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ANTICIPATION OF FUTURE CONSUMPTION

A MONETARY PERSPECTIVE

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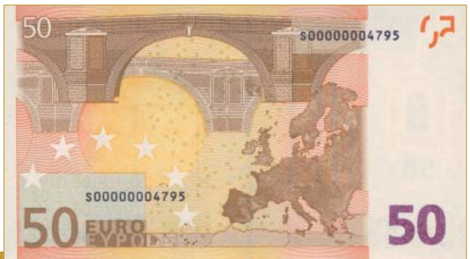

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Abstract

We adapt the (Sidrauski, 1967) monetary model to study the hypothesis of anticipation of future consumption. We assume that anticipation of future consumption affects an agent's instantaneous utility and that all effects of future consumption on current wellbeing are captured by the stock of future consumption. Monetary policy effectiveness is thereby reduced and a zero nominal lower interest rate (and thus the Friedman Rule) is destabilizing. Given this, we can derive a "just stable" equilibrium nominal interest rate with matching definitions for inflation and monetary growth. We demonstrate that these implied lower bounds match their historical analogues well.
JEL Codes: E41, D91, O42

Keywords: Anticipation, consumption behavior, money demand, money and growth, Friedman rule, stability.

Non-Technical Summary

The hypothesis of Anticipation of Future Consumption (AFC) is based on the insight that future consumption impacts immediate wellbeing (or "instantaneous utility"). Countless studies have documented that whether people are optimistic or pessimistic or anxious about the future affects their motivation to perform non-trivial tasks, make plans, take risks etc. However, whilst influential in behavioral economics, this hypothesis has yet to be analyzed in a monetary context. This is in contrast to models of habit formation which are both well-known and widely incorporated into general equilibrium frameworks.

To this end, we adapt the Sidrauski money-in-the utility-function model to study the role of anticipation of future consumption on equilibrium monetary outcomes. Other than being one of the most popular and widely analyzed monetary frameworks, the Sidrauski model was chosen because of its simplicity and tractability - for example the steady state of the economy is independent of the rate of growth of money (in other words, it is characterized by super neutrality). Given this common steady state, it is straightforward to compare the outcomes of the conventional Sidrauski model with the model supplemented with habit formation and then AFC preferences. The only distinction between the outcomes of these models lies in money demand equation and the dynamic monetary transmission mechanism.

For modeling purposes, we assume that anticipation of future consumption affects an agent's instantaneous utility and that all effects of future consumption on current wellbeing are captured by the stock of future consumption. The implications are striking:

i) Whilst money necessarily remains super neutral, the Friedman rule (i.e., that the optimal monetary policy rule is to set nominal interest rates to zero) which is a common welfare benchmark in the monetary literature is infeasible. It is inconsistent with the stability conditions of the monetary equilibrium in the sense that in equilibrium nominal interest rates will necessarily be bound above zero;

ii) Given this, we can derive a "just stable" equilibrium nominal interest rate with matching definitions for inflation and monetary growth. We demonstrate that these implied lower bounds match their historical analogues well;

iii) Money demand as a proportion of steady-state output is smaller than the money demand of the Sidrauski model, which is smaller than the money demand with habit formation. Thus the liquidity needs of these economies differ sharply;

iv) Money demand is less sensitive to variations of nominal interest rates and output than in the Sidrauski and habit-formation models.

The basic intuition behind these points above is that since deferred consumption yields some positive utility, there will be effectively a ceiling to money demand, preventing money supply being expanded to such an extent that its marginal utility is zero – implying that its "price", the nominal interest rate, cannot be driven to zero either.

1 Introduction

Traditionally many economic models restrict an individual's instantaneous utility from consumption to only be a function of current consumption. More recently, emphasis has been put on models stressing the effect of *past* consumption on utility (e.g., Abel (1990) and subsequent literature). Such 'habit-formation' models have, for instance, proved popular in monetary and business cycle analysis (following the seminal work of Fuhrer (2000)).

However, it is clear that there are several important life events that involve *anticipation* of future consumption such as retirement, the final payment of a mortgage and so on. Loewenstein (1987) was the first to formalize the insight that the anticipation of future consumption (AFC), like consumption itself, yields utility.[1] It seems intuitive that pessimism or optimism regarding the future affects a person's present motivation and decisions: people may, for example, prolong a desirable experience to 'savor' it, or expedite an undesirable one to shorten the period of 'dread'. This concept of 'anticipal utility' has proved influential in some behavioral fields to explain anomalies in standard discounted utility theory, as well as to address empirical irregularities in, for example, financial decisions.[2]

In our analysis, we study the implications of such preferences specifically for monetary analysis. To our knowledge, we are the first to do so. That is perhaps surprising since holdings of money balances is one of the key ways in which agents transfer and anticipate resources across time. Moreover, agents' financial decisions are often contingent on the underlying monetary regime in place. It therefore seems an odd omission in the literature to analyze agents' risk aversion and savings behavior etc under AFC preferences without having determined the associated outcomes for monetary policy.

To this end, we adapt the well-known Sidrauski (1967) neoclassical money-in-the-utility-function (MIUF) model to study the role of AFC preferences on equilibrium monetary outcomes: i.e., on money demand, interest rates and the optimum quantity of money. We further make comparisons with the standard Sidrauski framework (i.e., without AFC preferences) and one supplemented with habit formation. The Sidrauski model is particularly convenient since money is superneutral, so the only distinction between the models lies in money demand.

[1]Although as he comments, the notion of "anticipal utility" has a surprisingly long lineage in economics, being found in the works of Bentham, the Jevons and Marshall.

[2]The next section discusses the relevant literature in more detail.

The adaptation of the model consists of two steps. The first, based on Loewenstein (1987), assumes the representative agent derives utility from AFC. The second step follows Kuznitz et al. (2008) in assuming that all effects of future consumption on current well being are captured by a single variable, the "stock of future consumption", analogous to habit formation models.

Our study fits well into the money and growth literature (for textbook treatments, see Bénassy (2011), Walsh (2010)), although our results are by comparison both striking and distinctive:

1. **Monetary Policy effectiveness is reduced and the equilibrium monetary transmission mechanism is weaker.**
   Money demand under AFC preferences turns out to be less sensitive to variations in the nominal interest rate and activity than otherwise and nominal interest rates are necessarily bound above zero (the latter, by definition, constituting another constraint on monetary policy).[3]

2. **A zero nominal lower interest rate is destabilizing in the AFC model economy.**
   The Friedman Rule is therefore sub optimal and there is no optimum quantity of money.

3. Given 2., **we can derive a "just stable" equilibrium nominal interest rate with matching definitions for inflation and monetary growth.**
   In a simple calibration exercise, we demonstrated that these implied lower bounds match their historical analogues well.

4. **The model is also able to match money demand ratios** better than habit or Sidrauski variants – habit formation, in particular, provides grossly counterfactual values.

The paper proceeds as following. In the following section, we discuss in more depth some of the related literature. Thereafter, we describe the introduction of AFC preferences in a dynamic neoclassical monetary framework, as well as its stability properties. These properties are then used in Section 4 to study the viability of the Friedman Rule.

[3] In the first instance, we assume monetary-policy effectiveness refers to the interaction between money demand and interest rates and output, and the limits to stabilization policy. More generally, effectiveness is often taken to indicate the degree to which monetary policy is neutral with respect to the evolution of real variables. These aspects are taken up in section 3.1.

In Section 5, we derive the money demands consistent with AFC, habit and no-habit utility preferences and show the relative sensitivities of money demand to the nominal rate of interest. Section 6 concludes.

2 Related Literature: A General Overview

The premise of anticipated future consumption is that utility is a function both of current consumption plus some fraction of the discounted utility of the consumption of future goods. Thus anticipated consumption yields both consumption utility (when future consumption is realized) as well as anticipal utility (before it is realized).

Countless studies have documented that whether people are optimistic or pessimistic or anxious about the future affects their motivation to perform non-trivial tasks, make plans, take risks etc, e.g., Harris (2012). Indeed, it is interesting to observe that while the origins of the habit-formation hypothesis are related to empirical properties of the consumption function (e.g., Duesenberry (1949); Pollak and Wales (1969)), the intellectual roots of the AFC hypothesis are psychological, and theoretical in nature, and are related to anticipated utility (see Frederick et al. (2002) for a detailed survey; and Caplin and Leahy (2001) on the psychological literature on anticipation).

One can perhaps ascribe the motivation for AFC preferences – other than compelling psychological evidence – to two aspects: namely, addressing anomalies in traditional discounted utility theory and addressing some empirical puzzles.

Traditional discounted utility analysis suggests that agents prefer to consume desired outcomes as soon as possible, and to delay undesirable ones as much as possible. Yet it is trivial to imagine everyday counter examples.[4] Anticipatory utility also provides a motive for a preference for improvements over time in goods and experiences, and essentially points to the fact that people discount differently over different goods (which again stands counter to traditional analysis). The latter observation may explain why, in practice, inferring discount rates from savings and consumption behaviors has proven to be so problematic (see the discussion in Loewenstein (1987).)

Anticipation of future consumption has also been used to study important problems of actual consumption behavior, such as incentives for savings (e.g., Kuznitz et al.

[4]For instance, most people would prefer to be informed of medical results sooner rather than later, to both shorten the period of dread and to engage in remedial behaviors. On the other hand, people may intrinsically enjoy the act of looking forward to a vacation or else storing (and thus delaying the consumption of) vintage wine. Loewenstein (1987) provides many more such expressive examples.

(2008)), and can also be useful, through proper adaption, to study issues linked to retirement (e.g., Haider and Stephens (2007)) and mortgage re-payments (e.g., Coulibaly and Li (2006)). One of the most puzzling aspects of savings behavior is the sometimes absence of dis-saving in retirement: agents often raise their savings after retirement and amass wealth until death.[5]

Kuznitz et al. (2008) in particular is a pioneering paper in the literature applying the AFC to portfolio choice. Such preferences imply a new dimension of risk in financial markets, since investors wish to smooth not only consumption, but also their wealth-to-consumption ratio. Kuznitz et al. (2008) find that these preferences reduce the mean allocation to stocks, i.e., agents save more and invest less in risky assets.

So far it has been assumed that AFC influences present decisions, however, the converse is also true, since present activity affects anticipatory utility from future events. Consumers contrast the consumption outcome with relevant reference points, which are in part determined by outcomes they had anticipated receiving, Köszegi and Rabin (2006).

Given this selective survey, we can now proceed with the incorporation of AFC preferences into a monetary economy. This starts by deriving the stock of future consumption, which yields an additional dynamic constraint in the representative agent problem. We then show that this drives a wedge between the marginal rate of substitution between money and consumption, and the nominal interest rate. We then derive the systems of equations of the canonical model and examine stability conditions. These turn out to yield a number of interesting restrictions: restrictions on the form of admissible utility (its form and curvature) and between the rate of time preference and the degree of anticipation. These elements in turn lead to more concrete restrictions on monetary policy: namely the inadmissibility of zero equilibrium interest rates with empirical implications for inflation, monetary growth and money demand.

[5] This makes sense if agents derive utility from wealth, which finances and anticipates future consumption. Of course to explain this behavior one could also appeal to myopia, bequest motives, incomplete insurance mechanisms etc.

3 Anticipation of Future Consumption in the Sidrauski Monetary Model

The modeling strategy for considering the hypothesis of AFC in a dynamic neoclassical framework is the following. First the stock of future consumption is derived, which yields an additional dynamic constraint in the representative agent problem. Then the stock of future consumption is introduced in the instantaneous utility function. The derivation of the stock of future consumption is analogous to habit formation in consumption as derived by Ryder and Heal (1973), in which the stock of habits is a function of past consumption.

Defining the stock of future per-capita consumption, A, as a function of future consumption, we have,[6]

$$A = \rho e^{\rho t} \int_{t}^{\infty} e^{-\rho\tau} c(\tau)\, d\tau \tag{1}$$

where $\rho > 0$ represents the speed of adjustment of the stock of future consumption (the relative weights of future consumption at different times). The formulation gives positive but exponentially declining weight to consumption in future periods. The larger is ρ, the less weight is given to future consumption in determining A.

In our deterministic environment, the expected and actual stocks of future consumption are the same and the stock of future consumption is equal to current consumption in equilibrium. This has the virtue of keeping the model and our analysis parsimonious. In reality, though, future consumption streams are inherently uncertain. Kuznitz et al. (2008), however, point out that there is no single way to construct the variable "stock of future consumption". They use existing market prices of stochastic future consumption to formulate their stock of future consumption.[7] More generally in the consumption literature, uncertainty is variously handled by assuming certainty equivalence, by assuming that adjacent consumption streams follow a log-normal distribution and so on.

Differentiating (1) with respect to time yields the dynamics of the stock of future

[6] We call this the *stock* of future consumption in deference to Ryder and Heal (1973). Of course, it is also conceivable that agents have both habit formation and AFC preferences. But we abstract from the former for the sake of clarity and to isolate those effects due to the presence of AFC features.

[7] In an interesting approach, Willman (2007) posits density functions for future labor income streams which embody assumptions about how much period-specific information exists (e.g., regarding bonuses, overtime) and to what extent these expectations are front loaded into current decisions.

consumption, which will be used as an additional dynamic constraint in the representative agent problem:

$$\dot{A} = \rho \left(A - c \right) \tag{2}$$

where $\dot{A} = \frac{dA}{dt}$.

Given the derivation of A, the stock of future consumption, the next step is its introduction as an argument in the instantaneous utility function: $U = U(c, m, A)$, where c stands for current consumption, and $m = M/PN$ is real per-capita money balances (P is the price level, N is population, M is the quantity of money).

The properties of the utility function, U, will be determined by the stability conditions of the steady-state equilibrium, as shown below; as we shall see, expressing utility in implicit form initially appears more attractive than imposing a priori functional characteristics.[8,9]

In the Sidrauski model real per capita money balances are introduced in the utility function and in the budget constraint of the representative agent. The problem of the representative consumer with AFC preferences and money is then the following:

$$\underset{c,m}{Max} \int_{0}^{\infty} U\left(c, m, A\right) e^{-\theta t} dt \qquad \text{s.t.} \tag{3}$$

$$\dot{a} = \left(r - n\right) a + w + x - \left[c + \left(\pi + r\right) m\right] \tag{4}$$

$$\dot{A} = \rho \left(A - c \right) \tag{5}$$

$$\lim_{t \to \infty} a_t \left[e^{-\int_0^t (r_\tau - n) d\tau} \right] \geq 0 \tag{6}$$

where $\theta > 0$ is the rate of time preference, a is real wealth held in the form of either real money balances or capital: $a = m + k$; w is the real wage; π is the inflation rate, r is the real interest rate, consequently $i \approx \pi + r$ is the nominal interest rate through the *Fisher equation*; n is the population growth rate; and x is government lump-sum transfers. The variables are in per capita terms.

[8] As, for example, U being concave in A, and current consumption and the stock of future consumption being substitutes in the utility function.

[9] In a detailed examination, available upon request, of the Ramsey model (where m is not an argument) with anticipation of future consumption, the stability conditions suggest that the utility function should have the following characteristics: $U_{Ac}, U_{AA} < 0$, and $U_{AA} + U_{Ac} < 2U_{cc} < 0$, i.e., the utility function is concave in A, and c and A are substitutes in the utility function.

Equation (4) shows that wealth evolves as a function of past balances accumulated at the population-corrected real interest rate, wages and transfers and de-cumulates with consumption and money balances (the opportunity cost of which is the nominal interest foregone). The last equation, (6), is the usual no-Ponzi game condition.

The current value Hamiltonian associated with this problem is:

$$\mathcal{H} = U(c, m, A) + \lambda \{(r - n) a + w + x - [c + (\pi + r) m]\} + \mu \{\rho (A - c)\} \tag{7}$$

where λ and μ are the costate variables associated with the state variables a and A, respectively.[10]

The first order conditions are:

$$U_c(c, m, A) = \lambda + \mu\rho \tag{8}$$

$$U_m(c, m, A) = \lambda(\pi + r) \tag{9}$$

$$\dot{\lambda} = \lambda(\theta + n - r) \tag{10}$$

$$\dot{\mu} = \mu(\theta - \rho) - U_A(c, m, A) \tag{11}$$

Plus the transversality conditions:

$$\lim_{t\to\infty} a(t)\lambda(t)e^{-\theta t} = \lim_{t\to\infty} A(t)\mu(t)e^{-\theta t} = 0 \tag{12}$$

Condition (8) states that, for the agent to be in equilibrium, the marginal utility of consumption must equal the marginal utility of wealth plus the marginal utility of anticipated consumption μ (weighted by its speed of adjustment, ρ). In other words, at the margin the agent will consume up to the point where the additional utility from an extra unit of consumption matches the "cost" (or shadow price) of foregone wealth accumulation plus the effective shadow price of anticipated consumption.

Condition (9) equates the marginal utility of real money balances to the product of the nominal rate of interest and the shadow price of wealth. Equation (10) is the familiar Keynes-Ramsey Rule (KRR). Finally, (11) is the law of motion for the shadow value of the dynamic AFC constraint.

[10]We assume that the agent fully commits to his optimality conditions at *t=0*. Given that monetary policy is set by an exogenous (i.e., non-discretionary) constant money growth rule, there is no dynamic-game interaction between the policy maker and the agent which might give rise to time-inconsistency outcomes. In addition, we have a deterministic environment, so there is no sense in which the agent has to update his information set.

As we know, in the Sidrauski model the marginal rate of substitution between consumption and real money balances equals the nominal interest rate. This is because the opportunity cost of holding wealth in the form of real money balances is the nominal interest rate forgone evaluated at the marginal utility of consumption. Here, this is no longer true since from equations (8) and (9) we see that the term $\left(1 - \frac{\mu\rho}{U_c}\right)$ drives an intervening wedge (which reflects an additional benefit from holding money balances):[11]

$$\frac{U_m}{U_c} = (\pi + r)\left(1 - \frac{\mu\rho}{U_c}\right) \tag{13}$$

A more intuitive re-formulation of (13) is,

$$U_m = U_c\,(\pi + r) - \mu\rho\,(\pi + r) \tag{14}$$

The left hand side of (14) measures the marginal utility from holding an extra unit of real money balances. The first element on the right represents the cost: the loss of interest on interest-bearing assets evaluated at the marginal utility of consumption. As regards the final right hand term, an additional *benefit* of holding real money balances is that consumption is necessarily deferred and deferred consumption yields anticipal utility. This latter benefit accrues as the marginal utility of anticipated consumption, μ, (weighted as before by ρ) times the prevailing nominal interest rate. For the agent to be in equilibrium, both sides of (14) must match.

To close the model it is assumed that lump-sum government transfers are equal to the seigniorage from money issue:

$$x = \frac{dM/dt}{M}\frac{M}{PN} = \Phi m \tag{15}$$

where Φ is the growth rate of nominal money balances which we can assume to be constant.

We assume factor markets are competitive and that firms use constant returns technology,

$$r = f_k\,(k) \tag{16}$$

$$w = f\,(k) - f_k\,(k)\,k \tag{17}$$

[11] The wedge can also be expressed as the ratio of the marginal utilities of wealth to consumption: λ/U_c.

where $f_k > 0$, $f_{kk} < 0$ for all $k > 0$.

3.1 Short-Run Dynamic Considerations

A common working assumption among economists is that while monetary policy may or may not have long-run effects on real variables, short-run effects are likely (e.g., see the discussions in Wang and Yip (1992), Rotemberg and Woodford (1997), Zhang (2009), Levine et al. (2012)).

In that regard, Fuhrer (2000) includes habit formation in the consumer's utility function and shows that it improves the short-run dynamics of the model, both qualitatively and statistically. His formulation also generates the well-known empirical regularity of the monetary transmission mechanism: the "hump-shaped" gradual response of spending and inflation to shocks.

In our model monetary policy can also have short-run real effects. Recall that condition (10) is the Keynes-Ramsey Rule of optimal consumption growth. Solving this out in our model yields,

$$\hat{c} = \frac{1}{\psi}\left[r - \theta - n + \eta_{c,m}\hat{m} + \eta_{c,A}\hat{A} + \frac{\rho\left[(\rho-\theta)\,\mu + U_A\right]}{U_c}\right] \tag{18}$$

where $\psi = -c\frac{U_{cc}}{U_c}$, $\eta_{c,m} = \frac{U_{cm}m}{U_c}$, $\eta_{c,A} = \frac{U_{cA}A}{U_c}$ and where $\hat{c} = \frac{\dot{c}}{c}$ etc. This expression makes clear that in the transition money interacts with consumption (and thus other real variables such as capital accumulation). Moreover, growth in the AFC series will also have real effects.[12]

Direct comparisons with Fuhrer, though, are difficult. His analysis is in discrete time and in linear form, and omits money (and thus aspects of non-separability). Fuhrer also essentially looks at the consumption function in isolation whereas we emphasize system-wide stability properties.

In general terms, though, the way to look at the presence of hump-shaped dynamics is to think of them as being driven by a stock-flow interaction. Specifically, with habit formation, utility is not only driven by current (*flow*) consumption but also by past (*stock*) consumption. Further, the way habits are specified, i.e., $h_t = h(h_{t-1}, c_{t-1})$,[13]

[12]For reference, the "textbook" KRR arises if money is separable and there is no AFC ($U_{cm} = U_A = \rho = 0$). Optimal consumption growth is then determined by the difference between the real interest rate (adjusted for time preference) and population growth: $\frac{\dot{c}}{c} = \frac{1}{\psi}\left[r - \theta - n\right]$.

[13]Fuhrer (2000, equation 2).

is dynamically very rich and guaranteed to increase the number of stable roots which underpin gradual responses.

The analysis of the conditions that KRR (18) generates a hump-shaped response to monetary-policy shocks depends directly on the term $\eta_{c,m}$ and indirectly on the impact of money growth rate, $\hat{m}$ on the marginal utility (on U_c and U_A) as well as through the impact of $\hat{m}$ on the last term which is weighted by the speed of adjustment of the stock of future consumption ρ. Thus ρ plays in our model an analogous role that the persistence parameter of the habit-formation reference level plays in Fuhrer's case.

In effect, the AFC model is symmetric to the habit formation case in the sense that it does with future consumption what habit formation does to past consumption, i.e., by considering consumption in other periods of time besides the current consumption the agent aims at smoothing the consumption profile. This is the main rationale for Fuhrer's result, this means that instead of a jump resulting from a shock in income or in the interest rate, the consumer will tend (albeit conditional on particular parameter combinations and system dynamics) to react to shocks in a smooth, gradual way.

3.2 The Steady State

In the steady state the condition $dm/dt = 0$ implies,

$$\dot{m} = 0 \Rightarrow \frac{\dot{M}}{PN} - \frac{\dot{P}}{P}\frac{M}{PN} - \frac{\dot{N}}{N}\frac{M}{PN} = 0 \Rightarrow \Phi = \pi + n \tag{19}$$

As a consequence from (15) and (19), we have,

$$x = (\pi + n)\, m \tag{20}$$

Combining equations (10) and (16)-(17), yields in the steady state (denoted by an asterisk):

$$r = n + \theta \Rightarrow k^* = f_k^{-1}\,(n + \theta) \tag{21}$$

This is exactly the steady state equilibrium obtained in the Ramsey model without AFC, i.e., the *Modified Golden Rule* holds.

Substituting (20) into the budget constraint yields:

$$\dot{k} = f\,(k) - nk - c \tag{22}$$

which in the steady state yields the equilibrium value of per-capita consumption:

$$c^* = f(k^*) - nk^* \tag{23}$$

Further, from equation (5) we have,

$$A^* = c^* \tag{24}$$

and for steady-state inflation, we have from (19),

$$\pi^* = \Phi - n \tag{25}$$

Inflation thus occurs in the steady state if the growth of nominal balances exceeds population growth.

From (10) and (21) the steady state shadow price of capital, λ^*, is free, and from (8) and (11), we have,

$$\mu^* = \frac{1}{\theta}\left[U_A(c^*, m^*, A^*) + U_c(c^*, m^*, A^*) - \lambda^*\right] \tag{26}$$

To derive the steady state value of real money balances and the shadow price of A, notice that, given c^* and A^*, equations (13) and (26) determine μ and m implicitly. One consequence of the block recursiveness of this model is that it yields Sidrauski's superneutrality result. Thus, the money growth rate (given in (19)) does not affect the capital stock, consumption and the stock of future consumption.

Our superneutrality result is not the only possible outcome when AFC is taken into account. Money can be introduced in several ways besides the MIUF approach. In Appendix A we consider the cash-in-advance (CIA) and the transactions-costs (TC) approaches. The CIA economy with AFC preferences implies that money is not superneutral. Whilst an economy in which money is introduced via a transaction technology and AFC preferences will exhibit superneutrality.

The classic paper of Wang and Yip (1992) demonstrates that all these approaches with Pareto complementarity between consumption and leisure, and between consumption and money and Pareto substitutability between leisure and money and weakly dominant consumption effect of money growth compared to the real balance effect, higher money growth leads to lower steady-state capital, labor, consumption and welfare.

3.3 Stability Conditions

Prior to dealing with the canonical system of this model we analyze equations (8) and (9). Total differentiation of these two equations and the use of the Cramer rule yields the following multipliers:

$$c_A = \xi^{-1}\left(U_{cm}U_{mA} - U_{mm}U_{cA}\right); c_\lambda = \xi^{-1}\left(U_{mm} - U_{cm}\left(\pi + r\right)\right); c_\mu = \xi^{-1}\left(\rho U_{mm}\right) \quad (27)$$

$$m_A = \xi^{-1}\left(U_{cm}U_{cA} - U_{cc}U_{mA}\right); m_\lambda = \xi^{-1}\left(U_{cc}\left(\pi + r\right) - U_{cm}\right); m_\mu = \xi^{-1}\left(-\rho U_{cm}\right) \quad (28)$$

where $c_A = \frac{dc}{dA}$ etc and where the familiar concavity condition $\xi = U_{cc}U_{mm} - U_{cm}^2 > 0$ pertains. According to Fischer (1979) if real money and consumption are normal goods, then $c_\lambda < 0$. The signs of the remaining terms will be determined and retrieved according to the stability conditions below.

The canonical system of the Sidrauski model supplemented with AFC is:

$$\dot{k} = f(k) - nk - c(A, \lambda, \mu) + x - (\pi + n)\, m(A, \lambda, \mu) - \dot{m} \quad (29)$$

$$\dot{A} = \rho\left(A - c(A, \lambda, \mu)\right) \quad (30)$$

$$\dot{\lambda} = \lambda\left(\theta + n - f_k(k)\right) \quad (31)$$

$$\dot{\mu} = -U_A\left(c(A, \lambda, \mu), m(A, \lambda, \mu), A\right) + \mu\left(\theta - \rho\right) \quad (32)$$

Note that from equations (8) and (9), c and m are functions of A and the two costate variables λ and μ, so substituting them into the two dynamic constraints of the problem and in the two euler equations yields the canonical system (29)-(32).

The sufficient conditions for stability of the steady state equilibrium can be shown to be the following (categorized, respectively, into conditions on preferences, multipliers, and the utility function):

$$\textbf{Condition A} \ : \ \theta < \rho$$

$$\textbf{Condition B} \ : \ 1 > c_A > 0; \ c_\lambda, c_\mu < 0; \ m_A, m_\lambda, m_\mu > 0$$
$$(\pi + n) m_\lambda + c_\lambda < 0, c_\lambda m_A + (1 - c_A) m_\lambda < 0 \text{ and}$$
$$c_\mu m_\lambda - c_\lambda m_\mu > 0$$

$$\textbf{Condition C} : \ U_{Ac}, U_{Am,} U_{AA}, U_{cm}, U_{cc}, U_{mm} < 0;$$
$$U_{Ac} c_\mu + U_{Am} m_\mu > 0, U_{Am} - (\pi + n)(U_{Ac} + U_{AA}) > 0$$

Proof. See Appendix B. ■

3.3.1 Specifying Utility

Stability condition **C)** characterizes the utility function: U is non-separable and strictly concave in all its arguments, including the stock of future consumption A; plus c, m, and A are substitutes in the utility function.

A utility function consistent with conditions **B)** and **C)** is an adaptation of the instantaneous isoelastic utility function,

$$U(c, m, A) = \frac{1}{1 - \sigma} \left[\left(c A^\beta \right)^\alpha m^{1-\alpha} \right]^{1-\sigma} \tag{33}$$

where $\sigma > 0$ captures the curvature of the utility function, and parameters β and α lie in the unit interval. Parameter β indexes the importance of the stock of future consumption in instantaneous utility. $1 - \alpha$ gives the weight attached to real money balances in utility.[14]

Taking utility function (33) explicitly into account, the following proposition summarizes the required stability conditions:

[14]Equation (33) nests the Ramsey and Sidrauski case as $\beta = 1 - \alpha = 0$, and, $\beta = 0$, respectively.

Proposition *The sufficient conditions for stability of the steady state equilibrium are:*

$$i)\ \theta < \rho$$
$$ii)\ 2 < \sigma < 1 + \frac{n+\pi}{\alpha\theta}$$
$$iii)\ \beta > 0$$

Proof. Although conceptually straightforward, the proof is long and tedious and is suppressed for brevity.[15] Condition *i)* is independent of the form of utility function and thus carries over unchanged from Condition A. We solve the problem with the utility function given by (33). All conditions *i)-iii)* are sufficient for A), B) and C). Note that to show B) from the first order conditions, c and m are functions of A, λ and μ, and we have to show that the partial derivatives of c and m satisfy the inequalities in B). The same holds for the utility function in (33), with the parameters as in *i)-iii)* they satisfy all inequalities in C) for the utility function.[16] ■

3.3.2 Discussion of Sufficient Conditions

Conditions *i)-iii)* can be motivated in economic terms:

i) This condition states that the rate at which future consumption is discounted should be stronger than that which overall utility is discounted. Intuitively, it can be seen as a condition that specifies that consumption cannot be deferred indefinitely.[17] A larger ρ means less weight is given to consumption in the distant future in determining the stock of future consumption A, and therefore the greater is the importance of consumption in the near future. $\theta < \rho$ indicates that the agent discounts future consumption more heavily when it comes to calculating A, than the agent discounts future utilities by time discounting θ.[18]

[15] It is available on request from the authors.

[16] Notice that the superneutrality result is obtained in spite of the Pareto substitutability between consumption and money, and Pareto substitutability between future consumption and money. Since Brock (1974) it is known that if money is non-separated in the other arguments in the utility function, the superneutrality result may not hold, see, e.g., Wang and Yip (1992), and Matheny (1998).

[17] As we shall see below, the inequality $\rho - \theta > 0$ also ensures that $dm^*/dc^* > 0$, $dm^*/d(\pi + r)^* < 0$ and $dm^*/d\theta < 0$. In other words, that money demand expands with income, contracts with rises in the nominal interest rate, and that money demand is decreasing in the rate of time preference (i.e., the more impatient the individual the less real balances s/he is willing to hold).

[18] The discount factor in our model is Samuelsonian (Samuelson (1937)). If we consider other forms of discounting future utilities, such as in a Benthamite formulation (in which the number of family members receiving the given utility level is taken into account), the time preference component is

ii) If $\alpha\theta$ is small, then $\frac{n+\pi}{\alpha\theta}$ will be large. Moreover, $\sigma > 2$ values are comfortably within the support of empirical estimates, e.g., Campbell (2003). In the following section we analyze this sufficient condition through the lens of the Friedman Rule. Notice, that even though condition *ii)* does not explicitly include the β term, the condition nonetheless depends on the impact of A on c and m (as can be gauged from condition B).

iii) $\beta > 0$ implies that the average flow of the stock of future consumption yields utility.

4 The Optimum Quantity of Money (OQM) and the Friedman Rule

The Friedman Rule (FR) applies Pareto efficiency criteria to the provision of money: namely, that the opportunity cost of holding money faced by agents should equal the social cost of creating additional money. Under a fiat regime, the latter cost is essentially zero. From (9) we see that if the government decides to satiate individuals with money, making real money balances sufficiently large that their marginal utility equals zero, then we retrieve the rule,

$$\pi = -r \tag{34}$$

From the *Fisher relationship*, this implies a zero nominal interest rate, $i = 0$.

Although often challenged, the FR has proved to be a remarkably robust and influential normative prescription for monetary theory.[19] It has also gained particular currency in recent years due to the practise of many central banks (Japan since the mid-1990s, others since late 2008) of setting nominal policy rates around zero in response to slow growth and financial distress.

A startling feature of our model economy, however, is that the FR is *inconsistent* with the stability of the monetary equilibrium. To see this, substitute (34) into

reduced to $\theta - n$ (n is population growth), which makes the inequality more likely to be satisfied. The inequality of course holds in the original Ramsey formulation, since he famously declared discounting the future to be "ethically indefensible" (Ramsey, 1928, p. 543).

[19] Typical themes in the monetary literature challenging the FR include the introduction of transaction or shopping-time technologies; distortionary taxes; the existence of nominal rigidities; search-theoretic monetary models etc. See Walsh (2010) and Bénassy (2011) for discussions.

the *Modified Golden Rule* condition, (21). This turns stability condition *ii*) into the inadmissible form,

$$2 < \sigma < 1 - \frac{1}{\alpha} \ \ ! \tag{35}$$

By contrast, we can use stability condition *ii)* in isolation to determine a just-stable equilibrium nominal interest rate. Substituting the *Fisher* and *Modified Golden Rule* equations into *ii)*, would imply the lower bound,

$$i^L > \theta \left\{1 + \alpha \left(\sigma - 1\right)\right\} \tag{36}$$

which will be strictly positive (given $\sigma > 2$).

In short, *the model admits no optimum quantity of money and the equilibrium nominal interest rate is above zero.* This of course does not preclude the authorities from implementing the Friedman Rule, but it does imply that doing so will eventually lead the economy onto an explosive path.

4.1 Lower-Bound Values: A Calibration Exercise

Let us now give some empirical flavor to these issues. The literal lower bound for nominal interest rates and inflation are, respectively, zero and minus infinity. However, such values are either never or rarely observed. Statistically speaking, a more defensible lower-bound measure of a variable (say, variable q) is its mean minus its standard deviation: $\mu_{q_t} - \sigma_{q_t}$.

For the nominal quarterly US interest rate, this comes out at 2.0%.[20] Moreover, if, in the context of this model, we assume some reasonable parameter values – e.g., $\sigma = 2.5, \theta = 0.01, \alpha = 0.7$ – this would imply precisely the same "lower bound" of 2% when calculated through condition (36).

Likewise, utilizing (36), the FR, the Fisher equation and the steady-state inflation condition (25), we can also back out the growth of nominal balances consistent with that lower-bound interest rate,

$$\Phi^L > \theta\alpha \left(\sigma - 1\right) \tag{37}$$

For the same parameter values as above, this would yield $\Phi^L > 1.1\%$; the average

[20]Source: FRED series *FEDFUNDS* (Available sample: 1954:3-2011:2). The average quarterly Federal Funds rate over this period is 5.4%.

quarterly growth rate of narrow money was 1.4%.[21]

Finally, we can derive a lower bound for equilibrium inflation as,

$$\pi^{*,L} = \Phi^L - n \tag{38}$$

Given a value of $n \approx 0.4$,[22] this would imply $\pi^{*,L} \approx 1.0\%$. Again this value is consistent with its assumed empirical analogue: $\mu_{\pi_t} - \sigma_{\pi_t} \approx 1.0\%$. Thus, a simple calibration of some key relationships in our model are supported by the data.

5 Money Demand and Anticipation of Future Consumption

Our Proposition in section 3.3.1, fully characterizes the utility function in (33). Implementing (33) into the condition for the marginal rate of substitution between consumption and money, (13), and exploiting the steady-state condition between c and A, (24), allows us, after some algebra, to derive the following closed-form, equilibrium money demand:[23]

$$m^* = \frac{(1-\alpha)}{\alpha(\pi + r)} \frac{(\rho - \theta)}{\kappa} c^* \tag{39}$$

where $\kappa = \rho - \theta + \rho\beta > 0$.

Equilibrium money demand thus depends on consumption c^*, the rate of time preference, θ, the persistence of a speed of adjustment of the stock of future consumption, ρ, the weight of money in utility, α, and the nominal interest rate $\pi + r$.

Although there is no satiation point for real money balances and thus no OQM, we can substitute the lower-bound interest rate into equation (39), and so show the associated *level* of utility. Let us define the equilibrium lower bound nominal interest as $\theta\{1 + \alpha(\sigma - 1)\} + \epsilon$, where $\epsilon > 0$ is some perturbation sufficient to ensure (36) holds. Doing so, yields the level of utility consistent with the maximum (though finite)

[21] Source: FRED series *M1* (Available sample: 1975:1-2011:2). Notice, the appropriate comparison here is narrow money since the model contains no financial sector.

[22] Source: FRED series *POP* (Available sample: 1952:1-2010:4).

[23] The isoelastic form of the utility function (33) naturally yields a unitary intratemporal substitution elasticity between real money balances and consumption.

equilibrium real money balances,[24]

$$U = \frac{1}{1-\sigma}\left[\chi (c_t^*)^{1+\alpha\beta}\right]^{1-\sigma} \tag{40}$$

where $\chi = \left[\frac{(1-\alpha)}{\alpha(\theta\{1+\alpha(\sigma-1)\}+\epsilon)}\frac{(\rho-\theta)}{\kappa}\right]^{1-\alpha}$.

5.1 Comparisons of Money Demand Ratios

Before analyzing m^*, let us contrast it with the equilibrium money demands in the Sidrauski and habit formation models (respectively, denoted by superscripts **s** and **h**.

The results for habit utilize utility function,[25]

$$U(c, m, h) = \frac{1}{1-\sigma}\left[(c/h^{\gamma})^{\alpha} m^{1-\alpha}\right]^{1-\sigma} \tag{41}$$

where $\gamma \in (0, 1)$ indexes the importance of habits, h, and with the stock of habits given by (see Ryder and Heal (1973)):

$$h = \rho e^{-\rho t}\int_{-\infty}^{t} e^{\rho\tau} c(\tau)\, d\tau \tag{42}$$

Thus h is a weighted average of past consumption levels. The larger is ρ, the less weight is given to past consumption in determining the h series. Thus, specification (41) follows that used by Fuhrer (2000), albeit following Ryder and Heal (1973) in defining the law of motion for the stock of anticipated consumption. For notational convenience we assume a common value for ρ in the habit and AFC models (i.e., in equations (42) and (1)).

The equilibrium money demands in the Sidrauski and habit formation models when solving out the equivalent conditions for the marginal rate condition (13) are the fol-

[24] We thank an anonymous referee for this insight.

[25] It is worth stressing that in this framework, where all agents are equal, the aggregate consumption per capita is the same as the representative agent consumption, so the "catching up with the Joneses" utility function is the same as in the habit formation model.

lowing (see Faria (2001)):

$$m^{*,\mathbf{s}} = \frac{(1-\alpha)}{\alpha(\pi+r)}c^* \tag{43}$$

$$m^{*,\mathbf{h}} = \frac{(1-\alpha)}{\alpha(\pi+r)}\frac{(\rho+\theta)}{\theta}c^* \tag{44}$$

It can be easily shown that the model with habit formation and the model with AFC have the same steady state equilibrium capital stock and consumption.[26] However they differ regarding money demand, as can be checked by contrasting them with (39). Note, also, that the money demand without AFC in (39), equals the Sidrauski money demand, (43) when $\rho = 0$.

With the steady state money demand in the Sidrauski, habit formation, and AFC models derived, we can compare their size and characteristics. Knowledge of the size and characteristics of money demand is of fundamental importance for policy. Since Friedman (1956) the demand for real money balances has been related to the nominal interest rate and (some measure of) economic activity. Here, for instance, equilibrium money demand with AFC depends not only on activity (through consumption expenditures), and the nominal interest rate, but also on the speed of adjustment of the stock of future consumption.

To compare the size of the money demand in equations (39), (43), and (44), note that $0 < \frac{\rho-\theta}{\kappa} < 1$ in (39), and that $\frac{\rho+\theta}{\theta} > 1$, thus:

$$m^* < m^{*,\mathbf{s}} < m^{*,\mathbf{h}} \tag{45}$$

According to the inequalities in (45) money demand in the model with AFC is smaller than the money demand of the Sidrauski model, which in turn is smaller than the cash-rich, habit economy.[27]

The specific reason for a smaller equilibrium money demand in the case of AFC is the inverse impact of the relative importance of the stock of future consumption in

[26] This is because both economies are characterized by the superneutrality of money and thus by the same equilibrium conditions (21) and (23). See Faria (2001) for details.

[27] In the light of our discussion of the inadmissibility of the FR, this hierarchy is intuitive since if $i^* > 0$ characterizes the AFC economy, then agents would economize on money holdings.

utility on money demand:

$$\frac{dm^*}{d\beta} = -\frac{\rho\left(1-\alpha\right)\left(\rho-\theta\right)}{\alpha\left(\pi+r\right)\kappa^2}c^* < 0 \tag{46}$$

In other words, if the utility derived from anticipated consumption increases relative to current consumption, the less need there is for current transactions (given a preference for deferred consumption) and thus the less need there is for current money demand. Equation (46) also sheds light on the absence of an optimum quantity of money result in this model: the more anticipated consumption is valued in instantaneous utility, the lower are money balances and so the higher will be the marginal utility of money. This aspect prevents the marginal utility of money being driven to zero through satiation.

5.2 A Numerical Comparison of Equilibrium Money Demand

As in section 4.1, we can use our earlier calibrated parameter values to judge the plausibility (in this case) of equilibrium money demand to GDP ratios under our three maintained models (see **Table 1**) for the US, Japan and Germany. These calculations based on the money-to-expenditure ratios from (39), (39), and (44), where the money concept is, as before, M1:[28]

$$\begin{aligned} \text{Sidrauski}: & \frac{\left(1-\alpha\right)}{\alpha\left(\pi+r\right)} \\ \text{AFC}: & \frac{\left(1-\alpha\right)}{\alpha\left(\pi+r\right)}\frac{\left(\rho-\theta\right)}{\kappa} \\ \text{Habit}: & \frac{\left(1-\alpha\right)}{\alpha\left(\pi+r\right)}\frac{\left(\rho+\theta\right)}{\theta} \end{aligned}$$

In each case, we take the average historical nominal interest rate (central bank base rate, last column), the earlier calibration parameters (conditional on two β values, and two ρ values) and calculate the implied equilibrium (i.e., average) money demand ratio and compare it to the data (second last column). We see that habit formation produces a grossly counter-factual money demand. The table also shows that for low β the Sidrauski and AFC money demand ratios are similar (as might be expected). By contrast, the Sidrauski and, in particular, the AFC money demand ratios come

[28]In the case of the models, the ratios are to the level of equilibrium consumption rather than output.

considerably closer to the data.

[Insert Table 1 here]

5.3 Comparisons of Income and Interest Sensitivities

Naturally, recalling (39), an increase in consumption activity expands money demand, $\frac{dm^*}{dc^*} > 0$. From (21) and (23) it is also easy to see that $dc^*/d\theta < 0$ and thus that money demand is decreasing in the rate of time preference:[29]

$$\frac{dm^*}{d\theta} = \frac{(1-\alpha)(\rho-\theta)\left\{\kappa\frac{dc^*}{d\theta} - \rho\beta c^*\right\}}{\alpha(\pi+r)\kappa^2} < 0 \tag{47}$$

The impact of the speed of adjustment of the stock of future consumption on money demand is positive,

$$\frac{dm^*}{d\rho} = \frac{\theta(1-\alpha)\beta c^*}{\alpha(\pi+r)\kappa^2} > 0 \tag{48}$$

That is to say, the faster the speed of adjustment to anticipated consumption, the more this will this require front loading of money demand.

The nominal rate of interest has a negative non-linear monotonic impact on money demand which is consistent with its interpretation as the opportunity cost of holding money:[30]

$$\frac{dm^*}{d(\pi+r)} = -\frac{(1-\alpha)(\rho-\theta)c^*}{\alpha(\pi+r)^2\kappa} < 0 \tag{49}$$

An important result concerning these multipliers is that it can be easily shown that the AFC-related money demand has a lower interest-rate sensitivity than the money demand in the Sidrauski and habit-formation models. The same is true for the

[29] This is intuitive: the more impatient the individual, the less real money balances she is willing to hold.

[30] Although, in Dusansky and Koç (2009) the standard negative relationship between money demand and the bond interest rate is seen to be part of a larger economic reality which includes the possibility that the relationship may be positive.

derivative with respect to activity:

$$\frac{dm^{*,h}}{d(\pi+r)} > \frac{dm^{*,s}}{d(\pi+r)} > \frac{dm^{*}}{d(\pi+r)} \tag{50}$$

$$\frac{dm^{*,h}}{dc^{*}} > \frac{dm^{*,s}}{dc^{*}} > \frac{dm^{*}}{dc^{*}} \tag{51}$$

Thus, the commensurate level of money demand and the interest and expenditure sensitivity of money demand will differ across model economies. In short, *the strength and nature of the monetary transmission mechanism* is related to whether agents value habit formation or consumption anticipation (or neither) in their instantaneous utility function.

Equations (46) and (49) show that $\frac{d(\pi+r)}{d\beta} = \frac{d(\pi+r)}{dm^*}\frac{dm^*}{d\beta} > 0$. This can shed light on why there is no optimum quantity of money in this model economy. A higher preference for future anticipated consumption, lowers the utility derived from current consumption and real money balances. Via the money demand equation, (39), this pushes up the cost of holding real money balances and thus expands future consumption possibilities through interest accumulation.

6 Conclusions

The hypothesis of anticipation of future consumption is based on the insight that future consumption impacts immediate well being. Anticipation of future consumption has been shown to be an empirically relevant phenomenon by countless survey evidence and has been used fruitfully to explain key macroeconomic decisions related to wealth accumulation, portfolio compositions, retirement planning etc.

Whilst habit formation has been extensively researched in money and growth models, our contribution has been to make the first step of integrating the insight of anticipal consumption into a monetary model. Even in the relatively simple neoclassical Sidrauski model, the effects are striking and distinctive:

1. Monetary Policy effectiveness is reduced and the equilibrium monetary transmission mechanism is weaker;

2. A zero equilibrium nominal interest rate is destabilizing in the AFC model economy;

3. There exists however a "just stable" equilibrium nominal interest rate with matching definitions for inflation and monetary growth;

4. Money to income ratios are better matched under AFC preferences plus implied lower bounds of key variables match their historical analogues well.

Notably, our paper has also allowed system-wide stability conditions to inform the form of utility and the restrictions on policy. This seems preferable to, say, simply positing a log separable utility function and assuming the Friedman Rule pertains by inspecting the U_m/U_c condition in isolation.

To generate these conclusions in such a simple framework is suggestive of future work in the area. Natural extensions of this framework would be its incorporation into fully-fledged general equilibrium models. A rich source of issues could then be addressed such as the properties of the dynamic monetary transmission mechanism under AFC; the implications for optimal monetary policy[31] and the additional effect of the (strictly positive) lower bound constraint; the impact of nominal and real rigidities; and uncertainty about the path of anticipated consumption. We leave these open for future research.

[31]For example a key difference between the welfare properties of commitment and discretionary monetary policy solutions depends on whether habit pertains to the model, Levine et al. (2008). An interesting exercise would be to revisit that topic in the context of anticipal preferences.

APPENDICES

A Alternative Approaches to Money and Growth

In section 3.2 we assumed the MIUF approach à la Sidrauski (1967) and obtained as one of the main results that money is superneutral, i.e., its growth rate does not affect the real variables of the model such as capital stock, output and consumption. One wonders how the result depends on the way money is introduced in the model. Would this result stand if money is introduced through a cash-in-advance (CIA) constraint or a transaction technology? In what follows we consider the anticipation of future consumption in a model with CIA constraint and in a model with transactions-costs.

A.1 The Cash-in-Advance Approach

In the cash-in-advance constraint approach, consumption and a fraction, $\Gamma \in [0,1]$, of investment have to be purchased out of existing real money balances:

$$m \geq c + \Gamma \dot{k} \tag{A.1}$$

where $\Gamma = 0$ and $\Gamma > 0$ captures, respectively, the Lucas (1980) and Stockman (1981) formulations. The representative consumer problem with CIA constraint and AFC is:

$$\underset{c,m}{Max} \int_0^\infty U(c, A) e^{-\theta t} dt \qquad \text{s.t.} \tag{A.2}$$

$$\dot{k} + \dot{m} = (r - n) k + w + x - [c + (\pi + r) m] \tag{A.3}$$

$$\dot{A} = \rho (A - c) \tag{A.4}$$

$$\dot{k} = \frac{m - c}{\Gamma} \tag{A.5}$$

The first order conditions are then:

$$U_c(c, A) = \lambda + \mu\rho + \eta/\Gamma \tag{A.6}$$

$$\dot{\lambda} - \theta\lambda = \lambda(\pi + n) - \eta/\Gamma \tag{A.7}$$

$$\dot{\mu} - \theta\mu = -[U_A(c, A) + \mu\rho] \tag{A.8}$$

$$\dot{\eta} - \theta\eta = -\lambda[r - n] \tag{A.9}$$

where λ, μ and η are the costate variables associated with the state variables m, A and k, respectively. In the steady-state it follows from (A.7) and (A.9):

$$r = n + \theta(\theta + \pi + n)\Gamma \tag{A.10}$$

Taking into account that $r = f_k(k)$ and $\Phi = \pi + n$ we have,

$$f_k(k^*) = n + \theta(\theta + \Phi)\Gamma \tag{A.11}$$

In comparison to equation (21) the modified golden rule depends directly on money growth rate, Φ, and therefore the steady-state equilibrium capital stock is negatively affected by money growth rate:

$$\frac{dk^*}{d\Phi} = \frac{\theta\Gamma}{f_{kk}} < 0 \tag{A.12}$$

Given that, it follows that equilibrium output $y^* = f(k^*)$ is also a negative function of Φ. The remainder of the model in steady-state also shows that equilibrium consumption and stock of future consumption are also affected by Φ:

$$c^* = f(k^*) - nk^* \tag{A.13}$$

$$A^* = c^* \tag{A.14}$$

It is clear that in the AFC model with CIA money is not superneutral.

A.2 The Transactions-Costs Approach

In the transactions-costs approach money appears as an argument in a shopping-time technology as in Saving (1971):

$$T = T(c, m) \tag{A.15}$$

where (see Wang and Yip (1992)) $T_c > 0$, $T_m < 0$, $T_{cc}, T_{mm} > 0$, $T_{cm} \leq 0$, $T(0, m) = 0$ and $\lim_{m \to 0} T_m(c, m) = -\infty$. The representative consumer problem with the transactions costs approach and AFC is:

$$\underset{c,m}{Max} \int_0^\infty U(c, A, 1 - T(c, m)) e^{-\theta t} dt \quad \text{s.t.} \tag{A.16}$$

$$\dot{a} = (r - n) a + w + x - [c + (\pi + r) m] \tag{A.17}$$

$$\dot{A} = \rho (A - c) \tag{A.18}$$

The term $1 - T(c, m)$ is an argument in the instantaneous utility function because time is normalized to 1 and $1 - T(c, m)$ captures time dedicated to work and leisure – recall that labor is supplied inelastically in our original model. The first order conditions are:

$$U_c(c, A, 1 - T(c, m)) - U_{1-T}(c, A, 1 - T(c, m)) T_c(c, m) = \lambda + \mu\rho \tag{A.19}$$

$$-U_{1-T}(c, A, 1 - T(c, m)) T_m(c, m) = \lambda (\pi + r) \tag{A.20}$$

$$\dot{\lambda} - \lambda (\theta + n - r) = 0 \tag{A.21}$$

$$\dot{\mu} - \mu (\theta - \rho) = -U_A(c, A, 1 - T(c, m)) \tag{A.22}$$

where λ, and μ are the costate variables associated with the state variables a and A, respectively. It is easy to see that the system (A.19)-(A.22) is analytically equivalent to the system (8)-(11) provided that,

$$U_c(c, A, 1 - T(c, m)) - U_{1-T}(c, A, 1 - T(c, m)) T_c(c, m) = U_c(c, m, A);$$
$$U_A(c, A, 1 - T(c, m)) = U_A(c, m, A);$$
$$-U_{1-T}(c, A, 1 - T(c, m)) T_m(c, m) = U_m(c, m, A).$$

Feenstra (1986) has shown the equivalence between MIUF and money as a medium of exchange that minimizes transactions costs. Therefore the AFC model with transactions-costs approach has the same properties as the MIUF approach and, as a consequence, money is superneutral.

B Sufficient Conditions

Linearizing the canonical system (29)-(32) at the steady state, we obtain the following Jacobian matrix:

$$J = \begin{bmatrix} f_k - n & -m_A(\pi+n) - c_A & -m_\lambda(\pi+n) - c_\lambda & -m_\mu(\pi+n) - c_\mu \\ 0 & \rho(1-c_A) & -\rho c_\lambda & -\rho c_\mu \\ -\lambda f_{kk} & 0 & 0 & 0 \\ 0 & -U_{Ac}c_A - U_{AA} - U_{Am}m_A & -U_{Ac}c_\lambda - U_{Am}m_\lambda & \theta - \rho - U_{Ac}c_\mu - U_{Am}m_\mu \end{bmatrix} \tag{B.1}$$

Defining Z as the sum of the principal minors of J of dimension $2 - \theta^2$:

$$Z = \begin{vmatrix} f_k - n & -m_\lambda(\pi+n) - c_\lambda \\ -\lambda f_{kk} & 0 \end{vmatrix} + \begin{vmatrix} \rho(1-c_A) & -\rho c_\lambda \\ -U_{Ac}c_A - U_{AA} - U_{Am}m_A & \theta - \rho - U_{Ac}c_\mu - U_{Am}m_\mu \end{vmatrix}$$
$$+ 2\begin{vmatrix} -m_A(\pi+n) - c_A & -m_\mu(\pi+n) - c_\mu \\ 0 & 0 \end{vmatrix} \tag{B.2}$$

Given that we have two control variables and two state variables, the equilibrium is saddle-point stable if there are two positive and two negative real roots. Following Dockner (1985) this arises if the following conditions are satisfied:

$$|J| \in \left(0, Z^2/4\right] \tag{B.3}$$

$$Z < 0 \tag{B.4}$$

where $|J|$ is the Jacobian determinant. These two above terms are given by,

$$|J| = -\lambda\rho f_{kk}\left\{(c_\mu m_\lambda - c_\lambda m_\mu)\left[U_{Am} - (n+\pi)(U_{AA} + U_{Ac})\right] + (\theta-\rho)\left[(n+\pi)(c_\lambda m_A + (1-c_A)m_\lambda) + \right.\right. \tag{B.5}$$

$$Z = -\lambda f_{kk}\left((\pi+n)m_\lambda + c_\lambda\right) + \rho(1-c_A)\left[\theta - \rho - U_{Ac}c_\mu - U_{Am}m_\mu\right] + \rho c_\mu\left(U_{Ac}c_A + U_{AA} + U_{Am}m_A\right) \tag{B.6}$$

The sign of many of these elements are known from the stated conditions of the model (e.g., $f_{kk} < 0$); the sign determination of the remainder through (B.3) and (B.4) yield the sufficient conditions *i)* to *iii)*.

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TABLE 1
MONEY/OUTPUT RATIOS (M1/Y) FOR SELECTED COUNTRIES
UNDER DIFFERENT MODEL ASSUMPTIONS

		$\rho = 0.1$				$\rho = 0.9$				Data	
		$\beta = 0.1$		$\beta = 0.5$		$\beta = 0.1$		$\beta = 0.5$			
	Sidr.	AFC	Habit	AFC	Habit	AFC	Habit	AFC	Habit	$\mu_{M1/Y}$	$\mu_{\pi+r}$
Germany	0.41	0.37	4.51	0.26	4.51	0.37	37.3	0.27	4.51	0.23	1.04
Japan	0.41	0.37	4.53	0.26	4.53	0.37	37.5	0.27	4.53	0.49	1.04
US	0.40	0.37	4.43	0.26	4.43	0.37	36.7	0.27	4.43	0.14	1.06

NOTE: Samples are 1950-2010 (Germany), 1957-2010 (Japan), 1975-2011 (US). Data Source: US (FRED), Non-US sources (IFS).