EUROPEAN CENTRAL BANK

EUROSYSTEM

Working Paper Series

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Government guarantees and financial stability

No 2032 / February 2017

Abstract

Banks are intrinsically fragile because of their role as liquidity providers. This results in under-provision of liquidity. We analyze the effect of government guarantees on the interconnection between banks'liquidity creation and likelihood of runs in a model of global games, where banks'and depositors' behavior are endogenous and affected by the amount and form of guarantee. The main insight of our analysis is that guarantees are welfare improving because they induce banks to improve liquidity provision although in a way that sometimes increases the likelihood of runs or creates distortions in banks'behavior.

Keywords: panic runs, fundamental runs, government guarantees, bank moral hazard

JEL classifications: G21, G28

Non-Technical Summary

Government guarantees to financial institutions are common all over the world. They come in different forms, ranging from standard deposit insurance schemes to the promise of an ex-post bailout in the case of a bank's failure. The recent financial crisis was characterized by a massive use of government guarantees, which lead to a renewed interest and debate about government intervention in the financial sector. While public intervention proved to be effective in restoring confidence and preserving financial stability during the crisis, it also had significant negative consequences in terms of sovereigns' fiscal positions, and, in turn, banks' and firms' health and cost of funding.

In the current academic and policy debate, government guarantees are considered to be an effective tool to prevent the occurrence of panic crises and mitigate their negative effects. However, their provision can distort banks' risk-taking incentives and induce them to take excessive risk. Because of this moral hazard problem, the provision of guarantees can lead to the perverse outcome of increasing the overall instability in the banking sector (Demirguc-Kunt and Detraigiache, 1998) and magnify the costs for the government providing them.

Are guarantees effective in preventing banking crises? What are the implications they have for banks' role as liquidity providers and their risk-taking decisions? How do guarantees affect the interaction between liquidity provision and risk-taking?

This paper tackles these questions and provides new insights about the desirability of government guarantees, their effectiveness in preventing runs, as well as the type and severity of the associated moral hazard problem.

In the paper, we develop a theoretical framework where banks raise funds from risk-averse depositors in the form of demandable deposit contracts and, thus provide liquidity and risk-sharing to them by allowing depositors to access risky but profitable long-term investment opportunities, while still being able to obtain liquidity when needed. As a result, in our framework, banks are exposed to two sources of risk. One the one hand, they face insolvency risk, in that they can fail as a consequence of a bad realization of bank's investment projects (fundamental crises). On the other hand, they are exposed to liquidity risk, as they can fail as a consequence of large premature withdrawals by depositors, driven by the fear that others would withdraw and, thus deplete banks' resources (panic crises).

In our framework, the probability of both crises is endogenous and depends on the banks' risk choice, as well as on the (type and size of) government guarantees. This allows analysing how the bank's risk choice and the guarantees interact with the probability of fundamental and panic crises, as well as, with each other. In our model, the effect of the guarantees on the probability of a crisis is twofold. On the one hand, guarantees have a positive direct effect, since, by increasing depositors' repayments, they reduce their incentive to withdraw early and thus, banks' exposure to liquidity risk. On the other hand, they affect banks' risk-taking decisions and, thus have a negative indirect effect on the probability of a banking crisis.

To fully exploit these effects, we consider two guarantee schemes. The first one is only meant to prevent the occurrence of panic-driven crises and so depositors are guaranteed to receive at least a

minimum repayment if the project of the bank is solvent irrespective of what other depositors do. The second one, which resembles a standard deposit insurance scheme, guarantees depositors to receive a minimum repayment whenever the bank is not able to repay them the promised repayment, thus affecting both the probability of fundamental- and panic-driven crises.

The main insight of our analysis is that guarantees are welfare improving because they induce banks to improve liquidity provision although in a way that sometimes increases the likelihood of runs or creates distortions in banks' behavior. This result hinges on the fact that some risk taking by banks is desirable, as it is an inherent feature of the liquidity transformation function that banks perform in the economy, and it might be suppressed by the concern for financial fragility. By relaxing these concerns to some extent, guarantees allow banks to provide greater liquidity transformation and so are desirable. This effect, which is captured by our model thanks to the endogenization of banks' and depositors' behavior, is missing from the current debate and academic literature and generates a more nuanced assessment of government guarantees.

Another interesting result of our analysis regards the direction of the distortions in banks' behavior (i.e., moral hazard) induced by the guarantees. As typical in models where private agents, enjoy the benefits of a public form of insurance without internalizing its costs, also in our model the introduction of guarantees creates a wedge between the deposit rate chosen by banks and the one that the government would like to choose. However, unlike conventional wisdom, we show that government guarantees do not always lead to more risk-taking by banks, and, in turn, to an excessively high exposure to runs. Sometimes their introduction leads to the exactly opposite effect: banks choose to be less exposed to runs than what it would be socially optimal. The important detail is whether the government ends up paying depositors more in the case the bank ends up failing in the longer term for fundamental reasons or in case there is a run and the bank faces a shortage of liquidity. If the former holds, then the cost of a run from the point of view of banks is higher than from the point of the government and the banks choose to limit their exposure to run. The result that banks choose to be less exposed to runs than it would be desirable resembles the idea of prompt corrective actions. Liquidating banks early rather than letting them operate longer and intervene when banks' resources are completely deployed may be desirable when it allows to minimize the costs associated with public intervention.

1 Introduction

Government guarantees to financial institutions are common all over the world. They come in different forms, such as deposit insurance provided to depositors who put their money in commercial banks, or implicit guarantees for a bailout provided ex post upon the bank' s failure. The recent financial crisis has led to renewed interest and debate about the role of government guarantees and their desirability. On the one hand, government guarantees are thought to have a positive role in preventing panic among investors, and hence help stabilize the financial system. On the other hand, the common belief is that they might create adverse incentives for banks to engage in excessive risk taking, which might lead to an increase in financial fragility.[1]

In light of this trade-off, evaluating the overall effects of government guarantees to banks is challenging, as it requires a framework in which the behavior of banks and their investors interacts with the amount and form of guarantees. Such a model is known to be notoriously rich and hard to solve. It needs to endogenize the probability of runs and how it is affected by banks' risk choices and government guarantees. It also needs to endogenize banks' risk choices and how they vary with the guarantee, taking into account investors' expected run behavior. We make technical progress in this paper by putting all these ingredients together in a tractable framework.

Our framework generates some surprising results on the effects of government guarantees. Most notably, we show that the increase in bank risk taking following the introduction of government guarantees may sometimes be a desirable consequence of their introduction. Hence, the downside of guarantees that is often brought up in the policy debate might be exaggerated. To the extent that banks perform a welfare-enhancing role with their liquidity transformation activities and that such liquidity transformation is inherently risky, they might provide too little liquidity in the face of run risk. As they alleviate run risk, government guarantees may in turn induce banks to increase the scope of these activities, thus improving welfare. This effect, which is captured by our model thanks to the endogenization of banks' and depositors' behavior, is missing from the current debate and academic literature and generates a more nuanced assessment of government guarantees.

Our starting point is the seminal Diamond and Dybvig (1983) economy, which has served researchers for years in studying runs and financial fragility. In this framework, banks offer deposit contracts to investors, who might face early liquidity needs, and by that provide liquidity transformation enabling risk sharing

[1] See, e.g., Calomiris (1990), Demirguc-Kunt and Detragiache (1998), Gropp, Gruendl and Guettler (2014), and Acharya and Mora (2015).

among depositors. While banks may improve investors' welfare due to the risk sharing they provide, the deposit contracts also expose banks to the risk of a bank run, where many depositors panic and withdraw early out of the self-fulfilling belief that other depositors will do so and the bank will fail. In the original framework, Diamond and Dybvig (1983) propose a deposit insurance scheme that eliminates runs altogether and restores full efficiency. In their model, banking crises happen only due to a coordination failure. Then, by ensuring that depositors will receive the promised payment independently of the other depositors' withdrawal decisions, deposit insurance prevents the bank-run equilibrium without entailing any disbursement for the government and so the first best allocation is achieved.

The literature that followed Diamond and Dybvig (1983) recognized that the effects of deposit insurance are more complicated and that there might be tradeoffs involved with increasing the amount of coverage. When runs are not purely driven by panics but sometimes occur as a result of deteriorating fundamentals of the banks' assets (see evidence in Gorton (1988), Calomiris and Gorton (1991) and Calomiris and Mason (2003)), deposit insurance may not fully prevent runs. This implies that actual costs of paying for failed banks will be incurred, and these might be increased by the fact that banks elevate their risk taking when they know they are insured. Despite enriching Diamond and Dybvig analysis with more realistic features, this literature cannot fully evaluate the effects of government guarantees as, by and large, it does not endogenize the probability of runs and, thus, cannot capture how this is affected by the risk taking decisions of banks.[2] This is what we do in this paper.

To conduct our analysis, we build on the model developed in Goldstein and Pauzner (2005), in which depositors' withdrawal decisions are uniquely determined using the global-game methodology, and so the probability of a run and how it is affected by the banking contract and by government policy can be determined. Goldstein and Pauzner (2005) study the interaction between the demand deposit contract and the probability of a run. In their model the run probability depends on the banking contract, and the bank decides on the banking contract taking into account its effect on the probability of a run. We add a government to this model to study how the government guarantee policy interacts with the banking contract and the probability of a run.

In our model, there are two periods. Banks raise funds from risk-averse consumers in the form of deposits and invest them in risky projects whose return depends on the fundamentals of the economy. Depositors derive utility from consuming both a private and a public good. At the interim date, each depositor learns

[2] We provide a literature review in the next section.

whether he needs to consume early or not and receives an imperfect signal regarding the fundamentals of the bank. Impatient depositors withdraw at that point and patient ones decide when to withdraw based on the information received. In deciding whether to run or not, depositors compare the payoff they would get from going to the bank prematurely and waiting until maturity. These payoffs depend on the fundamentals and the expectation about the proportion of depositors running.

In this setting, the equilibrium outcome is that runs occur when the fundamentals are below a unique threshold. Within the range where they occur, they can be classified into *panic-based runs* or *fundamental-based runs*. The former type of run is one that is generated by the self-fulfilling belief of depositors that other depositors will run. The latter type of run happens at the lower part of the run region, where the signal on the fundamentals is low enough to make running a dominant strategy for depositors. Overall, the probability of the occurrence of a run (and of both types of runs) is uniquely determined and depends on the deposit contract offered to depositors. As in Diamond and Dybvig (1983), there is perfect competition with free entry in the banking sector, and so banks offer a contract that maximizes depositors' expected utility. Unlike in Diamond and Dybvig (1983), however, they recognize the implications that the contract has for the possibility of a run and take them into account when deciding on the contract.

As in Goldstein and Pauzner (2005), we first show that the decentralized solution, i.e., without government intervention, is inefficient. There are two sources of inefficiency. First, in equilibrium, banks choose to offer deposit contracts that lead to inefficient fundamental-based and panic-based runs. While they internalize the cost of the runs, the benefit from risk sharing is large enough to lead banks to offer contracts that entail some inefficient runs. Second, since they internalize the probability of inefficient runs, banks reduce the amount of liquidity they offer to depositors demanding early withdrawal. Hence, in equilibrium, the amount of risk sharing that is offered to depositors is lower than what depositors would have liked if there was no concern of a run.

Then, we enrich the model by adding the government, which attempts to alleviate these inefficiencies by guaranteeing depositors to receive a minimum repayment through the transfer of resources from the public good to the banking sector. We start by considering a simple scheme of guarantees that is the analog to the one in Diamond and Dybvig (1983), in that it is only meant to prevent the occurrence of panic runs due to coordination failures. To this end, the scheme foresees that depositors are guaranteed to receive a minimum payment if the bank's project is successful irrespective of what the other depositors do. By eliminating the negative externality that a run imposes, this scheme prevents the occurrence of panic runs with a mere announcement effect and thus it does not entail any actual disbursement in equilibrium for the government. Hence, it does not lead to distortions in the bank's choice of the deposit contract. The contract chosen by the bank internalizes all the equilibrium effects and so is identical to the one that the government would have chosen. However, unlike in Diamond and Dybvig (1983), fundamental runs still occur in our framework, as depositors are not protected against the risk that the assets of their bank fail to produce the required return.

An important result is that when this guarantee scheme is in place, banks increase the amount they offer to depositors in case of early withdrawal, and so create more liquidity. This leads to an increase in the probability of fundamental-based runs. This is one form of what researchers may refer to as an increase in risk taking following the introduction of guarantees (e.g., Calomiris, (1990)). However, in our model, this scheme always promotes welfare relative to the decentralized solution. The increase in welfare results from addressing both inefficiencies in the decentralized case. The guarantee scheme eliminates panic-based runs and encourages banks to perform more liquidity transformation. Intuitively, banks provide contracts that maximize depositors' expected welfare under the constraints. With this guarantee scheme in place, the implications of increasing the short-term rate for the probability of a run are less severe, and so banks choose to increase it more, reducing the extent of the inefficiency of the decentralized solution due to inefficient risk sharing mentioned above. The apparent increase in risk taking is in fact a desirable outcome. Interestingly, we can show that this guarantee scheme sometimes leads to an increase in the overall probability of a run, that is, the increase in the probability of fundamental-based runs is large enough to more than offset the elimination of panic-based runs. This is consistent with evidence presented by Demirguc-Kunt and Detragiache (1998) that crises might become more likely in the presence of deposit insurance. However, welfare is still higher under this insurance scheme than in the decentralized solution. The fact that banks increase the amount they offer for early withdrawals and might increase the likelihood of runs overall does not imply they are acting against depositors' interests. Hence, the model demonstrates the need for caution in interpreting often-mentioned empirical results.

The above guarantee scheme still exhibits inefficiency. The facts that depositors are not protected against the failure of the banks' projects and that inefficient fundamental-based runs might be triggered as a result limit the efficiency increase coming from this guarantee scheme.[3] This scheme might also be hard to implement in the real world as in principle the payment is only triggered in case of panic, which is perhaps

[3]Fundamental runs can also be inefficient: even though it is a dominant strategy to run, a run might still be collectively inefficient.

not easily verifiable. Hence, we also consider a second guarantee scheme, in which depositors receive a minimum guaranteed payment, irrespective of what the others do and irrespective of the bank's available resources. That is, they get some protection against panic runs and fundamental failures either in the form of fundamental-based runs or bank insolvency. This scheme resembles the real-world deposit insurance. We show that, for a given short-term rate set by the banking contract, this guarantee scheme reduces the probability of both panic-based and fundamental-based crises. But, crises still occur, leading to actual disbursements, and so leading to non-trivial costs of increasing the level of guarantees. Hence, the government is limited here in how much it helps the banking system.

As with the previous scheme, we show that the introduction of guarantees addressing both panic and fundamental runs leads banks to increase the short-term payment they offer to depositors, which acts to improve welfare because of the increase in risk sharing. However, due to the fact that disbursement actually happens in equilibrium, there is a wedge between the optimal short-term rate (that the government would like to set) and the one that banks set in their contracts. Banks internalize the effect of the short-term rate on the probability of a run among their depositors, but they do not internalize the effect it has on the amounts that the government needs to spend and so on the level of public good. This is because overall government spending and the amount left for the public good are determined by the decisions of all banks combined and are very slightly affected by the decision of each particular bank. This is where the intuition of moral hazard often featured in the public debate – according to which banks' incentives are distorted by guarantees– starts to show up in our model.

Interestingly, however, while it is commonly thought that banks set short-term deposit rates too high in response to guarantees, our framework shows that the distortion can go in both directions. The important detail determining the direction of the distortion is whether the government ends up paying to depositors more in case there is no run and the bank ends up failing in the longer term for fundamental reasons or in case there is a run and the bank faces a shortage of liquidity. If the former holds, then the cost of a run from the point of view of banks is higher than from the point of view of the government and the banks set too low of a deposit rate (generating the opposite of the common wisdom). If the latter holds, then the cost of a run from the point of view of banks is lower than from the point of view of the government and the banks set too high of a deposit rate (generating the common wisdom). In any case, however, the government can choose the amount of guarantee such that welfare will always increase, despite the distortion.

In summary, a careful analysis of the effects of government guarantees shows that they have an important role helping the financial system to provide risk sharing to investors while mitigating the problems associated with coordination failures and inefficient liquidations. The common criticism against guarantees – that they encourage excessive risk taking – neglects to consider that some risk taking by banks due to liquidity transformation is desirable and might be suppressed by the concern for financial fragility. Guarantees, in turn, relax these concerns to some extent allowing banks to provide greater liquidity transformation, which is welfare improving. Of course, our paper does not cover all possible aspects of government guarantees; for example, we do not model the choice of assets by banks, but rather all risk taking in our model is captured on the liability side. It is possible that extending the model further will uncover darker sides of government guarantees. Also, we make several simplifying assumptions on the form of the banking contract and government guarantees, which keep the analysis tractable, but might prevent additional implications from being revealed. Still, our framework, to the best of our knowledge, is the first one that allows studying the endogenous probability of runs and the endogenous risk choice by banks and how they interact with each other and with the government's guarantees policy.

The paper proceeds as follows. Section 2 contains a literature review. Section 3 describes the model without government intervention. Section 4 derives the decentralized equilibrium. Section 5 analyzes the guarantee schemes. It first characterizes a scheme against panic runs and then one protecting depositors against both panic runs and fundamental failures. Section 6 uses a parametric example to illustrate the properties of the model. Section 7 contains discussion and conclusion. All proofs are contained in the appendix.

2 Related literature

The analysis of our paper provides a step towards understanding the interconnection between guarantees, fragility and bank's behavior. Our starting point is the literature on deposit insurance originated with the seminal paper by Diamond and Dybvig (1983). That framework features multiple equilibria: one where banks provide optimal risk sharing and no run occurs; another one where a panic run occurs due to the coordination failure among depositors. The introduction of deposit insurance works as an equilibrium selection device. The guarantee of receiving the promised repayment removes depositors' incentives to run. As a consequence, only the good equilibrium survives and the maximum amount of risk-sharing is achieved. Deposit insurance neither entails any cost nor it affects banks' behavior.[4]

[4]Similar environments where runs are driven by agents' expectations and public intervention is desirable to eliminate the panic equilibrium are analyzed in subsequent papers including, recently, Cooper and Kempf (2015).

Subsequent contributions (see Allen, Carletti, Goldstein and Leonello (2015) for a survey) have instead looked at the effect of deposit insurance on banks' and investors' behavior in frameworks where banks invest in risky projects and runs are due to the expectation of bad bank fundamentals. For example, Cooper and Ross (2002) extend Diamond and Dybvig (1983) by allowing banks to invest in a risky technology and depositors to monitor banks' investment strategies. They show that, when deposit insurance is sufficiently generous, banks find it optimal to invest in excessively risky projects and depositors have no or little incentives to monitor banks. As a consequence, although it prevents runs, a complete deposit insurance scheme fails to achieve the first-best allocation because of the greater bank risk taking. Importantly, this literature considers runs as sunspot phenomena triggered by some exogenous shift in depositors' expectations and independent of bank's behavior. Hence, it does not endogenize the probability of a run and its interaction with banks' choices and government guarantees. Our model combines the two above described approaches to crises in that it features both fundamental and panic based runs. However, differently from previous studies, in our model the probability of either type of runs is fully endogenous and affected by banks' risk choice, as well as by the presence of guarantees.

Our public intervention resembles standard deposit insurance schemes and differs from bailouts, which represent a form of ex post intervention aimed at mitigating the negative consequences of a crisis rather than preventing it. Despite these differences, our analysis shares some features with the literature on bailouts (see, among others, Farhi and Tirole (2012), Nosal and Ordonez (2016), Keister (2016) and Keister and Narasiman, (2016)), in that also these contributions analyze how the (anticipation of) bailouts may adversely affect banks' risk taking incentives, and ultimately the desirability of public intervention. Among these contributions, the closest papers to ours are Keister (2016) and Keister and Narasiman (2016). Both contributions extend Diamond and Dybvig (1983) and consider the effect of public intervention on depositors' withdrawal decisions and banks' behavior. The anticipation of a bailout introduces a trade-off: on the one hand, it induces banks to engage in more liquidity creation, thus increasing depositors' incentives to run; on the other hand, it improves investors' payoffs, thus reducing their incentives to run if they expect others to do the same. Whether the bailout improves welfare and leads to more or less fragility depends on which of these two effects dominates.

In both frameworks, the occurrence of runs depends on the realization of a sunspot variable, whose probability is exogenous and not affected by the anticipation of bailouts, and there is always a no run equilibrium irrespective of the bailout policy chosen by the government. An advantage of our paper is that the probability of runs is fully endogenous in our model, and so we are able to better characterize the interconnection between fragility, public guarantees and bank behavior. Our results that guarantees enable banks to perform more welfare-improving liquidity transformation, which is true even if the probability of crisis increases, and the characterization of the direction of distortions caused by guarantees are not present in the other papers. The disadvantage of our model is perhaps the assumptions on the form of the banking contract and government guarantees, which are taken as given, but this enables us to fully endogenize investors' runs decisions, banks' risk choices, and the interaction between them and with the guarantees regime. This is something that the previous literature was not able to do.

The ability to endogenize the probability of panics- and fundamental-driven runs relies on the use of global games. This approach, which was first studied in the seminal paper by Carlsson and van Damme (1993), allows deriving unique equilibria in contexts where agents have private information on some random variable. In the first application, Morris and Shin (1998) use this feature to study the occurrence of currency crises. Since then, global games have been used in finance to analyze, among others, issues of contagion (Goldstein and Pauzner (2004)), the role of large traders on the occurrence of currency crises (Corsetti, Dasgupta, Morris and Shin (2004)), twin crises (Goldstein (2005)), central bank lending (Rochet and Vives (2004)) and the fragility of demand deposit contracts (Goldstein and Pauzner (2005)).[5]

Generally, in the global games literature the proof of the uniqueness of the equilibrium builds on the existence of global strategic complementarities between agents' actions, in that an agent's incentive to take a specific action increases with the number of other agents taking the same action. This is the case in all papers mentioned above besides Goldstein and Pauzner (2005). In their framework, a depositor's incentive to run does not monotonically increase with the proportion of other depositors running. As noted in the introduction, our paper extends Goldstein and Pauzner (2005) by studying how the provision of guarantees (their design and size) affects fragility in the banking sector and interacts with the bank's choice of the deposit contract. Our framework builds on theirs and thus shares the same technical challenge of characterizing the existence of a unique equilibrium in a context in which there are no global strategic complementarities.

As they allow to endogenize the probability of crises and derive unique equilibria in contexts characterized by strategic complementarities, global games techniques have also been increasingly used in recent years to analyze relevant policy questions concerning financial regulation and public intervention (e.g., Bebchuk and Goldstein (2011), Choi (2014), Vives (2014) and Eisenbach (2016)). In this respect, they represent a powerful

[5] See also Morris and Shin (2003) for a survey on the theory and application of global games.

tool for policy analysis. As emerged in our analysis, having a unique equilibrium and being able to disentangle the various effects of a specific policy is key to evaluate its desirability, effectiveness and costs.

3 The basic model

The basic model is based on Goldstein and Pauzner (2005), augmented to include a government for the purpose of studying guarantee policies. There are three dates ($t = 0, 1, 2$), a continuum $[0, 1]$ of banks and a continuum $[0, 1]$ of consumers in every bank. There is perfect competition among banks, so that they make no profit.

Banks raise one unit of funds from consumers in exchange for a deposit contract as specified below, and invest in a risky project. For each unit invested at date 0, the project returns 1 if liquidated at date 1 and a stochastic return $\tilde{R}$ at date 2 given by

$$\tilde{R} = \begin{cases} R > 1 & \text{w.p. } p(\theta) \\ 0 & \text{w.p. } 1 - p(\theta). \end{cases}$$

The variable θ, which represents the state of the economy, is uniformly distributed over $[0, 1]$. We assume that $p(\theta) = \theta$ and $E_\theta[p(\theta)]R > 1$, which implies that the expected long term return of the project is superior to the short term return.

Each consumer is endowed with one unit at date 0 and nothing thereafter. At date 0, each consumer deposits his endowment at the bank. The bank promises a fixed payment $c_1 > 1$ to depositors withdrawing at date 1. Alternatively, depositors can choose to wait until date 2 and receive a risky payoff $\widetilde{c}_2$, as specified below.

Consumers are ex ante identical but can be of two types ex post: each of them has a probability λ of being an early consumer (impatient) and consuming at date 1, and a probability $1 - \lambda$ of being a late consumer (patient) and consuming at either date (we usually refer to them as early depositors and late depositors, respectively). Consumers privately learn their type at date 1.

The government has an endowment g, which, for the moment, it can only use to provide public goods to consumers in addition to the deposit payments they obtain from banks. Consumers' preferences are then given by

$$U(c, g) = u(c) + v(g),$$

where $u(c)$ represents the utility from the consumption of the payments obtained from banks and $v(g)$ is the utility from the consumption of the public good provided by the government. In what follows, we will refer to $u(c)$ and $v(g)$ also as private and public utility, respectively.[6] The function $U(c, g)$ satisfies $u'(c) > 0$, $v'(g) > 0$, $u''(c) < 0$, $v''(g) < 0$, $u(0) = v(0) = 0$ and relative risk aversion coefficient, $-cu''(c)/u'(c)$, greater than one for any $c \geq 1$.

The state of the economy θ is realized at the beginning of date 1, but is publicly revealed only at date 2. After θ is realized at date 1, each consumer receives a private signal x_i of the form

$$x_i = \theta + \varepsilon_i, \tag{1}$$

where ε_i are small error terms that are independently and uniformly distributed over $[-\varepsilon, +\varepsilon]$. After the signal is realized, consumers decide whether to withdraw at date 1 or wait until date 2.

The bank satisfies consumers' withdrawal demands at date 1 by liquidating the long term asset. If the liquidation proceeds are not enough to repay the promised c_1 to the withdrawing depositors, each of them receives a pro-rata share of the liquidation proceeds.[7]

Since the banking sector is perfectly competitive, banks choose the deposit contract $(c_1, \widetilde{c}_2)$ at date 0 that maximizes depositors' expected utility. As standard in the financial crisis literature (e.g. Diamond and Dybvig (1983) and numerous papers thereafter), the deposit contract involves a non-contingent date 1 payment c_1 and a date 2 payment $\widetilde{c}_2$ equal to the return of the non-liquidated units of the bank's project divided by the number of remaining late depositors. The payment c_1 must be lower than the amount $\frac{1-\lambda c_1}{1-\lambda} R$ that each late depositor receives at date 2 when only the λ early depositors withdraw early and the project succeeds. Otherwise, the deposit contract is never incentive compatible and late consumers always have an incentive to withdraw early and thus generate a run.

The timing of the model is as follows. At date 0, each bank chooses the promised payment c_1. At date 1, after realizing their type and receiving the private signal about the state of the fundamentals θ, depositors decide whether to withdraw early or wait until date 2. At date 2, the bank's project realizes and waiting late depositors receive a pro-rata share of the realized returns of the project units remaining at the bank. The model is solved backwards.

[6] Consumers receive the same amount of public good irrespective of their type. As with the good provided by the bank, early consumers enjoy the public good at date 1 while late consumers enjoy it at either date. Given there is no discounting, the timing of the provision does not matter for the late types.

[7] The assumption that depositors' repayments follow a pro-rata share rule rather than a sequential service constraint as in Goldstein and Pauzner (2005) simplifies the analysis without affecting the qualitative results.

4 The decentralized equilibrium

We first analyze the model without government guarantees and refer to the result as the decentralized equilibrium. We start by analyzing depositors' withdrawal decisions at date 1 for a given fixed payment c_1. We then move to date 0 and analyze the bank's choice of c_1.

Early consumers always withdraw at date 1 to satisfy their consumption needs. By contrast, late consumers compare the expected payoffs from going to the bank at date 1 or 2 and withdraw at the date when they expect to obtain the highest utility. Their expected payoffs depend both on the realization of the fundamentals θ as well as on the proportion n of depositors withdrawing early. Since the signal x_i provides information on both θ and other depositors' actions, late consumers base their withdrawal decision only on the signal they receive. When the signal is high, a late consumer attributes a high posterior probability to the event that the bank's project yields the positive return R at date 2. Also, upon observing a high signal, he infers that the others have also received a high signal. This lowers his belief about the likelihood of a run and thus his own incentive to withdraw at date 1. Conversely, when the signal is low, a late consumer has a high incentive to withdraw early as he attributes a high likelihood to the possibility that the project's date 2 return will be zero and that the other depositors run. This suggests that late consumers withdraw at date 1 when the signal is low enough, and wait until date 2 if, by contrast, the signal is sufficiently high.

To show this formally, we first assume that there are two regions of extremely bad or extremely good fundamentals, where each late consumer's action is based on the realization of the fundamentals irrespective of his beliefs about the others' behavior. The existence of these two extreme regions, no matter how small they are, guarantees the uniqueness of the equilibrium in depositors' withdrawal decisions. We then analyze the intermediate region where beliefs about the others' behavior play an important role in the determination of the equilibrium. We start with the lower region.

Lower Dominance Region. When the fundamentals are very bad (θ is very low), the expected utility from waiting until date 2, $\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)$, is lower than that from withdrawing at date 1, $u(c_1)$, even if only the early depositors were to withdraw ($n = \lambda$). If, given his signal, a late depositor is sure that this is the case, running is a dominant strategy. We then denote by $\underline{\theta}(c_1)$ the value of θ that solves

$$u(c_1) = \theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right), \tag{2}$$

that is

$$\underline{\theta}(c_1) = \frac{u(c_1)}{u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)}. \tag{3}$$

We refer to the interval $[0, \underline{\theta}(c_1))$ as the lower dominance region, where runs are only driven by bad fundamentals. For the lower dominance region to exist for any $c_1 \geq 1$, there must be feasible values of θ for which all late depositors receive signals that assure them to be in this region. Since the noise contained in the signal x_i is at most ε, each late depositor withdraws at date 1 if he observes $x_i < \underline{\theta}(c_1) - \varepsilon$. It follows that all depositors receive signals that assure them that θ is in the lower dominance region when $\theta < \underline{\theta}(c_1) - 2\varepsilon$. Given that $\underline{\theta}$ is increasing in c_1, the condition for the lower dominance region to exist is satisfied for any $c_1 \geq 1$ if $\underline{\theta}(1) > 2\varepsilon$.

Upper Dominance Region. The upper dominance region of θ corresponds to the range $\left(\overline{\theta}, 1\right]$ in which fundamentals are so good that no late depositors withdraw at date 1. As in Goldstein and Pauzner (2005), we construct this region by assuming that in the range $(\overline{\theta}, 1]$ the project is safe, i.e., $p(\theta) = 1$, and yields the same return $R > 1$ at dates 1 and 2. Given $c_1 < \frac{1-\lambda c_1}{1-\lambda} R \leq R$, this ensures that the bank does not need to liquidate more units than the n depositors withdrawing at date 1. Then, when a late depositor observes a signal such that he believes that the fundamentals θ are in the upper dominance region, he is certain to receive his payment $\frac{1-\lambda c_1}{1-\lambda} R$ at date 2, irrespective of his beliefs on other depositors' actions, and thus he has no incentives to run. Similarly to before, the upper dominance region exists if there are feasible values of θ for which all late depositors receive signals that assure them to be in this range. This is the case if $\overline{\theta} < 1 - 2\varepsilon$.

The Intermediate Region

The two dominance regions are just extreme ranges of fundamentals in which late depositors have a dominant strategy that depends only on the fundamentals θ. When the signal indicates that θ is in the intermediate range $\left[\underline{\theta}(c_1), \overline{\theta}\right]$, a depositor's decision to withdraw early depends on the realization of θ as well as on his beliefs regarding other late depositors' actions.

To determine late depositors' withdrawal decisions in this region, we calculate their utility differential between withdrawing at date 2 and at date 1 as given by

$$v(\theta, n) = \begin{cases} \theta u\left(\frac{1-nc_1}{1-n} R\right) - u(c_1) & \text{if } \lambda \leq n \leq \widehat{n} \\ \\ 0 - u(\frac{1}{n}) & \text{if } \widehat{n} \leq n \leq 1, \end{cases} \tag{4}$$

where n represents the proportion of depositors withdrawing at date 1 and

$$\widehat{n}=1/c_1 \tag{5}$$

is the value of n at which the bank exhausts its resources if it pays $c_1 \geq 1$ to all withdrawing depositors. The expression for $v(\theta, n)$ states that as long as the bank does not exhaust its resources at date 1, i.e., for $n \leq \widehat{n}$, late depositors waiting until date 2 obtain the residual $\frac{1-nc_1}{1-n}R$ with probability θ while those withdrawing early obtain c_1. By contrast, for $n \geq \widehat{n}$ the bank liquidates its entire project at date 1. Each late depositor receives nothing if he waits until date 2 and the pro-rata share $1/n$ when withdrawing early.

Insert Figure 1

As Figure 1 illustrates, the function $v(\theta, n)$ decreases in n for $n \leq \widehat{n}$ and increases with it afterwards. This implies that a late depositor's incentive to withdraw early is highest when $n = \widehat{n}$ rather than when $n = 1$, as it is usually the case in standard global games where the equilibrium builds on the property of global strategic complementarity (e.g., Morris and Shin (1998)). However, since $v(\theta, n)$ decreases in n whenever it is positive and crosses zero only once for $n \leq \widehat{n}$ remaining always below zero afterwards, the model exhibits the property of *one-sided strategic complementarity* as in Goldstein and Pauzner (2005). This implies that there still exists a unique equilibrium in which a late depositor runs if and only if his signal is below the threshold $x^*(c_1)$. At this signal value, a late depositor is indifferent between withdrawing at date 1 and waiting until date 2.

Formally, $x^*(c_1)$ is such that, conditional on this signal, the expected utility $\int_{\lambda}^{\widehat{n}} u(c_1)dn + \int_{\widehat{n}}^{1} u(\frac{1}{n})dn$ from withdrawing at date 1 is equal to the expected utility $\int_{\lambda}^{\widehat{n}} \theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) dn + \int_{\widehat{n}}^{1} u(0)dn$ from waiting until date 2. Here, the marginal depositor expects the proportion n of running depositors to be uniformly distributed between λ and 1. This is a result of the fact that impatient depositors always run and patient depositors receive signals with uniformly distributed noise and in equilibrium they run below $x^*(c_1)$ and do not run above it. The following result holds.

Proposition 1 *The model has a unique equilibrium in which late depositors run if they observe a signal below the threshold $x^*(c_1)$ and do not run above. At the limit, as $\varepsilon \to 0$, $x^*(c_1)$ simplifies to*

$$\theta^*(c_1) = \frac{u(c_1)\left[1 - \lambda c_1\right] + c_1 \int_{n=\widehat{n}}^{1} u(\frac{1}{n})}{c_1 \int_{n=\lambda}^{\widehat{n}} u\left(\frac{1-nc_1}{1-n}R\right)}. \tag{6}$$

The proposition states that even in the intermediate region a late depositor's action depends uniquely on the signal he receives as this provides information both on the fundamentals θ and the other depositors' actions. For θ in the interval $[\underline{\theta}(c_1), \theta^*(c_1))$ there is strategic complementarity in consumers' withdrawal decisions: If $c_1 > 1$, the bank has to liquidate more than one unit for each withdrawing depositor. This implies that late depositors' date 2 payoff is decreasing in the proportion n of early withdrawing depositors and so their incentives to run increases with n. In the limit case when $\varepsilon \to 0$, all late depositors behave alike as they receive approximately the same signal and take the same action. This implies that only complete runs, where all late depositors withdraw at date 1, occur. In what follows, we will focus on this limit case.

Insert Figure 2

Proposition 1 implies that a run occurs for any $\theta < \theta^*(c_1)$, but for different reasons as also illustrated in Figure 2. For θ in the interval $[0, \underline{\theta}(c_1))$ runs are *fundamental-based*: Late depositors withdraw early because they expect the fundamentals to be bad so that running is a dominant strategy. For θ in the interval $[\underline{\theta}(c_1), \theta^*(c_1))$ runs are *panic-based*: Late depositors withdraw because they expect the others to do the same. The two types of runs differ significantly in terms of efficiency. Panic runs are always inefficient as they result from a coordination failure among depositors. By contrast, fundamental runs can be efficient if they lead to the early liquidation of unprofitable investments. For each unit that the bank liquidates at date 1 to repay the withdrawing depositors, the return R is foregone with probability θ. Liquidating the project is then inefficient for any $\theta > \underline{\theta}(1)$ since the utility $u(1)$ that a depositor obtains from the liquidated unit is lower than the expected utility $\theta u(R)$ he would obtain from the same unit if invested until date 2. If $c_1 > 1$, then $\underline{\theta}(1) < \underline{\theta}(c_1) < \theta^*(c_1)$. Thus, fundamental runs are efficient in the range $[0, \underline{\theta}(1))$ but inefficient in the range $[\underline{\theta}(1), \underline{\theta}(c_1))$.

The likelihood of both types of run$-$ as given by the thresholds $\underline{\theta}(c_1)$ and $\theta^*(c_1)-$ is affected by the promised payment c_1 offered by the bank to early withdrawers. We have the following result.

Corollary 1 *Both thresholds* $\underline{\theta}(c_1)$ *and* $\theta^*(c_1)$ *are increasing in* c_1 $\left(\textit{i.e.}, \frac{\partial \underline{\theta}(c_1)}{\partial c_1} > 0 \textit{ and } \frac{\partial \theta^*(c_1)}{\partial c_1} > 0\right)$ *with* $\frac{\partial \theta^*(c_1)}{\partial c_1} > \frac{\partial \underline{\theta}(c_1)}{\partial c_1}$.

The corollary suggests that both run thresholds increase with the deposit rates offered by banks, although their sensitivity is different. The reason is that the higher c_1, the lower is the payoff $\widetilde{c}_2$ accrued by the late depositors at date 2 and thus the stronger is the incentive for each late depositor to withdraw early. The panic threshold $\theta^*(c_1)$ is more sensitive to changes in c_1 than the fundamental threshold $\underline{\theta}(c_1)$ because in the case of panic runs an increase in c_1 also changes the beliefs that each depositor has on the others' behavior and on the damage that their withdrawals will cause to the remaining investors' returns. This reinforces each late depositor's incentive to run, thus making $\theta^*(c_1)$ more sensitive to changes in c_1 than $\underline{\theta}(c_1)$.

Now that we have characterized depositors' withdrawal decisions at date 1, we turn to date 0 and compute the optimal deposit contract c_1. Each bank chooses c_1 at date 0 to maximize the expected utility of a representative depositor, which is given by

$$\int_0^{\theta^*(c_1)} u(1)\, d\theta + \int_{\theta^*(c_1)}^1 \left[\lambda u(c_1) + (1-\lambda)\theta u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta + \int_0^1 v(g) d\theta. \tag{7}$$

The first term represents depositors' expected utility from depositing at the bank for $\theta < \theta^*(c_1)$ when, given that a run occurs, the bank liquidates its entire project and each depositor obtains 1 instead of the promised payment c_1. The second term is depositors' expected utility for $\theta \geq \theta^*(c_1)$ when the bank continues until date 2. The λ early consumers withdraw early and obtain c_1, while the $(1-\lambda)$ late depositors wait and receive the payment $\frac{1-\lambda c_1}{1-\lambda}R$ with probability θ. The last term is the utility that depositors receive from the consumption of the public good. Since the entire government's endowment g is used to provide the public good, depositors' utility $v(g)$ is unaffected by the occurrence of runs. We have the following result.

Proposition 2 *The optimal deposit contract $c_1^D > 1$ in the decentralized solution solves*

$$\begin{aligned} &\lambda \int_{\theta^*(c_1)}^1 \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta + \\ &-\frac{\partial \theta^*(c_1)}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right] = 0 \end{aligned} \tag{8}$$

In choosing the promised payment to early depositors the bank trades off the marginal benefit of a higher c_1 with its marginal cost. The former, as represented by the first term in (8), is the better risk sharing obtained from the transfer of consumption from late to early depositors. The latter, which is captured by the second term in (8), is the loss in expected utility $\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right]$ due to the increased probability of runs, as measured by $\frac{\partial \theta^*(c_1)}{\partial c_1}$. At the optimum, the bank chooses $c_1^D > 1$ even if this entails panic runs. The reason is that when $c_1 = 1$, the difference between early and late depositors' expected payment is maximal. A slight increase of c_1 provides a large benefit in terms of risk sharing given that depositors have a relative risk aversion coefficient greater than 1. Furthermore, at $c_1 = 1$ the loss in terms of expected utility in the case of a run approaches zero.[8] Thus, increasing c_1 slightly above 1 entails

[8] When $c_1 = 1$, the term $\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right]$ simplifies to $(1-\lambda)\left[\theta^*(1) u(R) - u(1)\right] = 0$ with

a second-order cost and a first-order benefit and so it is always optimal. The optimal c_1^D is chosen so that runs occur only for $\theta < \theta^*(c_1) < \overline{\theta}$. If this was not the case and runs occurred for any θ, consumers would obtain a utility $u(1)$, which would be lower than the utility they reach with the optimal c_1^D.

The bank's choice of $c_1^D > 1$ entails a trade-off in our model. On the one hand, c_1 represents the amount of risk sharing banks offer to depositors. On the other hand, given that the probability of runs is endogenous and is linked to the parameters of the deposit contract, c_1 represents a form of risk as it determines banks' exposure to runs. Hence, the higher payment c_1 the greater is banks' liquidity creation but also their fragility. Since banks anticipate the effect of a higher c_1 on their fragility, they reduce deposit rates thus scaling down liquidity creation. In equilibrium, banks offer too little risk sharing relative to the level they would choose if runs were not considered. In the latter case, each bank would in fact choose c_1 to maximize

$$\int_0^1 \left[\lambda u(c_1) + (1-\lambda)\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta + \int_0^1 v(g) d\theta.$$

The solution to this problem, which we denote c_1^{NR}, is obtained from the following equation:

$$\lambda \int_0^1 \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta = 0. \tag{9}$$

The solution c_1^{NR} maximizes the gains from risk sharing and it coincides with the promised repayment to depositors in Diamond and Dybvig (1983), with the difference that the bank's project is now risky and thus the utility of late consumers at date 2 includes the probability θ that the project succeeds. Comparing (8) and (9), it can be easily seen that $c_1^{NR} > c_1^D$. The intuition is simple: c_1^{NR} maximizes the gain from risk sharing while ignoring the possibility of bank runs. As such, it does not consider the effect of c_1 on the probability of runs and the associated costs.

To sum up, the decentralized equilibrium is characterized by too little risk sharing and the possibility of inefficient runs for $[\underline{\theta}(1), \theta^*(c_1))$. Both inefficiencies depend on the bank's choice of c_1. A higher c_1 improves liquidity creation but also enlarges the range $[\underline{\theta}(1), \theta^*(c_1))$ where runs destroy good investments. The question is whether and how the decentralized solution can be improved upon with the help of public intervention.

5 Introducing government guarantees

In this section, we analyze whether the introduction of government guarantees can improve upon the decentralized allocation. The starting point to understand the role of public intervention in our model is Diamond

$\theta^*(1) = \left[\frac{u(1)}{u(R)}\right]$.

and Dybvig (1983), where the decentralized economy features multiple equilibria: A good equilibrium, where banks are stable and provide full insurance to depositors, and a bad equilibrium, where a panic run occurs. Since there is no fundamental risk and runs are sunspot phenomena, the introduction of deposit insurance works simply as an equilibrium selection device: Only the good equilibrium survives and the maximum level of liquidity creation is attained. In other words, deposit insurance allows to restore the full role of banks as liquidity providers without entailing any fragility.

While being powerful, the analysis of deposit insurance in Diamond and Dybvig (1983) has two important shortcomings. First, deposit insurance removes the bad equilibrium, but it does not affect banks' behavior. Second, as it has only an announcement effect, deposit insurance does not entail any disbursement in terms of public funds. It follows that banks do not increase their risk taking in an attempt to exploit the benefit of public intervention. These features of the Diamond and Dybvig framework contrast with numerous studies (e.g., Calomiris (1990), and Cooper and Ross (2002)) arguing that deposit insurance worsens banks' risk taking incentives and thus increases their fragility at the expenses of public resources.

The richness of our model, where runs can be both fundamental and panic driven and the probability of runs is endogenously determined as a function of the parameters of the demand deposit contract, allows us to analyze the interconnections between government guarantees, depositors' withdrawal decisions and banks' behavior. The main insight is that guarantees are welfare improving because they induce banks to increase deposit rates and thus liquidity provision although in a way that sometimes increases the likelihood of runs or creates distortions in banks' behavior.

To illustrate our results in a simple way, we start by considering a simple form of guarantees that, in the spirit of Diamond and Dybvig, addresses only panic runs due to coordination failure. Although it is pedagocically useful, this scheme exhibits inefficiency as it does not prevent fundamental-based runs, and it may be difficult to implement in practice as it requires observing the nature of runs. Thus, we analyze a second guarantee scheme that addresses both fundamental and panic runs. In both schemes, guarantees have a twofold effect on the probability of runs. First, they have a *direct* positive effect as they reduce late depositors' incentives to run. Second, they have a negative *indirect* effect as they induce banks to increase deposit rates. Depending on which effect dominates, guarantees can either decrease or increase bank fragility. Withstanding this similarity, the two schemes differ in terms of the cost they entail for the government and the consequent distortions they may introduce in banks' behavior.

The introduction of guarantees modifies the timing of the model as follows. At date 0, the government chooses the amount to guarantee $\bar{c}$ and then the bank chooses c_1. At date 1, after learning their types and receiving the signal about the state of fundamentals θ, depositors decide whether to withdraw early or wait until date 2. As before, for each guarantee scheme considered, we solve the model backward. We first characterize depositors' withdrawal decisions for given $\bar{c}$ and c_1 and obtain the thresholds $\underline{\theta}(c_1, \bar{c})$ and $\theta^*(c_1, \bar{c})$ for the fundamental and panic runs, respectively. Then, we characterize the bank's choice of c_1, for given $\bar{c}$, and finally the government's choice of $\bar{c}$. In both guarantee schemes, the government finances the promised guarantee $\bar{c}$ through the transfer of resources from its endowment g to the banking sector.

5.1 Guarantees against panic runs

We first analyze a scheme that guarantees depositors to receive a minimum payment if the bank's project is successful irrespective of what the other depositors do. We first show that this scheme removes the coordination problem among depositors but fails to prevent the occurrence of fundamental runs because depositors are not protected against the risk that the projects of their bank fail. Second, we show that, as it removes panic runs, the guarantee induces banks to increase deposit rates. This in turn increases the probability of fundamental runs and possibly even the probability of runs overall.

As highlighted in the analysis of the decentralized economy, panic runs arise in our model because of the strategic complementarity between depositors' actions. The greater the number of depositors withdrawing at date 1, the more units of the long term asset the bank needs to liquidate prematurely. This, in turn, increases the incentive for a late depositor to run since his repayment in the case he waits is reduced. The coordination failure among depositors leads to a panic-driven run for θ in the interval $[\underline{\theta}(c_1), \theta^*(c_1))$.

As in Diamond and Dybvig (1983), panic runs can be prevented in our model through a guarantee scheme that eliminates the strategic complementarity between depositors' actions. To achieve this, consider that now the government promises depositors to always receive a repayment $\bar{c}$ equal to the amount they would obtain in case of no runs, i.e., $\bar{c} = \frac{1-\lambda c_1}{1-\lambda} R$, when the bank project succeeds at date 2.[9] This removes late depositors' incentives to withdraw at date 1 for θ in the range $[\underline{\theta}(c_1), \theta^*(c_1))$ because their date 2 payoff when the bank's project succeeds becomes independent of the other depositors' withdrawal decisions. This scheme has a pure announcement effect and does not entail any disbursement for the government.

Under this scheme, late depositors still choose to run when they expect bad fundamentals, that is when

[9] It is easy to show formally that setting $\bar{c} = \frac{1-\lambda c_1}{1-\lambda} R$ is optimal given that this guarantee scheme does not entail any disbursement for the government and banks maximize depositors' expected utility. The formal proof is available from the authors upon request.

$\theta < \underline{\theta}(c_1)$, where $\underline{\theta}(c_1)$ is determined in (3). Importantly though, the elimination of panic runs affects the bank's choice of c_1 and thus the probability of fundamental runs in equilibrium will be different from the decentralized economy. This is what we turn next.

Given that runs occur now for $\theta \leq \underline{\theta}(c_1)$, each bank chooses c_1 to maximize depositors' expected utility as given by

$$\underset{c_1}{Max} \int_0^{\underline{\theta}(c_1)} u(1)d\theta + \int_{\underline{\theta}(c_1)}^1 \left[\lambda u(c_1) + (1-\lambda)\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta + \\ + \int_0^1 v(g)d\theta \tag{10}$$

The terms in (10) have the same meaning as in (7), with the difference that runs are now only fundamental-driven and thus the relevant threshold is $\underline{\theta}(c_1)$ instead of $\theta^*(c_1)$. Since the insurance scheme does not entail any cost, the government still provides g units of public good and depositors still obtain public utility $v(g)$ as in the decentralized case. As in the decentralized solution, the solution to the problem in (10) must be such that $\underline{\theta}(c_1) < \overline{\theta}$ at the equilibrium c_1. We have the following result.

Proposition 3 *The optimal deposit contract $c_1^{DD} > c_1^D$ in the case of a guarantee scheme against panic runs solves*

$$\lambda \int_{\underline{\theta}(c_1)}^1 \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta + \\ - \frac{\partial \underline{\theta}(c_1)}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\underline{\theta}(c_1) u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(1)\right] = 0. \tag{11}$$

As in the decentralized economy, the bank chooses the deposit contract that trades off the marginal benefit of a higher c_1 with its marginal cost. The former, as represented by the first term in (11), is the better risk sharing obtained from the transfer of consumption from late to early depositors. The latter, as captured by the second term in (11), is the loss in expected utility $\left[\lambda u(c_1) + (1-\lambda)\underline{\theta}(c_1) u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(1)\right]$ due to the increased probability of fundamental runs as measured by $\frac{\partial \underline{\theta}(c_1)}{\partial c_1}$. The solution c_1^{DD} is now larger than c_1^D in the decentralized economy as given by the solution to (8). The reason is that both the run threshold $\underline{\theta}(c_1)$ and its sensitivity to changes in c_1, as represented by $\frac{\partial \underline{\theta}(c_1)}{\partial c_1}$, are lower than $\theta^*(c_1)$ and $\frac{\partial \theta^*(c_1)}{\partial c_1}$ as shown in Corollary 1. As a result, the marginal benefit of an increase in c_1 in terms of better risk sharing is higher than the one in the decentralized economy, while its marginal cost is lower. Thus, the bank has an incentive to choose a higher c_1 and improve liquidity creation relative to the case without government intervention.

Insert Figure 3

The proposition has important implications in terms of bank stability, as illustrated in Figure 3. The guarantee scheme eliminates panic runs, but increases the probability of fundamental runs due to the increased deposit rate. Given Corollary 1 and $c_1^{DD} > c_1^D$, $\underline{\theta}(c_1^{DD}) > \underline{\theta}(c_1^D)$.

If the difference between c_1^{DD} and c_1^D is large enough, it can even happen that the bank becomes more fragile than in the decentralized solution, that is $\underline{\theta}(c_1^{DD}) > \theta^*(c_1^D)$. The bank chooses to do so if the benefit of the increased c_1 in terms of risk sharing outweights the cost in terms of higher probability of runs. After comparing the expected overall utilities with and without guarantees in equilibrium as given in (10) and (7) and rearranging the terms, we can express the condition for this to happen as follows:

$$\begin{aligned} \int_{\underline{\theta}(c_1^{DD})}^{1} \left[\left(\lambda u(c_1^{DD}) + (1-\lambda)\theta u\left(\tfrac{1-\lambda c_1^{DD}}{1-\lambda} R\right)\right) - \left(\lambda u(c_1^{D}) + (1-\lambda)\theta u\left(\tfrac{1-\lambda c_1^{D}}{1-\lambda} R\right)\right)\right] d\theta > \\ \int_{\theta^*(c_1^D)}^{\underline{\theta}(c_1^{DD})} \left[\lambda u(c_1^D) + (1-\lambda)\theta u\left(\tfrac{1-\lambda c_1^D}{1-\lambda} R\right) - u\left(1\right)\right] d\theta. \end{aligned} \tag{12}$$

The term on the LHS represents the benefit in terms of greater risk sharing deriving from a larger c_1 in the range $\theta > \underline{\theta}(c_1^{DD})$ where no run occurs both in the decentralized economy and with guarantees. The term on the RHS represents instead the loss in terms of foregone risk sharing when in the range $\theta^*(c_1^D) \leq \theta < \underline{\theta}(c_1^{DD})$ a run occurs in the presence of the guarantee because of the higher c_1. Characterizing when (12) holds is not straightforward as several effects are at play. We will provide an example in which this happens and the guarantee leads to greater overall instability in Section 6.

Despite the negative effect on bank fragility, guarantees are welfare improving as, by eliminating panic runs, they allow risk sharing to be improved. Banks internalize all the effects of a change in the repayment c_1 on depositors' expected utility. Hence, depositors' expected utility is maximized, although the improved liquidity provision can lead to a higher probability of runs.

5.2 Guarantees against panic and fundamental runs

We now analyze a second scheme of guarantees where depositors receive a minimum promised amount irrespective of the time they withdraw and the realization of the bank's project return. This scheme resembles a standard deposit insurance mechanism, which entails a fix minimum payment whenever the bank is unable to repay the promised returns.

This scheme differs from the previous one in two respects. First, as it affects depositors' payoffs at date 1 and 2 for any possible θ, it influences the probability of both fundamental and panic runs. Second, it entails a disbursement of public funds and hence a lower provision of public good. Banks choose deposit rates without internalizing the effect of changes in c_1 on the provision of public good. Hence, guarantees introduce now distortions in banks' behavior. Despite this, they remain welfare improving thanks to their positive effect on liquidity provision.

Consider that the government promises depositors to always receive a minimum amount $\overline{c}$. This modifies depositors' payoffs as follows: in case of no run, early depositors still obtain c_1 at date 1 while late depositors obtain $\frac{1-\lambda c_1}{1-\lambda}R$ with probability θ and $\overline{c}$ with probability $1-\theta$ at date 2. In case of a run, all depositors obtain $\overline{c}$ at date 1 if $\overline{c} > 1$ and 1 if $\overline{c} \leq 1$. The government finances the provision of the guarantee by transferring part of its resources g to the banking sector, thus reducing the provision of the public good.

To study this new form of guarantee, we first characterize the equilibrium in depositors' withdrawal decisions and then the bank's deposit contract choice and the government's choice of the amount guaranteed. To make the analysis meaningful, we focus on the parameter space where $\overline{c} < c_1$ in equilibrium so that depositors are not fully insured and have still incentives to run.

We start by characterizing depositors' withdrawal decisions at date 1 for given c_1 and $\overline{c}$. As before, we first analyze the two extreme regions of fundamentals. The upper dominance region is as in the decentralized economy. The upperbound of the lower dominance region $\underline{\theta}(c_1, \overline{c})$ is the solution to

$$u(c_1) = \theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + (1-\theta)u(\overline{c}).$$

The terms have the same meaning as in (2) with the difference that depositors receive now utility $u(\overline{c})$ at date 2 when, with probability $1-\theta$, the bank's project fails and the guarantee $\overline{c}$ is paid. The solution is then equal to

$$\underline{\theta}(c_1, \overline{c}) = \frac{u(c_1) - u(\overline{c})}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\overline{c})}. \tag{13}$$

It is immediate to see that, differently from the previous scheme analyzed, the threshold of fundamental runs does now depend on both c_1 and $\overline{c}$, as we explain further below.

To characterize the intermediate region, we need to distinguish whether $\overline{c}$ is smaller or greater than 1. If $\overline{c} \leq 1$, the guarantee is paid only at date 2 and a late depositor's utility differential between withdrawing at date 2 versus date 1, denoted $v(\theta, n, \overline{c})$, is given by

$$v(\theta, n, \overline{c}) = \begin{cases} \theta u\left(\frac{1-nc_1}{1-n}R\right) + (1-\theta)u\left(\overline{c}\right) - u(c_1) & \text{if } \lambda \leq n \leq \overline{n} \\ u\left(\overline{c}\right) - u(c_1) & \text{if } \overline{n} \leq n \leq \widehat{n} \\ u\left(\overline{c}\right) - u(\frac{1}{n}) & \text{if } \widehat{n} \leq n \leq 1. \end{cases} \tag{14}$$

The expression for $v(\theta, n, \overline{c})$ has three intervals. In the first interval, for $\lambda \leq n \leq \overline{n} = \frac{R-\overline{c}}{Rc_1-\overline{c}}$, depositors waiting until date 2 receive $\frac{1-nc_1}{1-n}R > \overline{c}$ with probability θ and $\overline{c}$ with probability $1-\theta$, while those withdrawing early obtain c_1. As n reaches $\overline{n}$, the repayment of the bank at date 2 falls below $\overline{c}$ so that late depositors always receive $\overline{c}$ for $n \geq \overline{n}$. Depositors withdrawing at date 1 receive the promised repayment c_1 as long as $n \leq \widehat{n} = \frac{1}{c_1}$, that is as long as the bank has enough resources to pay c_1 from the liquidation of the project at date 1. As n grows further (i.e., for any $\widehat{n} \leq n \leq 1$), the bank liquidates its entire project for a value of 1 and each depositor receives the pro-rata share $\frac{1}{n}$ when withdrawing at date 1. Since $\frac{1}{n} \geq 1 \geq \overline{c}$, the guarantee is never paid to depositors withdrawing at date 1.

If $\overline{c} > 1$ the pro-rata share $\frac{1}{n}$ can fall below $\overline{c}$ when a large number of depositors withdraw at date 1. Then, in the case $\overline{c} > 1$, the expression for $v(\theta, n, \overline{c})$ becomes

$$v(\theta, n, \overline{c}) = \begin{cases} \theta u\left(\frac{1-nc_1}{1-n}R\right) + (1-\theta)u\left(\overline{c}\right) - u(c_1) & \text{if } \lambda \leq n \leq \overline{n} \\ u\left(\overline{c}\right) - u(c_1) & \text{if } \overline{n} \leq n \leq \widehat{n} \\ u\left(\overline{c}\right) - u(\frac{1}{n}) & \text{if } \widehat{n} \leq n \leq \widetilde{n} \\ u\left(\overline{c}\right) - u(\overline{c}) & \text{if } \widetilde{n} \leq n \leq 1 \end{cases}, \tag{15}$$

The function $v(\theta, c_1, \overline{c})$ has now four intervals. The first three are the same as in (14) and the terms have the same meaning as there. The fourth interval for $\widetilde{n} = \frac{1}{\overline{c}} \leq n \leq 1$ is where the pro-rata share $\frac{1}{n}$ falls below $\overline{c}$ and depositors withdrawing at date 1 start receiving the guarantee $\overline{c}$.

Insert Figures 4a and 4b

The functions $v(\theta, n, \overline{c})$ are illustrated in Figures 4a and 4b for the case $\overline{c} \leq 1$ and $\overline{c} > 1$, respectively. When $\overline{c} \leq 1$, the function $v(\theta, n, \overline{c})$ crosses zero once and remains strictly below zero afterwards. By contrast, when $\overline{c} > 1$, it crosses zero for $n < \widehat{n}$, it stays below zero for $\widehat{n} \leq n \leq \widetilde{n}$ and it is equal to zero for $\widetilde{n} \leq n \leq 1$. Despite this, in both cases it still exists a unique threshold equilibrium.

As in the decentralized economy, the threshold signal $x^*(c_1, \overline{c})$ can be found as the solution to the indifference condition that equates a depositor's expected utility from withdrawing early with the one from waiting until date 2. We have the following result.

Proposition 4 *The model with a guarantee scheme against panic and fundamental failures has a unique threshold equilibrium in which late depositors run if they observe a signal below the threshold $x^*(c_1, \overline{c})$ and do not run above. At the limit as $\varepsilon \to 0$, the equilibrium threshold $x^*(c_1, \overline{c})$ simplifies to*

$$\theta^*(c_1, \overline{c}) = \frac{\int_{n=\lambda}^{\widehat{n}} u\left(c_1\right) + \int_{n=\widehat{n}}^{min(\widetilde{n},1)} u(\frac{1}{n}) - \int_{n=\lambda}^{min(\widetilde{n},1)} u\left(\overline{c}\right)}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u\left(\overline{c}\right)\right]}. \tag{16}$$

The proposition characterizes the equilibrium threshold $\theta^*(c_1, \overline{c})$ as a function of the deposit contract c_1 chosen by the bank and the level of guarantees $\overline{c}$ set by the government. The expression for $\theta^*(c_1, \overline{c})$ depends on whether the level of guarantees $\overline{c}$ is above or below 1 as this determines when depositors enjoy the guarantee.

The likelihood of runs, as given by $\underline{\theta}(c_1, \overline{c})$ and $\theta^*(c_1, \overline{c})$, depends as before on the promised repayment c_1 but also on the guaranteed amount $\overline{c}$ as follows.

Corollary 2 *Both thresholds $\underline{\theta}(c_1, \overline{c})$ and $\theta^*(c_1, \overline{c})$ are increasing in c_1 and decreasing in $\overline{c}$, i.e.,* $\frac{\partial \underline{\theta}(c_1,\overline{c})}{\partial c_1} > 0$, $\frac{\partial \theta^*(c_1,\overline{c})}{\partial c_1} > 0$, $\frac{\partial \underline{\theta}(c_1,\overline{c})}{\partial \overline{c}} < 0$ *and* $\frac{\partial \theta^*(c_1,\overline{c})}{\partial \overline{c}} < 0$.

As in the decentralized economy, for a given $\overline{c}$, a higher c_1 leads to more fundamental and panic runs as it increases depositors' payoff at date 1, while lowering that at date 2. Thus, as before, greater risk sharing entails more runs also in the presence of this guarantee scheme. By contrast, for a given c_1, a higher $\overline{c}$ reduces the probability of a run as it increases the expected payment that late depositors receive if they wait until date 2. Hence, for a given c_1, both $\underline{\theta}(c_1, \overline{c})$ and $\theta^*(c_1, \overline{c})$ are lower than the correspondent thresholds in the decentralized economy. This represents the positive direct effect of government intervention on bank fragility.

Having characterized depositors' withdrawal decisions, we can now turn to date 0 and analyze the bank's choice of c_1 and the government's choice of $\overline{c}$. We start with the former. Given $\overline{c}$ and anticipating depositors' withdrawal decisions, each bank chooses c_1 to maximize depositors' expected utility:

$$\underset{c_1}{Max} \int_0^{\theta^*(c_1,\overline{c})} u(\max(1,\overline{c}))d\theta + \int_{\theta^*(c_1,\overline{c})}^1 \left[\lambda u(c_1) + (1-\lambda)\left(\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + (1-\theta)\,u\left(\overline{c}\right)\right)\right] d\theta + \\ + E\left[v\left(g, c_1^*, \overline{c}\right)\right] \tag{17}$$

where c_1^* denotes the equilibrium value of c_1 chosen by all banks, and $E\left[v\left(g, c_1^*, \overline{c}\right)\right]$ is the expected utility from the public good as given by

$$E\left[v\left(g, c_1^*, \overline{c}\right)\right] = \int_0^{\theta^*(c_1^*,\overline{c})} v\left(g\right) d\theta + \int_{\theta^*(c_1^*,\overline{c})}^1 \left[\theta v(g) + (1-\theta) v(g-(1-\lambda)\overline{c})\right] d\theta \tag{18}$$

when $\overline{c} \leq 1$, and by

$$E\left[v\left(g, c_1^*, \overline{c}\right)\right] = \int_0^{\theta^*(c_1^*,\overline{c})} v\left(g-\overline{c}+1\right) d\theta + \int_{\theta^*(c_1^*,\overline{c})}^1 \left[\theta v(g) + (1-\theta) v\left(g-(1-\lambda)\overline{c}\right)\right] d\theta \tag{19}$$

when $\overline{c} > 1$.

The first term in (17) represents depositors' expected utility when a run occurs for $\theta < \theta^*(c_1, \bar{c})$ and depositors obtain either $u(1)$ if $\bar{c} \leq 1$ or $u(\bar{c})$ if $\bar{c} > 1$. The second term in (17) represents depositors' expected utility for $\theta \geq \theta^*(c_1, \bar{c})$ when there is no run. In this case early depositors obtain the promised repayments and late depositors receive either the promised payment when the project succeeds or the guarantee $\bar{c}$ when the bank turns out to be insolvent, which occurs with probability $1 - \theta$.

The last term in (17) represents the utility from the public good. Differently from the previous scheme, the provision of the guarantee entails now a disbursement for the government, which depends on whether $\bar{c}$ is greater or smaller than 1. The government finances such a disbursement by transferring the necessary amount of resources from its endowment g to the banking sector. As they are atomistic, banks do not internalize the cost of the guarantee in their choice of c_1 and, consequently, the expressions for $E\left[v\left(g, c_1^*, \bar{c}\right)\right]$ depend on the equilibrium choice c_1^* of other banks rather than c_1 that the individual bank is making a decision on.

When $\bar{c} \leq 1$, the guarantee is paid only to the late depositors waiting until date 2 when the bank's project fails. The first term in (18) represents the utility that depositors obtain from the entire provision of g in the form of public good when, for $\theta < \theta^*(c_1^*, \bar{c})$, a run occurs; while the second term represents the utility from the provision of public good when, for $\theta \geq \theta^*(c_1^*, \bar{c})$, there is no run. In this case, depositors obtain a level of utility $v(g)$ with probability θ and $v(g - (1-\lambda)\bar{c})$ with probability $1 - \theta$ when the bank's project fails and the government uses $(1-\lambda)\bar{c}$ units of its endowment to repay $\bar{c}$ to each of the $(1-\lambda)$ waiting late depositors, thus reducing the provision of public good accordingly.

By contrast, when $\bar{c} > 1$, the government pays the guarantee also to depositors running at date 1. The first term in (19) represents the utility to depositors when, for $\theta < \theta^*(c_1^*, \bar{c})$, a run occurs and the government transfers $\bar{c} - 1$ resources to the banking sector, thus providing only $g - \bar{c} + 1$ units of the public good. The second term is the same as in (18).

As in the previous sections, the solution for c_1 is such that $\theta^*(c_1, \bar{c}) < \overline{\theta}$ at the equilibrium choice of c_1. This maximizes depositors' expected utility as runs do not always occur. We have the following result.

Proposition 5 *The deposit contract $c_1^{IN} > c_1^D$ in the case of a guarantee scheme against panic and fundamental runs solves*

$$\lambda \int_{\theta^*(c_1,\bar{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta + \tag{20}$$
$$-\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\left(\theta^*(c_1,\bar{c}) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) + \left(1-\theta^*(c_1,\bar{c})\right) u\left(\bar{c}\right)\right) - u(max(1,\bar{c}))\right] = 0,$$

and is increasing in the amount of the guarantee, i.e., $\frac{\partial c_1^{IN}}{\partial \overline{c}} > 0$.

As usual, in choosing the promised payment to early depositors the bank trades off the marginal benefit of a higher c_1 with its marginal cost. The former, as represented by the first term in (20), is the better risk sharing obtained from the transfer of consumption from late to early depositors. The latter, as captured by the second term in (20), is the loss in expected utility due to the increased probability of panic runs as measured by $\frac{\partial \theta^*(c_1,\overline{c})}{\partial c_1}$. The bank's choice of c_1 increases with the size of the guarantee $\overline{c}$ so that it is higher than in the decentralized solution.

The proposition suggests that this guarantee scheme has the same implications for bank behavior and run probability as the first scheme analyzed above: The promise to repay $\overline{c}$ has the positive direct effect of reducing the probability of runs. This induces the bank to increase deposit rates and offer better liquidity insurance, even if at the cost of re-increasing the probability of runs. The welfare implications of the guarantees are however more ambiguous now because besides the negative impact on the run probability, they are costly and introduce distortions in the bank's choice of deposit rates. Anticipating these effects, the government chooses to introduce the guarantee only if its effect on liquidity creation outweights its costs in terms of lower provision of the public good. To see this, we now turn to the government's choice of $\overline{c}^{IN}$.

Given the bank's choice of c_1^{IN}, at date 0 the government chooses $\overline{c}^{IN}$ to maximize depositors' total expected utility, which is given by the expression in (17) evaluated at $c_1 = c_1^{IN}(\overline{c})$.[10] The problem here is very complex and does not admit a sharp characterization. Still, we can elaborate on the forces behind the government's choice of the level of guarantees. We distinguish the case when $\overline{c} \leq 1$ and when $\overline{c} > 1$. We start with the former.

When $\overline{c} \leq 1$, the level of guarantee $\overline{c}^{IN}$ is the solution to

$$\begin{aligned}
&\int_{\theta^*(c_1,\overline{c})}^{1} (1-\lambda)\,(1-\theta)\,[u'(\overline{c}) - v'(g-(1-\lambda)\overline{c})]\,d\theta \,+\\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[\lambda u(c_1) + (1-\lambda)\left(\theta^*(c_1,\overline{c})u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + (1-\theta^*(c_1,\overline{c}))u(\overline{c})\right) - u(1)\right] +\\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[\theta^*(c_1,\overline{c})v(g) + (1-\theta^*(c_1,\overline{c}))v(g-(1-\lambda)\overline{c}) - v(g)\right] +\\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\frac{\partial c_1}{\partial\overline{c}}\left[\theta^*(c_1,\overline{c})v(g) + (1-\theta^*(c_1,\overline{c}))v(g-(1-\lambda)\overline{c}) - v(g)\right] = 0.
\end{aligned} \tag{21}$$

The solution to the first order condition gives the choice of $\overline{c}$ as a function of the amount of public resources available g. There are three effects to consider. First, increasing the amount of guarantees is a direct

[10] In order to keep the notation simple, we simply use c_1 instead of $c_1^{IN}(\overline{c})$ in the formulas below.

transfer from the public consumption to the private consumption, as reflected in the first term in (21). The net marginal benefit can be either positive or negative depending on the difference between $u'(\bar{c})$ and $v'(g-(1-\lambda)\bar{c})$. Of course, it is more likely to be positive when g is high. Second, increasing the amount of guarantees acts directly to reduce the probability of a run by $-\frac{\partial\theta^*(c_1,\bar{c})}{\partial\bar{c}}$ and this has direct implications for the utility from private and from public consumption as captured by the second and the third term in (21). In particular, the decline in the probability of a run increases utility from private consumption and decreases utility from public consumption. The decreased utility from public consumption results from the fact that the government pays when there is no run and the bank's asset fails. Third, increasing the guarantee leads the bank to increase the amount it promises in the deposit contract, which has consequences for consumers' expected utility. Interestingly, since c_1 is chosen by each bank so as to maximize depositors' expected utility from the private good, the Envelope Theorem implies that only the implications of the endogenous choice of c_1 on the utility from public consumption should be taken into account by the government in setting the level of guarantee. This is reflected in the fourth term in (21): The increase in the probability of a run due to the greater amount promised by the bank for early withdrawals leads to an increase in expected public consumption, which is not internalized by the bank.

For the case when $\bar{c}>1$, the amount $\bar{c}^{IN}$ chosen by the government solves

$$\begin{aligned}
&\int_0^{\theta^*(c_1,\bar{c})}\left[u'(\bar{c})-v'\left(g-\bar{c}+1\right)\right]d\theta+\int_{\theta^*(c_1,\bar{c})}^{1}(1-\lambda)(1-\theta)\left[u'\left(\bar{c}\right)-v'\left(g-(1-\lambda)\bar{c}\right)\right]d\theta+\\
&-\frac{\partial\theta^*(c_1,\bar{c})}{\partial\bar{c}}\left[\lambda u(c_1)+(1-\lambda)\left(\theta^*(c_1,\bar{c})u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)+(1-\theta^*(c_1,\bar{c}))u\left(\bar{c}\right)\right)-u(\bar{c})\right]+\\
&-\frac{\partial\theta^*(c_1,\bar{c})}{\partial\bar{c}}\left[\theta^*(c_1,\bar{c})v(g)+(1-\theta^*(c_1,\bar{c}))v\left(g-(1-\lambda)\bar{c}\right)-v\left(g-\bar{c}+1\right)\right]+\\
&-\frac{\partial\theta^*(c_1,\bar{c})}{\partial c_1}\frac{\partial c_1}{\partial\bar{c}}\left[\theta^*(c_1,\bar{c})v(g)+(1-\theta^*(c_1,\bar{c}))v\left(g-(1-\lambda)\bar{c}\right)-v\left(g-\bar{c}+1\right)\right]=0.
\end{aligned}\tag{22}$$

The differences in (22) relative to (21) originate from the fact that now the guarantee is also paid to depositors running at date 1. This leads to an increase in the utility from the private good and to a corresponding reduction in the utility from the public good when a run occurs. Overall, now an increase in the probability of a run may either increase or decrease the expected public consumption, depending on the sign of $[\theta^*(c_1,\bar{c})v(g)+(1-\theta^*(c_1,\bar{c}))v\left(g-(1-\lambda)\bar{c}\right)-v\left(g-\bar{c}+1\right)]$. As we will see below, this has different implications for the distortions that the guarantee scheme against panic runs and fundamental failures introduces in the bank's choice of the deposit contract.

Given it maximizes depositors' total expected utility, the government chooses to provide a positive amount of guarantee only if it is welfare improving.

Proposition 6 *The government chooses* $\overline{c} > 0$ *if* $u'(0) - v'(g) > 0$.

The proposition suggests that the government chooses to offer a positive level of guarantees if the marginal benefit in terms of increased private utility outweights the marginal cost in terms of reduced utility from the public good. Given the concavity of the utility functions, this condition will be satisfied unless the government is very constrained in its endowment g. As we will show in Section 6, the size of g will also determine whether $\overline{c} \lesseqgtr 1$.

It is worth noting that this guarantee scheme is welfare improving even if it entails distortions in bank's behavior. To see this, we compare the equilibrium just described with the one emerging in the case the government would choose both c_1 and $\overline{c}$. The main difference is that now, when choosing the deposit contract, the government takes explicit account of the disbursements needed to provide the guarantee.

The government chooses both c_1 and $\overline{c}$ to maximize depositors' total expected utility from the private and public goods as given in (17). The expressions for the utility of the public good $E\left[v\left(g, c_1, \overline{c}\right)\right]$ are still as in (18) and (19) with the difference that, since the government is not atomistic, they depend on c_1 rather than on c_1^*. Thus, the government chooses the level of c_1^G that solves

$$\begin{aligned}
&\lambda \int_{\theta^*(c_1,\overline{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta + \\
&-\frac{\partial \theta^*(c_1,\overline{c})}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\left(\theta^*(c_1,\overline{c}) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) + (1-\theta^*(c_1,\overline{c})) u\left(\overline{c}\right)\right) - u(1)\right] + \\
&-\frac{\partial \theta^*(c_1,\overline{c})}{\partial c_1}\left[\theta^*(c_1,\overline{c}) v(g) + (1-\theta^*(c_1,\overline{c})) v\left(g - (1-\lambda)\overline{c}\right) - v(g)\right] = 0
\end{aligned} \tag{23}$$

when $\overline{c} \leq 1$, and

$$\begin{aligned}
&\lambda \int_{\theta^*(c_1,\overline{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta + \\
&-\frac{\partial \theta^*(c_1,\overline{c})}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\left(\theta^*(c_1,\overline{c}) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) + (1-\theta^*(c_1,\overline{c})) u\left(\overline{c}\right)\right) - u(\overline{c})\right] + \\
&-\frac{\partial \theta^*(c_1,\overline{c})}{\partial c_1}\left[\theta^*(c_1,\overline{c}) v(g) + (1-\theta^*(c_1,\overline{c})) v(g - (1-\lambda)\overline{c}) - v(g - \overline{c} + 1)\right] = 0
\end{aligned} \tag{24}$$

when $\overline{c} > 1$. As before, the two expressions above differ as the guarantee is paid only at date 2 when $\overline{c} < 1$, and also at date 1 when $\overline{c} > 1$.

We can now compare the choice of c_1 of the government c_1^G with that of the banks c_1^{IN}. We have the following result.

Proposition 7 *For a given $\bar{c}$, the guarantee scheme entails a distortion in the bank's behavior. Specifically,*

i) If $\bar{c} \leq 1$, $c_1^{IN} < c_1^G$;

ii) If $\bar{c} > 1$, $c_1^{IN} < (>)\ c_1^G$ when $\left[\theta^*(c_1^{IN}, \bar{c})v(g) + (1 - \theta^*(c_1^{IN}, \bar{c}))v(g - (1-\lambda)\bar{c}) - v(g - \bar{c} + 1)\right] < (>) 0$.

The proposition shows that the bank's choice of c_1 differs from that of the government as the bank does not internalize the costs of providing the guarantee in terms of lower provision of the public good. This is a typical result in models where private agents, here the banks and their depositors, enjoy the benefits of a public form of insurance without internalizing its costs. Despite these distortions, the guarantee scheme remains welfare improving as the government anticipates the effect of the guarantee on bank behavior when choosing the guaranteed amount to provide.

Interestingly, the proposition highlights also that the distortion in the bank's choice of c_1 can go either way in our model, as the bank can choose either a lower or a higher level of c_1 than the level the government would choose. The case $c_1^{IN} < c_1^G$ is opposite to common wisdom as it suggests that banks can be less exposed to runs than what would be desirable. The direction of the distortion depends on whether the government ends up paying to depositors more in case there is no run and the bank ends up failing in the longer term for fundamental reasons or in case there is a run and the bank faces a shortage of liquidity. If the former holds, then the cost of a run from the point of view of banks is higher than from the point of the government and the banks set too low of a deposit rate. This is always the case if $\bar{c} \leq 1$ because no guarantee is provided at date 1 and when $[\theta^*(c_1, \bar{c})v(g) + (1 - \theta^*(c_1, \bar{c}))v(g - (1-\lambda)\bar{c}) - v(g - \bar{c} + 1)] > 0$ if $\bar{c} > 1$. If the latter holds, the opposite occurs and the bank chooses $c_1^{IN} > c_1^G$ in line with common wisdom. The result that banks choose to be less exposed to runs than it would be desirable (i.e., $c_1^{IN} < c_1^G$) resembles the idea of prompt corrective actions. Liquidating banks early rather than letting them operate longer and intervene when banks' resources are completely deployed may be desirable when it allows to minimize the costs associated with public intervention.

6 A numerical example

In this section we illustrate the properties of the model with the use of a numerical example. The goal is to demonstrate that our main results hold in a reasonable space and provide some comparison of the two guarantee schemes analyzed in terms of liquidity creation, banks' fragility and welfare.

In the example depositors' utility functions from the private good $u(c)$ and from the public good $v(g)$

are specified to be given by

$$u(c) = \frac{(c+f)^{1-\sigma}}{1-\sigma} - \frac{(f)^{1-\sigma}}{1-\sigma},$$

and

$$v(g) = \frac{(g+f)^{1-\sigma}}{1-\sigma} - \frac{(f)^{1-\sigma}}{1-\sigma},$$

respectively. The two functions are normalization of a standard CRRA function with σ being the risk aversion coefficient. The parameter f ensures that the assumption $u(0) = v(0) = 0$ is always satisfied; and it can be interpreted as consumption that depositors enjoy from resources not invested in the bank. We maintain $p(\theta) = \theta$, with the upper dominance region corresponding to $\theta = 1$, and set the parameters as follows $\sigma = 3$; $R = 5$; $\lambda = 0.3$ and $f = 4$. To show how the equilibrium of the model −and specifically the size of $\bar{c}-$ changes with the size of public resources, we consider two values of government endowment: $g = 0.7$ and $g = 1.5$.

In each table, we compare the decentralized allocation with the equilibrium in the two guarantee schemes analyzed. The intervention labelled "Guarantees against panic runs" corresponds to the one analyzed in Section 5.1, where the government guarantees depositors waiting until date 2 only in the case no runs have occurred and the bank project succeeds. The intervention labelled "Guarantees against panic and fundamental runs" corresponds to the one analyzed in Section 5.2, where depositors always receive a minimum amount $\bar{c}$. Finally, the last row labelled "Government choosing both c_1 and $\bar{c}$" describes the case when in the second guarantee scheme the government would choose directly both c_1 and $\bar{c}$. The comparison between c_1 in this row and in the row above highlights the distortion the guarantee induces in banks' behavior.

The columns of the tables report, in order, the probability of panic and fundamental runs (θ^*, $\underline{\theta}$), the equilibrium values for the deposit contract (c_1, c_2), the equilibrium level of guarantee $\bar{c}$, the expected utility from the private and public good ($E[u(c_1, c_2, \bar{c})]$ and $E[v(g, \bar{c})]$) and the percentage change in social welfare, as given by the sum $E[u(c_1, c_2, \bar{c})] + E[v(g, \bar{c})]$, in the various interventions relative to the decentralized economy. The first table is drawn for $g = 0.7$; the second one for $g = 1.5$. As we will see, in the first case $\bar{c} < 1$, while in the second case $\bar{c} > 1$.

Table $1 : g = 0.7$

	$\underline{\theta}$ θ^*	c_1 c_2	$\overline{c}$	$E\left[u\left(c_1, c_2, \overline{c}\right)\right]$ $E[v(g,\overline{c})]$	$\Delta SW\left(c_1, c_2, g, \overline{c}\right)$ (%)
Decentralized economy without guarantees	0.451436 0.463204	1.0076 4.98372	0	0.0139202 0.00861532	–
Guarantees against panic runs	0.488273 $\underline{\theta}$	1.10762 4.7694	$\frac{1-\lambda c_1}{1-\lambda}R$	0.013945 0.00861532	0.11
Guarantees against panic and fundamental runs	0.355442 0.373496	1.01445 4.96905	0.27	0.0144034 0.0082353	0.45
Government choosing both c_1 and $\overline{c}$	0.170141 0.331056	1.12229 4.73796	0.497291	0.0147207 0.00799576	0.80

The table shows that both guarantee schemes lead to higher deposit rates ($c_1^{DD} = 1.10762 > c_1^D = 1.0076$ and $c_1^{IN} = 1.01445 > c_1^D = 1.0076$) and thus greater risk sharing. In the guarantee scheme, panics are eliminated but the overall probability of runs ($\underline{\theta} = 0.488273$) is higher than in the decentralized solution ($\theta^* = 0.463162$) because of the higher deposit rate. This does not occur in the second scheme where the probability of runs is now $\theta^* = 0.373496$.

Interestingly, in the scheme against panic and fundamental runs banks choose a level of c_1 that is still low compared to the efficient one that the government would choose ($c_1^{IN} = 1.01445 < c_1^G = 1.12229$). This represents the case where the distortion in banks' behavior induced by the guarantee is opposite to the standard moral hazard problem usually associated with government intervention.

As they are chosen optimally, both guarantee schemes entails higher welfare than the decentralized solution, but the broader scheme against panic and fundamental runs performs better ($\Delta SW^{IN} = 0.45 > \Delta SW^{DD} = 0.11$).

The example with $g = 1.5$ is qualitatively similar to the previous one with the exception that now $\overline{c} = 1.0549 > 1$ in the second guarantee scheme. As before, guarantees improve risk sharing, although they lead to more runs in the first scheme and to distortions in banks' behavior in the second scheme. Concerning the latter, as with $g = 0.7$, the distortion goes again in the direction that the government would like banks to choose a higher c_1.

Table 2 : $g = 1.5$

	$\underline{\theta}$ θ^*	c_1 c_2	$\overline{c}$	$E\left[u\left(c_1, c_2, \overline{c}\right)\right]$ $E[v(g,\overline{c})]$	$\Delta SW\left(c_1, c_2, g, \overline{c}\right)$ (%)
Decentralized economy without guarantees	0.451436 0.463204	1.0076 4.98372	0	0.0139202 0.0147211	–
Guarantees against panic runs	0.488273 $\underline{\theta}$	1.10762 4.7694	$\frac{1-\lambda c_1}{1-\lambda}R$	0.013945 0.147211	0.08
Guarantees against panic and fundamental failures	0.0576375 0.0790397	1.15389 4.67023	1.0549	0.0163803 0.0123519	0.31
Government choosing both c_1 and $\overline{c}$	0.170141 0.331056	1.41144 4.11878	1.12702	0.016183 0.013117	2.29

7 Discussion and concluding remarks

In this paper we develop a model where both panic and fundamental runs are possible and both banks' and depositors' decisions are endogenously determined. We show that government guarantees are beneficial in that they improve depositors' welfare, as a result of the induced greater risk-sharing, even if this comes sometimes at the cost of greater fragility. This result holds also in the context of deposit insurance schemes that entail an actual disbursement for the government and so distortions in banks' risk-taking.

The paper offers a convenient framework to evaluate the implications of government guarantees because it allows to endogenize the probability of runs, bank's behavior and the amount of guarantee offered by the government. The framework builds on some assumptions regarding the type of contract banks offer and the set of feasible actions of the various players involved (i.e., depositors, banks and the government) that are typical in the standard literature of financial crises. Below, we discuss these assumptions. Attempting to relax them and enrich the framework is, in our view, a fruitful path for future research.

First, our framework assumes that banks offer non-contingent deposit contracts, which cannot be ex post renegotiated. Similarly, the amounts promised in the guarantee schemes we consider are non-contingent and non-renegotiable as typical in deposit insurance schemes. This framework allows us to study the interaction between government guarantees, fragility and banks' behavior. Extending this framework to consider the optimal contracts would certainly be an interesting direction, albeit the tractability of the framework is clearly a constraint.

Second, for simplicity, we do not allow banks to store liquidity between the intermediate and the final date as a way to insure themselves against the possibility that their project fails at the final date. The only choice banks make is the amount they offer to investors that withdraw early. As Ahnert and Elamin (2015) show, if storage is possible, banks will choose to hold a positive amount of liquidity. This reduces depositors' incentives to run and may improve allocation. However, as storage is costly, they show that banks do not fully self insure themselves and runs still occur in equilibrium. Hence, there is an underprovision of liquidity insurance and scope for public guarantees to improve welfare as in our framework. Integrating the analysis in Ahnert and Elamin (2015) with the one conducted in our paper is again an interesting direction for future research.

Third, we also do not model banks' asset choice. In the literature, many authors link the moral hazard associated with government guarantees to changes on the assets-side of banks' balance sheets towards more risky loans. The only risk choice we model is the decision of the bank on the short-term rate offered to depositors, which amounts to a leverage decision. By focusing on one dimension of risk choice, we can keep the framework tractable and endogenize the banks' and investors' behavior. It would be interesting to consider extensions where the bank's choice of risky assets is also modeled.

Fourth, as standard in models analyzing the role of public interventions and regulation in banking, in our framework guarantees represent the only way to transfer resources from the public sector to the private one. Such transfers take place through banks when a crisis occurs, either in the form of a run or of failure of bank projects. Another possibility would be to transfer public resources directly to consumers irrespective of the health of the banking sector. To the extent that consumers store the transfer received by the government, there would still be scope for guarantee schemes as the ones we analyze since runs would still occur. Future research can consider this possibility more fully.

Finally, still in line with the existing literature on public intervention, we consider that public funds are always sufficient for the provision of the guarantee, although their size affects the optimal amount of guarantee that the government offers in equilibrium. In other words, depositors are sure to receive the transfers announced by the government. The events in the recent Eurozone crisis proved that guarantees may not be always feasible and their provision may even have negative implications for the solvency of the country, their credibility and in turn their effectiveness. Allowing for such a possibility in our model would require to modify the framework significantly so as to be able to analyze the effect of the guarantees on government's solvency. This would be an interesting analysis. However, the basic trade-off between fragility and liquidity creation triggered by the provision of the guarantee we highlight in the paper would still be present in the extended framework.

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Appendix

Proof of Proposition 1: The proof follows Goldstein and Pauzner (2005). The arguments in their proof establish that there is a unique equilibrium in which depositors run if and only if the signal they receive is below a common signal $x^*(c_1)$. The number n of depositors withdrawing at date 1 is equal to the probability of receiving a signal x_i below $x^*(c_1)$ and, given that depositors' signals are independent and uniformly distributed over the interval $[\theta - \varepsilon, \theta + \varepsilon]$, it is given by:

$$n(\theta, x^*(c_1)) = \begin{cases} 1 & \text{if } \theta \leq x^*(c_1) - \varepsilon \\ \lambda + (1-\lambda)\left(\frac{x^*(c_1)-\theta+\varepsilon}{2\varepsilon}\right) & \text{if } x^*(c_1) - \varepsilon \leq \theta \leq x^*(c_1) + \varepsilon \\ \lambda & \text{if } \theta \geq x^*(c_1) + \varepsilon \end{cases} \quad (25)$$

When θ is below $x^*(c_1) - \varepsilon$, all patient depositors receive a signal below $x^*(c_1)$ and run. When θ is above $x^*(c_1) + \varepsilon$, all late depositors wait until date 2 and only the λ early consumers withdraw early. In the intermediate interval, when θ is between $x^*(c_1) - \varepsilon$ and $x^*(c_1) + \varepsilon$, there is a partial run as some of the late depositors run. The proportion of late consumers withdrawing early decreases linearly with θ as fewer agents observe a signal below the threshold.

Having characterized the proportion of agents withdrawing for any possible value of the fundamentals θ, we can now compute the threshold signal $x^*(c_1)$. A patient depositor who receives the signal $x^*(c_1)$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $x^*(c_1)$ can be then found as the solution to

$$f(\theta, c_1) = \int_{n=\lambda}^{\frac{1}{c_1}} \left[\theta(n) u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1)\right] + \int_{n=\frac{1}{c_1}}^{1} \left[u(0) - u\left(\frac{1}{n}\right)\right] = 0, \tag{26}$$

where, from (25), $\theta(n) = x^*(c_1) + \varepsilon - 2\varepsilon\frac{(n-\lambda)}{1-\lambda}$. Equation (26) follows from (4) and requires that a late depositor's expected utility when he withdraws at date 1 is equal to that when he waits until date 2. Note that at the limit, when $\varepsilon \to 0$, $\theta(n) \to x^*(c_1)$, and we denote it as $\theta^*(c_1)$. Solving (26) with respect to $\theta^*(c_1)$ gives the threshold as in the proposition.□

Proof of Corollary 1: The expression for $\frac{\partial \underline{\theta}(c_1)}{\partial c_1}$ can be obtained simply deriving $\underline{\theta}(c_1)$, as given in (3) with respect to c_1. It is equal to

$$\frac{\partial \underline{\theta}(c_1)}{\partial c_1} = \frac{u'(c_1) + \underline{\theta}(c_1)\left(\frac{\lambda R}{1-\lambda}\right) u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)} > 0 \tag{27}$$

since $u'(c) > 0$.

To prove that $\theta^*(c_1)$ is increasing in c_1, we apply the implicit function theorem to the expression for $f(\theta^*, c_1)$ as given by (26) in proof of Proposition 1. We obtain

$$\frac{\partial \theta^*(c_1)}{\partial c_1} = -\frac{\frac{\partial f(\theta^*, c_1)}{\partial c_1}}{\frac{\partial f(\theta^*, c_1)}{\partial \theta^*}}.$$

It is easy to see that $\frac{\partial f(\theta^*, c_1)}{\partial \theta} > 0$. Thus, the sign of $\frac{\partial \theta^*(c_1)}{\partial c_1}$ is given by the opposite sign of $\frac{\partial f(\theta^*, c_1)}{\partial c_1}$, where

$$\frac{\partial f(\theta^*, c_1)}{\partial c_1} = -\frac{1}{c_1^2}\left[\theta^*(c_1) u \quad \frac{1-\frac{1}{c_1}c_1}{1-\frac{1}{c_1}}R\right) - u(c_1)\bigg] + \frac{1}{c_1^2}\left[u(0) - u(c_1)\right] - \int_{n=\lambda}^{\frac{1}{c_1}} \left[u'(c_1) + \theta^*(c_1)\left(\frac{nR}{1-n}\right) u'\left(\frac{1-nc_1}{1-n}R\right)\right] =$$

$$= -\int_{n=\lambda}^{\frac{1}{c_1}} \left[u'(c_1) + \theta^*(c_1)\left(\frac{nR}{1-n}\right) u'\left(\frac{1-nc_1}{1-n}R\right)\right] < 0.$$

Thus,

$$\frac{\partial \theta^*(c_1)}{\partial c_1} = \frac{\int_{n=\lambda}^{\frac{1}{c_1}} \left[u'(c_1) + \theta^*(c_1)\left(\frac{nR}{1-n}\right) u'(\frac{1-nc_1}{1-n}R)\right]}{\int_{n=\lambda}^{\frac{1}{c_1}} u\left(\frac{1-nc_1}{1-n}R\right)} > 0, \tag{28}$$

To complete the proof, we need to show that $\frac{\partial \theta^*(c_1)}{\partial c_1} > \frac{\partial \underline{\theta}(c_1)}{\partial c_1}$. To see this, substitute the expression for each derivative from (27) and (28). After a few manipulations, we can rewrite the condition $\frac{\partial \theta^*(c_1)}{\partial c_1} > \frac{\partial \underline{\theta}(c_1)}{\partial c_1}$ as follows

$$\begin{aligned} & u'(c_1) \int_{n=\lambda}^{\frac{1}{c_1}} u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + \int_{n=\lambda}^{\frac{1}{c_1}} \theta^*(c_1) u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\left(\frac{nR}{1-n}\right) u'\left(\frac{1-nc_1}{1-n}R\right) \\ > \; & u'(c_1) \int_{n=\lambda}^{\frac{1}{c_1}} u\left(\frac{1-nc_1}{1-n}R\right) + \int_{n=\lambda}^{\frac{1}{c_1}} \underline{\theta}(c_1) u\left(\frac{1-nc_1}{1-n}R\right) u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\left(\frac{\lambda R}{1-\lambda}\right), \end{aligned}$$

which holds since $\frac{1-\lambda c_1}{1-\lambda}R > \frac{1-nc_1}{1-n}R$ and $\frac{nR}{1-n} > \frac{\lambda R}{1-\lambda}$ for any $n > \lambda$ and $\theta^*(c_1) > \underline{\theta}(c_1)$. This completes the proof of the corollary. □

Proof of Proposition 2: Differentiating (7) with respect to c_1 gives the deposit contract c_1^D as the solution to (8).

To show that $c_1^D > 1$, we evaluate (8) at $c_1 = 1$. From (6), at $c_1 = 1$ the threshold $\theta^*(c_1)$ simplifies to

$$\theta^*(1) = \frac{(1-\lambda)u(1)}{(1-\lambda)u(R)},$$

and, from (3), it is then

$$\theta^*(1) = \underline{\theta}(1).$$

Thus, when $c_1 = 1$, (8) can be rewritten as follows:

$$\lambda \int_{\underline{\theta}(1)}^{1} \left[u'(1) - \theta R u'(R)\right] d\theta - \frac{\partial \underline{\theta}(c_1)}{\partial c_1}|_{c_1=1}(1-\lambda)\left[\underline{\theta}(1)u(R) - u(1)\right]$$

The second term is equal to zero because of the definition of $\underline{\theta}(c_1)$ in (3), and thus the expression simplifies to

$$\lambda \int_{\underline{\theta}(1)}^{1} \left[u'(1) - \theta R u'(R)\right] d\theta.$$

Since the relative risk aversion coefficient is bigger than 1, it holds

$$1 \cdot u'(1) > R u'(R),$$

so that $\lambda \int_{\underline{\theta}(1)}^{1} \left[u'(1) - \theta R u'(R)\right] d\theta > 0$ and thus $c_1^D > 1$. □

Proof of Proposition 3: Denote $FOC_{c_1}^{DD}(c_1)$ as the first order condition in (11) which implicitly determines the deposit contract c_1^{DD} chosen by the banks. To show that $c_1^{DD} > c_1^D$, we need to compare (8) with (11) and show that that $FOC_{c_1}^{DD}(c_1)$ evaluated at $c_1 = c_1^D$ is greater than (8) evaluated at $c_1 = c_1^D$, which is equal to zero. The first term in each expression only differ in the lower extreme of the integrals and it is easy to see that the first term in (8) is smaller than that in (11) since $\theta^*(c_1) > \underline{\theta}(c_1)$. Thus, we only need to compare $\frac{\partial \theta^*(c_1^D)}{\partial c_1}\left[\lambda u(c_1^D) + (1-\lambda)\theta^*(c_1^D)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(1)\right]$ with $\frac{\partial \underline{\theta}(c_1^D)}{\partial c_1}\left[\lambda u(c_1^D) + (1-\lambda)\underline{\theta}(c_1^D)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(1)\right]$ and show that the former is larger than the latter. It is easy to see that

$$\left[\lambda u(c_1^D) + (1-\lambda)\theta^*(c_1^D)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(1)\right] > \left[\lambda u(c_1^D) + (1-\lambda)\underline{\theta}(c_1^D)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(1)\right],$$

since $\theta^*(c_1^D) > \underline{\theta}(c_1^D)$. Moreover, $\frac{\partial \theta^*(c_1^D)}{\partial c_1} > \frac{\partial \underline{\theta}(c_1^D)}{\partial c_1}$ holds from Corollary 1. Thus, the proposition follows. □

Proof of Proposition 4: We need to distinguish two cases depending on whether $\overline{c}$ is larger or smaller than 1. Consider first the case where $\overline{c} \leq 1$. The proof is analogous to the one of Proposition 2. A patient depositor who receives the signal $x^*(c_1, \overline{c})$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $x^*(c_1, \overline{c})$ can be then found as the solution to

$$\begin{aligned} f(\theta, c_1, \overline{c}) \quad &= \quad \int_{n=\lambda}^{\overline{n}} \left[\theta(n) u\left(\frac{1-nc_1}{1-n}R\right) + (1-\theta(n))u(\overline{c}) - u(c_1)\right] + \int_{n=\overline{n}}^{\widehat{n}} \left[u(\overline{c}) - u(c_1)\right] \\ &+ \int_{n=\widehat{n}}^{1} \left[u(\overline{c}) - u\left(\frac{1}{n}\right)\right] = 0 \end{aligned} \tag{29}$$

where, still from (25), $\theta(n) = x^*(c_1, \overline{c}) + \varepsilon - 2\varepsilon\frac{(n-\lambda)}{1-\lambda}$. Equation (29) follows from (14) and requires that a late depositor's expected utility when he withdraws at date 1 is equal to that when he waits until date 2. At the limit, when $\varepsilon \to 0$, $\theta(n) \to x^*(c_1, \overline{c})$, and the threshold $\theta^*(c_1, \overline{c})$ solves (29).

The case where $\bar{c} > 1$ is more involved since we need first to show that, despite the fact that the function $v(\theta, n, \bar{c})$ is zero in the range $\widetilde{n} \leq n \leq 1$, a unique threshold equilibrium exists. We then split the proof in two parts. First, we prove that a unique threshold equilibrium exists and then, we compute the equilibrium threshold.

Existence of a unique threshold equilibrium

The proof follows Goldstein and Pauzner (2005). Recall that the proportion of depositors running $n(\theta, x^*)$ when they behave according to the same threshold strategy x^* is given by (25). Denote as $\Delta(x_i, \dot{n}(\theta))$ an agent's expected difference in utility between withdrawing at date 2 rather than at date 1 when he holds beliefs $\dot{n}(\theta)$ regarding the number of depositors running. The function $\Delta(x_i, \dot{n}(\theta))$ is given by

$$\Delta(x_i, \dot{n}(\theta)) = \frac{1}{2\varepsilon} \int_{x_i-\varepsilon}^{x_i+\varepsilon} E_n \left[v(\theta, \dot{n}(\theta))\right] d\theta.$$

Since for any realization of θ, the proportion of depositors running is deterministic, we can write $n(\theta)$ instead of $\dot{n}(\theta)$ and the function $\Delta(x_i, n(\theta))$ simplifies to

$$\Delta(x_i, \dot{n}(\theta)) = \frac{1}{2\varepsilon} \int_{x_i-\varepsilon}^{x_i+\varepsilon} v(\theta, n(\theta)) d\theta.$$

Notice that when all depositors behave according to the same threshold strategy x^*, $\dot{n}(\theta) = n(\theta, x^*)$ defined in (25). The following lemma states a few properties of the function $\Delta(x_i, \dot{n}(\theta))$.

Lemma 1 *i) The function $\Delta(x_i, \dot{n}(\theta))$ is continuos in x_i; ii) for any $a > 0$, $\Delta(x_i + a, \dot{n}(\theta) + a)$ is nondecreasing in a, iii) $\Delta(x_i + a, \dot{n}(\theta) + a)$is strictly increasing in a if there is a positive probability that $n < \overline{n}$ and $\theta < \overline{\theta}$.*

Proof of Lemma 1. The proof follows Goldstein and Pauzner (2005). The function $\Delta(.)$ is continuous in x_i as a change in x_i only changes the limits of integration in the computation of $\Delta(.)$. The function $\Delta(x_i + a, \dot{n}(\theta) + a)$ is nondecreasing in a since, as a increases, depositors see the same distribution of n but expect θ to be higher. Since $v(\theta, n)$ is nondecreasing in θ, $\Delta(.)$ is nondecreasing in a. In order for $\Delta(x_i + a, \dot{n}(\theta) + a)$ to be strictly increasing in a, we need that $\theta < \overline{\theta}$ and that there is a positive probability that $n < \overline{n}$. This is the case because, when $n < \overline{n}$ and $\theta < \overline{\theta}$, $v(\theta, n)$ is strictly increasing in θ, and, thus, $\Delta(x_i + a, \dot{n}(\theta) + a)$ is strictly increasing in a. ■

A threshold equilibrium with the threshold signal x^* exists, if and only if no depositor finds it optimal to run if he receives a signal higher than x^* and to wait if he receives a signal below x^*:

$$\Delta(x_i, n(\theta, x^*)) < 0 \ \ \forall x_i < x^*; \tag{30}$$

$$\Delta(x_i, n(\theta, x^*)) > 0 \ \ \forall x_i > x^*. \tag{31}$$

By continuity, a depositor must be indifferent between withdrawing at date 1 rather than date 2 when he receives the signal $x_i = x^*$

$$\Delta(x^*, n(\theta, x^*)) = 0. \tag{32}$$

In the lower and upper dominance regions, $\Delta(x^*, n(\theta, x^*)) < 0$ and $\Delta(x^*, n(\theta, x^*)) > 0$, respectively. Thus, by continuity of $\Delta(x^*, n(\theta, x^*))$ in x_i, there exists some x^* at which it equals to zero. To prove that the x^* is unique, we use the property stated in Lemma 1 that $\Delta(x^*, n(\theta, x^*))$ is strictly increasing in x_i in the range

$\theta \in [x^* - \varepsilon, x^* + \varepsilon]$ since from (25), there is always a positive probability that $n < \overline{n}$ in that range. Thus, there is only one value of x^*, which is a candidate to be a threshold equilibrium. To show that it is indeed an equilibrium we have to show that no depositor has an incentive to deviate. This means that we have to show that, given that (32) holds, (30) and (31) also hold.

Let's start from (30). Decompose the intervals $[x_i - \varepsilon, x_i + \varepsilon]$ and $[x^* - \varepsilon, x^* + \varepsilon]$ over which the integrals $\Delta(x_i, n(\theta, x^*))$ and $\Delta(x^*, n(\theta, x^*))$ into a common part $c = [x_i - \varepsilon, x_i + \varepsilon] \bigcap [x^* - \varepsilon, x^* + \varepsilon]$ and two disjoint parts $d_i = \frac{[x_i - \varepsilon, x_i + \varepsilon]}{c}$ and $d^* = \frac{[x^* - \varepsilon, x^* + \varepsilon]}{c}$. We can then rewrite the integrals $\Delta(x_i, n(\theta, x^*))$ and $\Delta(x^*, n(\theta, x^*))$ as follows:

$$\Delta(x_i, n(\theta, x^*)) = \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, x^*)) + \frac{1}{2\varepsilon} \int_{\theta \in d_i} v(\theta, n(\theta, x^*))$$

$$\Delta(x^*, n(\theta, x^*)) = \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, x^*)) + \frac{1}{2\varepsilon} \int_{\theta \in d^*} v(\theta, n(\theta, x^*))$$

For any $\theta \in d_i$, $n = 1$ since $\theta \leq x^* - \varepsilon$. Thus, $v(\theta, n(\theta, x^*)) = 0$ and, in turn $\Delta(x_i, n(\theta, x^*)) = 0$ in that interval. In order to show that $\Delta(x_i, n(\theta, x^*)) < 0$, we need to show that $\frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, x^*)) < 0$. This is the case because (32) holds and the fundamentals in the range d^* are better than those in the range d_i, which implies that $\frac{1}{2\varepsilon} \int_{\theta \in d^*} v(\theta, n(\theta, x^*)) > \frac{1}{2\varepsilon} \int_{\theta \in d_i} v(\theta, n(\theta, x^*)) = 0$. The proof for (31) is analogous.

The equilibrium threshold

Having proved the existence of a unique threshold equilibrium, we can now compute $x^*(c_1, \overline{c})$. A patient depositor who receives the signal $x^*(c_1, \overline{c})$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $x^*(c_1, \overline{c})$ can be then found as the solution to

$$\begin{aligned} f(\theta, c_1, \overline{c}) \quad &= \quad \int_{n=\lambda}^{\overline{n}} \left[\theta(n) u\left(\frac{1 - nc_1}{1 - n} R \right) + (1 - \theta(n)) u(\overline{c}) - u(c_1) \right] + \int_{n=\overline{n}}^{\widehat{n}} [u(\overline{c}) - u(c_1)] \\ &+ \int_{n=\widehat{n}}^{\widetilde{n}} \left[u(\overline{c}) - u\left(\frac{1}{n} \right) \right] + \int_{n=\widetilde{n}}^{1} [u(\overline{c}) - u(\overline{c})] = 0. \end{aligned} \tag{33}$$

As before, $\theta(n) = x^*(c_1, \overline{c}) + \varepsilon - 2\varepsilon \frac{(n-\lambda)}{1-\lambda}$ and, at the limit, when $\varepsilon \to 0$, $\theta(n) \to x^*(c_1, \overline{c})$, the threshold $\theta^*(c_1, \overline{c})$ solves (33). This completes the proof of the proposition. □

Proof of Corollary 2: The expression for $\frac{\partial \underline{\theta}(c_1, \overline{c})}{\partial c_1}$ and $\frac{\partial \underline{\theta}(c_1, \overline{c})}{\partial \overline{c}}$ can be obtained simply deriving $\underline{\theta}(c_1, \overline{c})$, as given in (13) with respect to c_1 and $\overline{c}$, respectively. The former is equal to

$$\frac{\partial \underline{\theta}(c_1, \overline{c})}{\partial c_1} = \frac{u'(c_1) + \underline{\theta}(c_1, \overline{c}) \left(\frac{\lambda R}{1-\lambda} \right) u' \left(\frac{1-\lambda c_1}{1-\lambda} R \right)}{u\left(\frac{1-\lambda c_1}{1-\lambda} R \right) - u(\overline{c})} > 0. \tag{34}$$

The latter is equal to

$$\frac{\partial \underline{\theta}(c_1, \overline{c})}{\partial \overline{c}} = -\frac{(1 - \underline{\theta}(c_1, \overline{c})) u'(\overline{c})}{u\left(\frac{1-\lambda c_1}{1-\lambda} R \right) - u(\overline{c})} < 0. \tag{35}$$

Now we turn to show to the threshold of panic runs. To prove that $\theta^*(c_1, \overline{c})$ is increasing in c_1, we use the implicit function theorem and obtain

$$\frac{\partial \theta^*(c_1, \overline{c})}{\partial c_1} = -\frac{\frac{\partial f(\theta^*,\, c_1,\, \overline{c})}{\partial c_1}}{\frac{\partial f(\theta^*,\, c_1,\, \overline{c})}{\partial \theta^*}},$$

where the expression for $f(\theta^*, c_1, \bar{c})$ is given in (29) for the case $\bar{c} \leq 1$ and (33) for the case $\bar{c} > 1$. We start with the case $\bar{c} \leq 1$. It is easy to see that the denominator is positive since

$$\frac{\partial f(\theta^*, c_1, \bar{c})}{\partial \theta^*} = \int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u\left(\bar{c}\right) \right] > 0.$$

Thus, the sign of $\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}$ is given by the opposite sign of $\frac{\partial f(\theta^* v, c_1, \bar{c})}{\partial c_1}$. After some manipulations, we obtain:

$$\frac{\partial f(\theta^*, c_1, \bar{c})}{\partial c_1} = -\int_{n=\lambda}^{\widehat{n}} u'(c_1) - \int_{n=\lambda}^{\overline{n}} \theta^*(c_1, \bar{c}) u'\left(\frac{1-nc_1}{1-n}R\right)\left(\frac{nR}{1-n}\right) < 0.$$

This implies

$$\frac{\partial \theta^*(c_1, \bar{c})}{\partial c_1} = \frac{\int_{n=\lambda}^{\widehat{n}} u'(c_1) + \int_{n=\lambda}^{\overline{n}} \theta^*(c_1, \bar{c}) u'\left(\frac{1-nc_1}{1-n}R\right)\left(\frac{nR}{1-n}\right)}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u\left(\bar{c}\right) \right]} > 0. \quad (36)$$

We now turn to the effect of $\bar{c}$ on the threshold. To prove that $\theta^*(c_1, \bar{c})$ is decreasing in $\bar{c}$, we use again the implicit function theorem and obtain

$$\frac{\partial \theta^*(c_1, \bar{c})}{\partial \bar{c}} = -\frac{\frac{\partial f(\theta^*, c_1, \bar{c})}{\partial \bar{c}}}{\frac{\partial f(\theta^*, c_1, \bar{c})}{\partial \theta}}.$$

The denominator is as before and it is positive. Thus, he sign of $\frac{\partial \theta^*(c_1,\bar{c})}{\partial \bar{c}}$ is given by the opposite sign of $\frac{\partial f(\theta^*, c_1, \bar{c})}{\partial \bar{c}}$. After some manipulations, we obtain:

$$\frac{\partial f(\theta^*, c_1, \bar{c})}{\partial \bar{c}} = \int_{n=\lambda}^{1} u'(\bar{c}) - \int_{n=\lambda}^{\overline{n}} \theta^*(c_1, \bar{c}) u'(\bar{c}) > 0,$$

which implies that

$$\frac{\partial \theta^*(c_1, \bar{c})}{\partial \bar{c}} = -\frac{\int_{n=\lambda}^{1} u'(\bar{c}) - \int_{n=\lambda}^{\overline{n}} \theta^*(c_1, \bar{c}) u'(\bar{c})}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u\left(\bar{c}\right) \right]} < 0. \quad (37)$$

The case $\bar{c} > 1$ is analogous. In this case, the effect of c_1 and $\bar{c}$ on the threshold are given by

$$\frac{\partial \theta^*(c_1, \bar{c})}{\partial c_1} = \frac{\int_{n=\lambda}^{\widehat{n}} u'(c_1) + \theta^*(c_1, \bar{c}) \int_{n=\lambda}^{\overline{n}} u'\left(\frac{1-nc_1}{1-n}R\right)\left(\frac{nR}{1-n}\right)}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u\left(\bar{c}\right) \right]} > 0$$

and

$$\frac{\partial \theta^*(c_1, \bar{c})}{\partial \bar{c}} = -\frac{\int_{n=\lambda}^{\widetilde{n}} u'(\bar{c}) - \theta^*(c_1, \bar{c}) \int_{n=\lambda}^{\overline{n}} u'(\bar{c})}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u\left(\bar{c}\right) \right]} < 0,$$

respectively. □

Proof of Proposition 5: We consider first the case $\bar{c} \leq 1$. Denote $FOC_{c_1}^{IN}(c_1, \bar{c})$ the first order condition that implicitly determines c_1^{IN}. This is given by (20) evaluated at $\bar{c} \leq 1$ and, thus equal to

$$\lambda \int_{\theta^*(c_1,\bar{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right) \right] d\theta +$$
$$-\frac{\partial \theta^*(c_1, \bar{c})}{\partial c_1} \left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1, \bar{c}) \left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\bar{c}) \right] + (1-\lambda)u(\bar{c}) - u(1) \right] = 0. \quad (38)$$

To compute $\frac{\partial c_1}{\partial \bar{c}}$ the we use the implicit function theorem. Thus, $\frac{\partial c_1}{\partial \bar{c}} = -\frac{\frac{\partial FOC_{c_1}^{IN}(c_1,\bar{c})}{\partial \bar{c}}}{\frac{\partial FOC_{c_1}^{IN}(c_1,\bar{c})}{\partial c_1}}$. Since c_1^{IN} is an interior solution, $\frac{\partial c_1^{IN}}{\partial \bar{c}} > 0$ if and only if $\frac{\partial FOC_{c_1}^{IN}(c_1,\bar{c})}{\partial \bar{c}} > 0$. We have

$$\begin{aligned}
\frac{\partial FOC_{c_1}^{IN}(c_1,\bar{c})}{\partial \bar{c}} &= -\lambda \frac{\partial \theta^*(c_1,\bar{c})}{\partial \bar{c}}\left[u'(c_1) - \theta^*(c_1,\bar{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] + \\
&-\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1 \partial \bar{c}}\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1,\bar{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\bar{c})\right] + (1-\lambda)u(\bar{c}) - u(1)\right] + \\
&-\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}(1-\lambda)\frac{\partial \theta^*(c_1,\bar{c})}{\partial \bar{c}}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\bar{c})\right] + \\
&-\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}(1-\lambda)\left(1-\theta^*(c_1,\bar{c})\right)u'(\bar{c}).
\end{aligned}$$

Recall that $\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1} > 0$ and $\frac{\partial \theta^*(c_1,\bar{c})}{\partial \bar{c}} < 0$. Deriving $\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}$, as given in (36), with respect $\bar{c}$, after a few manipulations, the cross derivative $\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1 \partial \bar{c}}$ becomes

$$\begin{aligned}
\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1 \partial \bar{c}} &= \frac{1}{\int_{n=\lambda}^{\bar{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right) - u(\bar{c})\right]}\left\{\frac{-R(c_1-1)}{(Rc_1-\bar{c})^2}\theta^*(c_1,\bar{c})\left(\frac{\bar{n}R}{1-\bar{n}}\right)u'\left(\frac{1-\bar{n}c_1}{1-\bar{n}}R\right) + \right. \\
&\left. + \frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}\int_{n=\lambda}^{\bar{n}} u'(\bar{c}) + \frac{\partial \theta^*(c_1,\bar{c})}{\partial \bar{c}}\int_{n=\lambda}^{\bar{n}} u'\left(\frac{1-nc_1}{1-n}R\right)\left(\frac{nR}{1-n}\right)\right\}
\end{aligned}$$

Substituting the expression for $\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1 \partial \bar{c}}$ into that for $\frac{\partial FOC_{c_1}(c_1^{IN},\bar{c})}{\partial \bar{c}}$, after a few manipulations, we obtain:

$$\begin{aligned}
\frac{\partial FOC_{c_1}(c_1,\bar{c})}{\partial \bar{c}} &= -\lambda \frac{\partial \theta^*(c_1,\bar{c})}{\partial \bar{c}}\left[u'(c_1) - \theta^*(c_1,\bar{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] + \\
&-\frac{\partial \theta^*(c_1,\bar{c})}{\partial \bar{c}}\frac{\int_{n=\lambda}^{\bar{n}} u'\left(\frac{1-nc_1}{1-n}R\right)\left(\frac{nR}{1-n}\right)}{\int_{n=\lambda}^{\bar{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right) - u(\bar{c})\right]}\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1,\bar{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\bar{c})\right] + \right. \\
&\left. -(1-\lambda)u(\bar{c}) - u(1)\right] - \frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}\left[(1-\lambda)\frac{\partial \theta^*(c_1,\bar{c})}{\partial \bar{c}}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\bar{c})\right] + (1-\lambda)(1-\theta^*(c_1,\bar{c}))u'(\bar{c})\right] + \\
&-\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}\frac{\int_{n=\lambda}^{\bar{n}} u'(\bar{c})}{\int_{n=\lambda}^{\bar{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right) - u(\bar{c})\right]}\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1,\bar{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\bar{c})\right] + \right. \\
&\left. -(1-\lambda)u(\bar{c}) - u(1)\right] + \frac{\frac{R(c_1-1)}{(Rc_1-\bar{c})^2}\theta^*(c_1,\bar{c})\left(\frac{\bar{n}R}{1-\bar{n}}\right)u'\left(\frac{1-\bar{n}c_1}{1-\bar{n}}R\right)}{\int_{n=\lambda}^{\bar{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right) - u(\bar{c})\right]}\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1,\bar{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\bar{c})\right] + \right. \\
&\left. -(1-\lambda)u(\bar{c}) - u(1)\right].
\end{aligned} \tag{39}$$

All the terms in the expression above are positive beside the bracket

$$-\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}\left[(1-\lambda)\frac{\partial \theta^*(c_1,\bar{c})}{\partial \bar{c}}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\bar{c})\right] + (1-\lambda)(1-\theta^*(c_1,\bar{c}))u'(\bar{c})\right] \tag{40}$$

and

$$-\frac{\partial \theta^*(c_1,\bar{c})}{\partial c_1}\frac{\int_{n=\lambda}^{\bar{n}} u'(\bar{c})}{\int_{n=\lambda}^{\bar{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right) - u(\bar{c})\right]}\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1,\bar{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\bar{c})\right] - (1-\lambda)u(\bar{c}) - u(1)\right] \tag{41}$$

Let' s start to show that (40) is positive. In order for this to be true, we need to show that the term in the square bracket is negative. Substituting the expression for $\frac{\partial\theta^*(c_1^{IN},\overline{c})}{\partial\overline{c}}$ from (37), after a few manipulations, the term in the square bracket simplifies to

$$\frac{-\int_{n=\lambda}^{1}u'(\overline{c})+\theta^*(c_1,\overline{c})\int_{n=\lambda}^{\overline{n}}u'(\overline{c})}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]+\int_{n=\lambda}^{1}(1-\theta^*(c_1,\overline{c}))u'(\overline{c}),$$

which can be rearranged as

$$-\int_{n=\lambda}^{1}u'(\overline{c})\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}+\theta^*(c_1,\overline{c})\int_{n=\lambda}^{\overline{n}}u'(\overline{c})\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}+$$
$$+\int_{n=\lambda}^{1}(1-\theta^*(c_1,\overline{c}))u'(\overline{c}).$$

After a few manipulations, the expression above can be rewritten as follows

$$u'(\overline{c})\left[\begin{array}{c}-(\overline{n}-\lambda)(1-\theta^*(c_1,\overline{c}))\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}-(1-\overline{n})\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}+\\+(1-\lambda)(1-\theta^*(c_1,\overline{c}))\end{array}\right].$$

To show that (40) is positive it suffices to show that

$$(\overline{n}-\lambda)(1-\theta^*(c_1,\overline{c}))\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}>(1-\lambda)(1-\theta^*(c_1,\overline{c})).$$

Rewriting the condition above as follows

$$(1-\lambda)(1-\theta^*(c_1,\overline{c}))\frac{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}>(1-\lambda)(1-\theta^*(c_1,\overline{c})),$$

and it is easy to see that it always holds since $\frac{1-\lambda c_1}{1-\lambda}R>\frac{1-nc_1}{1-n}R$ for any $n>\lambda$.

In order to prove that $\frac{\partial c_1^{IN}}{\partial\overline{c}}>0$, we are left to show that (41) is dominated by some other term in (39). Thus, we show that (41) is smaller than the positive term $-\lambda\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[u'(c_1)-\theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]$. To do this, first, recall that from (38) it holds

$$\begin{aligned}&\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\lambda u(c_1)+(1-\lambda)\theta^*(c_1,\overline{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]-(1-\lambda)u(\overline{c})-u(1)\right]\\=&\ \lambda\int_{\theta^*(c_1,\overline{c})}^{1}\left[u'(c_1)-\theta Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]d\theta.\end{aligned}$$

Thus, a sufficient condition for $\frac{\partial c_1^{IN}}{\partial\overline{c}}>0$ is that

$$\begin{aligned}&-\lambda\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[u'(c_1)-\theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]>\\&\lambda\int_{\theta^*(c_1,\overline{c})}^{1}\left[u'(c_1)-\theta Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]d\theta\frac{\int_{n=\lambda}^{\overline{n}}u'(\overline{c})}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}.\end{aligned}$$

Substituting the expression for $\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}$, after a few manipulations, the condition above becomes

$$\lambda\left[u'(c_1)-\theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]\left[\int_{\lambda}^{1}u'(\overline{c})-\theta^*(c_1,\overline{c})\int_{\lambda}^{\overline{n}}u'(\overline{c})\right]>$$
$$\lambda\int_{\theta^*(c_1,\overline{c})}^{1}\left[u'(c_1)-\theta Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]d\theta\int_{n=\lambda}^{\overline{n}}u'\left(\overline{c}\right),$$

which can be simplified to

$$\left[u'(c_1)-\theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]u'(\overline{c})\left[(1-\lambda)-(\overline{n}-\lambda)\theta^*(c_1,\overline{c})\right]>$$
$$(1-\theta^*(c_1,\overline{c}))\left[u'(c_1)-E[\theta\mid\theta>\theta^*(c_1,\overline{c})]Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right](\overline{n}-\lambda)u'\left(\overline{c}\right).$$

Since $\theta^*(c_1,\overline{c})<E[\theta\mid\theta>\theta^*(c_1,\overline{c})]$ and $[(1-\lambda)-(\overline{n}-\lambda)\theta^*(c_1,\overline{c})]>(1-\theta^*(c_1,\overline{c}))(\overline{n}-\lambda)$, the condition above holds and the proposition follows. The proof for the case $\overline{c}\geq 1$ is analogous.□

Proof of Proposition 6: In order to prove that the government chooses a positive level of guarantees, we need to show that the first order condition (21) is positive for $\overline{c}=0$.

Evaluating (21) for $\overline{c}=0$, we obtain

$$\int_{\theta^*(c_1,0)}^{1}(1-\lambda)\left(1-\theta\right)\left[u'\left(0\right)-v'(g)\right]d\theta+$$
$$-\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\right|_{\overline{c}=0}\left[\lambda u(c_1)+(1-\lambda)\left(\theta^*(c_1,0)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)+(1-\theta^*(c_1,0))u\left(\overline{c}\right)\right)-u(1)\right]+$$
$$-\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\right|_{\overline{c}=0}\left[\theta^*(c_1,0)v(g)+(1-\theta^*(c_1,0))v\left(g\right)-v(g)\right]+$$
$$-\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\right|_{\overline{c}=0}\frac{\partial c_1}{\partial\overline{c}}\left[\theta^*(c_1,0)v(g)+(1-\theta^*(c_1,0))v\left(g\right)-v(g)\right],$$

with $\theta^*(c_1,0)$ being equal to the threshold $\theta^*(c_1)$ in the decentralized economy, where $\overline{c}=0$. It is easy to see that $[\theta^*(c_1,0)v(g)+(1-\theta^*(c_1,0))v\left(g\right)-v(g)]=0$ and the expression simplifies further to

$$\int_{\theta^*(c_1,0)}^{1}(1-\lambda)\left(1-\theta\right)\left[u'\left(0\right)-v'(g)\right]d\theta+ \tag{42}$$
$$-\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\right|_{\overline{c}=0}\left[\lambda u(c_1)+(1-\lambda)\left(\theta^*(c_1,0)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)+(1-\theta^*(c_1,0))u\left(\overline{c}\right)\right)-u(1)\right].$$

Since $\overline{n}=\widehat{n}$ when $\overline{c}=0$ and, as a result, $\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\right|_{\overline{c}=0}$ is equal to

$$-\frac{\int_{n=\lambda}^{1}u'(0)-\int_{n=\lambda}^{\widehat{n}}\theta^*(c_1,0)u'(0)}{\int_{n=\lambda}^{\widehat{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u\left(0\right)\right]}<0,$$

it follows that the second term in (42) is positive. Thus, a sufficient condition for (42) to be positive and, in turn, $\overline{c}>0$, is $u'\left(0\right)-v'(g)>0$. The proposition follows. □

Proof of Proposition 7: Evaluating (20) taking $\overline{c}\leq 1$ and comparing it with (23), it is easy to see that they only differ in the last term of (23) that is

$$-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\theta^*(c_1,\overline{c})v(g)+(1-\theta^*(c_1,\overline{c}))v(g-(1-\lambda)\overline{c})-v(g)\right]$$

Since $[\theta^*(c_1, \overline{c})v(g) + (1 - \theta^*(c_1, \overline{c}))v(g - (1 - \lambda)\overline{c}) - v(g)] < 0$, for given c_1 and $\overline{c}$, the expression in (20) is smaller than that in (23), thus implying that $c_1^{IN} < c_1^G$.

Evaluate now (20) taking $\overline{c} > 1$ and compare it with (24). They only differ in the last term in (24), which is equal to

$$-\frac{\partial \theta^*(c_1, \overline{c})}{\partial c_1} [\theta^*(c_1, \overline{c})v(g) + (1 - \theta^*(c_1, \overline{c}))v(g - (1 - \lambda)\overline{c}) - v(g - \overline{c} + 1)]$$

The bracket $[\theta^*(c_1, \overline{c})v(g) + (1 - \theta^*(c_1, \overline{c}))v(g - (1 - \lambda)\overline{c}) - v(g - \overline{c} + 1)]$ can be either positive or negative. Using the same argument as in the case with $\overline{c} \leq 1$, it follows that $c_1^{IN} < c_1^G$ if the bracket is negative, and $c_1^{IN} > c_1^G$ if it is positive. The proposition follows. □

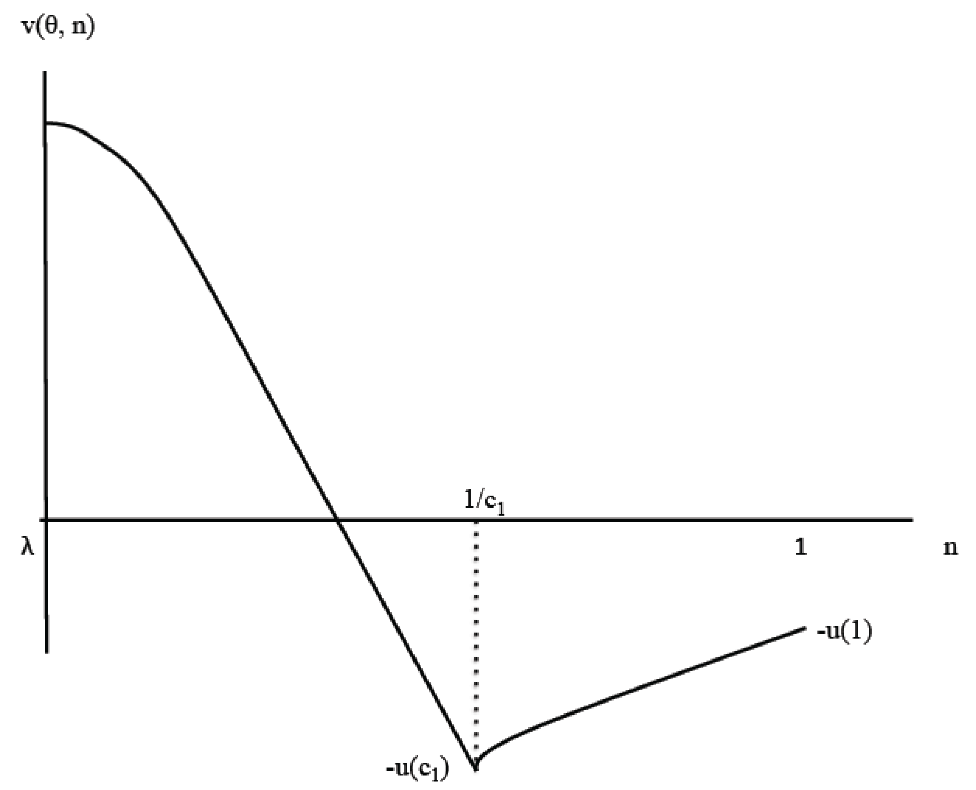

Figure 1: Depositors' utility differential in the decentralized economy. The figure shows how the utility differential for a late depositor between withdrawing at date 2 versus date 1 changes with the number n of depositors withdrawing at date 1. The function is decreasing in n for $\lambda \leq n < -$ and increasing for

$-$ It crosses zero only once for n$< -$ and remains below zero afterwards.

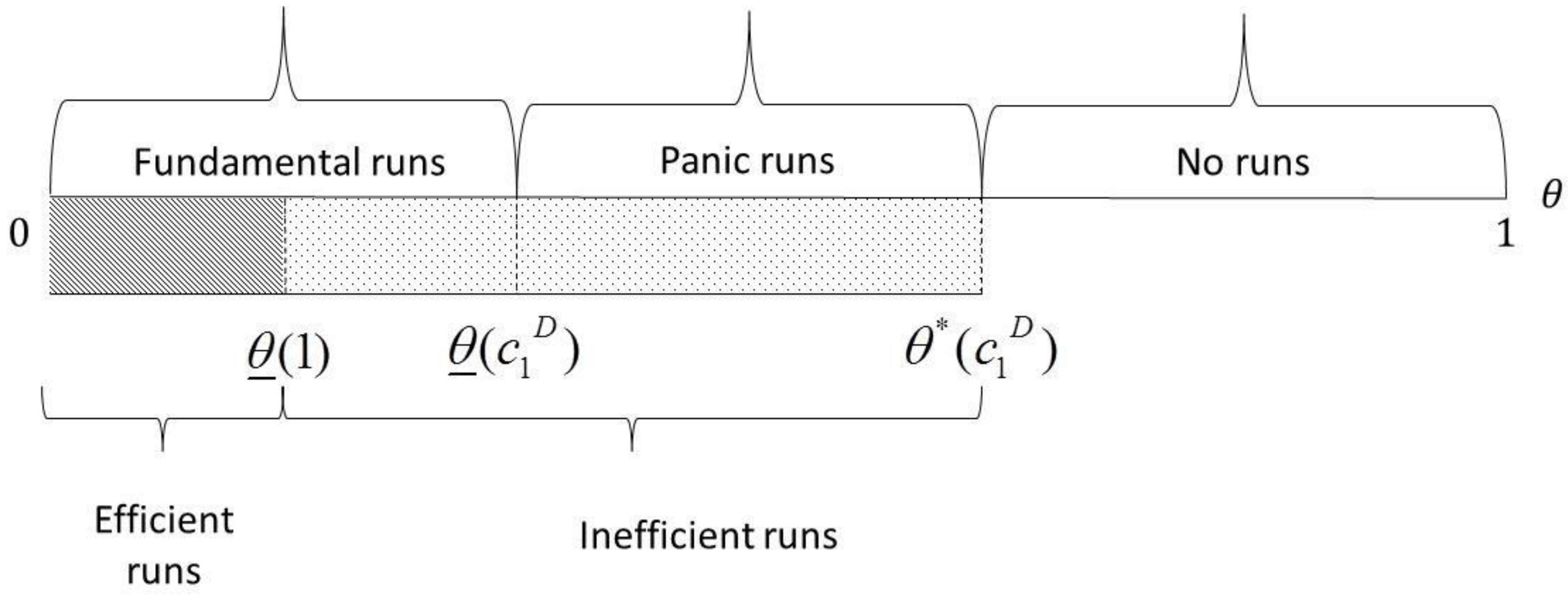

Figure 2: Depositors' withdrawal decision. The figure characterizes depositors' withdrawal decision as a function of the fundamental of the economy θ. Depositors run if $\theta^*(c_1^D)$ and do not run otherwise. In the region in which they run, two types of crisis can be distinguished. If $\underline{\theta}(c_1^D)$, runs are fundamentals-driven. If $\underline{\theta}(c_1^D) < \theta$ $\theta^*(c_1^D)$, runs are panic-driven. While all panic runs are inefficient, fundamental runs are inefficient only in the range $\underline{\theta}(1) < \theta$ $\underline{\theta}(c_1^D)$. Otherwise they entail an efficient liquidation of the banks' asset.

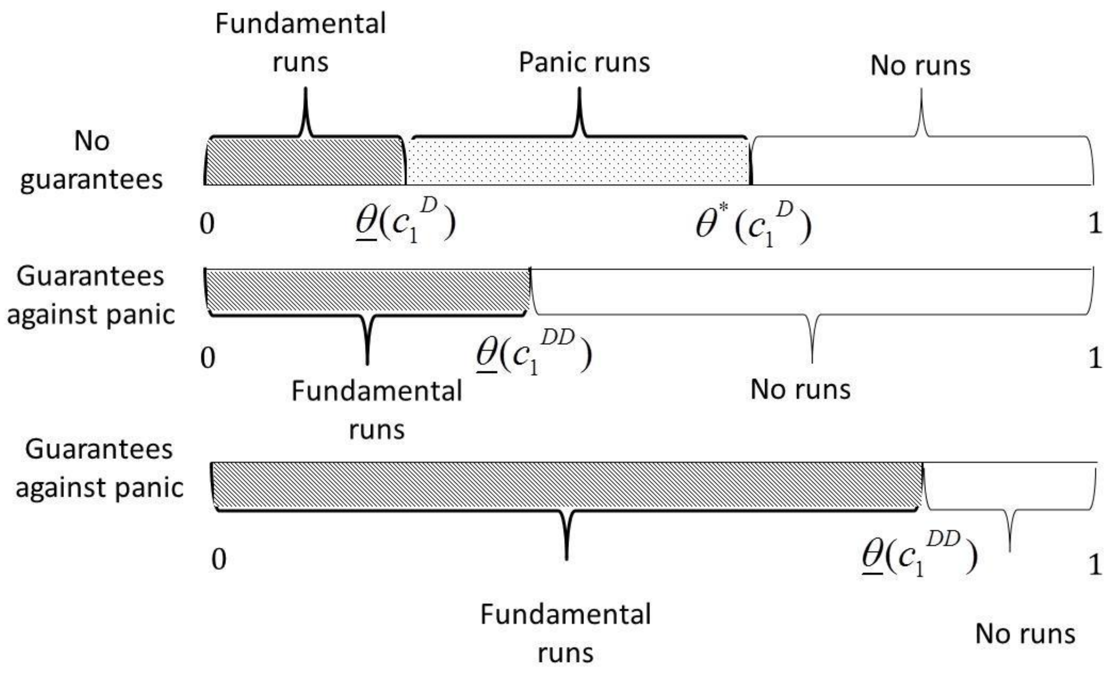

Figure 3: Guarantee against panic runs and financial stability. The figure shows the effect of the introduction of the guarantee scheme against panic runs on the stability of the banking sector. The guarantee scheme removes completely the occurrence of panic-driven runs, but fundamental runs become more likely as a result of an increase in the repayment offered by banks to early withdrawing depositors. If the increase in c1 is very large, then the overall probability of runs can be larger in the economy with guarantees than without it (i.e., $\underline{\theta}(c_1^{DD}) > \theta^*(c_1^D)$).

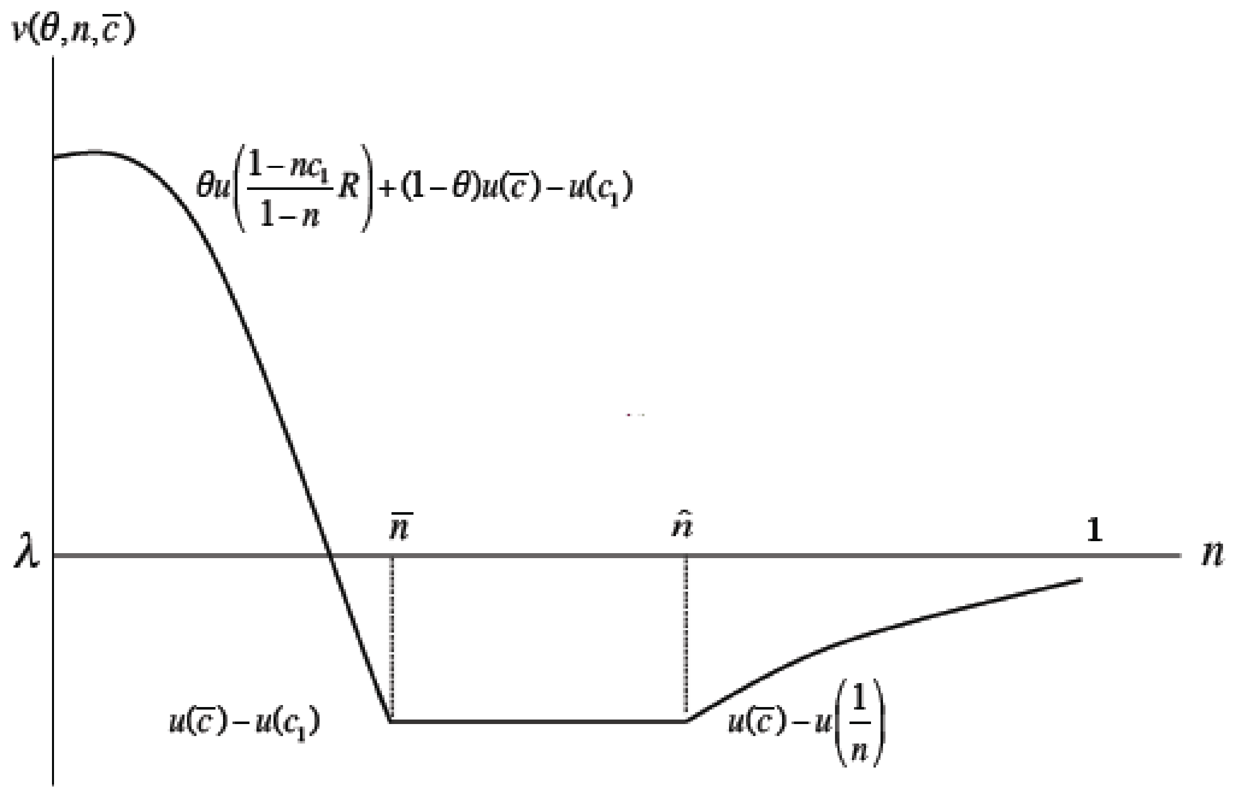

Figure 4a: Depositor's utility differential with a guarantee against runs and bank failure when $\bar{c} \leq 1$. The figure shows how the utility differential for a late depositor between withdrawing at date 2 versus date 1 changes with the number n of depositors withdrawing at date 1 for a given guarantee $\bar{c}$ chosen by the government. The function is decreasing in n for $\lambda \leq n < \bar{n}$, constant in the range $\bar{n} \leq n < \hat{n}$ and increasing for $\hat{n} \leq n \leq 1$. It crosses zero only once for $n < \bar{n}$ and remains below zero afterwards.

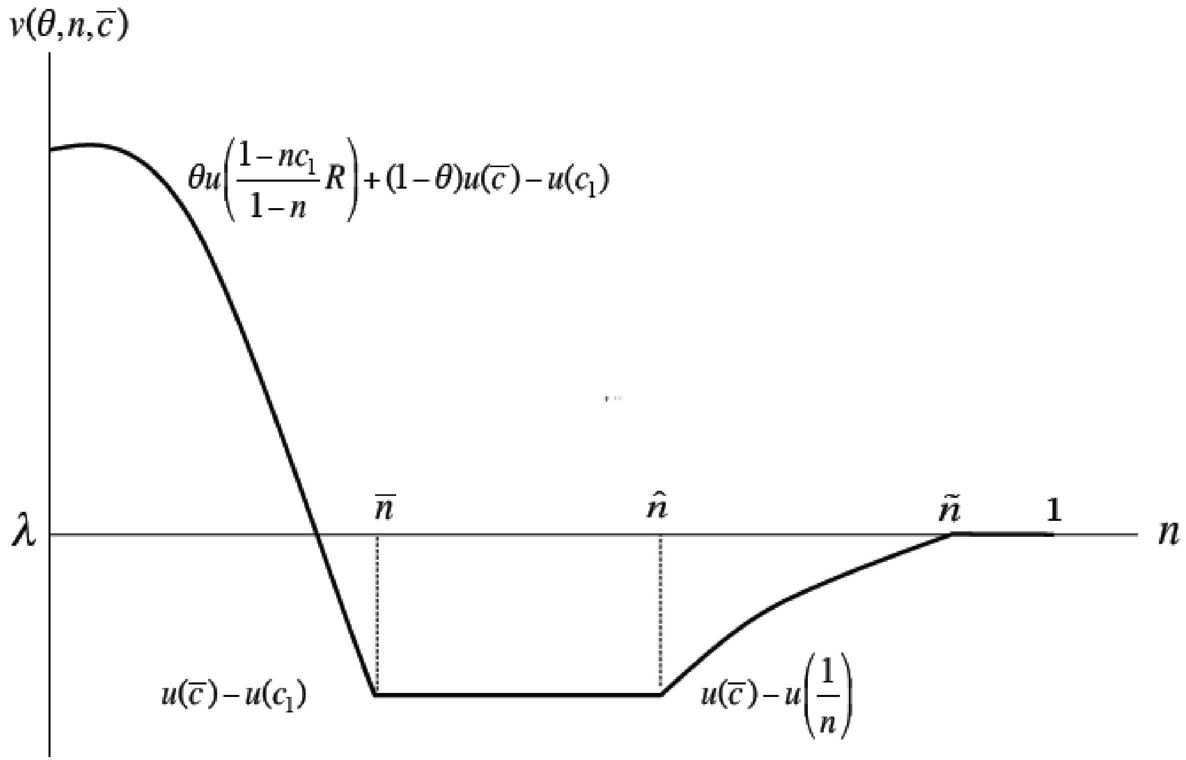

Figure 4b: Depositor's utility differential with a guarantee against runs and bank failure when $\bar{c} > 1$. The figure shows how the utility differential for a late depositor between withdrawing at date 2 versus date 1 changes with the number n of depositors withdrawing at date 1 for a given guarantee $\bar{c}$ chosen by the government. The function is decreasing in n for $\lambda \leq n < \bar{n}$, constant in the range $\bar{n} \leq n < \hat{n}$, increasing for $\hat{n} \leq n \leq \tilde{n}$ and again constant in the range $\tilde{n} \leq n \leq 1$. It crosses zero only once for $n < \bar{n}$ and takes value zero in the interval $\tilde{n} \leq n \leq 1$.

Acknowledgements

We are grateful to participants at the Western Finance Association (WFA) conference, the SFS Finance Cavalcade conference, the IDC summer finance conference, the IMFS conference on monetary and financial stability, the Arne Ryde Memorial Lectures workshop on "Financial Stability, Regulation and Public Intervention", the Isaac Newton Institute workshop on "Systemic Risk Models and Mechanisms", the Oxford Financial Intermediation Theory (OxFIT) conference, the European Finance Association (EFA) conference, the NBER summer workshop "Risk of Financial Institutions", the New York Fed/NYU Stern Conference on Financial Intermediation, the Financial Intermediation Research Society (FIRS) conference, the IESE/CEPR conference on "Financial Stability and Regulation", the WU Gutmann Symposium on "Sovereign Credit Risk and Asset Management", the Fifth Bank of Portugal Conference on Financial Intermediation, the Norges Bank Workshop "Understanding Macroprudential Regulation", the European Economic Association (EEA) conference; and to seminar participants at Birbeck College, the Riksbank, the European University Institute and the Wharton School. We also wish to thank for useful comments and suggestions Viral Acharya, Michael Gofman, Piero Gottardi, Frank Heinemann, Hendrik Hakenes, Anton Korinek, Xuewen Liu, Gregor Matvos, Robert McDonald, Alan Morrison, Adolfo DeMotta, Rafael Repullo, Jean-Charles Rochet, Javier Suarez, Anjan Thakor, Xavier Vives, Ernst-Ludwig von Thadden and Wolf Wagner. We thank Deeksha Gupta for excellent research assistance.

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ISSN	1725-2806 (pdf)	DOI	10.2866/46276 (pdf)
ISBN	978-92-899-2754-3 (pdf)	EU catalogue No	QB-AR-17-044-EN-N (pdf)