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Corporate Debt Structure and the Financial Crisis

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Abstract

We present a DSGE model where firms optimally choose among alternative instruments of external finance. The model is used to explain the evolving composition of corporate debt during the financial crisis of 2008-09, namely the observed shift from bank finance to bond finance, at a time when the cost of market debt rose above the cost of bank loans. We show that the flexibility offered by banks on the terms of their loans and firms' ability to substitute among alternative instruments of debt finance are important to shield the economy from adverse real effects of a financial crisis.

JEL Classification: E32, E44, C68, G23.

Keywords: Corporate debt, financial crisis, risk shocks, firms' heterogeneity.

Non-Technical Abstract

During the financial crisis of 2008-09, non-financial corporations in the euro area reacted to tightening bank lending conditions by shifting their debt composition from bank loans towards debt securities. At the same time, the cost of market debt rose together with (and above) the cost of bank loans. In this paper, we propose a model that can account for these facts. We use it to evaluate the role played by the debt composition in determining the response of investment and output to financial shocks.

We develop a stochastic dynamic general equilibrium model where lenders and borrowers face agency costs, and where heterogeneous firms in the risk of default optimally decide among alternative instruments of external finance. Depending on their productivity and default risk, entrepreneurs choose between bond finance, bank finance or not raising external finance. The model delivers an endogenous distribution of firms among these three choices that evolves over time depending on aggregate conditions.

We obtain two main sets of results. First, we show that our model requires a combination of financial shocks in order to account for the main facts about corporate finance observed during the crisis: an increase in the "iceberg" cost of obtaining bank financing (or a deterioration in bank efficiency), and two shocks to the uncertainty faced by firms concerning their own productivity and risk of default. The first uncertainty shock affects bank-financed firms only and aims at capturing, for instance, the difficulties faced by the U.S. sub-prime market at the beginning of the crisis. The second shock equally affects all debt-financed firms and captures a surge in the overall uncertainty, as reflected by the sharp increase in stock market volatility observed in 2008-09.

Our second main finding is that bank flexibility and firms' ability to shift among alternative instruments of external finance have important implications for the effects of shocks on aggregate activity. When firms have no access to the bond market, and banks cannot provide the flexibility needed by firms, the negative real effects of a shock to bank operating costs are greatly amplified. We draw the lesson that flexibility in the financial system, be it alternative access to financial markets or be it continual evaluation of project progress in banking relationships, is crucial for understanding the overall economic impact of distress in the financial sector.

1 Introduction

According to the standard narrative, European banks experienced major difficulties to finance themselves in money markets during the financial crisis of 2008-09. Concerns about their exposure to the US sub-prime market enhanced the perception of counterparty risk in the interbank market and triggered a drying-up of liquidity. These difficulties were soon passed on to Euro area non-financial corporations - traditionally heavily dependent on bank-finance - through progressively tightening lending standards.

Indeed, non-financial corporations started shifting the composition of their debt from bank loans towards debt securities early in 2008 (figure 1, panel a). At the same time, the cost of market debt rose together with (and above) the cost of bank loans (figure 1, panel b), while the default rate of non-financial corporations increased sharply. These events were accompanied by an aggregate drop in investment and output that was unprecedented since the introduction of the euro.

In this paper, we propose a model that can account for some of these key facts, incorporating the narrative above. We use it to evaluate the role played by the debt composition in determining the response of investment and output to financial shocks. In particular, we investigate the endogenously evolving debt structure, and the possibilities for companies to switch between bank financing and bond financing.

We develop and investigate a stochastic dynamic general equilibrium model where lenders and borrowers face agency costs, and where heterogeneous firms can choose among alternative instruments of external finance, building on De Fiore and Uhlig (2011, henceforth DFU). There, we focussed on the steady state analysis, while the emphasis here is on the dynamics and on the propagation of specific shocks, possibly accounting for the financial crisis.

As in DFU, our model generates an endogenous corporate debt structure. A key feature is the existence of two types of financial intermediaries, where banks (which intermediate loan finance) are willing to spend resources to acquire information about an unobserved productivity factor, while "capital market funds" (which intermediate bond finance) are not. Because information acquisition is costly, bond issuance is a cheaper - although riskier - instrument of external finance. We view banks as financial intermediaries that build a closer relationship with entrepreneurs than dispersed investors. They assess and monitor information about firms' uncertain productive prospects and are ready to adapt the terms of the loans accordingly, in line with the literature stressing the flexibility of bank finance, see Chemmanur and Fulghieri (1994) and Boot, Greenbaum and Thakor (1993). It is also consistent with the role taken by banks as originators of asset-backed securities, which requires screening of applicants' projects.

As in DFU, entrepreneurs (or firms) differ at the beginning of the period regarding their prediction about final productivity. Based on this information, they choose between bond finance, bank finance or abstaining from production. The model delivers a endogenous distribution of firms among these three financing choices.When they choose bank finance, a further, but costly investigation of the proposed production reveals additional information, and provides the entrepreneur with the option of not proceeding with the loan. In equilibrium, firms experiencing high risk of default choose to abstain from production and not to raise external finance, thereby safeguarding their net worth from a potential bankruptcy. Firms with relatively low risk of default choose to issue bonds because this is the cheapest form of external finance. Firms with intermediate risk of default approach banks, because they sufficiently value the option of abstaining based on further information.

We extend the analysis in DFU by adding shocks and examining the full dynamic behavior. In particular, we investigate the dynamic shift of the financing boundaries in response to aggregate shocks, requiring us to calculate the distributional dynamics. We do so per log-linearizing, including the integral equations which aggregate firm-individual choices. We show that our model requires a combination of three shocks in order to account for the main facts about corporate finance observed during the crisis: an increase in the "iceberg" cost of obtaining bank financing (or a deterioration in bank efficiency), and two shocks to the uncertainty faced by firms concerning their own productivity and risk of default. The first uncertainty shock affects bank-financed firms only and aims at capturing, for instance, the difficulties faced by the U.S. sub-prime market at the beginning of the crisis. The second shock affects all debt-financed firms and captures a surge in the overall uncertainty, as reflected by the sharp increase in stock market volatility observed at the end of 2008 and beginning of 2009 (Bloom (2009) and Christiano, Motto and Rostagno (2014)).

We obtain two sets of results. First, we show that our model can qualitatively replicate the observed changes in the composition of corporate debt in response to the first of these three shocks. This shock induces a fall in the ratio of bank loans to debt securities, as a larger share of firms with high ex-ante risk of default now finds the cost of external finance too high, and chooses to abstain from production. Similarly, a larger share of firms experiencing intermediate realizations of the first productivity shock finds the flexibility provided by banks too costly, and decides to issue bonds instead. Bond finance now becomes more costly as the average risk of default for the new pool of market-financed firms is higher. The cost of bank finance rises to a lower extent, because the share of firms with low risk of default that move from bank-finance to bond-finance is compensated by the share of firms with high risk of default that move out of banking and decide not to produce. Overall, as in the data, bond yields increase above lending rates.

The shock to banks' information acquisition, however, is unable to replicate the large movements observed during the crisis in the spreads, both on bonds and on loans, and in firms' default rate. For our "crisis scenario", we therefore add the two other shocks, capturing the general rise in uncertainty. This generates higher spreads and default rates and widens the distance between the spreads on bonds and the spreads on loans. Under this "crisis scenario" and with appropriately sized shocks, our model predictions are now broadly in line with the observations.

Our second main finding is that bank flexibility and firms' ability to shift among instruments of external finance have important implications for the effects of shocks on aggregate activity. This is related to the idea of efficient capital markets as a "spare tyre", see Greenspan (1999). To show that, we compare the real effects under the combined shocks to bank information acquisition and to the two types of uncertainty, under alternative scenarios.

In our benchmark model, when firms have full flexibility in substituting alternative instruments of debt finance, adverse shocks generate very mild effects on investment and output. The reason is that firms minimize the effect of the shocks on financing costs, expected profits and production by substituting among loans and bonds.

In an alternative scenario, where debt markets are shut down and no firm can issue debt securities, the real effects are only mildly amplified. This result is at a first glance surprising and may cast doubts on the idea that developed capital markets can alleviate the problems faced by firms when banks' role as financial intermediaries is impaired. The result is due to the flexibility of bank financing. While a share of firms are now forced to pay for the services of banking, they gain the flexibility of shutting down production in case of further unfavorable news about their situation. This remaining flexibility is still considerably large.

A different picture arises when capital markets are shut down and when banks are not able to provide the same flexibility to firms (possibly because they are no longer able to efficiently screen entrepreneurial projects, as during the sub-prime crisis). Under this scenario, the absence of bond finance has more severe consequences. The adverse shocks increase markups, reducing the return to the factors of production. The fall in investment (output) is five (three) times larger than in the case when firms can substitute among instruments of debt finance. We draw the lesson that flexibility in the financial system, be it alternative access to financial markets or be it continual evaluation of project progress in banking relationships, is crucial for understanding the overall economic impact of distress in the financial sector.

Our paper relates to recent work by Adrian, Colla and Shin (2012) and Crouzet (2014). Like us, both papers document the fall in the share of bank finance in corporate debt during the 2008-09 crisis and the rising cost of market debt. In order to account for this evidence, Adrian, Colla and Shin (2012) build a model of procyclical behaviour of bank leverage. In a recession, banks contract lending through deleveraging. Risk-averse bond investors need to increase their credit supply to fill the gap in demand, and this requires spreads to rise. Different from us, the authors do not address the response of the cost of bank finance relative to bond finance, nor the macroeconomic implications of debt substitution. Crouzet (2014) provide a model where banks offer more flexible lending arrangements than capital markets - in the form of debt restructuring offers that firms make to banks to avoid costly liquidation. In equilibrium, firms with low net worth use both bank and bond finance, while firms with large net worth use only bond finance. The macroeconomic implications of debt substitution differ from ours. Firms that move from bank finance to bond finance reduce borrowing and investment as they expect debt restructuring to be harder in the future. The possibility to access the bond market thus amplifies the negative real effects of a shock that increases bank lending costs.

Our work also relates to an older literature that models the endogenous choice between bank finance and market finance. Holstrom and Tirole (1997) and Repullo and Suarez (1999) model this choice for firms that are heterogeneous in the amount of available net worth. In those models, moral hazard arises because firms can divert resources from the project to their private use. In Holstrom and Tirole (1997), moral hazard applies to both firms and banks, while it applies only to firms in Repullo and Suarez (1999). In both cases, it is assumed that monitoring is more intense under bank finance. The papers find that, in equilibrium, firms with large net worth choose to raise market finance, firms with intermediate levels of net worth prefer to raise bank finance, and firms with little net worth do not obtain credit. One implication of their model is that a contraction in net worth, as observed during the crisis, leads to a reduction of bond finance, at odds with the evidence. In our model, firms financing choices depend on their risk of default. Hence, a fall in net worth needs not produce a reduction in the share of bond-financed firms. A second main difference is that we analyze the corporate finance choices in a fully dynamic general equilibrium model, relating them to real aggregate variables in the economy.

There is a literature that investigates empirically the choice of firms among bank debt and public debt. Denis and Mihov (2002) find that firms with highest credit quality borrow from public sources, firms with medium credit quality borrow from banks, while firm with the lowest credit quality borrow from non-bank private lenders. Our results on the aggregate importance of financial flexibility complement recent empirical evidence documented by Becker and Ivashina (2011). Using firm-level data on US firms over the period 1990Q2:2010Q4, the authors show that the effect of a reduction in loan supply on investment is positive and significant for firms that have access to both bond and loan markets. The contractionary effect is even larger for firms that are excluded from bond markets.

The paper proceeds as follows. Following a summary of the key facts about corporate finance of the 2008-09 financial crisis in the euro area in section 2, we describe the model in section 3. In section 4, we present the analysis and describe the equilibrium of the model. Section 5 describes our results. In section 6, we conclude.

2 The key facts

We aim at explaining the observed shift in the composition of corporate debt and the evolution of the cost of corporate bonds relative to bank loans. The data are described in detail in section A of the unpublished technical appendix (available in De Fiore and Uhlig 2015).

Figure 1, panel a, plots the growth rates of GDP, of bank loans (all maturities, outstanding amounts) extended by monetary and financial institutions to the euro area non-financial corporations, and of debt securities (outstanding amounts) issued by the same corporations. While the sharp reduction in GDP growth began in 2007, the growth rate of loans remained initially high and only started to decline in 2008, reaching negative levels in mid 2009. This contraction in bank loans was however partly compensated by an increase in the growth rate of debt securities. The counteracting development in these two instruments of external finance continued throughout and beyond the financial crisis of 2008-09.

Figure 1, panel b, shows the evolution of the cost of market debt relative to the cost of bank loans.[1] The shift in the composition of corporate debt occurred at the beginning of 2008, at the time when the cost of market debt financing increased above the cost of new bank loans. The gap between the cost of these two instruments only declined at the end of 2009.

Figure 2 shows that the substitution between bank finance and bond finance emerges as a noticeable feature of the financial crisis also when looking at cumulated flows rather than changes of outstanding amounts. As this figure shows, bank loans are the dominant source of debt finance for euro area corporations. The increase in bond issuance during the crisis was insufficient to compensate for the contraction in bank loans, although it was nonetheless an important valve to buffer the impacts of the crisis in the banking sector.

We obtain these key corporate finance facts for the 2008-09 financial crisis for the euro area (EA):[2] 1) The ratio of bank loans to debt securities (in outstanding amounts) fell by around 5 percent, from 6.2 to 5.9; 2) The spread between the cost of market debt and a German government bond yield with corresponding maturity rose by 120 percent, from 57 to 190 bps; 3) The spread between the cost of bank finance and a german government bond yield with corresponding maturity rose by 104 percent, from 23 to 64 bps; 4) The average default rate on debt of rated non-financial corporations increased by 110 percent, from 1.3 to 2.7 percent.

The financial crisis was also characterised by a stabilization of the ratio of corporate debt to GDP which ended the pre-crisis upward trend (with a peak increase of 17 percent), and by a dramatic fall in GDP and investment (whose peak drop was around 3 and 6 percent, respectively).

3 The model

We extend the model presented in DFU. There, we focussed on and calculated the steady state properties, while we wish to analyze the dynamic impact of key financial shocks here.

[1] The nominal cost of market-based debt is based on a Merrill Lynch index of the average yield of corporate bonds issued by euro area non-financial corporations with investment grade ratings and a euro-currency high-yield index. Average maturity of the bonds is five years. The measure of MFI lending rates is based on new business loans to non-financial corporations with maturities above 1 and up to 5 years, and amounts larger than 1 million EUR. See section A of the unpublished technical appendix for a description of the data.

[2] We compute "peak effects" observed during the crisis, which we define as the maximum percentage log deviation of each series over the period 2008Q1-2009Q4 relative to the post-EMU average.This latter refers to the the longest post-EMU, pre-crisis period on which the data are available.

To do so, we enrich our model along four dimensions. First, we add a number of shocks. Second, we adopt preferences that allow labor to react to movements in the real wage. Third, we assume that entrepreneurs die with a fixed probability, as this enables to proxy consumption and investment decisions of risk-averse entrepreneurs, despite the technical assumption of risk neutrality: the latter is needed to ensure the optimality of the costly state verification contract, see Carlstrom and Fuerst (2001) for further discussion. Fourth, we use a nominal model, which allows us to examine the role of monetary policy. Fifth, we calibrate the model quarterly in order to better characterise the dynamic path of each variable in reaction to shocks.

Before describing the details, it is useful to provide an overview of the model. Time is discrete, counting to infinity. There are entrepreneurs, regular households, capital market funds, banks and a central bank. Households enter the period, holding cash as well as securities, and owning capital. They receive payments on their securities and may receive a cash injection from the central bank. Then aggregate shocks are realized. Households deposit cash at banks, buy shares of capital market funds and keep some cash for transactions purposes. They rent capital to firms as well as supply labor, earning a wage. After receiving wages and capital rental payments, they purchase consumption goods and investments, subject to a cash-in-advance constraint. The deposits and capital market fund securities pay off at the end of the period: the household receives these payments at the beginning of the next period.

Entrepreneurs enter the period, holding capital. The (end-of-period) market value of the capital is their net worth. They can operate a production technology, employing capital and labor, but to do so, they need to have cash at hand to pay workers and capital rental rates up front. Entrepreneurs can borrow a fixed multiple of their net worth to do so. The productivity of entrepreneurs is heterogeneous, and only part of that information is public information ex ante. The final amount produced is observable to the entrepreneur, but not completely observable to lenders, unless they undertake costly verification. The contractual interest rate is determined endogeneously, taking into account verification costs and entrepreneur-specific repayment probabilities.

Capital market funds provide break-even costly state verification lending contracts to entrepreneurs based on the ex-ante publicly available productivity information. Banks are assumed to have closer relationships with entrepreneurs. At an iceberg cost to net worth, borne by the entrepreneur, they can obtain some additional information about the productivity. Based on that additional information, the banks offer break-even costly state verification contracts covering the remaining uncertainty. Given the initial publicly available information, entrepreneurs choose whether to approach capital market funds or banks for a loan, or abstain.[3] If they approach a bank, they can still abstain, after the banks have obtained the additional productivity information. If an entrepreneur obtained a loan, he proceeds to produce, learns the remaining uncertainty regarding his project, and then either repays the loan or defaults. In case of a default, there will be costly monitoring. The entrepreneur then splits end-of-period resources into consumption and capital held to the next period, as net worth.

It is important to recognize the limitations of our framework. First, there is no bank capital, and therefore, shocks to bank capital or bank insolvencies play no role in our analysis. We view our paper as complementary to a considerable literature which has investigated these channels. Our perspective here is one that sees reductions in bank capital and bank insolvencies as a consequence of asset deterioration and credit market turmoil, rather than their original cause. Second, firm financing is within-period: while financing contracts crossing periods would be desirable to analyze, they increase the technical challenges considerably. Our contracts are nonetheless not entirely static: we view the ability of banks to reassess projects in midstream as a crucial element of the dynamics between banks and entrepreneurs, which we have captured in our framework. Finally, there are no scale effects in our model: whether an entrepreneur approaches a bank or the bond market depends entirely on the quality of its project, not on the original net worth. Nonetheless, it turns out ex-post, that the entrepreneurs who end up producing the most are more likely to have received financing on the bond market rather than the bank market, in line with observations.

3.1 Households

At the beginning of period t, aggregate shocks are realized and financial markets open. We use P_t to denote the nominal price level in period t. Households receive the nominal payoffs on assets acquired at time $t-1$ and the monetary transfer $P_t\theta_t$ distributed by the central bank, where θ_t denotes the real value of the transfer. These payments plus their cash balances $\tilde{M}_{t-1}$

[3] In our model, it is not optimal for firms to contemporaneously raise loans and issue debt securities. This feature finds support in recent evidence by Colla, Ippolito and Li (2012). Using a panel data set involving 3,296 U.S. firms for the period 2002-2009, they show that around 85 percent of listed firms in the U.S. make use of only one type of debt.

carried over from the previous period are their nominal wealth. The households choose to allocate their nominal wealth among four types of nominal assets, namely cash for transactions M_t, nominal state-contingent bonds B_{t+1} paying a unit of currency in a particular state in period $t+1$, one-period deposits at banks D_t^B, and one-period deposits at capital market funds D_t^C. The deposits earn a nominal uncontingent return. In order for the households to be indifferent between these two deposits, the returns must be the same, a condition that we henceforth impose. Write $D_t = D_t^B + D_t^C$ for total deposits, R_t^d for the gross return to be earned per unit of deposit between period t and $t+1$, and Q_{t+1} for the nominal stochastic discount factor for pricing assets. The budget constraint is given by

$$M_t + D_t + E_t\left[Q_{t,t+1}B_{t+1}\right] \leq W_t, \tag{1}$$

and nominal wealth at the beginning of period t by

$$W_t = B_t + R_{t-1}^d D_{t-1} + P_t\theta_t + \widetilde{M}_{t-1}. \tag{2}$$

Households own capital k_t, which they rent to entrepreneurs at a real rental rate r_t. They also supply labor h_t ("hours worked") to entrepreneurs for a real wage w_t. After receiving rental payments and wage payments in cash, the goods market open, where the household purchases consumption goods c_t and new capital, using total available cash and the cash value of their existing capital, but not more. They thus face a cash-in-advance constraint, given by

$$\widetilde{M}_t \equiv M_t - P_t\left[c_t + k_{t+1} - (1-\delta)\,k_t\right] + P_t\left(w_t h_t + r_t k_t\right) \geq 0. \tag{3}$$

The household's problem is to maximize utility, given by

$$U = E_o\left\{\sum_0^\infty \beta^t\left[\log\left(c_t\right) - \frac{\eta}{1+1/\kappa}h_t^{1+1/\kappa}\right]\right\}, \tag{4}$$

subject to the constraints (1), (2) and (3), where β is the households' discount rate, η is a preference parameter, and κ denotes the Frisch elasticity of labor supply.

3.2 Entrepreneurs, banks and capital market funds

There are banks and capital market funds. They obtain deposits in the form of cash from households, make cash loans to producing entrepreneurs. The entrepreneurs use these loans to pay cash wages and cash rental rates to the factors of productions, then sell their output for cash in the goods market and repay their cash loan with interest. The financial intermediaries in turn repay their depositors in cash with interest.

There is a continuum $i \in [0,1]$ of entrepreneurs. They enter the period with capital z_{it}, which will earn a rental rate r_t and depreciate at rate δ. Entrepreneurs can post this capital as collateral, and therefore have net worth n_{it} given by the market value of z_{it},

$$n_{it} = (1 - \delta + r_t)\, z_{it}. \tag{5}$$

Each entrepreneur i operates a CRS technology described by

$$y_{it} = \varepsilon_{1,it}\varepsilon_{2,it}\varepsilon_{3,it} K_{it}^{1-\alpha} H_{it}^{\alpha}, \tag{6}$$

where K_{it} and H_{it} denote the capital and labor hired by the entrepreneur.

The shocks $\varepsilon_{1,it}, \varepsilon_{2,it}$ and $\varepsilon_{3,it}$ are random, strictly positive and mutually independent entrepreneur-specific disturbances with aggregate density functions $\varphi(\varepsilon_1;\sigma_1), \varphi(\varepsilon_2;\sigma_2)$ and $\varphi(\varepsilon_3;\sigma_3)$, and distribution functions $\Phi(\varepsilon_1;\sigma_1), \Phi(\varepsilon_2;\sigma_2)$ and $\Phi(\varepsilon_3;\sigma_3)$, respectively.[4]

The shocks are realized sequentially during the period, creating three stages of decision. In the first stage, $\varepsilon_{1,it}$ is publicly observed and realized at the time when the aggregate shocks occur, before the entrepreneur takes financial and production decisions. Conditional on its realization, the entrepreneur chooses between three alternatives. He can borrow cash from a capital market fund (henceforth: CMF) and produce. He can approach a bank and possibly receive nominal bank loans to produce. He can abstain from production. In the latter case, he will rent his capital z_{it} to other producing entrepreneurs, ending up with the rental rate plus the depreciated amount of capital $(1 - \delta + r_t)\, z_{it}$ or, more simply, with his net worth n_{it} at the end of the period. With linear preferences, an entrepreneur will thus pursue production, if he expects his net worth to increase as a result.

If the entrepreneur borrows from a CMF, he will obtain real total funds in fixed proportion to his net worth, $x_{it} = \xi n_{it}$, paid as a cash loan equal to $P_t x_{it}$, and learns about $\varepsilon_{2,it}$ and $\varepsilon_{3,it}$ once production has taken place.[5] If the entrepreneur approaches a bank, the bank

[4] Under the assumption that $\varepsilon_{1,it}$ is iid, firms could experience high volatility in ex-ante productivity and could frequently move from one instrument of external finance to the other. Assuming an AR1 process for $\varepsilon_{1,it}$ generates persistance both in firms' productivity and in the choice of the instrument of external finance. This, however, has no implications for the equilibrium allocations in the aggregate.

[5] In DFU, we discuss and defend in greater detail the assumption of a fixed proportion (otherwise, only the most productive entrepreneurs would receive all the funding) as well as ruling out actuarily fair gambles.

will investigate the quality of the project of the entrepreneur further, revealing $\varepsilon_{2,it}$ as public information. This investigation is costly to the entrepreur: his net worth shrinks from n_{it} to $\hat{n}_{it} = (1 - \tau_t)\, n_{it}$. Given the additional information as well as the new net worth, the entrepreneur then decides whether to proceed with borrowing or with abstaining. If the entrepreneur borrows, he obtains real funds $x_{it} = \xi \hat{n}_{it}$, paid as a cash loan equal to $P_t x_{it}$ from the bank. If the entrepreneur abstains either in the first or the second stage, the entrepeneur takes his (remaining) net worth to the end of the period, and splits it into a part to be consumed and into a part to be carried over as capital into the next period.

If the entrepreneur has obtained a loan, he proceeds with production, using the cash loans obtained in order pay the factors of production

$$P_t x_{it} = P_t \left(w_t H_{it} + r_t K_{it} \right). \tag{7}$$

Upon producing, the entrepreneur then learns about the remaining pieces of uncertainty, i.e. about $\varepsilon_{2,it}$ and $\varepsilon_{3,it}$, in case the loan came from a CMF, or $\varepsilon_{3,it}$, in case the loan came from a bank. These outcomes are not observable to the lender, however, unless the lender monitors the entrepreneur, destroying a fraction μ of the output in the process of doing so.

We assume that lending contracts are optimal and rely on revelation. As Townsend (1979) has shown, and as we discuss in DFU, the solution is a costly state verification contract, in which entrepreneurs promise to repay the cash loan $P_t x_{it} (\xi - 1) / \xi$ with a prior-information-dependent interest rate. They default if and only if they cannot repay, in which case the lender monitors the project. If the entrepreneur repaid the loan, he will split the reminder between current consumption and capital to be held to the next period, as net worth.

Entrepreneurs have linear preferences over consumption with rate of time preference β^e, and they die with probability γ. We assume β^e sufficiently high so that the return on internal funds is always higher than the preference discount, $\frac{1}{\beta^e} - 1$. It is thus optimal for entrepreneurs to postpone consumption until the time of death. When they die or default on the debt, they are replaced by newborn entrepreneurs: these new entrepreneurs receive an arbitrarily small transfer from the government to restart productive activity.

3.3 Monetary policy and equilibrium

Monetary policy occurs through central banks' liquidity injections, carried out with nominal transfers $P_t\theta_t$ to households. The total amount of liquidity injections in the economy is

$$P_t\theta_t = M_t^s - M_{t-1}^s, \tag{8}$$

where M_t^s denotes money supply. Money supply follows an exogenous process given by

$$\frac{M_t^s}{M_{t-1}^s} = \nu_t. \tag{9}$$

An equilibrium is defined in the usual manner as sequences so that all markets clear and so that all entrepreneurs, households and financial intermediaries take the optimal decisions, given the prices they are facing.

4 Analysis

4.1 Households

Define real balances as $m_t \equiv M_t/P_t$ and the inflation rate as $\pi_t \equiv P_t/P_{t-1}$. The safe nominal rate satisfies $R_t = (E_t[Q_{t,t+1}])^{-1}$. A comparison with the equation for the interest rate on deposits shows that $R_t = R_t^d$. Since we concentrate on equilibria with $R_t > 1$, we obtain the usual first-order conditions of the household,

$$\begin{aligned} \eta h_t^{1/\kappa} c_t &= w_t \\ \frac{1}{c_t} &= \beta R_t E_t \left[\frac{1}{c_{t+1}\pi_{t+1}}\right] \\ \frac{1}{c_t} &= \beta E_t \left[\frac{1}{c_{t+1}}\left(1-\delta+r_{t+1}\right)\right]. \end{aligned}$$

4.2 Entrepreneurs: production

We solve the decision problem of the entrepreneur "backwards", starting from the last stage: production. If the entrepreneur obtained a loan and commences production, he maximizes expected profits $\varepsilon_{it}^e H_{it}^{\alpha} K_{it}^{1-\alpha} - w_t H_{it} - r_t K_{it}$, subject to the financing constraint (7), where

$$\varepsilon_{it}^e \equiv \begin{cases} \varepsilon_{1,it} = E\left[\varepsilon_{1,it}\varepsilon_{2,it}\varepsilon_{3,it} | \varepsilon_{1,it}\right] & \text{if CMF finance} \\ \varepsilon_{1,it}\varepsilon_{2,it} = E\left[\varepsilon_{1,it}\varepsilon_{2,it}\varepsilon_{3,it} | \varepsilon_{1,it}, \varepsilon_{2,it}\right] & \text{if bank finance} \end{cases} \tag{10}$$

is the expected part of the entrepreneur-idiosynchratic productivity piece by the time the loan is obtained. A straightforward calculation shows that

$$w_t H_{it} = \alpha x_{it} \tag{11}$$

$$r_t K_{it} = (1-\alpha) x_{it} \tag{12}$$

Expected output at the time of loan contracting is given by

$$y_{it}^e \equiv \varepsilon_{it}^e q_t x_t \tag{13}$$

where $q_t \equiv \left(\frac{\alpha}{w_t}\right)^\alpha \left(\frac{1-\alpha}{r_t}\right)^{1-\alpha}$. Thus, $1/\left(\varepsilon_{it}^e q_t\right)$ is the marginal cost of producing one unit of expected output and hence it may be best to think of $1/q_t$ as the aggregate component of the marginal cost of production. Alternatively, one can think of q_t as the aggregate entrepreneurial markup over input costs or as the aggregate finance wedge. Actual output is given by

$$y_{it} \equiv \omega_{it} y_{it}^e \tag{14}$$

where ω_{it} is the remaining uncertain part of entrepreneur-specific productivity and is given by

$$\omega_{it} \equiv \begin{cases} \varepsilon_{2,it}\varepsilon_{3,it} & \text{if CMF finance} \\ \varepsilon_{3,it} & \text{if bank finance} \end{cases} . \tag{15}$$

4.3 Entrepreneurs: financial intermediaries and lending decisions

Much of the contracting problem and the resulting equations are derived and discussed in DFU: we shall thus be brief, but complete here. The optimal contract sets a threshold $\overline{\omega}_{it}$ corresponding to a fixed repayment of $P_t \varepsilon_{it}^e \overline{\omega}_{it} q_t x_{it}$ units of currency. If the entrepreneur announces a realization of the uncertain productivity factor $\omega_{it} \geq \overline{\omega}_{it}$, no monitoring occurs. If $\omega_{it} < \overline{\omega}_{it}$, the intermediary monitors the entrepreneur, at the cost of destroying a proportion $0 \leq \mu \leq 1$ of the firm output. Let Φ and φ be respectively the distribution and density function of ω_{it}, implied by our distributional assumptions for $\varepsilon_{2,it}$ and $\varepsilon_{3,it}$ as well as the lending decision of the entrepreneur. The residual uncertain productivity factor $\omega = \omega_{it}$ needs to be split across the entrepreneur, the lender and the monitoring costs. Given the treshold $\overline{\omega} = \overline{\omega}_{it}$, define

$$f(\overline{\omega};\sigma) = \int_{\overline{\omega}}^{\infty} (\omega - \overline{\omega})\, \varphi(\omega;\sigma)\, d\omega \tag{16}$$

as the expected share of final output accruing to the entrepreneur and

$$g(\overline{\omega};\sigma,\mu) = \int_0^{\overline{\omega}} (1-\mu)\, \omega\varphi(\omega;\sigma)\, d\omega + \overline{\omega}\left[1 - \Phi(\overline{\omega};\sigma)\right] \tag{17}$$

as the expected share of final output accruing to the lender, with $\overline{\omega}\Phi\left(\overline{\omega};\sigma\right)$ the share of final output lost due to monitoring. Competition between banks results in the break-even condition

$$g(\overline{\omega}_{it};\sigma_{it},\mu_t) = \frac{R_t}{\varepsilon^e_{it} q_t}\left(1-\frac{1}{\xi}\right). \tag{18}$$

where

$$\sigma_{it} \equiv \begin{cases} \sigma_{3t} & \text{if bank finance,} \\ \sqrt{\sigma_{2t}^2+\sigma_{3t}^2} & \text{if CMF finance.} \end{cases} \tag{19}$$

This is because the distribution of ω is either the distribution of $\varepsilon_{3,it}$ for bank finance or of $\varepsilon_{2,it}\ \varepsilon_{3,it}$ for capital market fund finance. We denote $\overline{\omega}_{it}$ as the minimal among all solutions to this equations and write it as

$$\overline{\omega}_{it} \equiv \begin{cases} \overline{\omega}^c(\varepsilon_{1,it}\varepsilon_{2,it};q_t,R_t,\sigma_{2t},\sigma_{3t}) & \text{if CMF finance} \\ \overline{\omega}^b(\varepsilon_{1,it};q_t,R_t,\sigma_{3t}) & \text{if bank finance} \end{cases} \tag{20}$$

It is easy to see that $\overline{\omega}_{it}$ is increasing in R_t and decreasing in ε^e_{it} and q_t.

If the entrepreneur has approached a bank for a loan, he has learned $\varepsilon_{2,it}$ and needs to decide whether to proceed with a loan or abstaining, by comparing his expected share of output when proceeding with a loan to the opportunity cost of holding the remaining net worth to the end of the period. The former is given by $F^d(\varepsilon_{1,it},\varepsilon_{2,it};q_t,R_t,\sigma_{3t})\hat{n}_{it}$, where

$$F^d(\varepsilon_1,\varepsilon_2;q,R,\sigma_3) = \varepsilon_1\varepsilon_2 q f(\overline{\omega}^b(\varepsilon_1\varepsilon_2;q,R,\sigma_3);\sigma_3)\xi \tag{21}$$

The entrepreneur will therefore proceed with the loan, if that second-phase value $\varepsilon_{2,it}$ exceeds a threshold $\varepsilon_{2,it} \geq \overline{\varepsilon}^d_{it} = \overline{\varepsilon}_d(\varepsilon_{1,it};q_t,R_t,\sigma_{3t})$, which satisfies

$$1 = F^d(\varepsilon_{1,it},\overline{\varepsilon}^d_{it};q_t,R_t,\sigma_{3t}). \tag{22}$$

Recall that an entrepreneur with linear preferences will pursue production, if he expects his net worth to increase as a result. In stage I and in light of $\varepsilon_{1,it}$ as well as aggregate information, the entrepreneur chooses whether or not to obtain a loan, and if so, whether to obtain it from a bank or from a capital market fund. Denote $\Phi(d\varepsilon_i) = \varphi\left(\varepsilon_i;\sigma_i\right)d\varepsilon_i$, for $i = 1,2,3$. The expected payoff for an entrepreneur, who proceeds with bank finance conditional on the realization of $\varepsilon_{1,it}$, is $F^b(\varepsilon_{1,it};q_t,R_t,\tau_t,\sigma_{2t},\sigma_{3t})n_{it}$, where

$$F^b(\varepsilon_1;q,R,\tau,\sigma_2,\sigma_3) \equiv (1-\tau)\left(\int_{\overline{\varepsilon}_d(\varepsilon_1;q,R,\sigma_3)} F^d(\varepsilon_1,\varepsilon_2;q,R,\sigma_3)\Phi(d\varepsilon_2) + \Phi(\overline{\varepsilon}_d(\varepsilon_1;q,R,\sigma_3);\sigma_2)\right) \tag{23}$$

is the expected payoff for each unit of net worth from either proceeding with a bank loan or abstaining, after learning ε_2. The expected payoff for an entrepreneur, who proceeds with CMF finance conditional on the realization of $\varepsilon_{1,it}$, is $F^c(\varepsilon_{1,it}; q_t, R_t, \sigma_{2t}, \sigma_{3t})n_{it}$, where

$$F^c(\varepsilon_1; q, R, \sigma_2, \sigma_3) \equiv \varepsilon_1 q f(\overline{\omega}^c(\varepsilon_1; q, R, \sigma_2, \sigma_3))\xi. \tag{24}$$

Finally, the expected payoff for an entrepreneur, who abstains from production, is n_{it}. Per $\varepsilon_{1,it}$, each entrepreneur chooses his best option and overall payoff $F(\varepsilon_{1,it}; q_t, R_t, \tau_t, \sigma_2, \sigma_3)n_{it}$, where

$$F(\varepsilon_1; q, R, \tau, \sigma_2, \sigma_3) \equiv \max\{1; F^b(\varepsilon_1; q, R, \tau, \sigma_2, \sigma_3); F^c(\varepsilon_1; q, R, \sigma_2, \sigma_3)\}. \tag{25}$$

We assume that $(A1)\, \frac{\partial F^b(\cdot)}{\partial \varepsilon_1} \geq 0$ and $(A2)\, \frac{\partial F^b(\cdot)}{\partial \varepsilon_1} < \frac{\partial F^c(\cdot)}{\partial \varepsilon_1}$, for all ε_1. Under (A1), a threshold for ε_1, below which the entrepreneur decides not to raise external finance, exists and is unique. We denote it as $\overline{\varepsilon}_{bt}$. It is implicitly defined by the condition

$$F^b(\overline{\varepsilon}_{bt}; q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}) = 1. \tag{26}$$

The unique cutoff point is a function of aggregate variables only, $\overline{\varepsilon}_{bt} = \overline{\varepsilon}^b(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t})$, and hence is identical for all firms. Under (A1) and (A2), there exists a unique threshold $\overline{\varepsilon}_{ct}$ for $\varepsilon_{1,it}$ above which entrepreneurs sign a contract with the CMF, implicitly defined by

$$F^b(\overline{\varepsilon}_{ct}; q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}) = F^c(\overline{\varepsilon}_{ct}; q_t, R_t, \sigma_{2t}, \sigma_{3t}) \tag{27}$$

and thus identical across firms, $\overline{\varepsilon}_{ct} = \overline{\varepsilon}^c(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t})$.

Conditional on q_t, R_t, τ_t, σ_{2t} and σ_{3t}, entrepreneurs split into three sets that are intervals in terms of the first idiosyncratic productivity shock $\varepsilon_{1,it}$. Denote as s_t^a, s_t^b, s_t^c and s_t^{bp} respectively the shares of firms that abstain from producing, approach a bank, raise CMF finance, and produce conditional on having approached a bank. We have that

$$s_t^a = \Phi\left(\overline{\varepsilon}^b\left(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}\right); \sigma_{1t}\right) \tag{28}$$

$$s_t^b = \Phi\left(\overline{\varepsilon}^c(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}); \sigma_{1t}\right) - \Phi\left(\overline{\varepsilon}^b\left(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}\right); \sigma_{1t}\right) \tag{29}$$

$$s_t^c = 1 - \Phi\left(\overline{\varepsilon}^c(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}); \sigma_{1t}\right) \tag{30}$$

$$s_t^{bp} = \int_{\overline{\varepsilon}^b(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t})}^{\overline{\varepsilon}^c(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t})} \int_{\overline{\varepsilon}^d(\varepsilon_1; q_t, R_t, \sigma_{3t})} \Phi(d\varepsilon_2)\Phi(d\varepsilon_1). \tag{31}$$

Because the return on internal funds is always higher than the rate of time preference, entrepreneurs accumulate wealth and only consume before dying. It follows that in the aggregate, entrepreneurs consume each period a fraction γ of their accumulated wealth. Denote

$\varkappa_t \equiv [q_t, R_t, \tau_t, \sigma_{1,t}, \sigma_{2,t}, \sigma_{3,t}]$. Entrepreneurial consumption and capital are given by

$$e_t = (1-\gamma)\,\psi^f(\varkappa_t)\,n_t, \tag{32}$$

$$z_{t+1} = \gamma\psi^f(\varkappa_t)\,n_t, \tag{33}$$

where $\psi^f(\varkappa_t)\,n_t$ are aggregate profits of the entrepreneurial sector, and the expression for $\psi^f(\varkappa_t)$ is reported in section B of the technical appendix.

For comparison to the data, the following calculations are useful. The loan rate R^l_{it}, defined as the nominal interest rate that is charged for the use of external finance, is given by

$$R^l_{it} = \varepsilon^e_{it} q_t \overline{\omega}_{it} \frac{\xi}{\xi - 1}, \tag{34}$$

and the spread between the lending rate and the risk free rate for a firm i, is given by

$$\Lambda_{it} = \frac{R^l_{it}}{R_t} - 1. \tag{35}$$

4.4 Aggregation and market clearing

Aggregate demand for funds, x_t, output y_t, and output lost to agency costs y^a_t are given by:

$$x_t = \left[(1-\tau_t)s^{bp}_t + s^c_t\right]\xi n_t \tag{36}$$

$$y_t = \psi^y(\varkappa_t)\,\xi q_t n_t \tag{37}$$

$$y^a_t = \left[\tau_t s^b_t + \psi^m(\varkappa_t)\,\mu\xi q_t\right] n_t, \tag{38}$$

where the function $\psi^y(\varkappa)$ aggregates the realized productivity factors across all producing firms. The terms $\tau_t s^b_t$ and $\psi^m(\varkappa_t)\,\mu\xi q_t$ measure the loss of resources due respectively to bank information acquisition and to monitoring costs, per unit of net worth. These functions are also defined in section B of the technical appendix.[6]

Aggregate factor demands are given by aggregating (11) and (12) across i. Aggregate investment I_t is given by $I_t = k_{t+1} + z_{t+1} + (1-\delta)(k_t + z_t)$. Market clearing for money, assets,

[6] In the appendix, we also provide analytical expressions for the aggregate financial variables used in the numerical analysis, i.e. the ratio of bank finance to bond finance, ϑ_t, the average spread for bank-financed firms, Λ^b_t, and for CMF-financed firms, Λ^c_t, the aggregate debt to equity ratio, χ_t, the default rate on corporate bonds, ϱ^c_t, the average default across firms, ϱ_t, and the net expected return to entrepreneurial capital, r^z_t (section C). We then collect the equations that characterize a competitive equilibrium (section D), characterize the steady state and describe the procedure we use to compute it (section E). Finally, we show how to log-linearize the equilibrium conditions around the nonstochastic steady state (section F). A particular challenge arises from the need to aggregate across firms and from the presence of endogenously evolving regions of integration.

labor and capital requires that $M_t^s = M_t + D_t$, $B_t = 0$, $K_t = k_t + z_t$ and $H_t = l_t$, respectively; those for loans and output require that

$$D_t = P_t \left[(1-\tau_t) s_t^{bp} + s_t^c \right] (\xi - 1)\, n_t, \tag{39}$$

$$y_t^a = y_t - c_t - e_t - K_{t+1} + (1-\delta) K_t. \tag{40}$$

5 Results

We investigate the ability of the model to qualitatively and quantitatively account for the key facts, see section 2. We then use the model to evaluate the importance of financial flexibility for aggregate activity. The dynamics of the system is solved, using log-linearization and Uhlig (1999)'s toolkit, but considerably extending the realm of applying these methods. For example, output is given as an integral across the productivity distribution of firms who have received financing. In response to some of the shocks, the density of that distribution changes as do the boundaries of the integral. The appropriate coefficients for the linearized dynamics are obtained from per appropriate differentiation. The analysis of a risk shock requires log-linearizing the equilibrium conditions of our model with respect to the standard deviations of the idiosyncratic productivity shocks, see also Christiano, Motto and Rostagno (2010). Challenges arise, because the standard deviations also affect the boundaries $\bar{\varepsilon}_{b,t}$, $\bar{\varepsilon}_{c,t}$ and $\bar{\varepsilon}_{d,t}$. Details are available in section F of the technical appendix.

5.1 Calibration

The model is calibrated in line with the long-run evidence for the euro area documented in DFU. The procedure delivers a set of parameter values that differs from those in DFU because of the differences among the two models, see section 3.

One period is a quarter. We set the discount factor at $\beta = .99$ and the depreciation rate at $\delta = .02$. The Frisch elasticity[7] is $\kappa = 3$. The inflation rate is 0.5 percent per quarter in line with the euro area average over the period 1999-2010, implying a nominal risk-free rate $R = 1.015$. We set $\alpha = .64$. We normalize consumption to equal unity and calculate η to be consistent with that in steady state. We set $\mu = .15$, a common value in the related literature. The iid productivity shocks $v = \varepsilon_1, \varepsilon_2, \varepsilon_3$ are lognormally distributed. Also, $\log(v)$ is normally distributed with mean $-\sigma_v^2/2$ and variance σ_v^2, so that $E(v) = 1$.

[7] See e.g. Peterman (2012) for an overview of the available empirical estimates of the Frisch elasticity.

We choose values of $\xi, \tau, \gamma, \sigma_{\varepsilon_1}, \sigma_{\varepsilon_2}$ and σ_{ε_3} that jointly minimize the squared log-deviation of the model-based predictions from their empirical counterparts for the following six financial facts:[8] i) a ratio of aggregate bank loans to debt securities for non-financial corporations (measured in transactions), ϑ, of 5.5; ii) a ratio of aggregate debt to equity, χ, of .64; iii) an annual average spread on debt securities, Λ^c, of 143 bps; iv) an annual average spread on bank loans, Λ^b, of 119 bps; v) an annual default rate on debt securities, ϱ^c, of 5 percent; vi) an expected return to entrepreneurial capital, r_t^z, of 9.3 percent. The parameter values selected from our procedure are $\tau = .01$, $\gamma = .977$, $\xi = 3.19$, $\sigma_{\varepsilon_1} = .017$, $\sigma_{\varepsilon_2} = .023$, $\sigma_{\varepsilon_3} = .171$. The stochastic processes for $\tau_t, \varepsilon_{v,t}, \sigma_{\varepsilon_2,t}$ and $\sigma_{\varepsilon_3,t}$ are assumed to have a persistence of 0.95.

5.2 Numerical analysis

We seek to account for the observed fall in bank loans relative to debt securities, the simultaneous rise in the cost of market finance and bank finance, with the former increasing above the latter, and the sharp increase in the default rate, see section 2. We conjecture that the shift was induced by a positive shock to the bank information acquisition costs τ_t, reducing the efficiency of banks as financial intermediaries. The shock can be seen as capturing the difficulties in raising liquidity faced by euro area banks in 2008-2009 as well as a decrease in the efficiency with which banks evaluate projects, having perhaps lost some of their confidence in procedures used up to that point.

To understand the responses to a shock in τ_t, it is useful to consider how a permanent increase in τ affects firms' financing choices and spreads in the steady state. An increase in τ induces a change in the expected profits for firms that approach a bank, $F^b(\varepsilon_1; q, R, \tau, \sigma_2, \sigma_3)$. It makes it less attractive to approach a bank and obtain additional information on ε_2, before deciding whether or not to produce and raise external finance. From equations (26) and (27), it follows that $\bar{\varepsilon}_b$ increases, i.e. more firms abstain altogether, and $\bar{\varepsilon}_c$ decreases, i.e. more firms seek market finance rather than bank finance. Since the bank fee τ is a sunk cost, equation (22) shows that the level of τ does not affect firms' choice of proceeding with production, conditional on having approached a bank and learning ε_2.

Figure 3 encapsulates this intuition. It plots the effect of a permanent increase in τ on the share of firms choosing to abstain, to approach a bank and wait, and to raise CMF finance

[8] For each variable, the target is given by the average value computed over the longest post-EMU, pre-crisis period on which the data are available.

and produce. The black solid line shows the density function $\varphi(\varepsilon_1;\sigma_1)$. The red and purple dashed lines show respectively the threshold for bank-finance, $\overline{\varepsilon}_b$, and for CMF finance, $\overline{\varepsilon}_c$, when τ equals its initial value. The green and pink dashed-dotted lines show those thresholds when τ increases by 40 percent.

At the initial value of τ, firms experiencing a realization of ε_1 at the left of the red dashed line find it optimal to abstain from production and retain their net worth. Their risk of default at the end of the period in case of production is too high. Firms experiencing a value of ε_1 between $\overline{\varepsilon}_b$ and $\overline{\varepsilon}_c$ rather find it optimal to raise external finance from banks. Their risk of default is sufficiently high that the "wait and see" option provided by banks compensate the extra-fee being charged. Only firms at the right of $\overline{\varepsilon}_c$ are safe enough to choose CMF finance. As discussed, the thresholds $\overline{\varepsilon}_b$ and $\overline{\varepsilon}_c$ shift inwards for the higher value of τ.

Because the average creditworthiness (as measured by the realization of the first shock, ε_1) of CMF-financed firms falls, the average spread on bonds rises. The average spread on bank finance can increase or fall, depending on parameter values. Under the selected parameterization, the reduction in average creditworthiness due to some firms with high ε_1 moving to CMF-finance just more than compensates the improved risk prospects due to firms with low ε_1 moving out of banking. Overall, the average spread increases more for bonds than for loans.

5.3 The response to a temporary decrease in bank efficiency

We compute the response of the economy to a temporary shock and increase of τ_t by 2.5 percent. This is calibrated to generate a fall on impact of the ratio of loans to bonds of 5.5 percent, in line with the peak effect observed during the crisis. Figure 4 shows that the response of the economy is qualitatively consistent with the evidence. As the cost of information acquisition increases, firms move away from bank finance. A larger share of firms facing low realizations of ε_1 find the cost of external finance too high, and choose to abstain from production. A larger share of firms experiencing high realizations of ε_1 find the flexibility provided by banks too costly, and decides to issue bonds instead. The ratio of bank loans to corporate bonds falls.

As in the data, the cost of bond finance rises to a greater extent than the cost of bank finance. The former unambiguously increases because the pool of CMF-financed firms now presents a higher average risk of default. The latter instead moves by very little because the share of firms with low risk of default that move from bank-finance to CMF-finance compensates for the share of firms with high risk of default that moves out of banking and abstains.

The shock increases the default rate on bonds, as observed during the crisis, while the debt to GDP ratio remains broadly constant. More frequent bankruptcies for CMF-financed firms result from the larger spread on bonds. The aggregate debt to GDP remains constant because the reduction in corporate debt matches a similar reduction in aggregate production.

The real effects of the shock to bank costs have two sources. The first is the reduction in the fraction of producing firms, as more firms decide not to approach a financial intermediary (*share abstain* increases) and a larger share of bank-financed firms decide to drop out after obtaining information on ε_2 (the share of firms that decide to produce conditional on banking, *share bank/produce,* falls). The second is the increase in the markup q_t, which reflects the larger financial distortion induced by higher bank intermediation costs. This latter reduces - *ceteris paribus* - real wages and the remuneration of capital, with an adverse impact on households' labor and capital accumulation decisions.

Qualitatively, these responses all point in the right direction for thinking about the financial crisis. Nonetheless, the spreads on bonds and on loans, the default rates, as well as consumption, investment and output, move very little compared to the crisis observations, as summarised in section 2. Our model predicts that, when firms can freely adjust their debt structure, a shock that affects bank efficiency and shifts the composition of debt finance as observed during the crisis, does not produce sizeable effects on aggregate activity. We draw the lesson that flexibility in the financial system can substantially mitigate the overall economic impact of distress in the financial sector on overall economic performance. We furthermore conclude, that additional shocks played a key role, and that it was the confluence of events responsible for the large decline in economic activity.

5.4 The response to an increase in bank costs and uncertainty

In our model, the impact of a negative technology shock is similar to the impact in simpler benchmark real business cycle models, but it reduces the demand for external finance, the markup q, the share of firms seeking finance, and thus finally the default rates and the spreads, at odds with the key corporate finance facts of section 2. A monetary policy shock of increasing the expected monetary growth rate induces agents to demand an extra return on deposits increases the cost of external finance, the share of abstaining firms, and finally the ratio of bank finance to bond finance, again at odds with the evidence.

We therefore seek out shocks to uncertainty as additional and promising candidates to explain the observed facts. Empirical evidence shows that uncertainty dramatically increases after major economic and political shocks. During the 2008-2009 crisis, Bloom (2009) documents that the standard deviation of a monthly U.S. index of stock market volatility jumped from 10 percent to around 50 percent. Risk can be captured by a range of alternative measures, but they generally tend to move together (Bloom, Bond and Van Reenen (2007)). Shocks to uncertainty have also been found to be relevant drivers of business cycle fluctuations in DSGE models estimated on both US and euro area data (see e.g. Christiano, Motto and Rostagno (2014)).

To provide a fuller quantitative account for the key facts observed during the crisis, we shall therefore appeal to two additional shocks, aside from the shock to bank efficiency τ_t. , we add two shocks to the firm-specific risks. The first is an increase in the uncertainty faced by bank-financed firms, i.e. an increase in the standard deviation σ_{ε_2} of ε_2. This relates to the funding difficulties of the U.S. sub-prime market at the beginning of the crisis. The second is an increase in the uncertainty faced by all debt-financed firms, i.e. an increase in the standard deviation σ_{ε_3} of ε_3. This latter shock reflects the observed increase in stock market volatility.

The three shocks, τ, σ_{ε_2} and σ_{ε_3}, are jointly calibrated to broadly match the observed peak effect in the ratio of loans to bonds (a reduction of 5.5 percent), the spread on bonds (an increase of 120 percent) and the spread on loans (an increase of 104 percent). This requires an increase in τ by 123 percent, in σ_{ε_2} by 50 percent and in σ_{ε_3} by 17 percent.

The green solid line in figure 5 shows the response of the economy to the three combined shocks. By raising the uncertainty faced by all producing firms and thus their default risk, the increase in σ_{ε_3} contributes to generate large effects on spreads and default rates. A joint increase in τ and σ_{ε_2} generates the desired fall in the ratio of loans to bonds, together with a differential impact of higher uncertainty on the spreads on bonds and on loans. In our model, an increase in τ reduces the desirability of bank finance for firms, while an increase in σ_{ε_2} makes the disclosure of additional information provided as a service by banks more valuable. When the uncertainty faced by bank-financed firms increases, a large counteracting increase in bank cost is needed in order to induce the same fall in the ratio of loans to bonds. The increase in τ also generates an increase in the spread on bonds above the increase in the spread on loans, as discussed above. The reason is that the shift from bank finance to bond finance raises relatively more the average default risk of the pool of CMF-financed firms.

A combination of these three shocks generates responses similar to those observed during the crisis. The larger σ_{ε_3} increases the probability of extreme realizations of the productivity shock ε_3. This, together with the higher information acquisition cost, τ, induces some good firms to move from bank finance to bond finance, despite the contemporaneous increase in uncertainty about the second shock, σ_{ε_2}, which raises the attractiveness of banks as intermediaries. As a consequence, the share of CMF-financed firms (*share CMF*) increases. The share of firms that abstain conditional on observing ε_1 (*share abstain*) also rises because the high bank costs more than compensate for the increase in σ_{ε_2}. A lower share of risky firms prefer to pay the information acquisition cost and obtain additional information about potentially very high ε_2 (*share bank* falls). However, because the distribution of ε_2 has fatter tails, a larger share of firms experiences sufficiently large realizations of ε_2, and thus decides to raise bank loans and produce (*share bank/produce* increases). The high overall uncertainty explains the larger increase in default rates relative to the case with the τ shock only. The model predicts that the debt to GDP ratio slightly falls on impact. In the data it first kept increasing along its pre-crisis trend, before stabilizing to a constant (below trend) level.

The combination of the three shocks delivers responses that are also quantitatively in line with the peak effects documented in section 2. The spread on bonds increases by 120 percent, more than the spread on loans (which rises by 104 percent). The ratio of loans to bonds falls by 5.5 percent. The default rate on bonds increases by around 110 percent. The contraction of output and investment to GDP is still mild (0.2 and 0.1 percent, respectively) but ten times larger than under the τ shock only.

The impulse responses shown in Figure 5 require large shocks to bank costs and firms' idiosyncratic volatility. These magnitudes, however, are not far from those suggested by available empirical evidence. For instance, during the financial crisis, the component "total operating expenses" of the consolidated pre-provisioning profits of the euro area monetary and financial institutions increased by 85 percent relative to its averge over the period 2002-2010 (Financial Stability Review (2011)). These are expenses that arise during the ordinary course of running business for banks and can be interpreted as a measure of τ in our model.

Concerning the shocks to idiosyncratic volatility, Gilchrist et al (2010) provide estimates of firms' time-varying idiosyncratic uncertainty. The measure is constructed from daily firm-level data on stock returns for all U.S. nonfinancial corporations with a minimun of 5 years of trade. After the Lehman collapse in 2008, that measure increased by around 180 percent.

Under this crisis scenario, our model generates small real effects. These results are in line with the findings in Basu and Bundick (2012) that only non-competitive, one-sector models with countercyclical markups through sticky prices can generate large simultaneous drops in output and investment, in response to changes in uncertainty. In our framework, it would be possible to obtain larger real effects of financial shocks by adding nominal or real rigidities, or obtain additional real responses from technology shocks or monetary policy shocks. A surprise increase in the monetary growth rate leads to a substantial drop in investment and output in this model, due to the increase of the costs of external finance via depositors and the absence of a liquidity effect. The model would deliver larger real effects under the assumption that firms borrow to finance investment or the purchase of the entire capital stock - as often assumed in this literature - rather than to cover their operating costs. Finally, the prolonged crisis in Europe beyond 2010 is surely due to factors such as concerns about the sustainability of public finances and public debt, and country-specific developments rather than the financial crisis of 2008: one could extend the current model incorporating such features. While it would be possible to pursue one of more of these avenues, our aim here is rather to disentangle the role of debt substitutability in the transmission of financial shocks to real economy. Our results suggest that, when firms are able to flexibly substitute among alternative debt instruments, negative credit supply shocks have per-se a limited impact on investment and output.

5.5 Exogenous thresholds

We evaluate the importance for the aggregate economy of firms' ability to shift among alternative instruments of external finance. We do so by comparing the impulse responses to the combined shocks described above when firms can optimally choose whether to raise loans or issue securities, to those that would arise under two alternative scenarios.

The first is the case when bond finance is not available, which we capture by setting the steady state level of the threshold for bond finance, $\bar{\varepsilon}_b$, to an arbitrarily large value. The dashed figure 5 shows the results and compares them to the benchmark case represented by the solid green line. The effect of the combined shocks on most variables - and particularly on investment and output - is largely unchanged.

This result seems at odds with the idea that capital markets provide a "spare tyre" to the financial system, which can mitigate the adverse effects of financial crises. But it is not hard to understand this outcome. Firms for which the absence of the bond market is a relevant constraint are those with highest ex-ante productivity and lowest risk of default. For those firms, the optimal choice is therefore to raise external finance in the form of bank loans and to produce. This constrained choice entails a resource loss due to the information acquisition cost that firms are forced to pay. Nonetheless, the lack of substitutability also provides some advantages. Firms are given the flexibility to step out of production, an option which becomes more valuable when uncertainty about productive prospects increases - as in the scenario of combined shocks that we consider. Because σ_{ε_2} is higher, there is a larger incidence of high realizations of ε_2 for firms that approach banks. This explains why a larger share of firms now decides to raise loans and produce. As a consequence, the ratio of loans to bonds increases despite the reduction in net worth due to higher information acquisition costs. Overall, forcing firms to use bank finance also in reaction to a shock that deteriorates bank efficiency does not induce larger real effects than in the case when firms can substitute among debt instruments.

Considerably more devastating is a scenario where bond finance is not available *and* where banks are not able to provide the same flexibility on the terms of their loans, which one may view as resulting from the upheaval and confusion generated inside banks by the financial crisis. We capture this scenario by setting $\overline{\varepsilon}_b$ to an arbitrarily large value and by fixing the threshold for decisions in the second stage of production at its steady state value, $\overline{\varepsilon}_d$. This implies that some firms do not choose optimally whether to raise loans and produce, or whether they should abstain, after learning their ε_2. We do not fix the threshold $\overline{\omega}_t$, so the financial contract remains optimal. Figure 6 shows the results (the red dotted line) and compares them to our benchmark case described in figure 5 (the green solid line). Notice that the share of firms producing under banking moves despite $\overline{\varepsilon}_d$ being fixed. This movement is due to the endogeneity of the threshold for banking, $\overline{\varepsilon}_{bt}$, which adjusts optimally to aggregate conditions and affects the share of firms that decide to produce under banking. The ratio of loans to bonds falls slightly reflecting the overall decrease in the number of firms that approach a bank and decide to produce.

The negative effect of the combined shocks on real activity is exacerbated relative to the case when firms have full flexibility in financial decisions. The effects are nonetheless more short-lived, indicating a trade-off between depth and length of the recession. These results are due to the different response in the markup, which reflects the average financial distortion. In the absence of CMF-finance and bank flexibility, the markup increases substantially on impact - due both to the increase in information costs and to the exogeneity of the threshold for giving up production at banks. This has two opposite effects on real activity. On the one hand, it exacerbates the reduction in the real wage and rental rate on capital, reducing the supply of labor and demand for capital, with adverse effects on investment and output. On the other hand, a larger q_t increases the profits of producing firms and speeds up their accumulation of net worth. As firms are able to regain access to credit more quickly than in the benchmark economy, the adjustment of real activity back to the steady state is faster.

6 Conclusions

Our model points to an important role played by the composition of corporate debt in determining the response of real activity during the crisis. When firms have no access to the bond market, and banks cannot provide the flexibility needed by firms, the negative real effects of a shock to bank operating costs are amplified. These findings suggest that abstracting from an endogenous corporate debt structure and from the flexibility offered by financial intermediaries on existing contracts - an abstraction, which is generally done in models that assess the impact of financial shocks - may overstate the negative consequences of adverse shocks on real activity.

These results also suggest that the post-crisis policy debate in Europe needs to be broadened beyond banks and financial intermediaries, and needs to include considerations of shifts in firm financing from banks to capital markets. Notwithstanding the central role of banks for ensuring financial stability, policy measures aimed at achieving easier substitutability of bank loans for other instruments of external finance or generally more flexibility in funding markets may be equally important. Their differential dynamic impacts need to be carefully analyzed.

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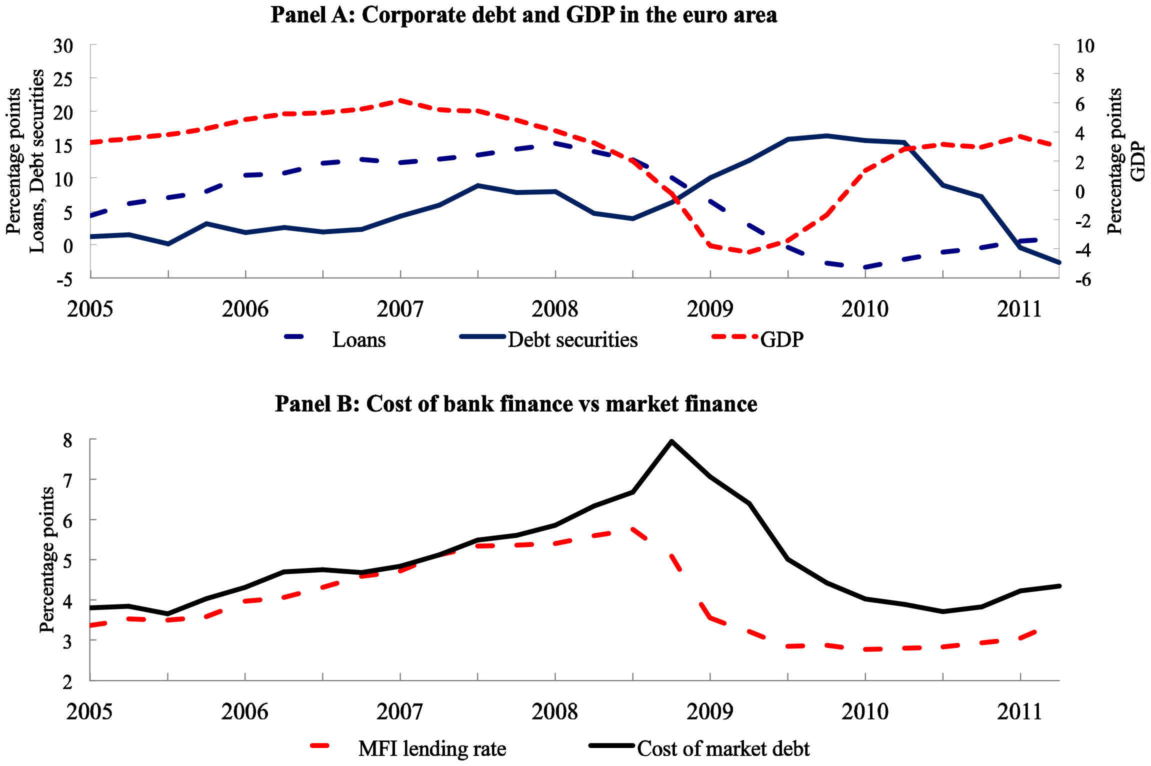

Figure 1: Corporate debt instruments and their costs during the financial crisis.

Note: Panel a shows the annualized growth rate of bank loans and debt securities (left hand scale, nominal values, outstanding amounts) and of GDP (right hand scale), in percentage points. Panel b shows measures of the nominal cost of market debt and of MFI loans. Notice that the shift away from loans towards debt securities starting in 2008, occurred at the time when the cost of market debt increased above the cost of bank loans. Source: ECB Statistical Data Warehouse, ECB calculations and ECB Area Wide Model database. See section A of the technical appendix for a detailed description of the data.

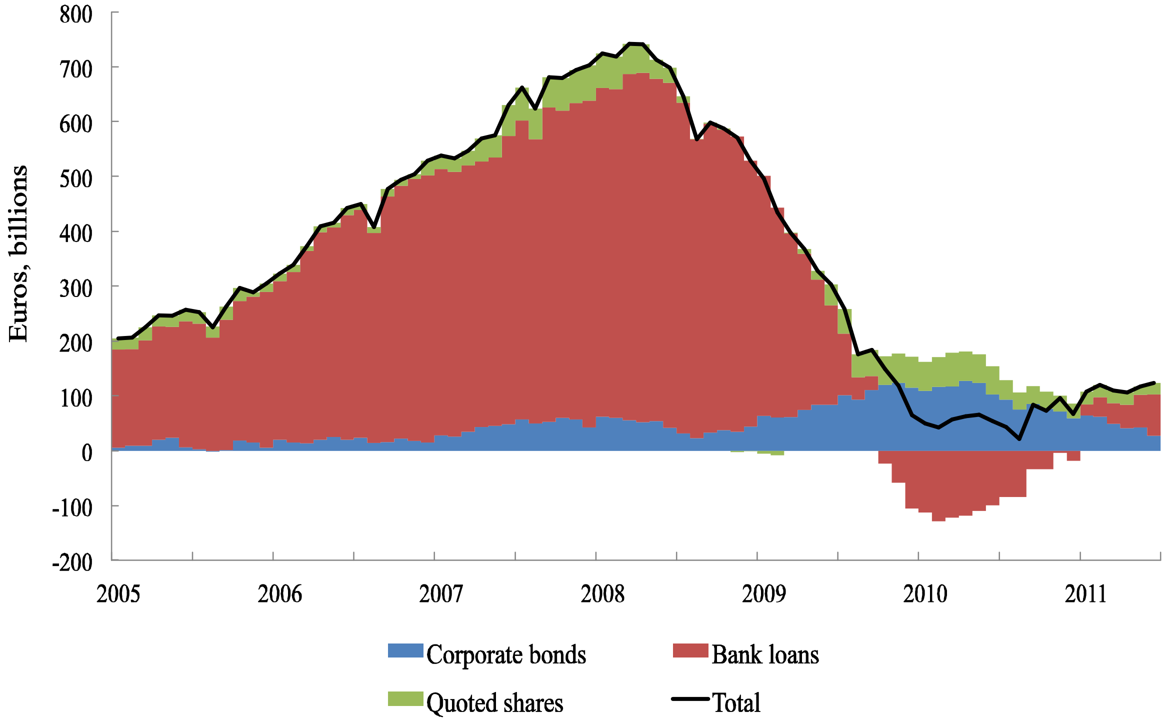

Figure 2: Sources of external finance for euro area non-financial corporations.

Note: 12-month cumulated flows for euro area non-financial corporations, in billions EUR. The figure shows that the substitution between bank finance and bond finance emerges as a noticeable feature of the financial crisis also when looking at cumulated flows rather than changes of outstanding amounts. While the increase in bond issuance during the crisis was insufficient to compensate for the contraction in the extension of bank loans, it is nonetheless an important valve to buffer the impacts of the crisis in the banking sector. Source: ECB Statistical Data Warehouse and Euro Area Balance of Payments and International Investment Position Statistics.

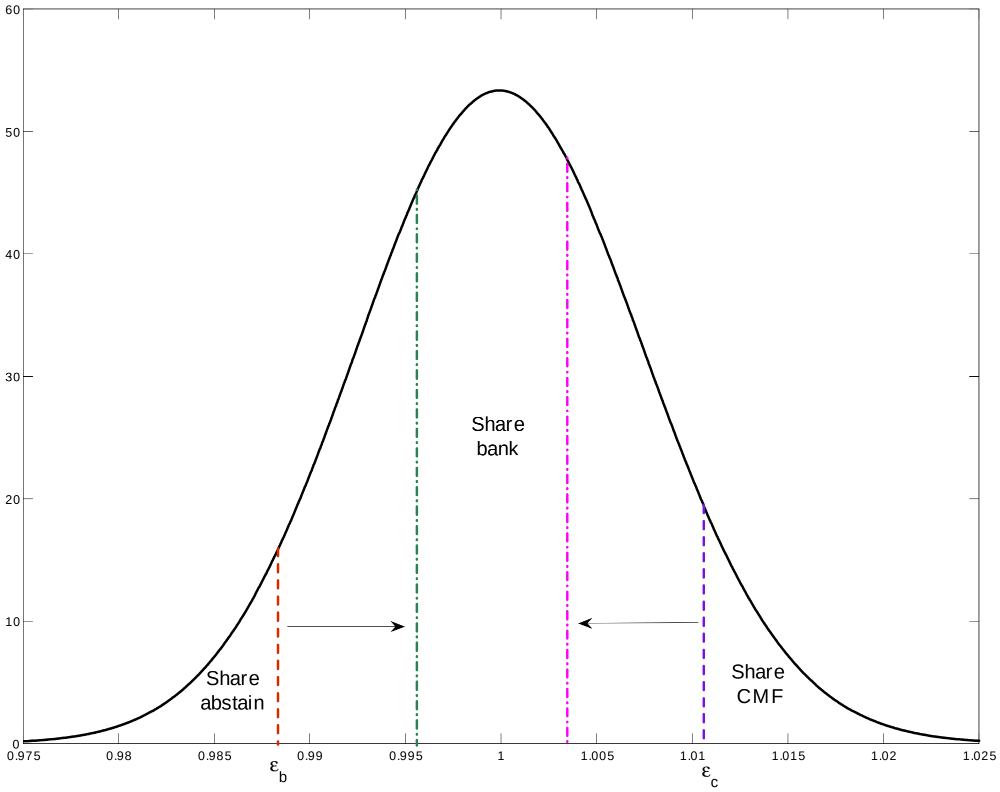

Figure 3: Impact on the steady state distribution of firms of an increase in information acquisition costs, τ.

Note: The red and purple dashed lines show respectively the threshold for bank-finance and the threshold for CMF finance, when the information acquisition cost parameter takes its initial value. The green and pink dashed-dotted lines show the same thresholds, now shifted inward, when that parameter is increased by 40 percent.

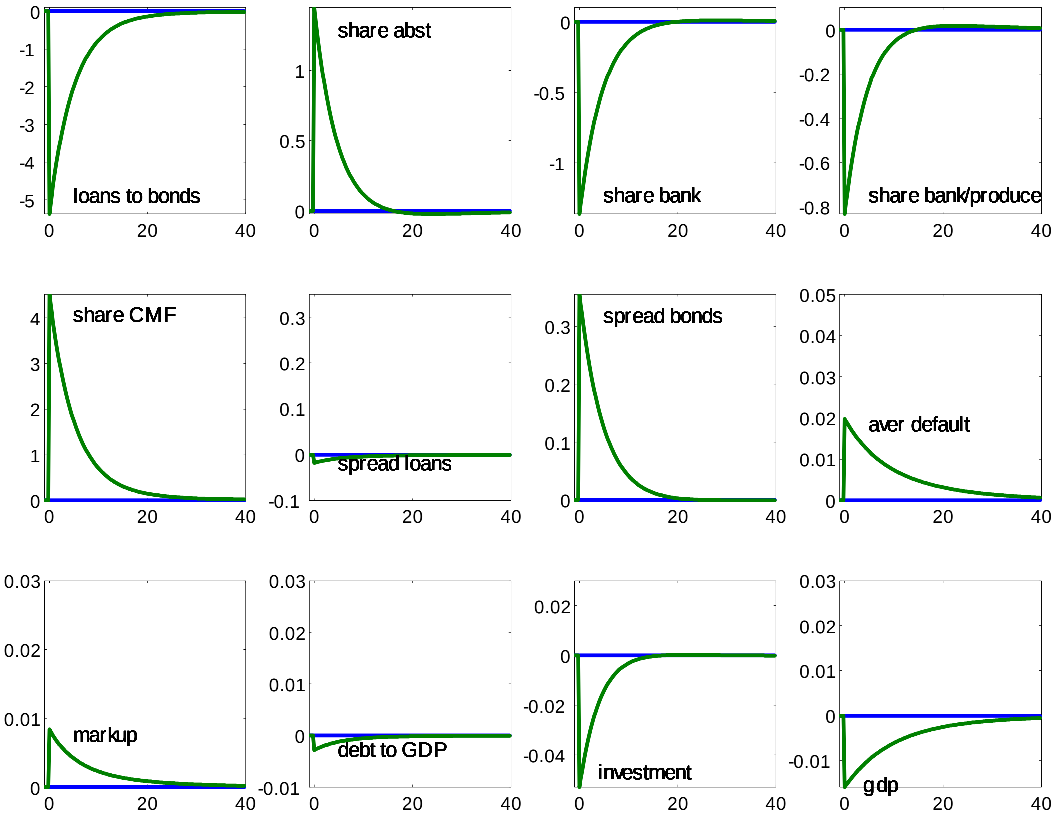

Figure 4: Impulse responses to an increase in banks' information acquisition costs, τ.

Note: In response to an increase in banks' information acquisition costs, some firms move out of bank finance and into capital market financing. As in the data, the cost of bond finance rises to a greater extent than the cost of bank finance.

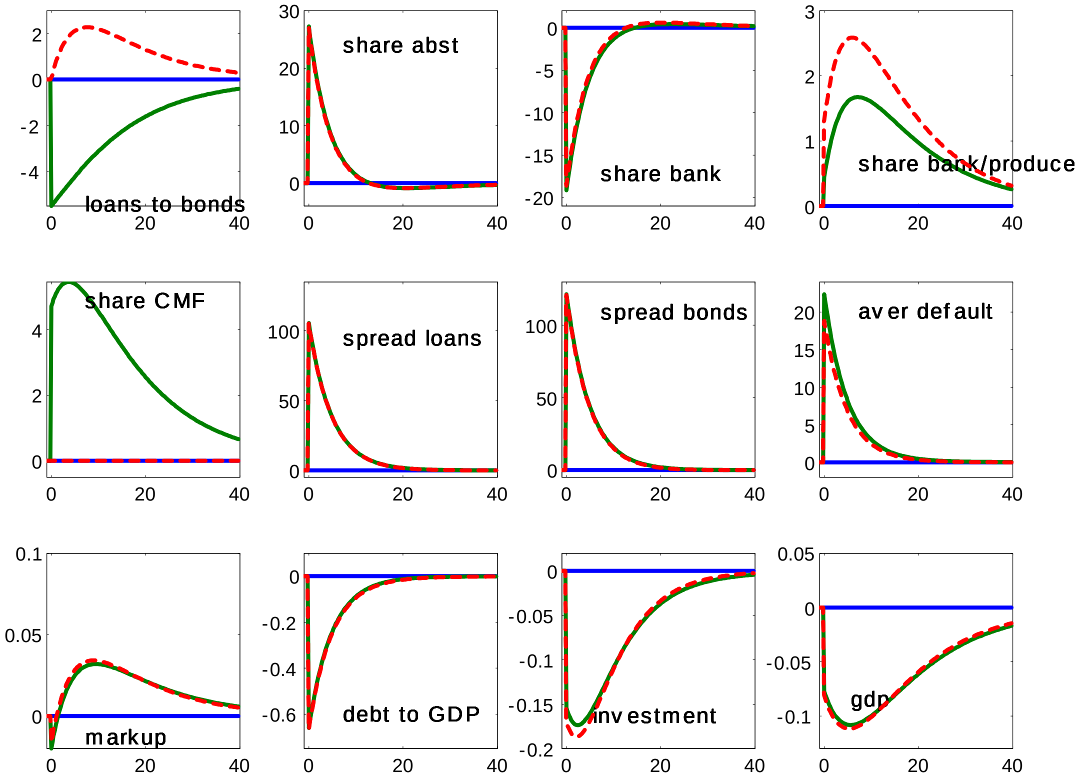

Figure 5: Impulse responses to a combined shock to bank costs (τ), the risk faced by bank-financed firms (σ_{ε_2}), and the risk faced by all debt-financed firms (σ_{ε_3}).

Note: The green solid line denotes the responses in the benchmark case when firms can substitute among bank loans and corporate bonds. The red dashed lines denote the case where firms have no access to bond finance.

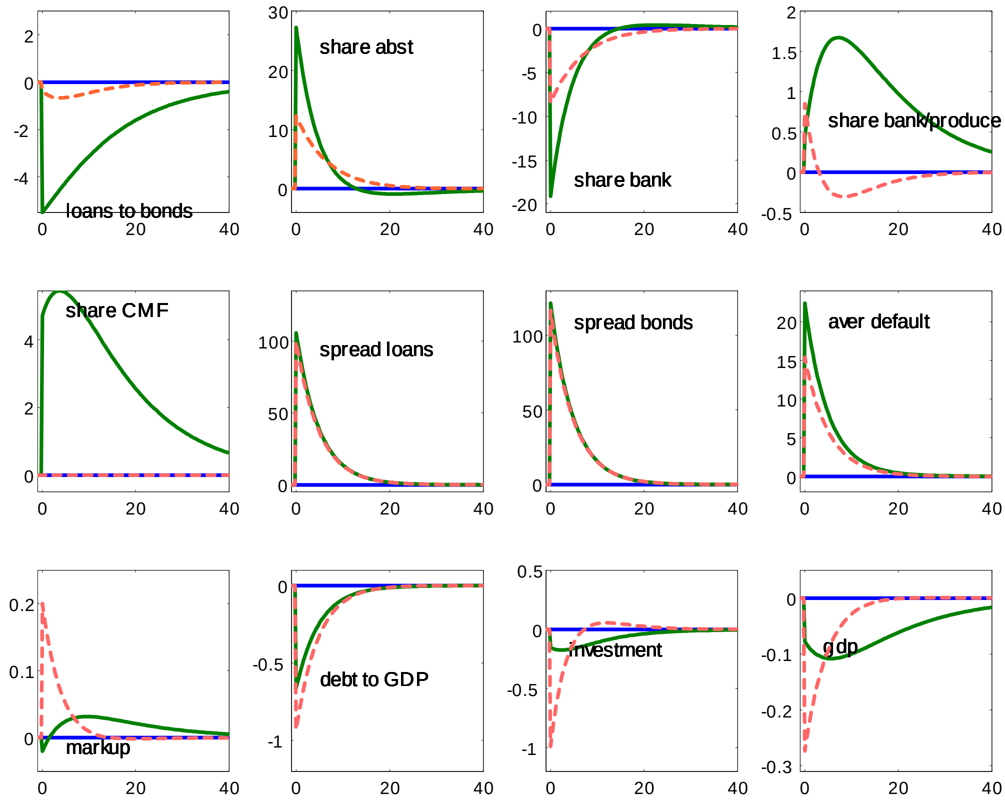

Figure 6: Impulse responses to the combined shock of figure 5, when the thresholds for firms' financial decisions are set exogenously.

Note: The red line shows the impulse responses to the combination of three shocks for a scenario where bond finance is not available and banks are not able to provide the same flexibility on the terms of their loans. The green line shows the benchmark case with perfect substitutability among loans and bonds, as in figure 5.

Appendix

A Data description

The peak effects reported in the paper have been computed using the data below, based on the period 1999Q1-2010Q2, unless stated below. Our choice of ending the sample on 2010Q2 is motivated by the fact that corporate spreads increased drastically after that date due to tensions in the sovereign market, an issue we don't address in our model.

Real GDP. GDP deflated using GDP deflator (reference year 1995), in millions of EUR. Seasonally adjusted. Source: ECB Area Wide Model database, update 12.

Investment. Gross investment, in millions of EUR. Source: ECB Area Wide Model database, update 12.

Bank loans. MFI loans with all maturities to the non-financial corporations sector in the euro area (changing composition), all currency combined, denominated in euro. MFIs exclude the ESCB reporting sector. Data are neither seasonally nor working day adjusted. Outstanding amounts and 12-months cumulated flows. ECB Statistical Data Warehouse.

Debt securities. Securities other than shares, excluding financial derivatives. Nominal value. Non-financial corporations issuing sector. All currencies combined, denominated in Euro. Euro area 17 (fixed composition). Outstanding amounts and 12-months cumulated flows. Source: ECB Statistical Data Warehouse.

Corporate debt to GDP ratio. Ratio of euro area non-financial corporations's debt to euro area GDP. Debt is in outstanding amount. Debt includes loans, debt securities issued and pension fund reserves. Source: Financial Stability Review Database.

Nominal cost of market debt. Measure based on a Merrill Lynch index of the average yield of corporate bonds with a maturity of more than one year issued by euro area NFCs with investment grade ratings, and a euro-currency high-yield index. National yields are aggregated using GDP weights corresponding to the purchasing power parities in 2001. The average duration of the corporate bonds is five years. Period 2003Q1-2010Q2. Source: ECB calculations.

Nominal cost of bank loans. MFI lending rates for new business loans to NFCs with maturities above 1 and up to 5 years, and amounts larger than 1 million EUR. Period 2003Q1-2010Q2. Source: ECB databank.

Risk free rate. Germany, Government Benchmarks, Public Debt Securities, 4-5 Years, Yield, Average, EUR. Period 2003Q1-2010Q2. Source: German Bundesbank.

Default rate on corporate bonds. Annual default rates for all non-financial corporations in Europe, period 1999-2010. Source: Standard & Poor's Global Fixed Income Research and Standard & Poor's CreditPro. The data refer to the occurrence of default for non-financial corporations that Standard & Poor's rated as of Dec. 31, 1980, or that were first rated between that date and Dec. 31, 2011. A default is recorded on the first occurrence of a payment default on any financial obligation, rated or unrated, other than a financial obligation subject to a bona fide commercial dispute. Our model does not distinguish firms in terms of ratings or size. Because rated firms are generally large and more likely to hit the bond market, the closest correspondant to these data in our model is the default rate on bonds.

Banks' total operating expenses. Expenses that arise during the ordinary course of running a business. Operating expenses consists of salaries paid to employees, research and development costs, legal fees, accountant fees, bank charges, office supplies, electricity bills, business licenses, and more. Annual observations, period 2002-2010. Source: ECB, Consolidated Banking Data database.

B Aggregating across firms

Aggregate profits of the entrepreneurial sector are given by $\psi^f(\varkappa_t)n_t$, where

$$\psi^f(\varkappa) \equiv \int F(\varepsilon_1; q, R, \tau, \sigma_2, \sigma_3)\Phi(d\varepsilon_1),$$

or, equivalently, by

$$\begin{aligned}\psi^f(\varkappa) = s^a \quad & + \int_{\overline{\varepsilon}_b(q,R,\tau,\sigma_2,\sigma_3)}^{\overline{\varepsilon}_c(q,R,\tau,\sigma_2,\sigma_3)} F^b(\varepsilon_1; q, R, \tau, \sigma_2, \sigma_3)\Phi(d\varepsilon_1) \\ & + \int_{\overline{\varepsilon}_c(q,R,\tau,\sigma_2,\sigma_3)} F^c(\varepsilon_1; q, R, \sigma_2, \sigma_3)\Phi(d\varepsilon_1).\end{aligned}$$

Entrepreneurial consumption and accumulation of capital can then be written as equations (32) and (33) in the text.

Define

$$\psi^y(\varkappa) = (1-\tau)\int_{\overline{\varepsilon}_b(q,R,\tau,\sigma_2,\sigma_3)}^{\overline{\varepsilon}_c(q,R,\tau,\sigma_2,\sigma_3)} \varepsilon_1 \int_{\overline{\varepsilon}_d(\varepsilon_1;q,R,\sigma_3)} \varepsilon_2 \Phi(d\varepsilon_2)\Phi(d\varepsilon_1) + \int_{\overline{\varepsilon}_c(q,R,\tau,\sigma_2,\sigma_3)} \varepsilon_1 \Phi(d\varepsilon_1) \tag{41}$$

and

$$\psi^m(\varkappa) = (1-\tau)\psi^{mb}(\varkappa) + \psi^{mc}(\varkappa),$$

where

$$\psi^{mb}(\varkappa) = \int_{\overline{\varepsilon}_b(q,R,\tau,\sigma_2,\sigma_3)}^{\overline{\varepsilon}_c(q,R,\tau,\sigma_2,\sigma_3)} \int_{\overline{\varepsilon}_d(\varepsilon_1;q,R,\sigma_3)} \Phi\left(\overline{\omega}^b(\varepsilon_1\varepsilon_2;q,R,\sigma_3);\sigma_3\right)\Phi(d\varepsilon_2)\Phi(d\varepsilon_1),$$

$$\psi^{mc}(\varkappa) = \int_{\overline{\varepsilon}_c(q,R,\tau,\sigma_2,\sigma_3)} \Phi\left(\overline{\omega}^c(\varepsilon_1;q,R,\sigma_2,\sigma_3);\sigma_2,\sigma_3\right)\Phi(d\varepsilon_1).$$

Then, total output, y_t, and total output lost to monitoring costs, y_t^a, are given by equations (37) to (38) in the text.

C Financial variables

We provide analytical expressions for financial variables used in the numerical analysis.

The ratio of bank finance to bond finance, ϑ_t, is defined as the ratio of the funds raised by bank-financed firms to the funds raised by CMF-financed firms, and is given by

$$\vartheta_t = \frac{(1-\tau_t)\, s_t^{bp}}{s_t^c}. \tag{42}$$

Recall that the credit spread for a firm i is given by

$$\Lambda_{it} = \frac{\varepsilon_{it}^e q_t \overline{\omega}_{it}}{R_t}\frac{\xi}{\xi-1} - 1.$$

Let $\psi^{rb}(\varkappa)$ and $\psi^{rc}(\varkappa)$ be

$$\psi^{rb}(\varkappa) = \int_{\overline{\varepsilon}_b(q,R,\tau,\sigma_2,\sigma_3)}^{\overline{\varepsilon}_c(q,R,\tau,\sigma_2,\sigma_3)} \int_{\overline{\varepsilon}_d(\varepsilon_1;q,R,\sigma_3)} \left[\frac{\left(\frac{\xi}{\xi-1}\right) q\varepsilon_1\varepsilon_2\overline{\omega}^b(\varepsilon_1\varepsilon_2;q,R,\sigma_3)}{R} - 1\right]\Phi(d\varepsilon_2)\Phi(d\varepsilon_1),$$

$$\psi^{rc}(\varkappa) \equiv \int_{\overline{\varepsilon}_c(q,R,\tau,\sigma_2,\sigma_3)} \left[\frac{\left(\frac{\xi}{\xi-1}\right) q\varepsilon_1\overline{\omega}^c(\varepsilon_1;q,R,\sigma_2,\sigma_3)}{R} - 1\right]\Phi(d\varepsilon_1).$$

The average spreads for bank-financed firms, Λ_t^b, and for CMF-financed firms, Λ_t^c, are then given by

$$\Lambda_t^b \equiv \frac{\psi^{rb}(\varkappa_t)}{s_t^{bp}}, \tag{43}$$

$$\Lambda_t^c \equiv \frac{\psi^{rc}(\varkappa_t)}{s_t^c}. \tag{44}$$

The debt to output ratio is the ratio of all debt instruments used by producing firms to aggregate output, y_t,

$$\chi_t = \left[(1-\tau_t)\, s_t^{bp} + s_t^c\right](\xi - 1)\,\frac{n_t}{y_t}. \tag{45}$$

The default rate on bonds, ϱ_t^c, is given by the share of firms which borrow from CMFs but cannot repay the debt,

$$\varrho_t^c = \frac{\psi^{mc}(\varkappa_t)}{s_t^c}. \tag{46}$$

The average default amounts to the share of firms which sign a contract with either a bank or a CMF but cannot repay the debt,

$$\varrho_t = \frac{\psi^{mb}(\varkappa_t) + \psi^{mc}(\varkappa_t)}{s_t^{bp} + s_t^c}. \tag{47}$$

Finally, we define the net expected return to entrepreneurial capital as

$$r_t^z = \psi^f(\varkappa_t)(1-\delta + r_t) - 1. \tag{48}$$

D Competitive equilibrium

For the convenience of further analysis, we collect the relevant equations here.

1. (a) Households:

$$m_{t+1} + d_{t+1} = \frac{R_{t-1}}{\pi_t} d_t + \theta_t \tag{49}$$

$$0 = m_{t+1} + w_t h_t + r_t k_t - c_t - k_{t+1} + (1-\delta)k_t \tag{50}$$

(b) Entrepreneurs:

$$n_t = (1-\delta + r_t) z_t \tag{51}$$

(c) Monetary authority:

$$\theta_t = \left(v_t^f + \nu - 1\right)\frac{m_{t-1}^s}{\pi_t} \tag{52}$$

$$m_t^s = \left(v_t^f + \nu\right)\frac{m_{t-1}^s}{\pi_t} \tag{53}$$

(d) Market clearing:

$$y_t^a = y_t - c_t - e_t - I_t \tag{54}$$

$$I_t = k_{t+1} + z_{t+1} + (1-\delta)(k_t + z_t) \tag{55}$$

$$m_t^s = m_t + d_t \tag{56}$$

$$d_t = \left[(1-\tau_t)s_t^{bp} + s_t^c\right](\xi - 1)\, n_t \tag{57}$$

(e) Production and aggregation:

$$x_t = \left[(1-\tau_t)s_t^{bp} + s_t^c\right]\xi n_t \tag{58}$$

$$y_t = \psi^y(\varkappa_t)\, q_t \xi n_t \tag{59}$$

$$y_t^a = \left[\tau_t s_t^b + \psi^m(\varkappa_t)\,\mu\xi q_t\right] n_t \tag{60}$$

2. First-order conditions.

(a) Household:

$$\eta h_t^{1/\kappa} c_t = w_t \tag{61}$$

$$\frac{1}{c_t} = \beta R_t E_t\left[\frac{1}{c_{t+1}\pi_{t+1}}\right] \tag{62}$$

$$\frac{1}{c_t} = \beta E_t\left[\frac{1}{c_{t+1}}(1-\delta + r_{t+1})\right] \tag{63}$$

(b) Entrepreneurs:

$$q_t = \left(\frac{\alpha}{w_t}\right)^{\alpha}\left(\frac{1-\alpha}{r_t}\right)^{1-\alpha} \tag{64}$$

$$r_t(k_t + z_t) = (1-\alpha)x_t \tag{65}$$

$$w_t h_t = \alpha x_t \tag{66}$$

$$e_t = \gamma\psi^f(\varkappa_t)\, n_t \tag{67}$$

$$z_{t+1} = (1-\gamma)\,\psi^f(\varkappa_t)\, n_t \tag{68}$$

$$1 = F^d(\varepsilon_{1t}, \overline{\varepsilon}_t^d; q_t, R_t, \sigma_{3t}) \tag{69}$$

$$1 = F^b(\overline{\varepsilon}_t^b; q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}) \tag{70}$$

$$F^b(\overline{\varepsilon}_{ct}; q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}) = F^c(\overline{\varepsilon}_t^c; q_t, R_t, \sigma_{2t}, \sigma_{3t}) \tag{71}$$

where the functions F^d, F^b and F^c are defined in equations (21), (23) and (24) in the paper. Note that these definitions require knowledge of the function $\bar{\omega}^b(\cdot)$ and

$\bar{\omega}^c(\cdot)$, which are defined in the main text in equation (20) as solution to equation (18).

3. Financial structure:

$$\vartheta_t = \frac{(1-\tau_t)\, s_t^{bp}}{s_t^c}, \tag{72}$$

$$rp_t^b \equiv \frac{\psi^{rb}(\varkappa_t)}{s_t^{bp}}, \tag{73}$$

$$rp_t^c \equiv \frac{\psi^{rc}(\varkappa_t)}{s_t^c}, \tag{74}$$

$$\chi_t = \frac{d_t}{y_t}, \tag{75}$$

$$\varrho_t^c = \frac{\psi^{mc}(\varkappa_t)}{s_t^c}, \tag{76}$$

$$\varrho_t = \frac{\psi^{mb}(\varkappa_t) + \psi^{mc}(\varkappa_t)}{s_t^{bp} + s_t^c} \tag{77}$$

$$s_t^a = \Phi\left(\overline{\varepsilon}^b(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}); \sigma_{1t}\right) \tag{78}$$

$$s_t^b = \Phi\left(\overline{\varepsilon}^c(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}); \sigma_{1t}\right) - \Phi\left(\overline{\varepsilon}^b(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}); \sigma_{1t}\right) \tag{79}$$

$$s_t^c = 1 - \Phi\left(\overline{\varepsilon}^c(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}); \sigma_{1t}\right) \tag{80}$$

$$s_t^{bp} = \int_{\overline{\varepsilon}^b(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t})}^{\overline{\varepsilon}^c(q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t})} \int_{\overline{\varepsilon}^d(\varepsilon_1; q_t, R_t, \sigma_{3t})} \Phi(d\varepsilon_2)\Phi(d\varepsilon_1). \tag{81}$$

4. Exogenous variables:

   (a) Information acquisition costs

$$\log \tau_t - \log \tau = \rho_\tau (\log \tau_{t-1} - \log \tau) + \varepsilon_{\tau,t},\ \ \varepsilon_{\tau,t} \sim \mathcal{N}\left(0, \sigma_\tau^2\right),$$

   (b) Standard deviation of the productivity factor $\varepsilon_{2,t}$

$$\log \sigma_{\varepsilon_2,t} - \log \sigma_{\varepsilon_2} = \rho_{\sigma_{\varepsilon_2}} (\log \sigma_{\varepsilon_2,t-1} - \log \sigma_{\varepsilon_2}) + \varepsilon_{\sigma_{\varepsilon_2},t},\ \ \varepsilon_{\sigma_{\varepsilon_2},t} \sim \mathcal{N}\left(0, \sigma_{\varepsilon_2}^2\right),$$

   (c) Standard deviation of the productivity factor $\varepsilon_{3,t}$

$$\log \sigma_{\varepsilon_3,t} - \log \sigma_{\varepsilon_3} = \rho_{\sigma_{\varepsilon_3}} (\log \sigma_{\varepsilon_3,t-1} - \log \sigma_{\varepsilon_3}) + \varepsilon_{\sigma_{\varepsilon_3},t},\ \ \varepsilon_{\sigma_{\varepsilon_3},t} \sim \mathcal{N}\left(0, \sigma_{\varepsilon_3}^2\right).$$

   (d) Money growth rate v_t^f

$$\log v_t^f = \rho_v \log v_{t-1}^f + \varepsilon_{v,t},\ \ \varepsilon_{v,t} \sim \mathcal{N}\left(0, \sigma_v^2\right)$$

Given the exogenous variables τ_t, $\sigma_{\varepsilon_2,t}$, $\sigma_{\varepsilon_3,t}$ and $\varepsilon_{v,t}$, equations (49) to (81) need to be solved for the variables characterizing the households choices, $(m_t, d_t, c_t, k_t, h_t)$, the entrepreneurs choices $(e_t, z_t, n_t, \bar{\varepsilon}_t^b, \bar{\varepsilon}_t^c, \bar{\varepsilon}_t^d)$, the choices of the monetary authority (θ_t, m_t^s), aggregate quantities (y_t, y_t^a, x_t), financial variables $(\vartheta_t, rp_t^b, rp_t^c, \chi_t, \varrho_t^c, \varrho_t, s_t^a, s_t^b, s_t^{bp}, s_t^c)$, and prices and returns $(\pi_t, R_t, r_t, q_t, w_t)$.

This is a system of 32 equations in 31 unknowns. Indeed, one equation is superfluous. By Walras' law, fulfillment of the budget constraints of the entrepreneurs and market clearing on all markets implies fulfillment of the budget constraints of the households as well.

E The steady state

We compute a steady state where we shut down the aggregate shocks, i.e. $\tau_t = \tau$ and $\sigma_{jt} = \sigma_j$, for all j and t. We denote steady state variables by dropping the time subscript.

We find it convenient to specify one of the endogenous variables, q, as exogenous and to treat γ as endogenous.Under the assumed specification of the utility function, the unique steady state can be obtained as follows. We normalize aggregate consumption to be unity, solving for the preference parameter η from the first-order condition for labor further below,

$$c = 1.$$

For each value of q, we can compute π, r and w by solving the equations

$$\begin{aligned} \pi &= \beta R \\ r &= \frac{1}{\beta} - 1 + \delta \\ w &= \left(\frac{1}{q}\right)^{\frac{1}{\alpha}} \alpha \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}}. \end{aligned}$$

To compute overall expected profits, given by

$$F(\varepsilon_1; q, R, \tau, \sigma_2, \sigma_3) \equiv \max\{1; F^b(\varepsilon_1; q, R, \tau, \sigma_2, \sigma_3); F^c(\varepsilon_1; q, R, \sigma_2, \sigma_3)\},$$

we use the following procedure. First, under our distributional assumptions about the productivity shocks $\varepsilon_1, \varepsilon_2$ and ε_3, and given some $\overline{\omega}$, we know that

$$\varphi(\overline{\omega}; \sigma) = \varphi(\zeta) \frac{1}{\overline{\omega}\sigma}$$

$$f(\overline{\omega}; \sigma) = 1 - \Phi(\zeta - \sigma) - \overline{\omega}[1 - \Phi(\zeta)],$$

$$g(\overline{\omega};\sigma,\mu) = (1-\mu)\,\Phi\left(\zeta-\sigma\right) + \overline{\omega}\left[1-\Phi\left(\zeta\right)\right].$$

where φ and Φ denote the standard normal, $\zeta = \frac{\log\overline{\omega}+\frac{\sigma^2}{2}}{\sigma}$ and σ is given by

$$\sigma \equiv \begin{cases} \sigma_3 & \text{if bank finance,} \\ \sqrt{\sigma_2^2+\sigma_3^2} & \text{if CMF finance.} \end{cases}$$

Second, we solve numerically the condition $\varepsilon^e q g(\overline{\omega};\sigma,\mu)\xi = R\left(\xi-1\right)$ to obtain the function $\overline{\omega}(\varepsilon^e;q,R,\sigma)$. The function $\overline{\omega}^b(\varepsilon_1\varepsilon_2;q,R,\sigma_3)$ for bank-financed firms is derived by using the variance $\sigma^2_{\varepsilon_3}$ of the log-normal distribution. The function $\overline{\omega}^c(\varepsilon_1;q,R,\sigma_2,\sigma_3)$ for CMF-financed firms is derived by using the variance $\sigma^2_{\varepsilon_2}+\sigma^2_{\varepsilon_3}$. The cutoff value $\overline{\varepsilon}^d$ for proceeding with the bank loan is found by solving numerically the condition $F^d(\varepsilon_{1,}\overline{\varepsilon}^d;q,R,\sigma_3) = 1$. Using $\overline{\varepsilon}^d$, it is then possible to compute the expected utility per unit of net worth for the bank-financed entrepreneur, $F^b(\varepsilon_1;q,R,\tau,\sigma_2,\sigma_3)$. The expected utility per unit of net worth for the CMF-financed entrepreneur can be computed as $F^c(\varepsilon_1;q,R,\sigma_2,\sigma_3) = \varepsilon_1 q f(\overline{\omega}^c(\varepsilon_1;q,R,\sigma_2,\sigma_3))\xi$. With this, it is possible to calculate the overall return $F(\varepsilon_1;q,R,\tau,\sigma_2,\sigma_3)$ to entrepreneurial investment, the thresholds $\overline{\varepsilon}^b$ and $\overline{\varepsilon}^c$, the shares s^{bp}, s^c, and the ratios $\frac{x}{z}$, $\frac{K}{x}$ and $\frac{h}{x}$, as given by

$$\frac{x}{z} = \left[(1-\tau)s^{bp}+s^c\right]\xi\left(1-\delta+r\right)$$
$$\frac{K}{x} = \frac{1-\alpha}{r}$$
$$\frac{h}{x} = \frac{\alpha}{w}.$$

Notice that in steady state,

$$m = \left(\frac{R}{\pi}-1\right)d+\theta = c+\delta k-(wh+rk)$$
$$d = \left[(1-\tau)s^{bp}+s^c\right]\left(\xi-1\right)\left(1-\delta+r\right)z$$
$$\theta = (\nu-1)\,\frac{m^s}{\pi} = \left(\frac{\pi-1}{\pi}\right)m^s,$$

and

$$m^s = m+d = c-wh-(r-\delta)\,k+\left[(1-\tau)s^{bp}+s^c\right]\left(\xi-1\right)\left(1-\delta+r\right)z.$$

Now write the budget constraint of the household as

$$1 = c = \left(\frac{R}{\pi}-1\right)d+\theta+wh+(r-\delta)\,k$$

or as

$$\frac{1}{z} = (R-1)\left[(1-\tau)s^{bp} + s^{c}\right](\xi - 1)(1-\delta + r) + w\frac{h}{z} + (r-\delta)\frac{k}{z}.$$

Using the solution obtained, calculate z and then compute the aggregate variables n, x, K, h and k. Then, use

$$z = \gamma\psi^{f}(\varkappa)\, n$$

to compute γ, the steady state version of equations (37) and (32) to compute y and e, and of the resource constraint (40) to compute y^{a}. The first order condition for labor can now be used to solve for the preference parameter η consistent with the normalization $c = 1$,

$$\eta = \frac{w}{h^{1/\kappa}}.$$

Finally, we use these results to compute the financial variables, given by (42)-(47), and the net expected return to entrepreneurial capital, given by (48), in steady state.

F Log-linearization

The equilibrium can be obtained by solving the system of equilibrium conditions, log-linearized around a nonstochastic steady state where $\pi = 1$ and the aggregate shocks are set to their steady state values, but where the idiosynchratic shocks are present. The log-linearized equations are standard and are therefore omitted here.

The difficulty arises in the computation of the coefficients multiplying the variables in the log-linearized equations. We illustrate here how they can be obtained. A detailed appendix with all the log-linearized equations and relative coefficients is available from the authors upon request.

First, note that

$$\ln \varepsilon_i \sim N\left(-\frac{\sigma_i^2}{2}, \sigma_i^2\right),$$

where $i = 1, 2, 3$.

Denote with Ξ the distribution function for a standard normal,

$$\Xi(z_i) = \int_{-\infty}^{z_i} \frac{1}{\sqrt{2\pi}} e^{-v^2/2} dv$$

where $z_i = \frac{\ln(\varepsilon_i)+\sigma_i^2/2}{\sigma_i}$. Note that $\varepsilon_i = \exp(\sigma_i z_i - \sigma_i^2/2)$. The density function of ε_i is obtained with the change of variable $d\ln(\varepsilon_i) = d\varepsilon_i/\varepsilon_i$ and $dz_i = d\ln(\varepsilon_i)/\sigma_i$, resulting in

$$\varphi(\varepsilon_i;\sigma_i) = \left(\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\frac{(\ln(\varepsilon_i)+\frac{\sigma_i^2}{2})^2}{\sigma_i^2}}\right)\frac{1}{\varepsilon_i\sigma_i}$$

The distribution function is

$$\Phi(\varepsilon_i;\sigma_i) = \int_{-\infty}^{\frac{\ln(\varepsilon_i)+\sigma_i^2/2}{\sigma_i}} \frac{1}{\sqrt{2\pi}}e^{-v^2/2}dv$$

From here, we can now calculate derivatives with respect to σ_i:

$$\varphi_2(\varepsilon_i;\sigma_i) = \left[-\frac{1}{\sigma_i} - \frac{(\ln(\varepsilon_i)+\frac{\sigma_i^2}{2})}{\sigma_i} + \frac{(\ln(\varepsilon_i)+\frac{\sigma_i^2}{2})^2}{\sigma_i^3}\right]\varphi(\varepsilon_i;\sigma_i)$$

as well as

$$\Phi_2(\varepsilon_i;\sigma_i) = \left(\frac{\sigma_i^2/2-\ln(\varepsilon_i)}{\sigma_i}\right)\varepsilon_i\varphi(\varepsilon_i;\sigma_i)$$

Now define, for $j = b, c$, $\zeta_j = \frac{\log\overline{\omega}^j + \frac{\sigma_j^2}{2}}{\sigma_j}$ and notice that $\sigma_b^2 = \sigma_3^2$, while $\sigma_c^2 = \sigma_2^2 + \sigma_3^2$.

From $g(\overline{\omega}^j, \sigma_j, \mu) = (1-\mu)\,\Phi\left(\zeta_j - \sigma_j\right) + \overline{\omega}^j\left[1-\Phi\left(\zeta_j\right)\right]$, we get

$$\begin{aligned} g_1(\overline{\omega}^j,\sigma_j,\mu) &= (1-\mu)\,\varphi\left(\zeta_j-\sigma_j\right)\frac{1}{\overline{\omega}^j\sigma_j} + 1 - \Phi\left(\zeta_j\right) - \overline{\omega}^j\varphi\left(\zeta_j\right)\frac{1}{\overline{\omega}^j\sigma_j}, \\ g_2(\overline{\omega}^j,\sigma_j,\mu) &= -(1-\mu)\,\varphi\left(\zeta_j-\sigma_j\right)\frac{\zeta_j}{\sigma_j} - \overline{\omega}^j\varphi\left(\zeta_j\right)\left(\frac{\frac{\sigma_j^2}{2}-\log\overline{\omega}^j}{\sigma_j^2}\right) \end{aligned}$$

and from $f\left(\overline{\omega}^j;\sigma_j\right) = \Phi\left(\sigma_j - \zeta_j\right) - \overline{\omega}^j\left[1-\Phi\left(\zeta_j\right)\right]$, we get

$$\begin{aligned} f_1\left(\overline{\omega}^j,\sigma_j\right) &= -\varphi\left(\sigma_j-\zeta_j\right)\frac{1}{\overline{\omega}^j\sigma_j} - \left[1-\Phi\left(\zeta_j\right)\right] + \overline{\omega}^j\varphi\left(\zeta_j\right)\frac{1}{\overline{\omega}^j\sigma_j}, \\ f_2\left(\overline{\omega}^j,\sigma_j\right) &= \varphi\left(\sigma_j-\zeta_j\right)\frac{\zeta_j}{\sigma_j} + \overline{\omega}^j\varphi\left(\zeta_j\right)\left(\frac{\frac{\sigma_j^2}{2}-\log\overline{\omega}^j}{\sigma_j^2}\right). \end{aligned}$$

Notice that, for $i = 2, 3$, $\frac{\partial f(\overline{\omega}^c;\sigma_c)}{\partial\sigma_i} = f_2\left(\overline{\omega}^c;\sigma_c\right)\frac{\sigma_i}{\sqrt{\sigma_2^2+\sigma_3^2}}$.

With this, we can proceed to derive the coefficients of the log-linearized equations. Consider the condition corresponding to equation (37),

$$\widehat{y_t} = \left(\frac{\psi_q^y q}{\psi^y}+1\right)\widehat{q}_t + \frac{\psi_R^y R}{\psi^y}\widehat{R}_t + \frac{\psi_\tau^y\tau}{\psi^y}\widehat{\tau}_t + \widehat{n}_t + \frac{\psi_{\sigma_1}^y\sigma_1}{\psi^y}\widehat{\sigma_1}_t + \frac{\psi_{\sigma_1}^y\sigma_2}{\psi^y}\widehat{\sigma_2}_t + \frac{\psi_{\sigma_3}^y\sigma_3}{\psi^y}\widehat{\sigma_3}_t.$$

From equation (41), evaluated at the steady state, we obtain

$$\psi_{\sigma_i}^y(\cdot) = \psi_{\sigma_i}^{y,A}(\cdot) + \psi_{\sigma_i}^{y,B}(\cdot)$$

for $i = 1, 2, 3$, where the task of taking the derivative has been split into two components "A" and "B".

The "A" component is the derivatives of the bounds and is given, for $i = 2, 3$, by

$$\begin{aligned}\psi_{\sigma_i}^{y,A}(\cdot) &= -\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\sigma_i}\overline{\varepsilon}_c\varphi(\overline{\varepsilon}_c;\sigma_1) \\ &+ (1-\tau)\left[\begin{array}{c} \frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\sigma_i}\overline{\varepsilon}_c\varphi(\overline{\varepsilon}_c;\sigma_1)\int_{\overline{\varepsilon}_d(\overline{\varepsilon}_c;\cdot)}\varepsilon_2\Phi(d\varepsilon_2) \\ -\frac{\partial\overline{\varepsilon}_b(\cdot)}{\partial\sigma_i}\overline{\varepsilon}_b\varphi(\overline{\varepsilon}_b;\sigma_1)\int_{\overline{\varepsilon}_d(\overline{\varepsilon}_b;\cdot)}\varepsilon_2\Phi(d\varepsilon_2) \\ -\int_{\overline{\varepsilon}_b(\cdot)}^{\overline{\varepsilon}_c(\cdot)}\frac{\partial\overline{\varepsilon}_d(\varepsilon_1;\cdot)}{\partial\sigma_i}\varepsilon_1\overline{\varepsilon}_d(\varepsilon_1;\cdot)\varphi(\overline{\varepsilon}_d(\varepsilon_1;\cdot);\sigma_2)\Phi(d\varepsilon_1)\end{array}\right]\end{aligned}$$

where it should be noticed that $\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\sigma_2} = 0$ and $\psi_{\sigma_i}^{y,A}(\cdot) = 0$ for $i = 1$.

The second part is specific to the standard deviations, only arises for σ_1 and σ_2 and is given by

$$\psi_{\sigma_1}^{y,B}(\cdot) = (1-\tau)\int_{\overline{\varepsilon}_b(\cdot)}^{\overline{\varepsilon}_c(\cdot)}\varepsilon_1\int_{\overline{\varepsilon}_d(\varepsilon_1;\cdot)}\varepsilon_2\,\varphi(\varepsilon_2;\sigma_2)\,d\varepsilon_2\,\varphi(\varepsilon_1;\sigma_1)\;d\varepsilon_1 + \int_{\overline{\varepsilon}_{c(\cdot)}}\varepsilon_1\,\varphi(\varepsilon_1;\sigma_1)\,d\varepsilon_1$$

and

$$\psi_{\sigma_2}^{y,B}(\cdot) = (1-\tau)\int_{\overline{\varepsilon}_b(\cdot)}^{\overline{\varepsilon}_c(\cdot)}\varepsilon_1\int_{\overline{\varepsilon}_d(\varepsilon_1;\cdot)}\varepsilon_2\,\varphi(\varepsilon_2;\sigma_2)\,d\varepsilon_2\,\varphi(\varepsilon_1;\sigma_1)\;d\varepsilon_1.$$

Define, for $\upsilon = q, R, \sigma_2, \sigma_3$,

$$\begin{aligned}\psi_{\upsilon}^{y,A}(\cdot) &= -\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\upsilon}\overline{\varepsilon}_c\varphi(\overline{\varepsilon}_c(\cdot);\sigma_1) \\ &+ (1-\tau)\left[\begin{array}{c} \frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\upsilon}\overline{\varepsilon}_c\varphi(\overline{\varepsilon}_c;\sigma_1)\int_{\overline{\varepsilon}_d(\overline{\varepsilon}_c;\cdot)}\varepsilon_2\Phi(d\varepsilon_2) \\ -\frac{\partial\overline{\varepsilon}_b(\cdot)}{\partial\upsilon}\overline{\varepsilon}_b\varphi(\overline{\varepsilon}_b;\sigma_1)\int_{\overline{\varepsilon}_d(\overline{\varepsilon}_b;\cdot)}\varepsilon_2\Phi(d\varepsilon_2) \\ -\int_{\overline{\varepsilon}_b(\cdot)}^{\overline{\varepsilon}_c(\cdot)}\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\upsilon}\right|_{(\varepsilon_1;\cdot)}\varepsilon_1\overline{\varepsilon}_d(\varepsilon_1;\cdot)\varphi(\overline{\varepsilon}_d(\varepsilon_1;\cdot);\sigma_2)\Phi(d\varepsilon_1)\end{array}\right].\end{aligned}$$

Then,

$$\begin{aligned}\psi_q^y(\cdot) &= \psi_q^{y,A}(\cdot), \\ \psi_R^y(\cdot) &= \psi_R^{y,A}(\cdot), \\ \psi_{\sigma_i}^y(\cdot) &= \psi_{\sigma_i}^{y,A}(\cdot) + \psi_{\sigma_i}^{y,B}(\cdot).\end{aligned}$$

and

$$\begin{aligned}\psi_{\tau}^y(\cdot) &= -\int_{\overline{\varepsilon}_b(\cdot)}^{\overline{\varepsilon}_c(\cdot)}\varepsilon_1\int_{\overline{\varepsilon}_d(\varepsilon_1;\cdot)}\varepsilon_2\Phi(d\varepsilon_2)\Phi(d\varepsilon_1) - \frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\tau}\overline{\varepsilon}_c\varphi(\overline{\varepsilon}_c;\sigma_1) \\ &+ (1-\tau)\left[\begin{array}{c} \frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\tau}\overline{\varepsilon}_c\varphi(\overline{\varepsilon}_c;\sigma_1)\int_{\overline{\varepsilon}_d(\overline{\varepsilon}_c;\cdot)}\varepsilon_2\Phi(d\varepsilon_2) \\ -\frac{\partial\overline{\varepsilon}_b(\cdot)}{\partial\tau}\overline{\varepsilon}_b\varphi(\overline{\varepsilon}_b;\sigma_1)\int_{\overline{\varepsilon}_d(\overline{\varepsilon}_b;\cdot)}\varepsilon_2\Phi(d\varepsilon_2)\end{array}\right]\end{aligned}$$

To compute the derivatives of $\psi^y(\cdot)$, we now need to compute the derivatives of the thresholds $\overline{\varepsilon}_b, \overline{\varepsilon}_c, \overline{\varepsilon}_d$.

Consider first the threshold at stage II, $\overline{\varepsilon}_d(\varepsilon_1; q, R, \sigma_3)$, which is implicitely defined by

$$F^d(\varepsilon_1, \overline{\varepsilon}_d; q, R, \sigma_3) = 1.$$

Using the implicit function theorem, we have that, for $\upsilon = q, R, \sigma_3$,

$$\left.\frac{\partial \overline{\varepsilon}_d(\cdot)}{\partial \varepsilon_1}\right|_{(\varepsilon_1; q, R, \sigma_3)} = -\frac{F_1^d(\varepsilon_1, \overline{\varepsilon}_d; q, R, \sigma_3)}{F_2^d(\varepsilon_1, \overline{\varepsilon}_d; q, R, \sigma_3)} \tag{82}$$

$$\left.\frac{\partial \overline{\varepsilon}_d(\cdot)}{\partial \upsilon}\right|_{(\varepsilon_1; q, R, \sigma_3)} = -\frac{F_\upsilon^d(\varepsilon_1, \overline{\varepsilon}_d; q, R, \sigma_3)}{F_2^d(\varepsilon_1, \overline{\varepsilon}_d; q, R, \sigma_3)}. \tag{83}$$

From $F^d(\varepsilon_1, \varepsilon_2; q, R, \sigma_3) = \varepsilon_1 \varepsilon_2 q f(\overline{\omega}^b(\varepsilon_1 \varepsilon_2; q, R, \sigma_3); \sigma_3)\xi$, we obtain

$$F_1^d(\varepsilon_1, \varepsilon_2; \cdot) = \varepsilon_2 q \xi \left[f(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot); \sigma_3) + \varepsilon_1 f_1(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot); \sigma_3) \frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial \varepsilon_1} \right]$$

$$F_2^d(\varepsilon_1, \varepsilon_2; \cdot) = \varepsilon_1 q \xi \left[f(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot); \sigma_3) + \varepsilon_2 f_1(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot); \sigma_3) \frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial \varepsilon_2} \right]$$

$$F_q^d(\varepsilon_1, \varepsilon_2; \cdot) = \varepsilon_1 \varepsilon_2 \xi \left[f(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot); \sigma_3) + q f_1(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot); \sigma_3) \frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial q} \right]$$

$$F_R^d(\varepsilon_1, \varepsilon_2; \cdot) = \varepsilon_1 \varepsilon_2 q \xi f_1(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot); \sigma_3) \frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial R}$$

$$F_{\sigma_3}^d(\varepsilon_1, \varepsilon_2; \cdot) = \varepsilon_1 \varepsilon_2 q \xi \left[f_1(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot); \sigma_3) \frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial \sigma_3} + f_2(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot); \sigma_3) \right]$$

Computation of the derivatives of $F^d(\cdot)$ requires computing also the partial derivatives of $\overline{\omega}^b(\varepsilon_1, \varepsilon_2; q, R, \sigma_3)$. Define

$$\widetilde{\omega}^b(\varepsilon_1, \varepsilon_2; q, R, \sigma_3) \equiv \frac{g(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; q, R, \sigma_3), \sigma_3, \mu)}{g_1(\overline{\omega}^b(\varepsilon_1, \varepsilon_2; q, R, \sigma_3), \sigma_3, \mu)}.$$

From condition $g(\overline{\omega}^b, \sigma_3, \mu) = \frac{R_t}{\varepsilon_1 \varepsilon_2 q}\left(1 - \frac{1}{\xi}\right)$, we get

$$\frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial \varepsilon_1} = -\frac{\widetilde{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\varepsilon_1} \tag{84}$$

$$\frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial \varepsilon_2} = -\frac{\widetilde{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\varepsilon_2} \tag{85}$$

$$\frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial q} = -\frac{\widetilde{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{q} \tag{86}$$

$$\frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial R} = \frac{\varepsilon_1 \varepsilon_2 q g(\overline{\omega}^b, \sigma_3, \mu)\xi}{\varepsilon_1 \varepsilon_2 q g_1(\overline{\omega}^b, \sigma_3, \mu)\xi R} \tag{87}$$

$$\frac{\partial \overline{\omega}^b(\varepsilon_1, \varepsilon_2; \cdot)}{\partial \sigma_3} = -\frac{g_2(\overline{\omega}^b, \sigma_3, \mu)}{g_1(\overline{\omega}^b, \sigma_3, \mu)}. \tag{88}$$

Define now $\Lambda^b(\varepsilon_1, \overline{\varepsilon}_d; \cdot) = 1 - \frac{f_1(\overline{\omega}^b(\varepsilon_1,\overline{\varepsilon}_d;\cdot);\sigma_3)}{f(\overline{\omega}^b(\varepsilon_1,\overline{\varepsilon}_d;\cdot);\sigma_3)}\widetilde{\omega}^b(\varepsilon_1, \overline{\varepsilon}_d; \cdot)$. We can then write

$$\begin{aligned}
F_1^d(\varepsilon_1, \overline{\varepsilon}_d; \cdot) &= \frac{F^d(\varepsilon_1, \overline{\varepsilon}_d; \cdot)}{\varepsilon_1}\Lambda(\varepsilon_1, \overline{\varepsilon}_d; \cdot) \\
F_2^d(\varepsilon_1, \overline{\varepsilon}_d; \cdot) &= \frac{F^d(\varepsilon_1, \overline{\varepsilon}_d; \cdot)}{\overline{\varepsilon}_d}\Lambda(\varepsilon_1, \overline{\varepsilon}_d; \cdot) \\
F_q^d(\varepsilon_1, \overline{\varepsilon}_d; \cdot) &= \frac{F^d(\varepsilon_1, \overline{\varepsilon}_d; \cdot)}{q}\Lambda(\varepsilon_1, \overline{\varepsilon}_d; \cdot) \\
F_R^d(\varepsilon_1, \overline{\varepsilon}_d; \cdot) &= \frac{F^d(\varepsilon_1, \overline{\varepsilon}_d; \cdot)}{R}\left[1 - \Lambda(\varepsilon_1, \overline{\varepsilon}_d; \cdot)\right] \\
F_{\sigma_3}^d(\varepsilon_1, \overline{\varepsilon}_d; \cdot) &= \varepsilon_1\overline{\varepsilon}_d q\xi\left[f_2(\overline{\omega}^b(\varepsilon_1, \overline{\varepsilon}_d; \cdot); \sigma_3) - f_1(\overline{\omega}^b(\varepsilon_1, \overline{\varepsilon}_d; \cdot); \sigma_3)\frac{g_2(\overline{\omega}^b, \sigma_3, \mu)}{g_1(\overline{\omega}^b, \sigma_3, \mu)}\right].
\end{aligned}$$

and

$$\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\varepsilon_1}\right|_{(\varepsilon_1;\cdot)} = -\frac{\overline{\varepsilon}_d}{\varepsilon_1} \tag{89}$$

$$\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial q}\right|_{(\varepsilon_1;\cdot)} = -\frac{\overline{\varepsilon}_d}{q} \tag{90}$$

$$\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial R}\right|_{(\varepsilon_1;\cdot)} = -\frac{\overline{\varepsilon}_d}{R}\left[\frac{1}{\Lambda(\varepsilon_1, \overline{\varepsilon}_d; q, R)} - 1\right] \tag{91}$$

$$\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\sigma_3}\right|_{(\varepsilon_1;\cdot)} = \frac{\overline{\varepsilon}_d}{f(\overline{\omega}^b(\varepsilon_1, \overline{\varepsilon}_d; \cdot); \sigma_3)\Lambda^b(\varepsilon_1, \overline{\varepsilon}_d; \cdot)}\left[f_2(\overline{\omega}^b(\varepsilon_1, \overline{\varepsilon}_d; \cdot); \sigma_3) - f_1(\overline{\omega}^b(\varepsilon_1, \overline{\varepsilon}_d; \cdot); \sigma_3)\frac{g_2(\overline{\omega}^b, \sigma_3, \mu)}{g_1(\overline{\omega}^b, \sigma_3, \mu)}\right] \tag{92}$$

We now need to obtain derivatives of the threshold $\overline{\varepsilon}_b(q, R, \tau)$. This latter is implicitely defined by condition (26) evaluated at the steady state. Using the implicit function theorem, we have that, for $\upsilon = q, R, \tau, \sigma_2, \sigma_3$

$$\frac{\partial\overline{\varepsilon}_b(\cdot)}{\partial\upsilon} = -\frac{F_\upsilon^b(\overline{\varepsilon}_b; q, R, \tau, \sigma_2, \sigma_3)}{F_1^b(\overline{\varepsilon}_b; q, R, \tau, \sigma_2, \sigma_3)}$$

Now, define $\Gamma(\varepsilon_1; \cdot) = \varepsilon_1\overline{\varepsilon}_d q f(\overline{\omega}^b(\varepsilon_1\overline{\varepsilon}_d(\cdot); \cdot); \sigma_3)\xi\varphi(\overline{\varepsilon}_d(\cdot); \sigma_2)$. Using condition $F^b(\overline{\varepsilon}_{bt}; q_t, R_t, \tau_t, \sigma_{2t}, \sigma_{3t}) = 1$, we get

$$F_1^b(\varepsilon_1; \cdot) = (1-\tau)\left(\begin{array}{c} -\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\varepsilon_1}\right|_{(\varepsilon_1;\cdot)}\Gamma(\varepsilon_1; \cdot) + \int_{\overline{\varepsilon}_d(\varepsilon_1;\cdot)} F_1^d(\varepsilon_1, \varepsilon_2; \cdot)\Phi(d\varepsilon_2) \\ +\varphi(\overline{\varepsilon}_d; \sigma_2)\frac{\partial\overline{\varepsilon}_d(\varepsilon_1;\cdot)}{\partial\varepsilon_1} \end{array}\right)$$

$$F_\upsilon^b(\varepsilon_1; q, R, \tau) = (1-\tau)\left(\begin{array}{c} -\frac{\partial\overline{\varepsilon}_d}{\partial q}\Gamma(\varepsilon_1; \cdot) + \int_{\overline{\varepsilon}_d(\varepsilon_1;\cdot)} F_q^d(\varepsilon_1, \varepsilon_2; \cdot)\Phi(d\varepsilon_2) \\ +\varphi(\overline{\varepsilon}_d(\cdot); \sigma_2)\frac{\partial\overline{\varepsilon}_d(\varepsilon_1;\cdot)}{\partial q} \end{array}\right)$$

$$F_\tau^b(\varepsilon_1; q, R, \tau) = -\frac{F^b(\varepsilon_1; \cdot)}{(1-\tau)}$$

$$F_{\sigma_2}^b(\varepsilon_1; \cdot) = (1-\tau)\left(\int_{\overline{\varepsilon}_d(\varepsilon_1;\cdot)} F^d(\varepsilon_1, \varepsilon_2; \cdot)\varphi(\varepsilon_2; \sigma_2)d\varepsilon_2 + \Phi(\overline{\varepsilon}_d(\varepsilon_1; \cdot); \sigma_2)\right)$$

$$F_{\sigma_3}^b(\varepsilon_1;\cdot) = (1-\tau)\left(-\frac{\partial\overline{\varepsilon}_d\left(\varepsilon_1;\cdot\right)}{\partial\sigma_3}\Gamma(\varepsilon_1;\cdot) + \varphi(\overline{\varepsilon}_d;\sigma_2)\frac{\partial\overline{\varepsilon}_d(\varepsilon_1;\cdot)}{\partial\sigma_3} + \int_{\overline{\varepsilon}_d(\varepsilon_1;\cdot)} F_{\sigma_3}^d(\varepsilon_1,\varepsilon_2;\cdot)\Phi(d\varepsilon_2)\right),$$

for $\upsilon = q, R$. Notice that the derivatives of the threshold $\overline{\varepsilon}_d\left(\cdot\right)$ are given by (82)-(83). Moreover, the derivatives of the threshold $\overline{\omega}^b$ are given by (84)-(88).

Consider now the threshold for the first stage, $\overline{\varepsilon}_c\left(q,R,\tau\right)$. It is implicitely defined by condition $F^b(\overline{\varepsilon}_c;q,R,\tau,\sigma_2,\sigma_3) = F^c(\overline{\varepsilon}_c;q,R,\sigma_2,\sigma_3)$. Using the implicit function theorem, we have that, for $\upsilon = q,R,\tau,\sigma_2,\sigma_3$

$$\frac{\partial\overline{\varepsilon}_c\left(\cdot\right)}{\partial\upsilon} = -\left(\frac{F_\upsilon^b(\overline{\varepsilon}_c;q,R,\tau,\sigma_2,\sigma_3) - F_\upsilon^c(\overline{\varepsilon}_c;q,R,\sigma_2,\sigma_3)}{F_1^b(\overline{\varepsilon}_c;q,R,\tau,\sigma_2,\sigma_3) - F_1^c(\overline{\varepsilon}_c;q,R,\sigma_2,\sigma_3)}\right),$$

Notice that $\sigma_c = \sqrt{\sigma_2^2+\sigma_3^2}$ and $\frac{\partial\sigma_c}{\partial\sigma_i} = \frac{\sigma_i}{\sqrt{\sigma_2^2+\sigma_3^2}}$, for $i = 2,3$. From $F^c(\varepsilon_1;q,R,\sigma_2,\sigma_3) \equiv \varepsilon_1 q f(\overline{\omega}^c(\varepsilon_1;q,R,\sigma_2,\sigma_3))\xi$, we get

$$\begin{aligned}
F_1^c(\varepsilon_1;\cdot) &= \frac{F^c(\varepsilon_1;\cdot)}{\varepsilon_1}\left[1+\varepsilon_1\frac{f_1(\overline{\omega}^c(\varepsilon_1;\cdot);\cdot)}{f(\overline{\omega}^c(\varepsilon_1;\cdot);\cdot)}\frac{\partial\overline{\omega}^c\left(\cdot\right)}{\partial\varepsilon_1}\right]\\
F_q^c(\varepsilon_1;\cdot) &= \frac{F^c(\varepsilon_1;\cdot)}{q}\left[1+q\frac{f_1(\overline{\omega}^c(\varepsilon_1;\cdot);\cdot)}{f(\overline{\omega}^c(\varepsilon_1;\cdot);\cdot)}\frac{\partial\overline{\omega}^c\left(\varepsilon_1;\cdot\right)}{\partial q}\right]\\
F_\tau^c(\varepsilon_1;\cdot) &= 0\\
F_R^c(\varepsilon_1;\cdot) &= \varepsilon_1 q\xi f_1(\overline{\omega}^c(\varepsilon_1;\cdot);\cdot)\frac{\partial\overline{\omega}^c\left(\cdot\right)}{\partial R}\\
F_{\sigma_2}^c(\varepsilon_1;\cdot) &= \varepsilon_1 q\xi\left[f_1(\overline{\omega}^c(\varepsilon_1;\cdot);\cdot)\frac{\partial\overline{\omega}^c\left(\varepsilon_1;\cdot\right)}{\partial\sigma_2} + f_2(\overline{\omega}^c(\varepsilon_1;\cdot);\cdot)\frac{\sigma_2}{\sqrt{\sigma_2^2+\sigma_3^2}}\right],\\
F_{\sigma_3}^c(\varepsilon_1;\cdot) &= \varepsilon_1 q\xi\left[f_1(\overline{\omega}^c(\varepsilon_1;\cdot);\cdot)\frac{\partial\overline{\omega}^c\left(\varepsilon_1;\cdot\right)}{\partial\sigma_3} + f_2(\overline{\omega}^c(\varepsilon_1;\cdot);\cdot)\frac{\sigma_3}{\sqrt{\sigma_2^2+\sigma_3^2}}\right].
\end{aligned}$$

Define $\widetilde{\omega}^c\left(\varepsilon_1;q,R,\sigma_2,\sigma_3\right) \equiv \frac{g(\overline{\omega}^c(\varepsilon_1;\cdot);\sqrt{\sigma_2^2+\sigma_3^2},\mu)}{g_1(\overline{\omega}^c(\varepsilon_1;\cdot);\sqrt{\sigma_2^2+\sigma_3^2},\mu)}$ and $\Lambda^c\left(\overline{\varepsilon}_c;q,R\right) = 1 - \frac{f'(\overline{\omega}^c(\overline{\varepsilon}_c;q,R))}{f(\overline{\omega}^c(\overline{\varepsilon}_c;q,R))}\widetilde{\omega}^c\left(\overline{\varepsilon}_c;q,R\right).$ From condition $g(\overline{\omega}^c;\sqrt{\sigma_2^2+\sigma_3^2},\mu) = \frac{R}{\varepsilon_1 q}\left(1-\frac{1}{\xi}\right)$, we get

$$\begin{aligned}
\frac{\partial\overline{\omega}^c}{\partial\varepsilon_1} &= -\frac{\widetilde{\omega}^c\left(\varepsilon_1;\cdot\right)}{\varepsilon_1}\\
\frac{\partial\overline{\omega}^c}{\partial q} &= -\frac{\widetilde{\omega}^c\left(\varepsilon_1;\cdot\right)}{q}\\
\frac{\partial\overline{\omega}^c}{\partial R} &= \frac{\widetilde{\omega}^c\left(\varepsilon_1;\cdot\right)}{R}\\
\frac{\partial\overline{\omega}^c}{\partial\sigma_2} &= -\frac{g_2(\overline{\omega}^c\left(\varepsilon_1;\cdot\right);\cdot)}{g_1(\overline{\omega}^c\left(\varepsilon_1;\cdot\right);\cdot)}\frac{\sigma_2}{\sqrt{\sigma_2^2+\sigma_3^2}}\\
\frac{\partial\overline{\omega}^c}{\partial\sigma_3} &= -\frac{g_2(\overline{\omega}^c\left(\varepsilon_1;\cdot\right);\cdot)}{g_1(\overline{\omega}^c\left(\varepsilon_1;\cdot\right);\cdot)}\frac{\sigma_3}{\sqrt{\sigma_2^2+\sigma_3^2}}.
\end{aligned}$$

It follows that

$$
\begin{aligned}
F_1^c(\overline{\varepsilon}_c;\cdot) &= \frac{F^c(\overline{\varepsilon}_c;\cdot)}{\overline{\varepsilon}_c}\Lambda^c\left(\overline{\varepsilon}_c;\cdot\right) \\
F_q^c(\overline{\varepsilon}_c;\cdot) &= \frac{F^c(\overline{\varepsilon}_c;\cdot)}{q}\Lambda^c\left(\overline{\varepsilon}_c;\cdot\right) \\
F_R^c(\overline{\varepsilon}_c;\cdot) &= \frac{F^c(\overline{\varepsilon}_c;\cdot)}{R}\left(1-\Lambda^c\left(\overline{\varepsilon}_c;\cdot\right)\right) \\
F_{\sigma_2}^c(\overline{\varepsilon}_c;\cdot) &= \frac{\sigma_2}{\sqrt{\sigma_2^2+\sigma_3^2}}\overline{\varepsilon}_c q\xi\left(f_2(\overline{\omega}^c(\overline{\varepsilon}_c;\cdot);\cdot)-f_1(\overline{\omega}^c(\overline{\varepsilon}_c;\cdot);\cdot)\frac{g_2(\overline{\omega}^c\left(\overline{\varepsilon}_c;\cdot\right);\cdot)}{g_1(\overline{\omega}^c\left(\overline{\varepsilon}_c;\cdot\right);\cdot)}\right) \\
F_{\sigma_3}^c(\overline{\varepsilon}_c;\cdot) &= \frac{\sigma_3}{\sqrt{\sigma_2^2+\sigma_3^2}}\overline{\varepsilon}_c q\xi\left(f_2(\overline{\omega}^c(\overline{\varepsilon}_c;\cdot);\cdot)-f_1(\overline{\omega}^c(\overline{\varepsilon}_c;\cdot);\cdot)\frac{g_2(\overline{\omega}^c\left(\overline{\varepsilon}_c;\cdot\right);\cdot)}{g_1(\overline{\omega}^c\left(\overline{\varepsilon}_c;\cdot\right);\cdot)}\right),
\end{aligned}
$$

from which we can compute $\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial q}$, $\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial R}$, $\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\sigma_2}$, $\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\sigma_3}$ and $\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\tau}$.