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WHAT HORIZON FOR PRICE STABILITY

BY FRANK SMETS

JULY 2000

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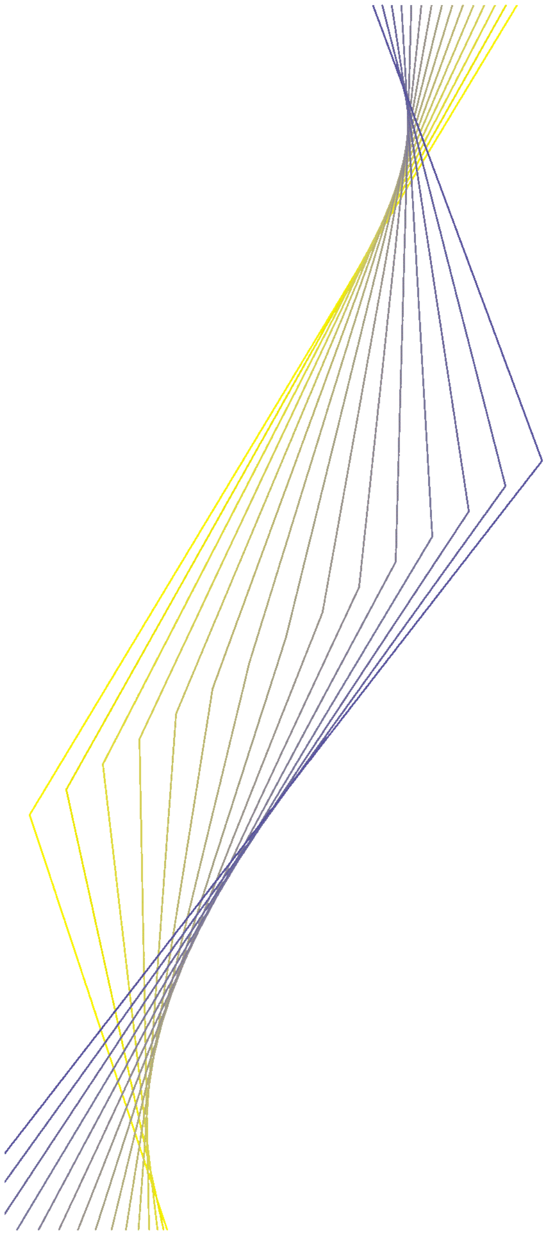

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WHAT HORIZON FOR PRICE STABILITY

BY FRANK SMETS*

JULY 2000

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Contents

Abstract

This paper analyses in a unified framework the twin issues of the appropriate horizon for achieving price stability in the face of unexpected disturbances and the choice of a price level versus an inflation objective. Using a small estimated forward-looking model of the euro area economy, the three main findings are: 1) The policy horizon becomes shorter the greater the weight on price stability in society's objective function, the higher the degree of "forward-lookingness" in the economy and the greater the slope of the Phillips curve; 2) The optimal policy horizon for a price level objective is generally greater than that for an inflation objective; 3) Even if society cares about inflation stabilisation (rather than the stabilisation of the price level), it often pays to give the central bank a price level objective (rather than an inflation objective), provided the horizon is optimally chosen to be somewhat longer and there is a small weight on interest rate stabilisation in the loss function. This result depends, however, on the structure of the economy.

JEL classification codes: E4-E5
Keywords: monetary policy, strategy, policy rules, euro area;

1. Introduction

In this paper I analyse two dimensions of the forward-looking nature of monetary policy. The first issue concerns the optimal policy horizon: Over what horizon should price stability be achieved in the face of unexpected disturbances to prices? Such a medium-term orientation in monetary policy typically arises because of two reasons. First, monetary policy affects prices only with a lag. Attempts at controlling short-term shocks to the price level, over which the central bank has little control, would therefore be counterproductive and may lead to instrument instability. Second, a medium-term orientation of monetary policy may be important to avoid excessive volatility in short-term interest rates or the real economy. A measured and gradualist response to some types of unforeseen shocks which may threaten price stability may avoid such excessive volatility, while ensuring that price stability is maintained over the medium term. [1] In this paper I analyse what factors determine the optimal policy horizon. In particular, I use a small estimated forward-looking model to examine how the length of the horizon depends on the structure of the economy and considerations such as avoiding excessive interest rate and output volatility.

The second issue, which recently has received increasing attention in both academic and policy circles, concerns the choice between a price level and an inflation objective.[2] Faced with a shock to prices, should a central bank attempt to revert the price level to a well-defined deterministic level, or should it allow base drift and aim at stabilising the inflation rate. In the first case, the central bank is expected to bring down inflation below its medium-term price stability objective in order to achieve price level stability following a positive shock to inflation. In the second case, bygones are bygones and shocks to the price level are accommodated. One reason for this renewed interest in price level objectives is that, in a regime of low inflation, credible price level objectives may alleviate the constraint on monetary policy that may arise due to the zero lower bound on nominal interest rates. Credible price level objectives appear superior than inflation objectives because an incipient decline in prices triggers expectations of a future increase in prices, i.e. inflation expectations, which reduces the ex-ante real interest rate and has an automatic equilibrating impact on the economy.

[1] For example, in the context of an inflation targeting strategy, Goodhart (1998) discusses how, by adjusting the instrument such as to stabilise the forecast of inflation at some appropriate horizon around the target level, the central bank can largely succeed in stabilising actual inflation, while avoiding destabilising effects on output. Similarly, the monetary policy strategy of the Eurosystem states that price stability "is to be maintained over the medium term". The reasons for such a medium-term orientation are explained in ECB (1999), p.47.

[2] See, for example, the contributions by Svensson (1999) and King (1999) at the Jackson Hole Conference organised by the Kansas Fed in September 1999 or Blinder (1999) at a recent ECB Conference.

The paper analyses the twin issues of the choice between a price level or an inflation objective and the appropriate horizon for achieving such price stability in a unified framework. The macro-economic stabilisation of the economy is analysed when the central bank maximises a set of secondary objectives such as interest rate and output gap stabilisation subject to the constraint that in expected terms it achieves either a price level or an inflation objective at a specific horizon in the future. Analysing both issues in the same framework is important because the relative performance of price level versus inflation objectives will depend on the horizon over which these objectives are being achieved. In particular, in the case of persistent inflation developments, the output cost of reverting the price level back to its objective following a positive shock may depend on the horizon over which this reversal is achieved. This point was made clear by King (1999), who, elaborating on the potential usefulness of price level targets, said: “Earlier, I suggested that it was useful to think in terms of the horizon over which inflation was brought back to its target level… Equally, one can think in terms of the horizon over which policy-makers wish to bring the price level back to some desired pre-determined path”. He continued: “Proponents of inflation targeting point out that to return prices to their previous level might imply significant volatility of output. I find this contrast somewhat artificial. The reason is that the dichotomy between the two approaches is analysed in models in which the target variable, whether inflation or the price level, is returned to its desired level in the following period”.

By analysing the issues of price level versus inflation objectives and the optimal policy horizon in a unified framework, the paper brings together two quite separate strands of the literature. The literature on price level versus inflation targeting typically analyses the potential trade-off between the advantage of lower price level uncertainty under price level targeting and the higher inflation and output variability that may result when prices are sticky. This literature generally abstracts from the important question of the policy horizon. One set of examples are the theoretical papers by Svensson (1999), Kiley (1998) and Vestin (1999). Svensson (1999) shows, among other things, that in a simple model with a Lucas supply curve and persistent output developments price level targeting may involve a free lunch. Compared to inflation targeting, it reduces both inflation and output variability. Kiley (1998) shows that this result depends on the form of the supply curve. With a new-Keynesian Phillips curve output variability will be higher under price level targeting. However, Vestin (1999) shows that in a model with a forward-looking Calvo-Taylor Phillips curve, price level targeting under discretion outperforms inflation targeting if one allows the relative weight on output variability to vary appropriately. To the extent that the weight on output variability in the central bank's loss function is related to the policy horizon, this result echoes some of the results in this paper.

Another set of papers in this literature compares inflation and price level targeting by simulating the effects of postulated reaction functions with a feedback on the price level in estimated or calibrated models of the economy. Examples are Lebow et al (1992), Fillion and Tetlow (1994), Williams (1999), Black et al (1998) and Reifschneider and Williams (1999). [3] Overall, these papers show that adding a small feedback term on past deviations of the price level from its objective may shift the output-inflation variability efficiency frontier inward, thus allowing for a better stabilisation of the economy. Finally, a number of papers, such as Fischer (1994), Duguay (1994) and McCallum (1997) compare the properties of simple postulated stochastic processes for inflation and the price level under both regimes.[4] Importantly, Duguay (1994) shows that with stationary price levels inflation variability need not be higher than when base drift is allowed.

The other strand of the literature focuses on the optimal horizon for inflation forecast targeting. Svensson (1997) and Ball (1999) emphasise in a small theoretical model the positive correspondence between the horizon over which the inflation forecast is brought back to target and the weight on output stabilisation in the central bank's objective function. Other papers, such as Batini and Haldane (1999), Black et al (1997), Amano et al (1999) and Levin et al (1999), analyse the performance of inflation forecast rules with different horizons for the forecasts. In general, the optimal forecast horizon appears to be relatively short and only rarely exceeds the lags in the transmission mechanism, although these results obviously depend on the properties of the model used. For example, Levin et al (1999) find that in most of the models they use the forecast horizon extends over less than a year. Amato and Laubach (1999) use an estimated optimisation-based model with staggered price and wage setting and analyse the welfare properties when the central bank is charged with minimising the deviations of forecasts at different horizons from their targets. They also find that the optimal forecast horizon is relatively short.[5] Finally, Batini and Nelson (1999) make the important distinction between the optimal policy horizon, i.e. the horizon over which inflation is brought back to target, and the optimal feedback horizon, i.e. the horizon over which the monetary authorities should form the inflation forecast that enters their policy rule. While the optimal feedback horizon may be relatively

[3] In a small-scale open-economy model for the United Kingdom, Batini and Yates (1999) examine both a set of simple rules feeding back from alternative combinations of price level and inflation deviations from target and a set of optimal control rules obtained assuming that policy makers minimise a loss function which penalises a mixed price level/inflation target.

[4] For an account of one of the few historical experiences with price level targeting, see Berg and Jonung (1998).

[5] In a backward-looking model, Nessen (1999) studies the effects of putting inflation calculated at different frequencies into the objective function of the central bank. Nessen (1999) studies the effects of putting inflation calculated at different frequencies into the objective function of the central bank.

short, the optimal policy horizon will depend on the weight of other objectives than price stability in the central bank's loss function and may therefore be longer. Among other things, Batini and Nelson (1999) find that the optimal policy horizon depends on the source of the shock that affects the economy.

The rest of the paper is structured as follows. Section 2 discusses the unified framework and methodology I use to analyse the two issues. Section 3 presents estimates of a forward-looking model for the euro area. In Section 4 the main results are discussed. Finally, Section 5 contains the main conclusions.

2. Methodology

In this section I explain the unified framework that is used to analyse the two issues mentioned above. Section 2.1. first presents the simple forward-looking model that is used to describe the euro area economy. Obviously, the structure of the economy will have an impact on the optimal policy horizon and the choice between a price level or inflation objective. Section 2.2. then discusses the central bank's decision problem and the solution method for calculating optimal monetary policy under commitment for a given forward-looking price level or inflation constraint. Finally, Section 2.3. describes the loss function that is used to evaluate the choice of the optimal mandate for price stability.

2.1. A forward-looking model of the economy

In order to keep the interpretation of the results manageable, I use a simple two equation version of a model that has recently been used quite extensively to analyse monetary policy issues. The model consists of the following output and price equation:

(1) $$y_t = \delta y_{t-1} + (1-\delta)E_t y_{t+1} + \sigma(r_t - E_t\pi_{t+1}) + \varepsilon_t$$

(2) $$\pi_t = \alpha\pi_{t-1} + (1-\alpha)E_t\pi_{t+1} + \kappa y_t + u_t$$

where y_t is output, π_t is the inflation rate, r_t is the nominal short-term interest rate, E_t is the expectations operator based on time *t* information, ε_t is a shock to the output equation and u_t is a shock to the inflation equation. In what follows I assume that both shocks follow a first-order autoregressive process with ρ_1 and ρ_2 being their respective autoregressive parameters.[6] All variables are written in terms of deviations from their steady state values.

For δ and α equal to zero, this model provides the basis of the so-called new neo-classical synthesis (See Goodfriend and King (1997)). As shown by, among others, Rotemberg and Woodford (1997), Goodfriend and King (1997) and McCallum and Nelson (1999), these equations can be derived from micro foundations.

However, in order to fit the data, I have replaced the forward-looking component in both equations by a weighted average of a backward and a forward-looking component. The empirical case for introducing such persistence in the model has been forcefully made by Fuhrer and Moore (1996) for the case of the inflation equation and by Estrella and Fuhrer (1998) for both the output and inflation equations. The presence of lagged output in equation (1) can be justified on the basis of a micro-based model in which agents' utility functions exhibit habit persistence (see, for example, Fuhrer (1998)). Similarly, the presence of lagged inflation in equation (2) can be justified on the basis of a model where agents care about relative wages (e.g. Garcia and Ascari (1999)). Gali and Gertler (1998) assume the existence of rule-of-thumb price setters to derive a similar inflation equation with both lagged and lead terms.

A number of characteristics of the admittedly very simple model of the euro area economy deserve some further considerations. First, the fact that both output and inflation equations contain forward-looking elements is of importance when analysing alternative policy regimes. While the Lucas critique could still apply to the extent that some of the "structural" parameters may not be invariant to the change in policy regime, the introduction of forward-looking expectations will at least ensure that the change in regime is taken into account in expectation formation.[7] The extent to which our results depend on these forward-looking features can be examined by allowing the coefficients α and δ to vary.

Second, in equations (1) and (2) there are no explicit lags in the transmission of the monetary policy instrument - which we take to be the short-term nominal interest rate - to output and inflation. One implication is that, if the central bank wanted to stabilise prices immediately, it could do so. This would, however, come at a large cost in terms of interest rate volatility. The reason for not introducing explicit lags is twofold. First, I want to focus on the role of other objectives for the optimal choice of a policy horizon. This may be easier when abstracting from explicit lags in the transmission mechanism. Second, model

[6] In the simulations these parameters are set to a very small value (0.01).

[7] The rational expectations assumption implies that the different price stability objectives considered will be perfectly credible.

(1) - (2) is estimated on the basis of annual data. Restricting the effects of monetary policy on output and inflation to be zero in the first year, appears to be equally restrictive.

2.2. The central bank's decision problem

Given the structure of the economy, the central bank's decision problem can now be analysed. It is assumed that the central bank minimises the loss from output and interest rate variability subject to the constraint that in expected terms it achieves either a price level or inflation objective at a specific horizon in the future. This representation of the central bank's behaviour has the advantage that it explicitly brings out the form of the price stability objective (in price levels or inflation rates) and the forward-looking policy horizon. This makes it easier to analyse the effect of changes in the objective and the policy horizon on interest rate policy and the performance of the economy.

This representation characterises actual central bank behaviour in two respects. First, it captures the observation that many central banks have what appears to be a lexicographic ordering in their mandated objectives. In this ordering, the central bank has no leeway in deviating from its primary objective price stability. For example, the Treaty on European Union defines the objectives of the Eurosystem as follows: "The *primary* objective of the ESCB shall be to maintain price stability. *Without prejudice* to the objectives of price stability, the ESCB shall support the general economic policies in the Community with a view of contributing to the achievement of the objectives of the Community as laid down in Article 2".[8] The Community's objectives (Article 2) consist of a long list of general and potentially conflicting goals which includes "sustainable and non-inflationary growth". In the central bank's decision problem, this lexicographic ordering is captured by modelling the price stability objective as a constraint on the central bank's behaviour. Other objectives, such as avoiding excessive output and interest rate volatility, can therefore only be pursued to the extent that they are consistent with this primary objective. Of course, as discussed in the introduction, the horizon will determine how binding the price stability constraint is at any moment in time.

Second, this representation of the central bank's behaviour can be seen as a reasonable description of what Svensson (1997) has called a flexible inflation targeting rule. For example, Svensson (1999) states that one such rule which "arguably can be inferred from the Bank of England's MPC" is to "select the instrument path such that the deviations of output from capacity are minimised, subject to the inflation forecast hitting the inflation target eight quarter ahead".

[8] Italics added.

More formally, equations (1) and (2) describing the dynamics of the economy can be written in state space form as:

$$\begin{bmatrix} p_t \\ y_t \\ \pi_t \\ \varepsilon_{t+1} \\ u_{t+1} \\ y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \rho_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_2 & 0 & 0 \\ 0 & -\delta/\gamma & -\sigma\alpha/(\gamma(1-\alpha)) & -1/\gamma & -\sigma/(\gamma(1-\alpha)) & (1-\alpha-\sigma k)/(\gamma(1-\alpha)) & \sigma/(\gamma(1-\alpha)) \\ 0 & 0 & -\alpha/(1-\alpha) & 0 & -1/(1-\alpha) & -\kappa/(1-\alpha) & 1/(1-\alpha) \end{bmatrix} \begin{bmatrix} p_{t-1} \\ y_{t-1} \\ \pi_{t-1} \\ \varepsilon_t \\ u_t \\ y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sigma/\gamma \\ 0 \end{bmatrix} r_t + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \varepsilon^y_{t+1} \\ \varepsilon^\pi_{t+1} \\ 0 \\ 0 \end{bmatrix}$$

where $\gamma = 1-\delta$. In short notation, this can be rewritten as:

(3) $$x_{t+1} = Ax_t + Br_t + \varepsilon_{t+1}$$

The central bank's mandate is then to minimise the following loss function:

(4) $$E_0 \sum_{i=0}^{\infty} \beta^i (\omega_2 y_i^2 + (1-\omega_2) r_i^2)$$

subject to equation (3) describing the dynamics of the economy and the constraint that the expected price level or inflation rate after H periods equal a target which is normalised to zero:

(5) $E_t p_{t+H} = 0$ or $E_t \pi_{t+H} = 0$

In addition, I assume that this optimisation occurs under commitment, i.e. the central bank can commit not to renege on its plans to achieve the price stability target as time goes on and the horizon shifts forward.

Because of the non-recursive nature of the constraint, this problem cannot be solved directly using standard methods. However, Marcet and Marimon (1999) show how such problems can be rewritten using Lagrange multipliers so that the recursive structure is retrieved (See the appendix). The transformed problem can then be solved using, for example, the algorithms explained in Söderlind (1999).

2.3 Evaluating the price stability mandate

In order to evaluate the choice of the price stability constraint discussed in Section 2.2., it is convenient to use the following loss function:

(6) $$E_0 \sum_{i=0}^{\infty} \beta^i (\omega_1(\omega_2 p_i^2 + (1-\omega_2)\pi_i^2) + (1-\omega_1)(\omega_3 y_i^2 + (1-\omega_3) r_i^2)).$$ [9]

[9] I thank my discussant Tony Yates for suggesting to put both inflation and price level variability in the loss function.

In this loss function, the parameters ω_1, ω_2 and ω_3 capture the relative weight society puts on the losses which result from variability in the main endogenous variables in the economy.

In the benchmark case, I will assume that ω_2 equals zero and ω_3 equals one, so that society only cares about inflation and output variability. Rotemberg and Woodford (1997) show that such a loss function can be interpreted as a quadratic approximation of the steady-state utility of the representative agent in sticky-price models similar to equations (1) and (2). In these models inflation variability enters the loss function because it creates relative price distortions between those firms that can adjust their prices and those that can not.[10]

In addition, it is interesting to see how the results vary when society also cares about volatility in the nominal interest rate. Interest rate variability may enter as an argument for various reasons. One, probably minor, reason is to minimise the distortions arising from the inflation tax on money balances. A more important reason (emphasised in Rotemberg and Woodford (1997)) is that the lower bound on nominal interest rates puts a premium on low interest rate volatility, particularly at low steady-state inflation rates. A final reason may be that big and unexpected changes in interest rates may cause problems for financial stability as discussed in Cukierman (1990). As will become clear in section 4, given the estimated structure of the economy putting no weight on nominal interest rate variability in society's loss function results in interest rate volatility which is several times higher than the actual volatility observed. In order to avoid this counterfactual outcome, I will assume a small weight on interest rate variability ($\omega_3 = 0.9$) in most of the sensitivity analysis of Section 4. [11]

Finally, while it may be difficult to formalise why society cares about unexpected volatility in the price level, it is clear that it is the stability of the long-run price level which creates confidence in the monetary standard and enables nominal contracts to play an important role in the economy (King (1999)). Long-term lenders know what their return will be in real terms, while long-term borrowers know what they will pay. In particular when credit constraints due to asymmetric information are prevalent and the costs of indexation are substantial, long-run price level stability may be beneficial. For example, it may avoid financial instability due to an unexpected redistribution of wealth associated with shifts in the price level. Below I therefore also analyse how the results

[10] The implications of rule-of-thumb behaviour (which may explain the lagged terms in both the output and inflation equations) for the form of the loss function and optimal monetary policy is discussed in Amato and Laubach (2000).
[11] This is a common finding in the literature on optimal monetary policy rules (see, for example, Rudebusch and Svensson (1999)). An alternative way of dealing with this empirical puzzle is to impose a constraint on interest rate variability directly as in Levin et al (1999).

about the optimal policy horizon and price stability mandate change when society cares about price level stability ($\omega_2 > 0$).

More formally, the value of society's loss function which results from the central bank's optimal policy can be denoted by $J(j,H)$, where $j = p,\pi$ denotes whether the central bank has a price level objective (p) or an inflation objective (π) and H denotes the corresponding policy horizon. The optimal price stability constraint can then be found by minimising this loss function with respect to both the horizon and whether the central bank should pursue a price level or inflation objective:

(7) $$\underset{j;H}{Min}\, J(j,H).$$

The outcome under such a mandate can then be compared with the first best outcome which can be derived from optimising the loss function (6) directly subject to constraint (3).[12]

3. An estimated version of the small-scale forward-looking model for the euro area.

To get benchmark parameters, equations (1) and (2) are estimated using synthetic annual data for the euro area over the period 1974 to 1998. The estimates which are obtained using GMM methods and lagged variables as instruments are reported in Table 1.

Overall the estimated parameters have the expected sign and are significant. The hypothesis that the sum of the lagged and lead variable in both the output and inflation equation is equal to one can not be rejected. In both cases the weight on the backward-looking variables is a bit less than one half, capturing the considerable degree of persistence in both output and inflation series. The sensitivity of the output gap to the real interest rate is significantly negative, whereas the slope of the Phillips curve is significantly positive. The standard deviation of the shocks to the annual output gap equation is about 65 basis points, whereas the standard deviation of the shocks to the inflation equation is somewhat higher at about 70 basis points. There is evidence of a small negative correlation between the two estimated residuals of minus 15 percent.

In order to get a feeling for the typical dynamics in this economy, Graph 1 plots the responses of output, inflation, the nominal interest rate and the price level to a typical output and inflation shock when the central bank's policy is characterised by the

[12] This minimisation problem under commitment can be solved using standard methods as explained, for example, in Söderlind (1999)).

minimisation of loss function (6) under commitment. In these simulations it is assumed that there is equal weight on the price stability and other objectives ($\omega_1 = 0.5$). In addition, I assume a small weight on interest rate variability ($\omega_3 = 0.9$) to get a reasonable interest rate response to both shocks. The left-hand panel depicts the case in which there is no weight on price level stability ($\omega_2 = 0.0$), whereas the right-hand panel depicts the case with a small positive weight on price level ($\omega_2 = 0.1$).

As one may expect, the impulse response functions indicate quite a lot of persistence in the economy in response to the two shocks. The interest rate cycle is about 6 to 7 years from peak to trough. Obviously, the main difference between the left and the right-hand panel is that, under the latter regime, the price level is forced back to its target level. This is particularly striking in the case of a price shock. In order to achieve mean reversion in the price level, monetary policy needs to tighten more. With price level stabilisation, the maximum rise in nominal interest rates is almost 10 basis points higher, while the output gap reaches a minimum of about 10 basis points lower after three years. One finding that at first sight may be surprising is that with inflation stabilisation, the long-run effect of a positive demand shock on the price level is slightly negative. This result is entirely due to the assumed monetary policy behaviour. As discussed in Clarida, Gali and Gertler (1999), the optimal policy under commitment in this type of models resembles a policy rule which responds to the price level rather than the inflation rate.[13] One reason is that with a forward-looking output equation the commitment to revert the rise in the price level has a stabilising impact on output variability as discussed in the introduction.

4. The optimal horizon: results for the forward-looking model.

4.1. The benchmark cases

Using the estimated model of the euro area, the optimal horizon for both a forward-looking inflation and price level objective can now be analysed. As a starting point, three benchmark cases are examined. In the first case society only cares about the variability in inflation and the output gap, which are equally weighted ($\omega_1 = 0.5$). In the second case a small weight (of 0.05) on interest rate volatility is incorporated. In the third case an additional small weight (of again 0.05) on price level stability is included. Table 2 and Graph 2 summarise the results in each of these cases. Graph 2 plots the losses as a function of the horizon. Note that a logarithmic scale is used for the y-axis.

[13] Similar simulations under a discretionary policy (not reported) confirm that an output shock has a positive long run impact on the price level.

A number of observations can be made. First, when society cares about other objectives than price stability, it is not optimal to stabilise inflation or the price level immediately in response to output and price shocks although in our annual model without explicit policy lags the central bank could do so. Given an equal weight on price stability and other objectives, the optimal horizon for a forward-looking inflation constraint is between three and four years in the three benchmark cases.

Second, in each of the three benchmark cases the optimal horizon for a price level objective is about twice as long as the one for an inflation objective. For example, in the first benchmark case the optimal horizon for a price level objective is six years compared to three years for an inflation objective. In order to understand the intuition behind this result, it is instructive to examine Graph 3 which plots the standard deviations of output, inflation and the nominal interest rate as a function of the horizon and the choice of the objective. For a given horizon the variance of output and interest rates is generally greater under a price level objective than under an inflation objective, while the variance of inflation is less. The first finding is quite intuitive in the presence of sticky prices and inflation. Take, for example, the case of a price shock. In order to bring not just inflation but also the price level back to its target within a given horizon, interest rates will have to rise by more, while output will have to fall by more. This will tend to raise the variance of output and interest rates under a price level target. This finding is consistent with the results in Kiley (1998) who shows that as long as there is some price stickiness, price level targeting will be costly in terms of output volatility. The finding that for a given horizon inflation variability is lower under a price level objective versus under an inflation objective is less intuitive.[14] However, it is consistent with the general point made in Duguay (1994) and Svensson (1999) that a stationary price level does not necessarily lead to higher inflation variability.

It now becomes clear why for a given loss function it is optimal to have a longer horizon for a price level objective compared to an inflation objective. Basically, lengthening the horizon for the price level objective compared to the one for an inflation objective allows the central bank to trade off less interest rate and output volatility for higher inflation variability.

Third, the outcome of choosing an appropriately longer horizon for the price level objective is that the overall loss under a price level objective comes close to or is (as in the second benchmark case) even lower than the loss under an inflation objective. In other

[14] See, for example, Fischer (1994, p283-3): "Price level targeting is thus a bad idea, one that would add unnecessary short-term fluctuations to the economy. It is also true, …, that there is more variability and uncertainty about short-term inflation rates with a price level target than with a target inflation rate."

words, even if society cares only about inflation stabilisation, it may be optimal for the central bank to choose a price level objective rather than an inflation objective provided the horizon is optimally chosen to be somewhat longer. This is particularly the case when there is some, be it small, weight on interest rate stabilisation in the loss function. As shown in the middle panel of Table 2, in both cases a very similar stabilisation of the output gap and inflation is achieved, but a price level objective is rather more beneficial in terms of the interest rate volatility it creates. The standard deviation of the nominal interest rate under an optimal price level objective is 75 basis points lower than under an optimal inflation objective.[15]

This finding is consistent with the analysis in Coulombe (1997) who shows that credible price level targeting may help to alleviate the lower zero bound because it reduces the need to move nominal interest rates. The reason for this is intuitive. When the price level target is credible, private agents will expect the price level to rise after an unexpected fall. For given nominal interest rates such positive inflation expectations will reduce the ex-ante real rate, which in turn will have an equilibrating effect on output and prices. As a result the need to reduce nominal interest rates to achieve a given adjustment is reduced. These results also confirm the analysis in Reifschneider and Williams (1999) and Wolman (1998) who show that when agents are forward-looking and the central bank credibly responds to deviations of the price level from some target, problems associated with the lower zero bound on interest rates and possible risks of deflationary spirals may become less important.

Fourth, the lower panel of Table 2 and Graph 2 show that with even a small weight on price level volatility the balance is completely tilted in favour of an appropriately defined price level objective. Obviously, a forward-looking inflation objective results in price level drift. The price level does not revert to baseline following an output or price shock and as a result becomes a non-stationary process with an infinite variance. This is very costly if society cares about price level volatility as illustrated in Graph 2c.[16]

A couple of additional smaller observations can be made. Graph 2 shows that in general for short horizons an inflation target is preferred, while for longer horizons a price level objective becomes optimal. One implication is that if a central bank is constrained in choosing a short policy horizon, for example for reasons of imperfect credibility, it will prefer an inflation objective. Reversely, a central bank gaining reputation may consider

[15] That it is reasonable to put some weight on interest rate volatility in the loss function is clear from comparing the first and second benchmark case. In the first benchmark case, with zero weight on interest rate volatility, the standard deviation of the nominal interest rate exceeds 10 percent. This is clearly not compatible with observed interest rate volatility. Even a small weight on interest rate volatility (as in the second benchmark case) is able to correct this discrepancy with the empirical data.

[16] Note that the loss is nevertheless finite because of discounting.

announcing a price level objective and lengthening its policy horizon appropriately. Another feature of the loss functions in Graph 2 is that the cost of deviating from the optimal horizon is often asymmetric. The cost of choosing a shorter horizon than optimal is larger than the cost of choosing a longer one. To some extent, this is again likely to be due to the assumption of perfect credibility. When a longer horizon is more likely to cause a deterioration of the central bank's credibility regarding the maintenance of price stability, this result may be reversed. It is also obvious that none of the optimal mandates achieves the first best outcome which could be achieved if the central bank could commit to society's loss function.

Finally, Graph 4 plots the impulse responses to a typical output and price shock for an optimal inflation and price level target in the second benchmark case. As noted by Coulombe (1997), one of the striking features is that the nominal interest rate follows the path of the price level rather than that of the inflation rate under price level targeting. Coulombe (1998) argues that this can explain Gibson's paradox, i.e. the positive correlation between the nominal interest rate and the price level in the gold standard period. One less appealing feature of these impulse responses is the volatility in the interest rate towards the end of the horizon which results from the need to exactly pin down inflation or the price level at that horizon. In future work, this feature could be alleviated by assuming a small target range.

4.2. The optimal horizon as a function of the weights in the loss function

To examine the dependence of the optimal horizon on the weights in the objective function, Table 3 presents the optimal horizon for an inflation objective (upper panel) and a price level objective (middle panel) as a function of ω_1 and ω_3. In this table it is assumed that the weight on price level variability in the loss function is zero. In addition, the lower panel reports the log difference between the loss under the optimal inflation and price level objective. A positive number means that the optimal price level objective is preferred.

A number of regularities can be observed. First, the higher the weight on price stability in society's objectives the shorter the optimal horizon is. This holds for both an inflation and price level objective. The optimal horizon converges to infinity as the weight on price stability approaches zero. It becomes the shortest possible when the weight on the other objectives is zero. Note again that there are no explicit lags in the effects of monetary policy in the annual model I have estimated. As a result, the central bank is able to stabilise inflation or the price level contemporaneously. Starting from a situation of price stability, price level and inflation targeting will then be equivalent. This result is consistent with the theoretical findings in Svensson (1997), who shows in a simple analytical model that a “strict” inflation targeter will set the inflation forecast at the shortest possible lag equal to target, while a “flexible” inflation targeter who also cares about output stabilisation will only gradually close the gap between the inflation forecast and the target. How gradual the central bank moves depends on the weight on output stabilisation.

Second, generally speaking the optimal horizon for a price level target is longer than that for an inflation target. The intuition for this result was discussed in Section 4.1. above. There are a number of cases (indicated as shaded areas) in which this result does not appear to be the case. Upon examination it appears that in most of these cases the loss function as a function of the horizon is very flat in the case of an inflation objective. To some extent these cases may therefore not be very interesting and the reversed ranking may be due to computational inaccuracies.

Third, generally speaking a higher weight on output stabilisation compared to interest rate stabilisation shortens the optimal horizon. In other words, a stronger desire to smooth interest rates compared to output will lead to a more gradual monetary policy response and a longer policy horizon for both an inflation and price level objective. The intuition for this result is clear. While a trade-off between interest rate and inflation stabilisation arises in the face of both price and output shocks, only price shocks give rise to a trade-off between output and inflation variability.

Fourth, for some small weight on the volatility in interest rates in the loss function (say, $\omega_3 \leq 0.9$), it is optimal for the central bank to stabilise the price level rather than the inflation rate at the appropriate horizon, even if society cares only about inflation volatility (and not price level volatility). Moreover, the welfare gains can be quite substantial. As discussed above, the balance will completely tilt in favour of a price level objective if there is only a small weight on price level variability in society’s loss function.

4.3. Some sensitivity analysis

In this section I analyse the sensitivity of the results with respect to some of the crucial parameters. As discussed above, the forward-looking elements in the output and inflation equations are important in determining the optimal horizon and for benefiting from the advantages of price level targeting.

First, as can be seen from Graph 5, a higher weight on the backward-looking component in the price equation will in general not only lead to a lengthening of the optimal horizon for both inflation and price level objectives (upper panel), it can also tilt the balance in favour of inflation objectives (lower panel). The lower panel plots the loss difference as a function of α under two assumptions regarding the weight on interest rate variability in the loss function. Even if there is no weight on interest rate stabilisation, price level targeting would be beneficial if the weight on the backward-looking component is less than 0.4.

The weight on the forward-looking component in the output equation is also important for the choice between price level and inflation targeting. Here, however, the weight on interest rate stabilisation in the loss function is crucial. The lower panel of Graph 6 shows that, when there is a small weight on interest rate volatility, the more backward-looking the output equation is, the less beneficial price level targeting will be. In contrast, if society does not care about interest rate stabilisation, then the difference in loss is not affected by the degree of "forward-looking-ness" in the output equation. The intuition is as follows. A higher degree of "forward-looking-ness" implies a higher effect of changes in the long-term real interest rate on output. As pursuing price level objectives will lead to an automatic variation in the ex-ante long-term real interest rate through its effect on inflation expectations, less action will be necessary through nominal interest rates. However, if one does not care about nominal interest rate volatility, then one can always compensate the absence of this automatic mechanism under an inflation objective by higher nominal interest rate volatility.

Finally, given the discussion above, it is unsurprising that the slope of the Phillips curve is also an important determinant of the optimal horizon and the choice between price level and inflation targeting. When the slope is steeper (in other words the output cost of disinflating is less), then the optimal horizon will become shorter for both the inflation and the price level target and the output cost of price level targeting will become less. This will tend to tilt the balance in favour of price level targeting. Somewhat surprisingly this mechanism only works when there is some weight on interest rate volatility in the loss function.

5. Conclusions

In this paper I have analysed the optimal policy horizon for price stability and some of its determinants. The conclusions from this analysis can be summarised as follows. First, the policy horizon becomes shorter the greater the weight on price stability in society's objective function, the higher the degree of "forward-lookingness" in the economy and the greater the slope of the Phillips curve. Second, the optimal policy horizon for an inflation objective will generally be less than that for a price level objective. Third, even if society cares about inflation stabilisation (rather than the stabilisation of the price level), it often pays to give the central bank a price level objective (rather than an inflation objective), provided the horizon is optimally chosen to be somewhat longer (and there is some weight on interest rate variability). This result depends, however, on the structure of the economy.

There are various directions in which this research needs to be extended. First, as the results very much depend on the model of the economy used, the robustness of the results with respect to different models needs to be checked. In particular, the model used in this paper is a very simple annual model and exhibits no lags in the transmission mechanism of monetary policy. As such lags are an important reason for the forward-looking nature of monetary policy, it would be appropriate to use a model in which such lags are present. A useful start would be the quarterly small-scale model of the euro area economy by Coenen and Wieland (2000).

Second, many of the results comparing inflation with price level objectives are likely to hinge on the assumption of perfect credibility. In particular, the results suggest that with the estimated degree of inflation stickiness the optimal horizon for a price level objective may extend over a full business cycle. The fact that such a long horizon allows the policy makers to avoid large output and interest rate volatility without endangering price stability is likely to be due to the full credibility of the nominal anchor. As in real-life monetary policy-making such credibility can not be taken for granted, it should be explored how the results change under conditions of imperfect credibility. All of the analysis comparing the different price stability mandates and policy horizons was done under the assumption that the central bank can commit to the policy necessary to achieve the mandate. It would be interesting to analyse how the results are affected when such commitment is not feasible and the central bank acts under discretion.

Finally, in this paper I have modelled the price stability constraint as an exact forward-looking constraint on either inflation or the price level at a particular horizon. In the first-best case, this price stability constraint will only be achieved asymptotically. Imposing an exact constraint at a particular horizon gives rise to unattractive interest rate volatility at that horizon. This could be avoided by modelling the forward-looking price stability constraint as a target zone rather than a point target. I leave this for future research.

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Table I

Estimated parameters of a forward-looking model for the euro area economy

Output equation (1):

$$y_t = \underset{(0.03)}{0.44} y_{t-1} + 0.56 E_t y_{t+1} - \underset{(0.03)}{0.06}(r_t - E_t \pi_{t+1}) + \varepsilon_t \qquad \sigma_\varepsilon = 0.65$$

Inflation equation (2):

$$\pi_t = \underset{(0.08)}{0.48} \pi_{t-1} + 0.52 E_t \pi_{t+1} + \underset{(0.03)}{0.18} y_t + u_t \qquad \sigma_u = 0.70$$

Notes: Estimated using annual data with GMM over the period 1974-1998. Standard errors are in parentheses. All data are in percent. The output gap is measured as the percentage deviation of a linear trend. The inflation rate is the annual $(Q4/Q4)$ percentage change in the GDP deflator. The nominal short-term interest rate is the quarterly average rate during a year.

Table 2

The optimal horizon in three benchmark cases

	Under commitment	With optimal inflation objective	With optimal price level objective
Benchmark 1: $\omega_1 = 0.5$, $\omega_2 = 0.0$ **and** $\omega_3 = 1.0$			
Horizon	-	3 years	6 years
Log Loss	16.7	19.9	21.22
Std output gap	0.71	0.66	0.88
Std inflation	0.94	1.11	1.06
Std interest rate	11.0	12.8	12.9
Std price level	-	-	2.01
Benchmark 2: $\omega_1 = 0.5$, $\omega_2 = 0.0$ **and** $\omega_3 = 0.9$			
Horizon	-	4 years	8 years
Log Loss	30.4	36.6	33.1
Std output gap	1.01	0.95	1.08
Std inflation	1.10	1.18	1.17
Std interest rate	2.18	3.06	2.30
Std price level	-	-	2.50
Benchmark 3: $\omega_1 = 0.5$, $\omega_2 = 0.1$ **and** $\omega_3 = 0.9$			
Horizon	-	3 years	8 years
Log Loss	34.8	100.3	37.8
Std output gap	1.12	1.02	1.14
Std inflation	1.06	0.97	1.06
Std interest rate	2.31	4.50	2.78
Std price level	1.95	-	2.05

Table 3

The optimal horizon as a function of the weights in the loss

Inflation objective

	ω_3									
ω_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	20	20	20	20	20	30	29	27	30	5
0.2	13	13	13	13	13	13	12	11	5	4
0.3	13	13	13	12	12	11	10	5	5	4
0.4	13	12	12	11	11	10	5	5	5	3
0.5	12	11	11	10	10	5	5	5	4	3
0.6	11	11	10	10	5	5	5	4	4	2
0.7	10	10	10	5	5	5	4	4	4	2
0.8	10	10	5	4	4	4	4	4	3	2
0.9	4	4	4	4	4	4	3	3	3	1
1.0	1	1	1	1	1	1	1	1	1	1

Price level objective

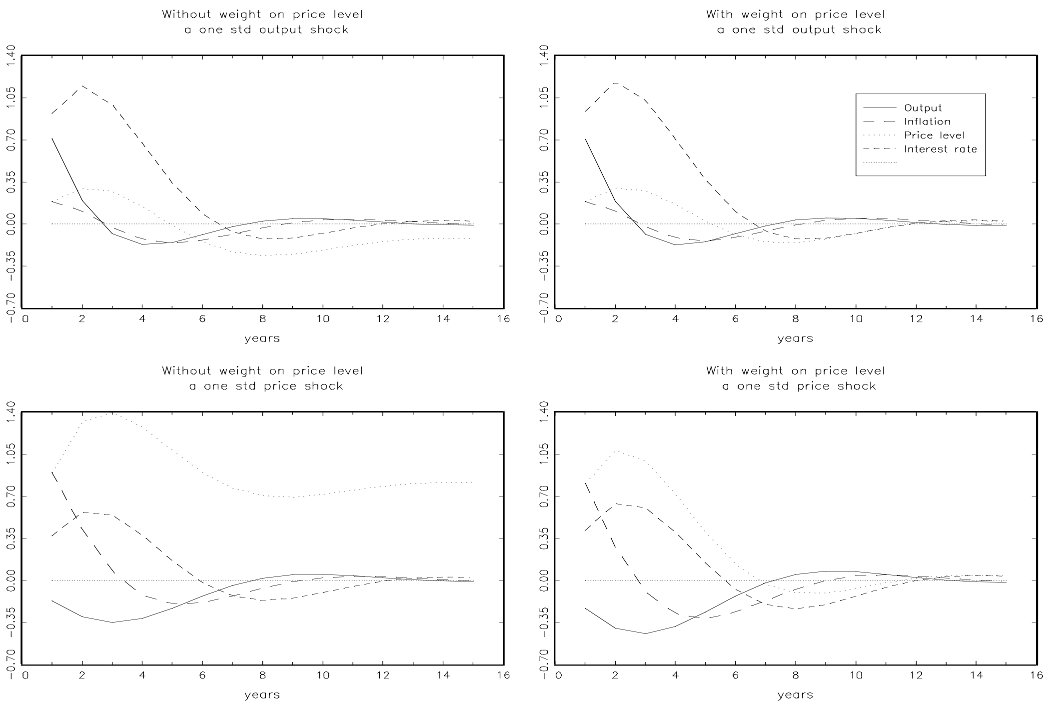
	ω_3									
ω_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	30+	30+	30+	30+	30+	30+	30+	18	16	13
0.2	30+	24	23	19	18	17	16	11	11	10
0.3	23	19	18	18	17	11	10	10	10	9
0.4	23	18	18	17	10	10	10	9	9	7
0.5	23	18	10	10	10	9	9	9	8	6
0.6	10	10	9	9	9	9	9	8	7	6
0.7	9	9	9	9	9	8	8	7	7	5
0.8	9	8	8	8	8	8	7	7	6	4
0.9	7	7	7	7	7	7	6	6	5	3
1.0	1	1	1	1	1	1	1	1	1	1

Log difference in loss

	ω_3									
ω_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	19	14	9	5	3	1	1	1	3	-3
0.2	28	23	18	14	12	11	11	13	10	-6
0.3	24	23	21	20	19	19	22	19	9	-5
0.4	21	23	24	24	25	27	26	18	10	-7
0.5	18	23	24	27	30	30	24	18	9	-6
0.6	18	24	27	31	33	28	24	19	9	-7
0.7	24	28	32	33	30	28	23	16	13	-8
0.8	28	33	34	31	27	23	20	18	11	-1
0.9	27	27	26	25	24	23	20	15	14	-9
1.0	116	116	116	116	116	116	116	116	116	116

Notes: The upper and middle panel give the optimal horizon with an inflation and a price level target respectively; the lower panel gives the log difference between the two associated losses. A positive number means a lower loss under a price level target. The shaded areas are cases in which the optimal horizon for a price level target is shorter than the optimal horizon for an inflation target.

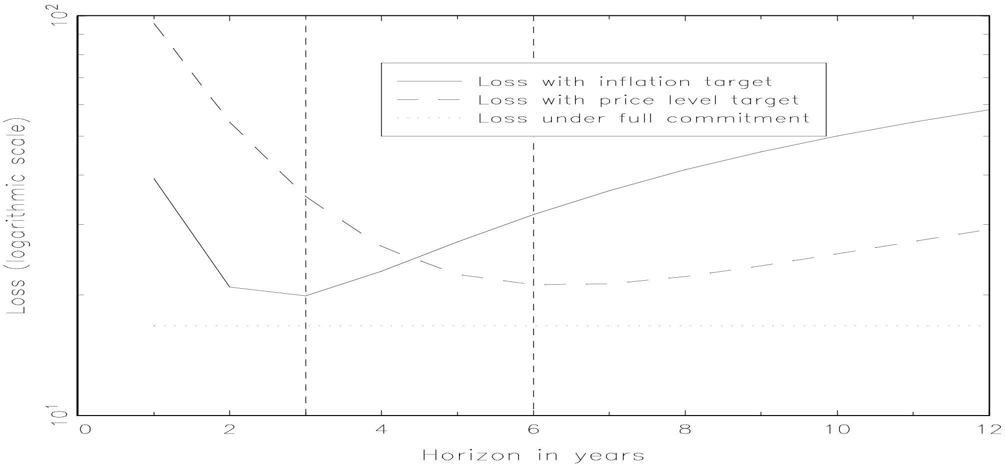
Graph 1
Impulse responses under commitment
An estimated annual forward-looking model for the euro area



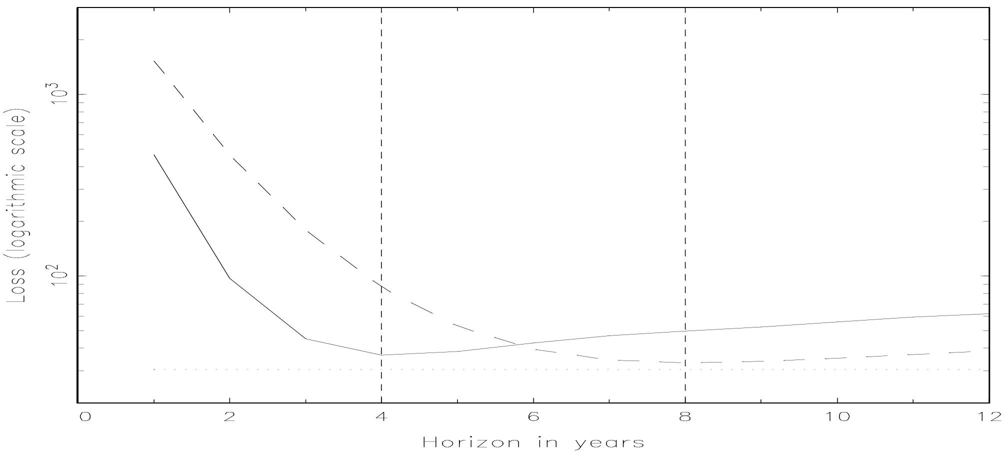
Graph 2

The optimal policy horizon (benchmark cases)

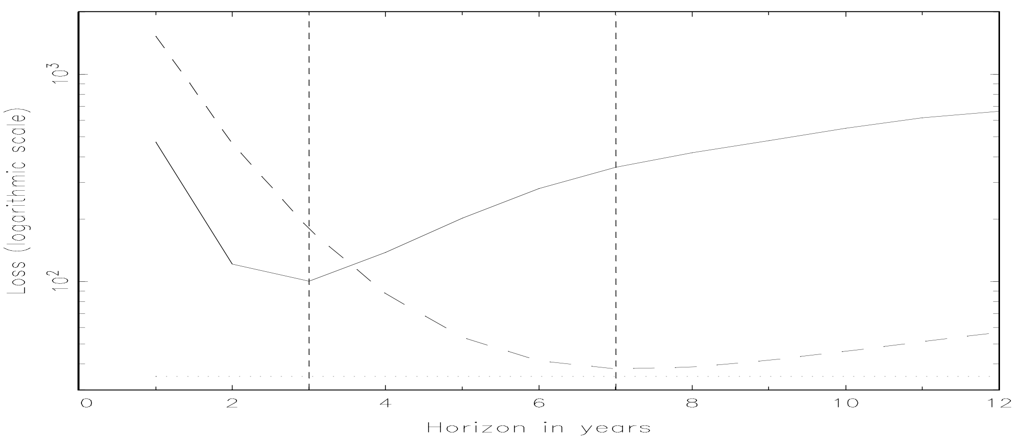
(ω_1=0.5; ω_2=0.0; ω_3=1.0)



(ω_1=0.5; ω_2=0.0; ω_3=0.9)



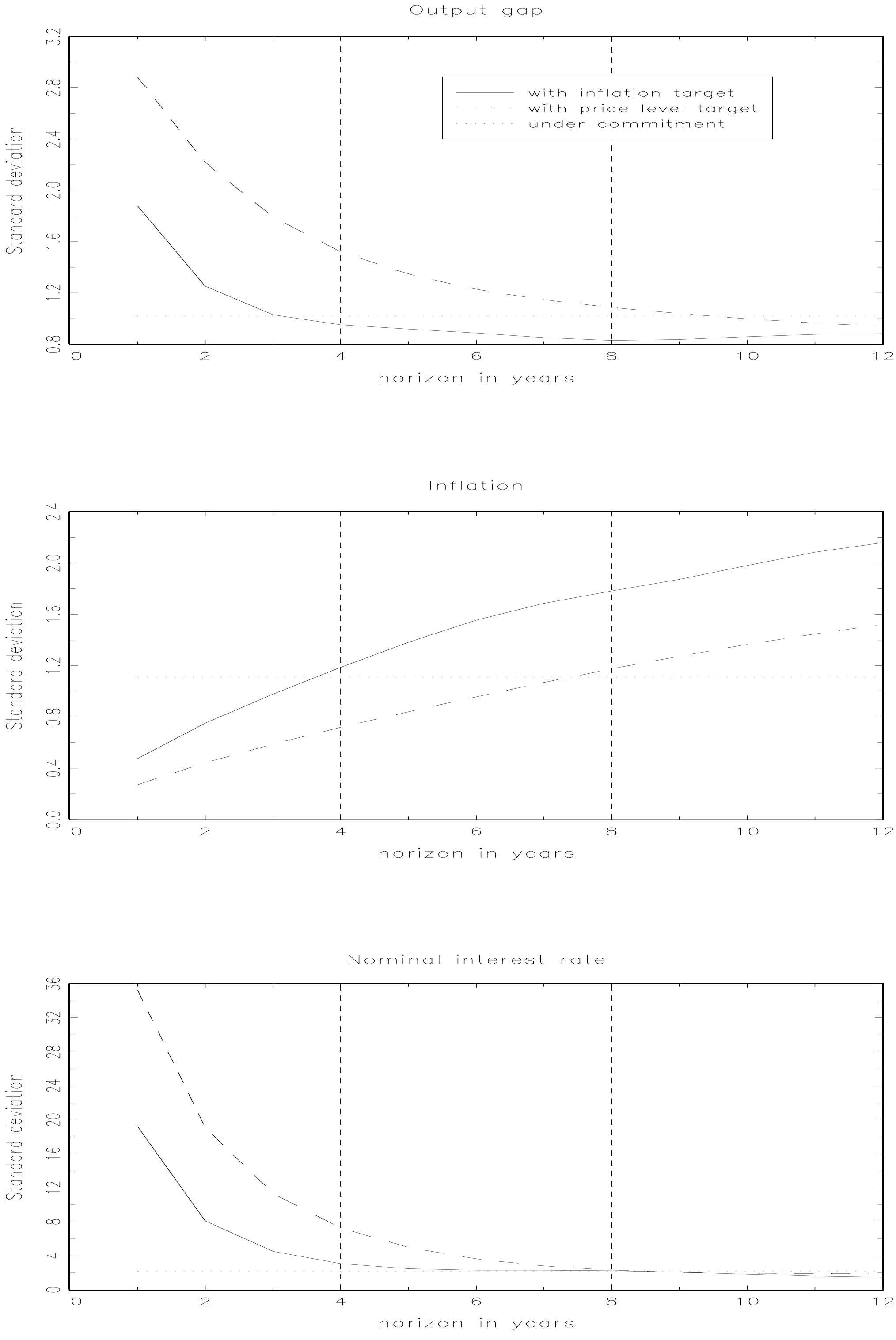
(ω_1=0.5; ω_2=0.1; ω_3=0.9)



Graph 3

The variability of the goal variables as a function of the horizon

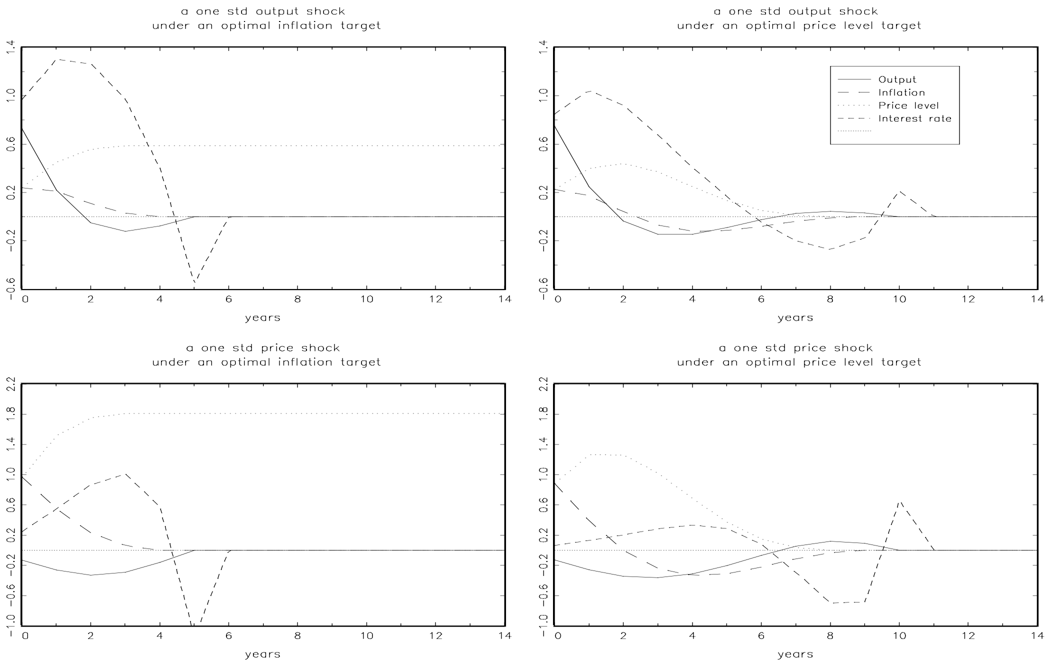
(ω_1=0.5; ω_2=0.0; ω_3=0.9)



Graph 4

Impulse responses with an optimal policy horizon

(ω_1=0.5; ω_2=0.0; ω_3=0.9)

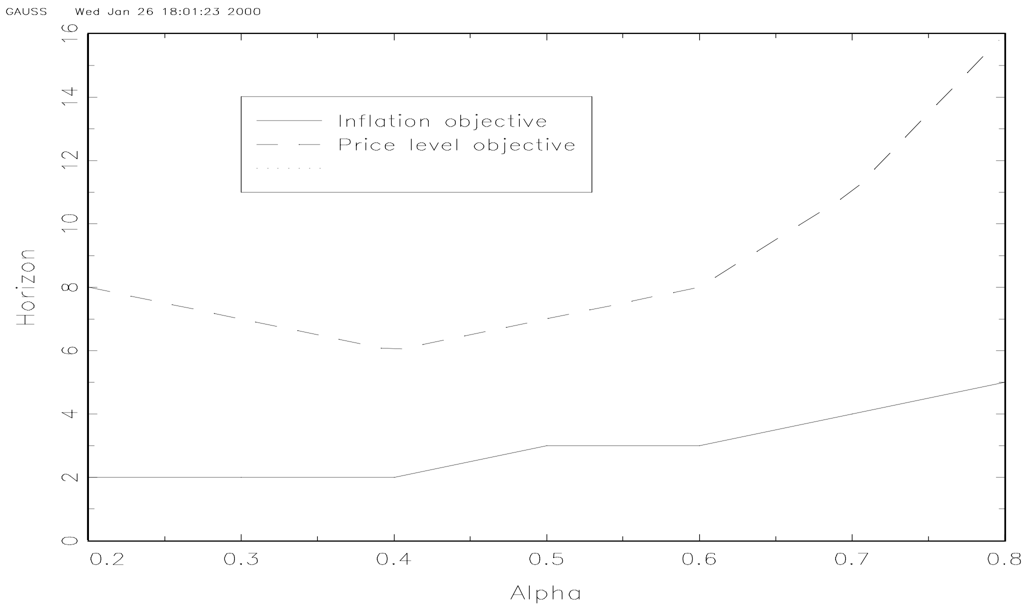


Graph 5

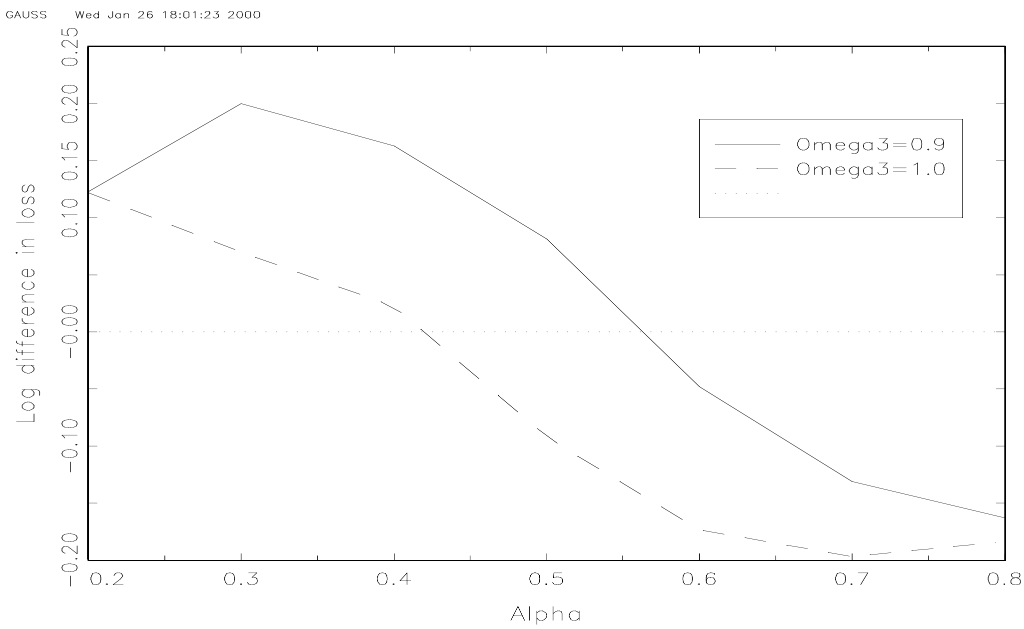
Sensitivity with respect to α

The optimal horizon

(ω_1=0.5; ω_2=0.0; ω_3=0.9)



Log difference in loss under an inflation and price level objective

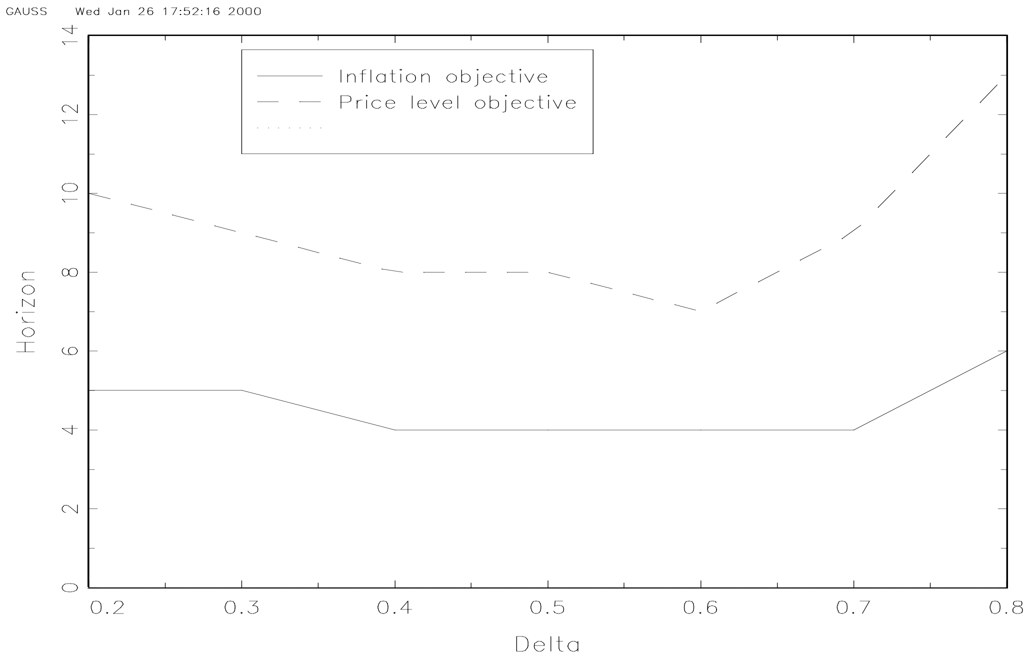


Graph 6

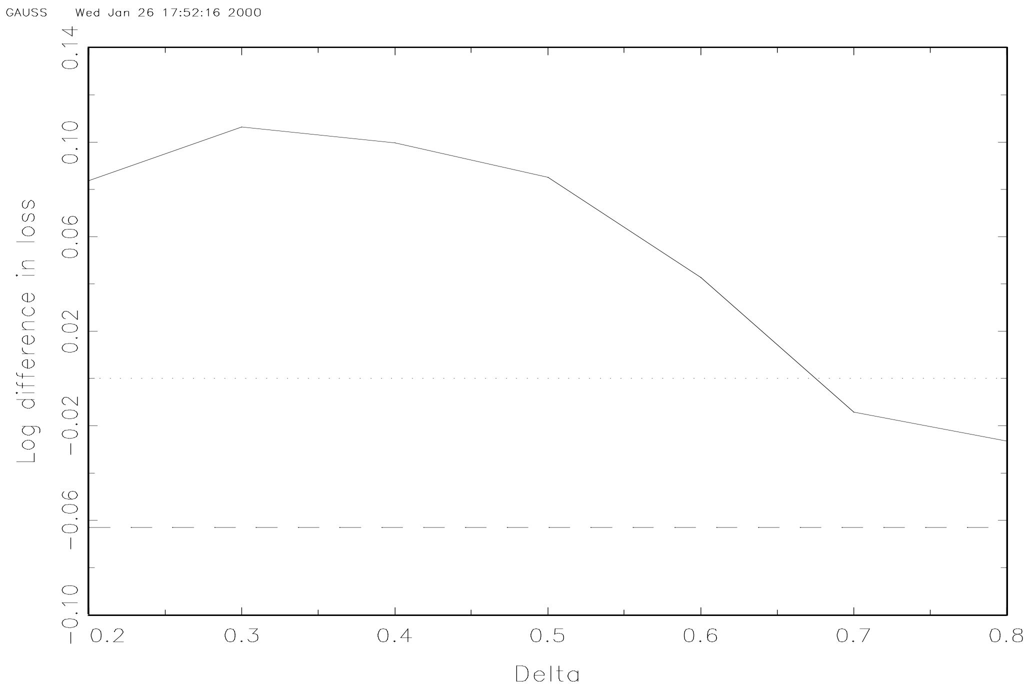
Sensitivity with respect to δ

The optimal horizon

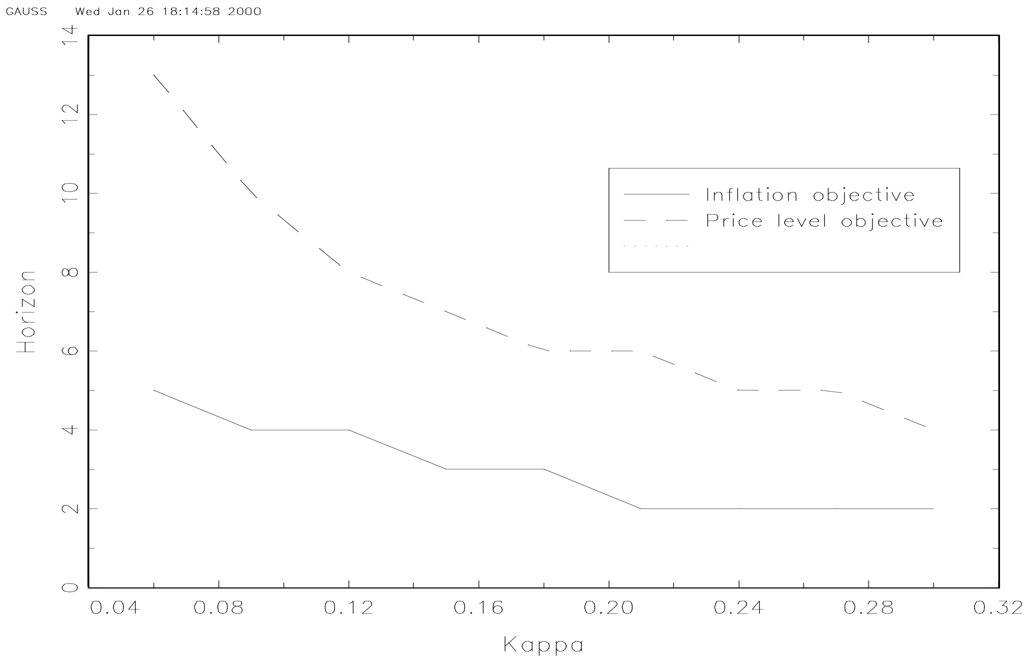
(ω_1=0.5; ω_2=0.0; ω_3=0.9)



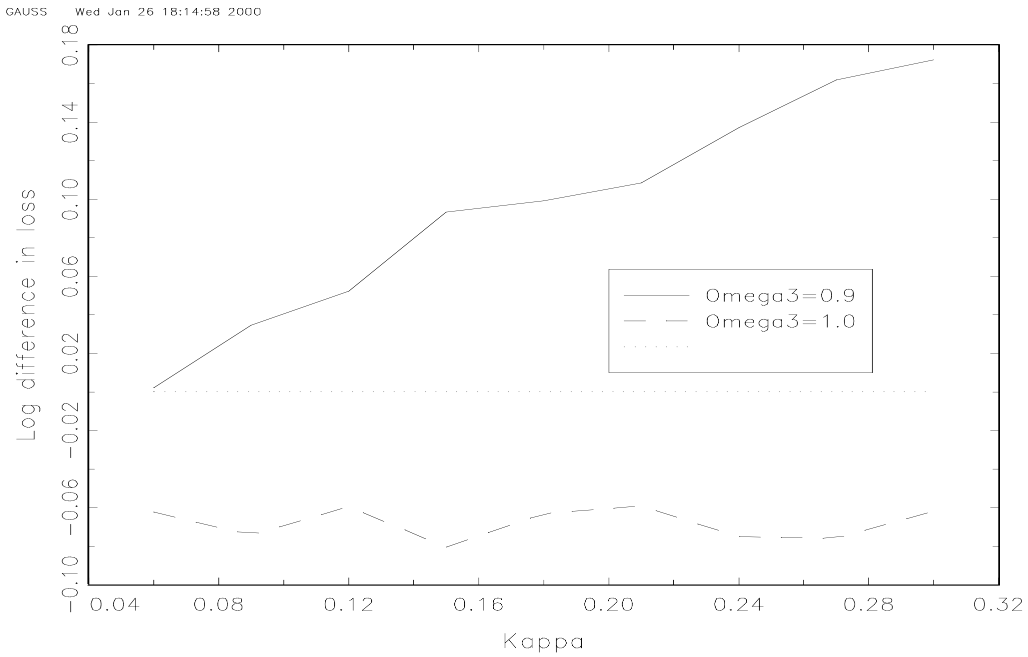
Log difference in loss under an inflation and price level objective



Graph 7
Sensitivity with respect to κ
The optimal horizon
(ω_1=0.5; ω_2=0.0; ω_3=0.9)



Log difference in loss under an inflation and price level objective



Appendix: Optimal Monetary Policy with a Forward Looking Inflation Constraint

In this appendix I show how to rewrite the minimisation problem with a forward-looking inflation constraint as a standard recursive problem following the suggestion made by Marimon & Marcet (1999).

The original problem is given by:

$$\underset{r_t}{Min}\, E_0 \sum_{t=0}^{\infty} \beta^t (\omega_3 y_t^2 + (1-\omega_3) r_t^2) \qquad (1)$$

subject to: $\begin{cases} x_{t+1} = Ax_t + Br_t + \varepsilon_{t+1} \\ E_t \pi_{t+H} = 0 \end{cases} \forall t \geq 0$

where H is the horizon.

The vector x_t consists of n_1 predetermined variables $(x_{1,t})$ and n_2 forward-looking variables $(x_{2,t})$

The matrices A and B can be partitioned according to $x_{1,t}$ and $x_{2,t}$ as follows:

$$A = \begin{bmatrix} A_{11} A_{12} \\ A_{21} A_{22} \end{bmatrix},\ B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

The forward-looking inflation constraint can be incorporated in the loss function using the Lagrange multiplier γ_t as follows:

$$\underset{r_t, \gamma_t}{Min}\, E_o \sum_{t=0}^{\infty} \beta^t (\omega_3 y_t^2 + (1-\omega_3) r_t^2 + \gamma_t \pi_{t+H}) \qquad (2)$$

subject to: $\begin{matrix} x_{t+1} = Ax_t + Br_t + \varepsilon_{t+1} \\ \gamma_t \geq 0 \end{matrix}$

This can rewritten in terms of time t and t+1 variables by using the following definition:

$$\mu_t^H = \beta^{-H} \gamma_{t-H}$$

In state space form this gives:

$$\underset{r_t, \gamma_t}{Min}\, E_0 \sum_{t=0}^{\infty} \beta^t (\omega_3 y_t^2 + (1-\omega_3) r_t^2 + \mu_t^H \pi_t) \qquad (3)$$

subject to $\begin{cases} x_{t+1} = Ax_t + Br_t + \varepsilon_{t+1} \\ \mu_{t+1} = C\mu_t + D\gamma_t \end{cases}$,

$$\text{where } \mu_{t+1} = \begin{bmatrix} \mu^1_{t+1} \\ . \\ . \\ . \\ . \\ . \\ \mu^H_{t+1} \end{bmatrix}; C = \begin{bmatrix} 0 & . & . & . & . & . & 0 \\ 1/\beta & 0 & . & . & . & . & 0 \\ 0 & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & 0 & 0 \\ 0 & . & . & . & 0 & 1/\beta & 0 \end{bmatrix}; D = \begin{bmatrix} 1/\beta \\ 0 \\ . \\ . \\ . \\ . \\ 0 \end{bmatrix}$$

Hx1 vector HxH matrix Hx1 vector

In order to use the standard solution programs as, e.g., discussed in Söderlind (1999), we redefine matrices in the following way.

$$\bar{x}_t = \begin{bmatrix} x_{1t} \\ \mu_t \\ x_{2t} \end{bmatrix} \begin{matrix} \} \; n_1 + H \\ \\ \} \; n_2 \end{matrix}$$

So, the new system has $n_1 + H$ predetermined variables and n_2 forward-looking variables.

The corresponding $\bar{A}, \bar{B}$ and $\bar{u}$ matrices are:

$$\bar{A} = \begin{bmatrix} A_{11} & 0 & A_{12} \\ 0 & C & 0 \\ A_{21} & 0 & A_{22} \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B_1 & 0 \\ 0 & D \\ B_2 & 0 \end{bmatrix} \quad \bar{u}_t = \begin{bmatrix} r_t \\ \gamma_t \end{bmatrix} \quad \bar{\varepsilon}_t = \begin{bmatrix} \varepsilon_{1,t} \\ 0 \\ \varepsilon_{2,t} \end{bmatrix}$$

The system (3) can now be rewritten as:

$$\underset{\bar{U}_t}{Min}\, E_0 \sum_{t=o}^{\infty} \beta^t (\omega_3 y_t^2 + (1-\omega_3) r_t^2 + \mu_t^H \pi_t)$$

subject to: $\bar{x}_{t+1} = \bar{A}\bar{x}_t + \bar{B}\bar{u}_t + \bar{\varepsilon}_{t+1}$

With a suitable definition of the Q and R matrices, this can be turned in the standard linear-quadratic problem with forward-looking variables:

$$\underset{\bar{u}_t}{Min}\, E_0 \sum_{t=0}^{\infty} \beta^t (\bar{x}_t^{`} Q \bar{x}_t + \bar{u}_t^{`} R \bar{u}_t)$$

subject to: $\bar{x}_{t+1} = \bar{A}\bar{x}_t + \bar{B}\bar{u}_t + \bar{\varepsilon}_{t+1}$

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