.

Figure 11b displays simulated individual welfare in The Model with and without CBDC, for all the possible q_i values for the case of remaining depositors (i.e., those consumers for which $q_i < \tilde{\tilde{q}}$). The adoption of a CBDC leads to a decline in the average belief of depositors, on which the deposit contract offered by the bank depends. Therefore, the individual welfare increases with the introduction of CBDC for all remaining depositors who were more optimistic about bank stability compared to the average depositor. All other remaining depositors are worse-off. These consumers optimally choose to remain as a depositor despite the welfare loss because, given their individual belief, the welfare loss of switching to CBDC would be larger.

Figure 11c offers an overview of how individual welfare is affected by the introduction of CBDC for all possible individual beliefs by representing simulated individual welfare in The Model with and without CBDC, for the entire range of possible q_i values.

Figure 11: Simulated individual welfare by consumer type in Case 2

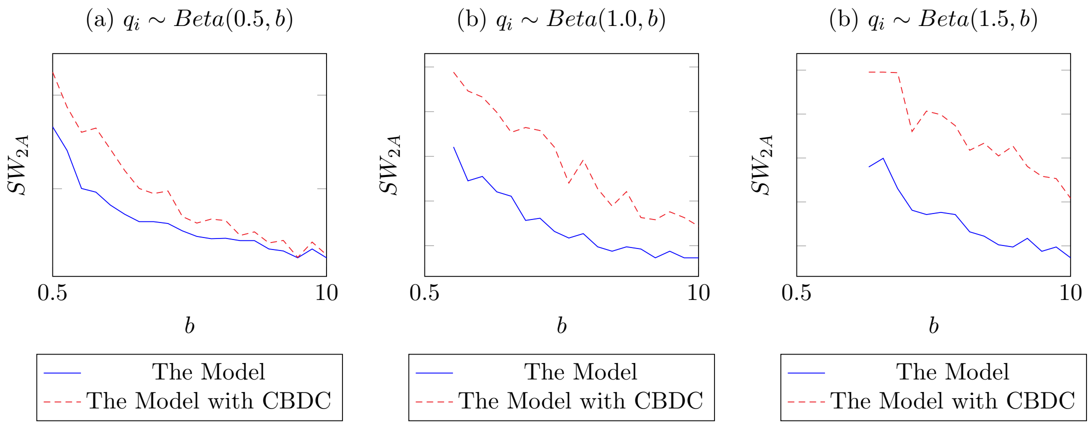
(a) Pre-existent and new public money holders | (b) Remaining depositors | (c) All consumers



Notes: The figure displays simulated individual welfare under Case 2 in The Model with and without CBDC for each type of consumer and the entire range of q_i values. simulations use $R = 1.5$, $\lambda = 0.3$, $\gamma = 1.5$, $f = 0.2$, $f^{DC} = 0$, and $q_i \sim Beta(2, 5)$. It follows that $D_0 = 0.71$ and $D_0^{DC} = 0.23$.

Case 2A: In this case, the social planner knows the distribution of individual beliefs. Given the definition of social welfare, the introduction of CBDC leads to an increase in social welfare as the majority of consumers are better-off with the availability of this new public storage technology (see Figure 11c). Figure 12 shows that this result is robust across different distributions of individual beliefs by displaying simulated social welfare in The Model with and without CBDC for a wide range of Beta distributions. In fact, we were unable to find a distribution for which the assumption of the deposit contract being run-prone holds and social welfare is not higher in The Model with CBDC. Thus, according to the Brunnermeier et al. (2014) welfare criterion, The Model with CBDC is associated with a belief-neutral superior allocation also in this case.

Figure 12: Simulated social welfare for different distributions in Case 2



Notes: The figure displays simulated social welfare under Case 2 in The Model with and without CBDC for a comprehensive range of individual belief distributions that accounts for the spectrum of cases for which the assumption of the run-prone deposit contract holds. Simulations use $R = 1.5$, $\lambda = 0.3$, $\gamma = 1.5$, $f = 0.2$, $f^{DC} = 0$.

Case 2B: In this case, the social planner does not know the distribution of individual beliefs. Consequently, she maximizes a measure of social welfare based on her estimate of this distribution. Based on Figure 12, we conclude that even if the estimation error of the planner is very large, the choice of the social planner will coincide with the one based on the maximization of actual social welfare.

7 Conclusion

This paper develops a banking model a la Diamond and Dybvig (1983) with public money as a store of value and heterogeneous beliefs about bank stability. The assumption of heterogeneous beliefs allows to rationalize how different consumers choose between bank deposits and cash holdings, which are safe but subject to storage costs. Our model accounts for the key empirical observations on public money as a store of value and is consistent with empirical findings highlighted in the behavioral finance literature.

Under certain conditions, the introduction of a CBDC that is a more attractive store of value compared to cash leads to bank disintermediation as it increases the demand for public money at the expense of bank deposits. As the demand for public money increases, the average depositor is more optimistic about bank stability. Consequently, the bank optimally re-balances its portfolio towards a larger share of long-term lending. Thus, while in absolute terms the issuance of CBDC as a store of value leads to a decline in bank funding and lending, in relative terms it translates into increased maturity transformation. The appropriate calibration of CBDC design features such as remuneration and quantity limits can mitigate any bank disintermediation and the extent to which CBDC is used as a store of value.

An utilitarian social planner would decide to introduce a CBDC as a store of value depending on aspects such as her information set, the distribution of individual beliefs, and whether individual welfare depends on the true probability of a bank run or on individual beliefs about such probability.

Our analysis hints at additional considerations that are important for understanding the implications of introducing a CBDC as a store of value but which are beyond the scope of this paper. We show how perceived bank stability affects demand for public money as a store of value. Throughout the analysis, we assume no endogenous feedback effects of bank's portfolio choices on (perceived) bank stability. Future work may endogenize the probability of a bank run, possibly by adopting a global-games approach as in Goldstein and Pauzner (2005) or by following the approach of Rochet and Vives (2004). Our analysis could also be extended to include regulatory or policy options that may affect the demand for public money as a store of value and its implications for banks, such as central bank lending or deposit insurance.

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8 Appendix A: proofs

8.1 Proposition 1

For any value for q, the deposit contract with the lowest possible expected return offered by the bank is a run-proof contract that includes no long-term lending ($x = 0$). In that case, Problem (A) reduces to

$$\max_{y} \lambda u(\frac{y}{\lambda}) + (1-\lambda)u(\frac{y^l}{1-\lambda}),$$

subject to

$$y + y^l = D_0, \tag{11}$$
$$0 \leq c_1^B \leq c_2^B. \tag{12}$$

The first order condition that characterizes the solution is given by

$$u'(\frac{y^l}{1-\lambda}) - u'(\frac{D_0 - y^l}{\lambda}) = 0. \tag{13}$$

This defines $y^l = (1-\lambda)D_0$ and $y = \lambda D_0$.

If a single atomistic consumer with endowment ϵ invests in deposits, her expected utility is

$$E[U^{deposit}] = u\Big[\epsilon\Big].$$

If she instead holds cash, her expected utility is

$$E[U^{cash}] = u\Big[\epsilon(1-f)\Big].$$

Since $f > 0$, she will not hold any cash. Any chosen deposit contract yields at least as high expected returns and, thus, there is never any demand for cash.

8.2 Corollary 1

The proof consists of three parts: i) the run-proof solution to Problem (A); ii) the run-prone solution to Problem (A); iii) the conditions under which each contract is offered. We will use that there is no cash demand (Proposition 1), but we include deposit funding D_0 in the bank's problem as it is useful for the proof of Corollary 2.

First, an optimal run-proof contract solves Problem (A) where the indicator function is zero and subject to an additional constraint that allows only for run-proof contracts:

$$y^l = c_1^B - y. \tag{14}$$

Let η_E and η_L be the Lagrange multipliers on constraints (2) and (3) of Problem (A), respectively, and η_R the multiplier on the additional constraint (14). Let γ and β be the multipliers on the non-negativity constraints for x and y^l, respectively. When first ignoring the incentive compatibility constraint, the first order conditions that characterize the solution are given by

$$c_1^B : \lambda u'(c_1^B) - \eta_E \lambda - \eta_R = 0, \tag{15}$$
$$c_2^B : (1-\lambda) u'(c_2^B) - \eta_L (1-\lambda) = 0, \tag{16}$$
$$x : -\eta_E + \eta_L R - \eta_R + \gamma = 0, \tag{17}$$
$$y^l : -\eta_E + \eta_L + \beta = 0. \tag{18}$$

Rewriting (15) gives

$$\eta_E = u'(c_1^B) - \frac{1}{\lambda}\eta_R,$$

and rewriting (16) gives

$$\eta_L = u'(c_2^B).$$

Since $y^l > 0$ must hold for any run-proof contract, $\beta = 0$ and thus expression (18) implies that $\eta_L = \eta_E$. This allows to solve for η_R as

$$\eta_R = \lambda\Big[u'(c_1^B) - u'(c_2^B)\Big].$$

Substituting for η_E, η_L, and η_R into expression (17) gives the following optimality condition

$$u'(c_1^B) = u'(c_2^B)\frac{R-1+\lambda}{\lambda}. \tag{19}$$

Since $R > 1$ and u is concave, the optimal run-proof contract is indeed incentive compatible.

The optimality condition (19) is restated as

$$u'\Big[\frac{y}{\lambda}\Big] = \frac{R-1+\lambda}{\lambda} u'\Big[\frac{R(D_0 - y)}{1-\lambda} - (R-1)\frac{y}{\lambda}\Big], \tag{20}$$

and when $D_0 = 1$ this results in a solution $y = y^{proof}$. We denote the resulting expected utility of a single consumer with endowment ϵ who deposits with the bank as

$$E[U^{proof}] = \lambda u\Big[\frac{\epsilon}{D_0}\frac{y^{proof}}{\lambda}\Big] + (1-\lambda) u\Big[\frac{\epsilon}{D_0}\Big[\frac{R(D_0 - y^{proof})}{1-\lambda} - (R-1)\frac{y^{proof}}{\lambda}\Big]\Big].$$

Second, an optimal run-prone contract solves Problem (A) where the indicator function is equal to one. Let η_E and η_L be the Lagrange multipliers on constraints (2) and (3) of Problem (A), and let γ and β be the multipliers on the non-negativity constraints for x and y^l, respectively. When first ignoring the incentive compatibility constraint, the first order conditions that characterize the solution are given by

$$c_1^B : (1-q)\lambda u'(c_1^B) + q\lambda u'(c_R^B) - \eta_E \lambda = 0, \tag{21}$$
$$c_2^B : (1-q)(1-\lambda) u'(c_2^B) + q(1-\lambda) u'(c_R^B) - \eta_L (1-\lambda) = 0, \tag{22}$$
$$x : -qRu'(c_R^B) - \eta_E + \eta_L R + \gamma = 0, \tag{23}$$
$$y^l : -\eta_E + \eta_L + \beta = 0. \tag{24}$$

We will first show that any optimal run-prone contract has no excess liquidity. To do so, suppose the opposite, so that $y^l > 0$. Then $\beta = 0$ must hold. From (23) and (24), it follows that

$\eta_E = \eta_L = \frac{qR}{R-1}u'(c_R^B)$, while from (22) we find that $\eta_L = (1-q)u'(c_2^B) + qu'(c_R^B)$. Combining these two expressions for η_L gives:

$$(R-1)(1-q)u'(c_2^B) = qu'(c_R^B).$$

From expressions (21) and (22), we also find that $c_1^B = c_2^B$. Thus, this implies the following relationship between c_1^B, c_2^B and c_R^B

$$c_1^B = c_2^B = Ac_R^B,$$

where constant A equals

$$A = [\frac{q}{(R-1)(1-q)}]^{-\frac{1}{\gamma}} > 0.$$

We can now rewrite the problem as

$$max_{c_1^B}(1-q)u(c_1^B) + qu(\frac{1}{A}c_1^B).$$

At the optimum, the following first order condition must apply

$$(1-q)u'(c_1^B) + q\frac{1}{A}u'(\frac{1}{A}c_1^B) = 0,$$

which is never satisfied since $A > 0$. Thus, we must have that $y^l = 0$ at the solution. Now, using that $y^l = 0$, the first order conditions reduce to

$$c_1^B : (1-q)\lambda u'(c_1^B) + q\lambda u'(c_R^B) - \eta_E\lambda = 0, \tag{25}$$
$$c_2^B : (1-q)(1-\lambda)u'(c_2^B) - \eta_L(1-\lambda) = 0, \tag{26}$$
$$x : -qRu'(c_R^B) - \eta_E + \eta_L R = 0. \tag{27}$$

Rewriting expression (25) gives

$$\eta_E = (1-q)u'(c_1^B) + qu'(c_R^B),$$

and rewriting expression (26) gives

$$\eta_L = (1-q)u'(c_2^B).$$

Substituting for η_E and η_L into expression (27) gives

$$-qu'(c_R^B) - (1-q)u'(c_1^B) + (1-q)Ru'(c_2^B) = 0,$$

which can be rewritten into the optimality condition

$$(1-q)\Big[Ru'(c_2^B) - u'(c_1^B)\Big] = qu'(c_R^B). \tag{28}$$

When $q = 0$, and since $R > 1$, the contract that satisfies the optimality condition (28) is incentive compatible as $u'(c_2^B) < u'(c_1^B)$ and hence $c_2^B > c_1^B$. When $q > 0$, the optimality condition (28) is restated as

$$u'(\frac{y}{\lambda}) = Ru'(\frac{R(D_0 - y)}{1-\lambda}) - \frac{q}{1-q}u'(y).$$

Using the assumed utility function, y is solved for as

$$y = \frac{\frac{RD_0}{1-\lambda}}{R^{\frac{1}{\gamma}}\left[\frac{1}{\lambda^{-\gamma}} + \frac{q}{1-q}\right]^{-\frac{1}{\gamma}} + \frac{R}{1-\lambda}}. \tag{29}$$

It follows that $\frac{\partial y}{\partial q} > 0$, and so $\frac{\partial c_1^B}{\partial q} > 0$ and $\frac{\partial c_2^B}{\partial q} < 0$. Thus, a critical bound q' exists such that when $q > q'$ it holds that the solution to the optimality condition (28) includes $c_2^B < c_1^B$ and when $q < q'$ it holds that $c_2^B > c_1^B$.

Since the solution to the optimality condition (28) when $q > q'$ contradicts with the incentive compatibility constraint, it cannot be an equilibrium contract. Instead, if $q > q'$ and the bank wishes to offer a run-prone contract, the best incentive compatible run-prone contract it could offer is the solution to optimality condition (28) when $y^* = y|_{c_1^B = c_2^B}$: the run-prone incentive compatible contract with the highest early repayment. Utility under this run prone contract with $y = y^*$ of a single consumer who deposits her endowment ϵ with the bank is given as

$$E[U^{prone}|y = y^*] = (1-q)\Big[\lambda u(\frac{\epsilon}{D_0}\frac{y^*}{\lambda}) + (1-\lambda)u(\frac{\epsilon}{D_0}\frac{R(D_0 - y^*)}{1-\lambda}\Big] + qu(\frac{\epsilon}{D_0}y^*). \tag{30}$$

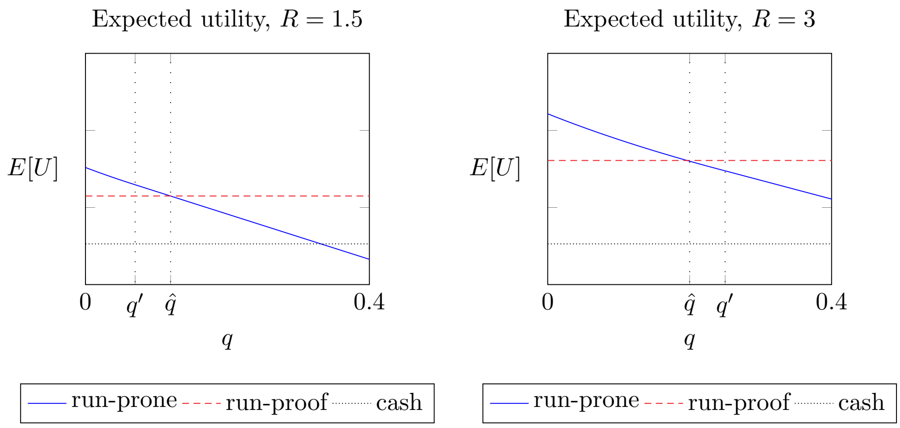
Finally, we solve for the unique equilibrium contract. From expressions (29) and (30) it follows that an optimal run-prone contract is such that $\frac{\partial U^{prone}}{\partial q} < 0$, both when $q < q'$ and when $q > q'$. Thus, we can derive a second critical bound $\hat{q}$ such that when $q = \hat{q}$ the utility obtained from the run-prone deposit contract is equal to the utility obtained from the run-proof contract. The cut-off value $\hat{q}$ is equal to

$$\hat{q} = \frac{\lambda u(\frac{1}{D_0}\frac{y}{\lambda}) + (1-\lambda)u(\frac{1}{D_0}\frac{R(D_0-y)}{1-\lambda})}{\lambda u(\frac{1}{D_0}\frac{y}{\lambda}) + (1-\lambda)u(\frac{1}{D_0}\frac{R(D_0-y)}{1-\lambda}) - u(\frac{1}{D_0}y)} - \ldots$$
$$\frac{\lambda u\left[\frac{1}{D_0}\frac{y^{proof}}{\lambda}\right] + (1-\lambda)u\left[\frac{1}{D_0}\left[\frac{R(D_0-y^{proof})}{1-\lambda} - (R-1)\frac{y^{proof}}{\lambda}\right]\right]}{\lambda u(\frac{1}{D_0}\frac{y}{\lambda}) + (1-\lambda)u(\frac{1}{D_0}\frac{R(D_0-y)}{1-\lambda}) - u(\frac{1}{D_0}y)}.$$

where when $\hat{q} < q'$, y is the solution to optimality condition (28) using $q = \hat{q}$, and when $\hat{q} > q'$, y is the solution to (28) using $q = q'$ and $c_1^B = c_2^B$.

Figure (13) illustrates the relative cut-off values by plotting the simulated expected utility of each type of contract as a function of q, using that $D_0 = 1$ and under different values for R.

Figure 13: Optimal deposit contract



Notes: The simulations use $\lambda = 0.3$, $f = 0.2$, and $\gamma = 1.5$.

8.3 Proposition 2

The run-prone contract offered by the bank satisfies expression (9). A consumer's expected utility when depositing its endowment ϵ with the bank depends on its belief q_i and equals

$$E[U^{deposits}] = (1 - q_i)\Big[\lambda u(\frac{\epsilon}{D_0} c_1^B) + (1 - \lambda) u(\frac{\epsilon}{D_0} c_2^B)\Big] + q_i u(\frac{\epsilon}{D_0} c_R^B).$$

If a consumer instead holds cash, her expected utility is

$$E[U^{cash}] = u\Big[\epsilon(1 - f)\Big].$$

From here it follows that positive cash demand requires $E[U^{deposits}] < E[U^{cash}]$, so when

$$q_i > \frac{\lambda u(\frac{c_1^B}{D_0}) + (1 - \lambda) u(\frac{c_2^B}{D_0}) - u(1 - f)}{\lambda u(\frac{c_1^B}{D_0}) + (1 - \lambda) u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})} = \tilde{q}.$$

8.4 Proposition 3

Cash demand equals

$$M_0 = \int_{\tilde{q}}^{1} f(q, \sigma) dq.$$

First, when σ increases, all else equal, clearly cash demand increases. Next, cash demand also depends on the bound $\tilde{q}$, given as

$$\tilde{q} = \frac{\lambda u(\frac{c_1^B}{D_0}) + (1 - \lambda) u(\frac{c_2^B}{D_0}) - u(1 - f)}{\lambda u(\frac{c_1^B}{D_0}) + (1 - \lambda) u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})}.$$

At lower deposit funding D_0, the average belief of bank depositors $\bar{q}$ decreases as only relatively optimistic depositors remain, or $\frac{\partial \bar{q}}{\partial \sigma} < 0$. This implies that the optimal $\frac{y}{D_0}$, as determined by expression (29) with $q = \bar{q}$, decreases (so $\frac{\partial \frac{y}{D_0}}{\partial \sigma} < 0$), and thus the bound $\tilde{q}$ is affected:

$$\frac{\partial \tilde{q}}{\partial \sigma} = \frac{\partial \frac{y}{D_0}}{\partial \sigma} u'(\frac{c_R^B}{D_0}) \frac{\Big[\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(1-f)\Big] - \frac{\bar{q}}{1-\bar{q}}\Big[u(1-f) - u(\frac{c_R^B}{D_0})\Big]}{\Big[\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})\Big]^2}.$$

The sign of this expression depends on $\bar{q}$. When $\bar{q} < \tilde{q}$ it is negative, and when $\bar{q} > \tilde{q}$ it is positive. Clearly, since only consumers with $q_i < \tilde{q}$ hold bank deposits, it must be that $\bar{q} < \tilde{q}$ and $\frac{\partial \tilde{q}}{\partial \sigma} < 0$: an increase in σ decreases the bound $\tilde{q}$, further increasing cash demand.

8.5 Proposition 5

Consider $\tilde{q}$:

$$\tilde{q} = \frac{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(1-f)}{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})}.$$

Holding bank pay-outs and deposits constant, the impact of cash storage cost equals

$$\frac{\partial \tilde{q}}{\partial f} = \frac{u'(1-f)}{\lambda u(c_1^B) + (1-\lambda)u(c_2^B) - u(c_R^B)} > 0.$$

Thus, a decrease in cash storage cost, all else equal, results in a decrease of $\tilde{q}$.

Next, a lower $\tilde{q}$ implies an increase in M_0 and, thus, a decrease in D_0. Lower deposit funding not only implies lower reserves and lower long-term lending, but also a lower share of reserves in the bank's portfolio as $\bar{q}$ decreases (similar to in Proposition 3).

9 Appendix B: a mixed portfolio

9.1 Baseline model

Let c_E denote the consumption of impatient depositors (who consume at $t = 1$), c_L the consumption of patient depositors (who consume at $t = 1$ or $t = 2$) in case of no bank run, c_R the consumption in case of a bank run. Consider a single atomistic consumer who considers holding a share $d_0 \in [0, 1]$ of her endowment ϵ as deposits and a share $(1 - d_0)$ as cash. Her portfolio allocation problem, given a run-prone deposit contract, is given by

$$\max_{d_0} (1-q)\Big[\lambda u(c_E) + (1-\lambda)u(c_L)\Big] + qu(c_R),$$

subject to

$$c_E = d_0 \epsilon \frac{c_1^B}{D_0} + (1-d_0)\epsilon(1-f), \qquad c_L = d_0 \epsilon \frac{c_2^B}{D_0} + (1-d_0)\epsilon(1-f),$$

$$c_R = d_0 \epsilon \frac{c_R^B}{D_0} + (1-d_0)\epsilon(1-f).$$

The first order condition for a given deposit contract equals

$$\lambda u'(c_E)\Big[\epsilon(\frac{c_1^B}{D_0}-(1-f))\Big]+(1-\lambda)u'(c_L)\Big[\epsilon(\frac{c_2^B}{D_0}-(1-f))\Big]+...$$
$$q\Big[u'(c_R)\Big[\epsilon(\frac{c_R^B}{D_0}-(1-f))\Big]-\lambda u'(c_E)\Big[\epsilon(\frac{c_1^B}{D_0}-(1-f))\Big]-...$$
$$(1-\lambda)u'(c_L)\Big[\epsilon(\frac{c_2^B}{D_0}-(1-f))\Big]\Big]=0. \tag{31}$$

Expression 31 can result in either corner solution or an interior choice for d_0, depending on q. Certainly, when q is sufficiently low, consumers only hold deposits.

However, the deposit contract is affected by deposit funding. The bank's problem in the version of the baseline model that allows for mixed portfolios is given by

$$\max_{c_1^B,c_2^B,c_R^B,x,y,y^l}(1-\bar{q}\mathbb{1}_{\bar{\lambda}<1})\Big[\lambda u(c_E)+(1-\lambda)u(c_L)\Big]+\bar{q}\mathbb{1}_{\bar{\lambda}<1}u(c_R),$$

subject to

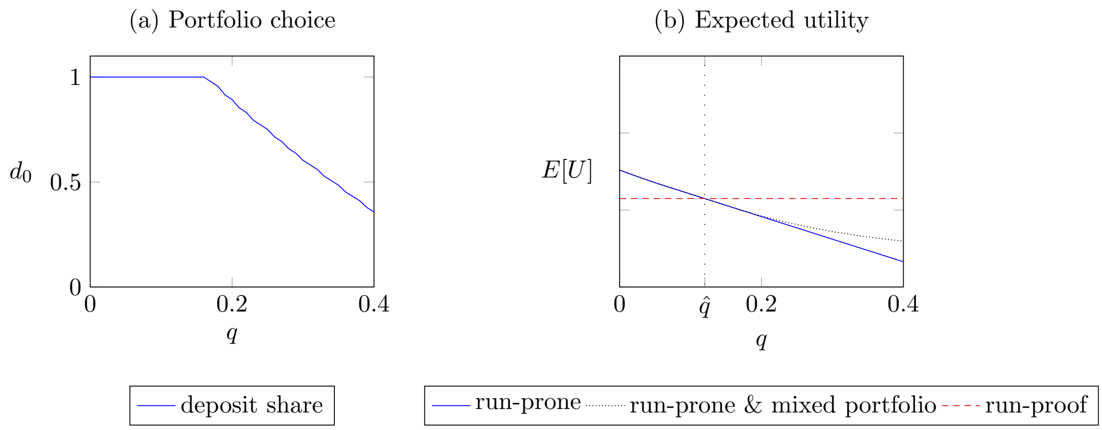
$$x+y+y^l=D_0, \qquad \lambda c_1^B=y,$$
$$(1-\lambda)c_2^B=Rx+y^l, \qquad c_R^B=y+y^l,$$
$$0\leq c_1^B\leq c_2^B, \qquad c_1^B,c_2^B,x,y,y^l\geq 0,$$

where

$$c_E=c_1^B+(1-f)(1-D_0), \tag{32}$$
$$c_L=c_2^B+(1-f)(1-D_0), \tag{33}$$
$$c_R=c_R^B+(1-f)(1-D_0), \tag{34}$$
$$\bar{\lambda}=\frac{y+y^l}{c_1^B}. \tag{35}$$

With an intermediate q, consumers may opt for some cash holdings (a mixed portfolio) in which case aggregate deposit funding D_0 would be lower and the bank optimally offers adjusted payoffs, similar to in Ennis and Keister (2003). In other words, in equilibrium the consumer's problem and the bank's problem are simultaneously determined. Panel (a) of Figure (14) uses a simulation to illustrate the run-prone contract consumers may choose for a mixed portfolio at an intermediate q. Panel (b) confirms that at some intermediate q a mixed portfolio results in a higher expected utility, given that the contract is run-prone. Panel (b) also shows that, under this calibration, the run-proof contract is always preferred over a linear combination of the run-prone contract and cash holdings and. That is, in this case, the same results apply regardless of whether mixed portfolios are allowed or not.

Figure 14: Baseline model mixed portfolio



Notes: The simulations use $R = 1.5$, $\lambda = 0.3$, $f = 0.2$, and $\gamma = 1.5$.

9.2 The Model

Consider a single atomistic consumer who considers holding a share d_0 of her endowment ϵ as deposits and a share $(1-d_0)$ as cash. Her portfolio allocation problem given a run-prone contract depends on her belief, and is given by

$$\max_{d_0}(1-q_i)\Big[\lambda u(c_E) + (1-\lambda)u(c_L)\Big] + q_i u(c_R),$$

subject to

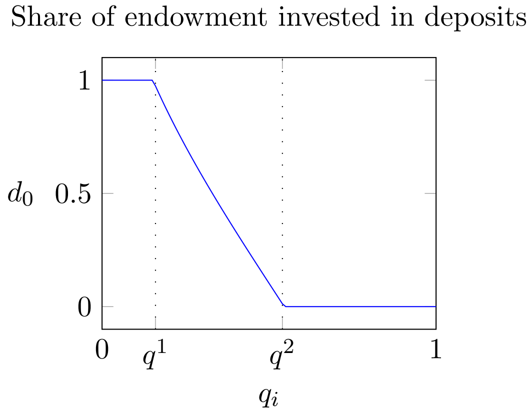
$$c_E = d_0\epsilon\frac{c_1^B}{D_0} + (1-d_0)\epsilon(1-f), \qquad c_L = d_0\epsilon\frac{c_2^B}{D_0} + (1-d_0)\epsilon(1-f),$$
$$c_R = d_0\epsilon\frac{c_R^B}{D_0} + (1-d_0)\epsilon(1-f).$$

The first order condition equals

$$\begin{aligned}
&\lambda u'(c_E)\Big[\epsilon(\frac{c_1^B}{D_0} - (1-f))\Big] + (1-\lambda)u'(c_L)\Big[\epsilon(\frac{c_2^B}{D_0} - (1-f))\Big] + ...\\
&q_i\Big[u'(c_R)\Big[\epsilon(\frac{c_R^B}{D_0} - (1-f))\Big] - \lambda u'(c_E)\Big[\epsilon(\frac{c_1^B}{D_0} - (1-f))\Big] - ...\\
&(1-\lambda)u'(c_L)\Big[\epsilon(\frac{c_2^B}{D_0} - (1-f))\Big]\Big] = 0. \qquad (36)
\end{aligned}$$

The first term is positive and increasing in d_0, whereas the second term is negative and decreasing in d_0. Thus, a q^1 exists such that when $q_i < q^1$ it follows that $d_0 = 1$, a $q^2 > q^1$ such that when $q^1 < q_i < q^2$ it follows that $0 < d_0 < 1$ where d_0 solves optimality condition (36), and when $q_i > q^2$ it follows that $d_0 = 0$. Figure (15) uses a simulation to illustrate how consumers with an interior belief choose a mixed portfolio.

Figure 15: Mixed portfolio



Notes: The simulations use $R = 1.5$, $\lambda = 0.3$, $f = 0.2$, $\gamma = 1.5$, $D_0 = 0.7$, $y = 0.26$, and $\bar{q} = 0.05$.

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