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ARE MONEY AND CONSUMPTION ADDITIVELY SEPARABLE IN THE EURO AREA?

A NON-PARAMETRIC APPROACH

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In 2006 all ECB publications will feature a motif taken from the €5 banknote.

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Abstract

We propose a numerical test of the non-parametric conditions for additive separability between consumption and real money balances, building on Varian (1983). If additive separability is rejected, then real balances enter into the theoretical IS curve. We test whether or not monetary assets and consumption are additively separable for the euro area using quarterly data from 1991 to 2005. Previous results using a parametric approach suggest that real balances can be excluded from the IS curve. We find that additive separability is violated over this sample period. After 1992, however, violations involve only a few observations and are in some instances related to measurement problems in the data. Overall, our results tend to support the claim that perfect non-separability between consumption and real balances is implausible, but that non-separabilities may not be very important empirically. At the same time, we reject additive separability throughout if we extend the sample period back to the 1980s, a period characterised by higher volatility in inflation and money growth.

Keywords: Non-parametric testing, Revealed Preference, Additive Separability, Money, IS Curve

JEL codes: C14, C63, E41

Non-technical summary

Recent research has emphasized the empirical success of using monetary aggregates to predict output fluctuations in several economies, even in econometric models that include real interest rates. The inclusion of money in an empirical specification of the IS curve, however, has to be clearly distinguished from money entering directly in a theoretical specification derived from an intertemporal optimization-based model. In a money-in-the-utility function model, where the household's lifetime utility function is strongly time separable, real balances can be excluded from the IS curve if the instantaneous utility function has a particular separable form.

Ireland (2004) explicitly tests for non-separability in an estimated stochastic dynamic general equilibrium model of the US economy. He shows that non-separability is equivalent in his model to a particular coefficient being different from zero, where the coefficient can be interpreted as measuring the degree of non-separability near the steady state. His empirical results indicate that this coefficient is statistically insignificant for the US. Using essentially the same method, Andres, Lopez-Salido and Valles (2001) find similar results for the euro area.

In this paper, we revisit this issue by using a non-parametric revealed preference approach to test for additive separability between money and consumption. Our approach fits within the non-parametric approach to demand analysis developed by Varian (1982, 1983) and others. Non-parametric methods have been widely used to test the conditions for weak separability, but the conditions for additive separability have not been tested previously. Therefore, our test is a methodological innovation compared with the previous literature. The main contribution of our paper is to develop and implement a numerical test for additive separability based upon Varian's conditions.

If additive separability is rejected, then real balances will enter into the IS curve, since no monotonic transformation of the instantaneous utility function can render it into the sum of two utility functions separating consumption and real balances. If the condition is not rejected, however, we cannot necessarily conclude that real balances can be excluded from the IS curve. In other words, the condition we test is necessary, but not sufficient, for excluding real balances from the IS curve.

The main advantage of using non-parametric revealed preference methods to test separability hypotheses relative to parametric methods is that they do not require any assumptions about the functional form of the utility function nor employ any approximations. On the other hand, the main disadvantage of such methods is that they are deterministic. Specifically, in contrast to the approach of Ireland (2004), we cannot incorporate LM curve shocks into our analysis. Thus, we do not view results from our test as being superior to existing results in the literature, but rather as complementing them with results obtained using an alternative methodology, which has different strengths and weaknesses.

We use the test to determine if the monetary assets in the M3 monetary aggregate and private consumption are additively separable for the euro area using quarterly data from 1991 to 2005. Unlike some previous studies, we use data for both the disaggregate components of the broad money measures as well as for monetary aggregates. We also account explicitly for interest paid on monetary assets.

Our main results are as follows:

- We reject additive separability of money and consumption using euro area data for the period 1991Q1 to 2005Q1, but the violations of separability are mainly attributable to observations in 1991 and 1992.
- While additive separability is still violated for the shorter period 1993Q1 to 2005Q1, the violations for this sub-sample involve just three pairs of quarterly observations one of which involves the launch of the euro banknotes and coins.
- Thus, strictly speaking, our test rejects additive separability for the euro area indicating that real balances enter the IS curve. Looking at our findings in terms of economic significance, however, we interpret the test results as meaning that additive separability is respected most of the time over the period covering the 1990s and beyond. Specifically, after 1992, violations of additive separability are attributable to a small number of observations including the currency changeover.
- For robustness, we also ran our test using data starting from the 1980s. Unlike the period after 1992, we find systematic violations of the hypothesis of additive separability especially in the first part of the 1980s, which corresponds to a period of relatively high inflation.
- Although highly speculative, one might conjecture that (exact) additive separability could be regime-dependent; i.e. it may prevail in a credible low-inflation regime, but not in a regime of high and volatile inflation. If true, this would reinforce the intuition that monetary variables contain more information and are, consequently, more interesting when there are more pronounced changes in inflation and money growth than in periods when such changes are subdued (as argued for example by Estrella and Mishkin, 1997).

1 Introduction

Recent research has emphasized the empirical success of using monetary aggregates to predict output fluctuations in several economies, even in econometric models that include real interest rates.[1] The inclusion of money in an empirical specification of the IS curve, however, has to be clearly distinguished from money entering directly in a theoretical specification derived from an intertemporal optimization-based model. In a money-in-the-utility function model, where the household's lifetime utility function is strongly time separable, real balances can be excluded from the IS curve if the instantaneous utility function has a particular separable form. Specifically, if the instantaneous utility function is the sum of two different utility functions, one containing only consumption and the other containing only money, then the Euler equation describing consumption will exclude current and future real money balances. In contrast, real balances will enter into the IS curve if the instantaneous utility function is non-separable across money and consumption (see McCallum and Nelson, 1999, McCallum, 2001, and Ireland, 2004, for further discussion).[2] In this case, the marginal rate of substitution between current and future consumption will depend on current and future real balances and money will play a role in explaining aggregate demand.

McCallum (2001, pp. 148-150) argues that perfect non-separability between real money balances and consumption seems "implausible", but calibration analysis indicates that "...although it is theoretically incorrect to specify a model without money, the magnitude of the error thereby introduced is extremely small" (see also McCallum, 2000 and Woodford, 2003).[3] Ireland (2004) explicitly tests for non-separability in an estimated stochastic dynamic general equilibrium model of the US economy. He shows that non-separability is equivalent in his model to a particular coefficient being different from zero, where the coefficient can be interpreted as measuring the degree of non-separability near the steady state. His empirical results indicate that this coefficient is statistically insignificant for the US. Using essentially the same method,

[1] See, for examples, Nelson (2002) and Leeper and Roush (2003) for the US; Brand, Reimers, and Seitz (2003) and Stracca (2004) for the euro area; and Nelson (2002) and Elger, Jones, Edgerton, and Binner (2006) for the UK. See also Nelson (2003).

[2] Similar results can be otained in money-in-the-utility function and shopping time models.

[3] The calibration analysis is based, however, on a simple (though fairly standard) linear, double log, money demand function with constant interest elasticity and unit consumption elasticity. See Fisher and Fleissig (1997) for more complex empirical specifications of money demand.

Andres, Lopez-Salido and Valles (2001) find similar results for the euro area, but Kremer, Lombardo, and Werner (2003) find contrary results for Germany. These papers all employ relatively broad monetary aggregates to proxy real balances (either $M2$ or $M3$).

In this paper, we revisit this issue by using a non-parametric revealed preference approach to test for additive separability between money and consumption. Our approach fits within the non-parametric approach to demand analysis developed by Varian (1982, 1983) and others. Specifically, Varian (1983) derives necessary and sufficient conditions for both weak separability and additive separability. Non-parametric methods have been widely used to test the conditions for weak separability, but the conditions for additive separability have not been tested previously.[4] Therefore, our test is a methodological innovation compared with the previous literature.

The instantaneous utility function is said to be additively separable between consumption and monetary assets if some monotonic transformation of it is the sum of two utility functions, one containing only consumption and the other containing only money (see Varian, 1983). Additive separability is a more restrictive condition than weak separability of the set of monetary assets from consumption.[5] If additive separability is rejected, then real balances will enter into the IS curve, since no monotonic transformation of the instantaneous utility function can render it into the sum of two utility functions separating consumption and real balances. If the condition is not rejected, however, we cannot necessarily conclude that real balances can be excluded from the IS curve. In other words, the condition we test is necessary, but not sufficient, for excluding real balances from the IS curve.

[4]In particular, a number of studies use non-parametric methods to test sets of monetary assets for weak separability, which is usually interpreted as an admissibility condition for aggregation. See, for examples, Patterson (1991), Swofford and Whitney (1994), Drake and Chrystal (1997), Spencer (1997), Fisher and Fleissig (1997), Swofford (2000), Jones, Dutkowsky, and Elger (2005), and Elger, Jones, Edgerton, and Binner (2006).

[5]Since the property is defined in terms of a monotonic transformation, it would be more correct to say that preferences are additively separable. In general terms, a group of goods is weakly separable from all other goods if there exists a sub-utility function, which separates the weakly separable group from all other goods in the utility function. Weak separability is necessary and sufficient for the second stage of a two-stage budgeting problem, meaning that the quantities purchased within the weakly separable group can always be written as a function of group expenditure and group prices; see Deaton and Muellbauer (1980). In contrast, preferences are additively separable if the set of all goods can be partitioned into n non-overlapping groups, such that a monotonic transform of the utility function equals the sum of n utility functions each of which is defined over one of the non-overlapping groups. Additive separability implies blockwise weak separability: *i.e.* each of the non-overlapping groups is weakly separable from all other goods.

The main contribution of our paper is to develop and implement a numerical test for additive separability based upon Varian's conditions.[6] We also develop a technique for identifying influential observations in cases where additive separability is violated. We use the test to determine if the monetary assets in the $M3$ monetary aggregate and private consumption are additively separable for the euro area using quarterly data from 1991 to 2005. Unlike some previous studies, we use data for both the disaggregate components of the broad money measures as well as for monetary aggregates. We also account explicitly for interest paid on monetary assets.

The main advantage of using non-parametric revealed preference methods to test separability hypotheses relative to parametric methods is that they do not require any assumptions about the functional form of the utility function nor employ any approximations. On the other hand, the main disadvantage of such methods is that they are deterministic (see Varian, 1985 and Swofford and Whitney, 1994).[7] Specifically, in contrast to the approach of Ireland (2004), we cannot incorporate LM curve shocks into our analysis. Thus, we do not view results from our test as being superior to existing results in the literature, but rather as complementing them with results obtained using an alternative methodology, which has different strengths and weaknesses.

We reject additive separability of money and consumption using euro area data for the period 1991Q1 to 2005Q1, but the violations of separability are mainly attributable to observations in 1991 and 1992.[8] While additive separability is still violated for the shorter period 1993Q1 to 2005Q1, the violations for this sub-sample involve just three pairs of quarterly observations one of which involves the launch of the euro banknotes and coins. Thus, strictly speaking, our test rejects additive separability for the euro area indicating that real balances enter the IS curve. Looking at our findings in terms of economic significance, however, we interpret the test results as meaning that additive separability

[6] The new test is similar in some respects to numerical procedures that test for weak separability, such as Swofford and Whitney (1994) and Fleissig and Whitney (2003).

[7] There are several approaches that have been developed to determine whether or not violations of revealed preference axioms, such as GARP and WACM, are statistically significant. See, for examples, Varian (1985), Epstein and Yatchew (1985), Gross (1995), Fleissig and Whitney (2005), de Peretti (2005), Jones and de Peretti (2005). See also Varian (1990).

[8] As a robustness analysis, we also examine an extended sample period going back to the 1980's, partly using data from the ECB's Area Wide Model. We find systematic violations of the hypothesis of additive separability especially in the first part of the 1980s, which is a period of relatively high inflation and money growth variability.

is respected most of the time over the period covering the 1990s and beyond. Specifically, after 1992, violations of additive separability are attributable to a small number of observations including the currency changeover. These results are particularly striking when we consider that our non-parametric test is entirely deterministic and that additive separability is a very restrictive assumption to impose on the data (relative to, for example, weak separability). Thus, our results could be seen as largely consistent with the claim that non-separability between consumption and real balances is implausible, but that non-separabilities may not be very important empirically and not systematic (see McCallum, 2001, for related discussion). Nevertheless, the fact that the failure of separability after 1992 is caused by only a few observations may not necessarily be seen as reassuring, especially since we have not yet developed a criterion to assess the importance/significance of violations of additive separability detected by our new non-parametric test.

The remainder of the paper is organized as follows: In Section 2, we present a model of expected lifetime utility maximization, which allows for payment of interest on money. In Section 3, we describe our test for additive separability. In Section 4, we discuss the advantages and disadvantages of our non-parametric test relative to other tests that have been used in the literature. In Section 5, we provide test results for the euro area. Section 6 concludes.

2 A Model with Interest-Bearing Money

In this section, we present a standard intertemporal model with money in the utility function adapted from Ireland (2004). Initially, for illustrative purposes, we consider a model with a single monetary variable. Then we consider an extension of the model that includes multiple monetary assets. Our test results are based on the latter.

2.1 A Single Monetary Variable

In this model, at each time period t, a representative household is assumed to maximize the expected value of a strongly time separable lifetime utility function defined as follows:

$$E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} \left(u(c_{t+\tau}, m_{t+\tau}) - \eta h_{t+\tau} \right) \right] \qquad (1)$$

where E_t denotes conditional expectations, β is a discount factor, c_t is real consumption, $m_t = M_t/P_t$ is real balances (M is nominal money balances and P is the price level), and h_t is labour supply ($\eta > 0$). The household faces a sequence of inter-temporal budget constraints of the following form:

$$M_{s-1}(1+R^M_{s-1})+B_{s-1}(1+R_{s-1})+W_s h_s+D_s=P_s c_s+B_s+M_s \quad (2)$$

for all $s=t,t+1,...,\infty$, where B represents nominal holdings of a non-monetary "benchmark" asset (which is only held in order to inter-temporally transfer wealth), D represents income from other sources (dividends in Ireland's model), W is the nominal wage, R^M is the nominal interest rate (possibly zero) on money holdings, and R is the nominal interest rate on the benchmark asset. The own rate of interest on money accounts for the fact that broader monetary aggregates include interest-bearing monetary assets. In real terms, the inter-temporal budget constraints are as follows:

$$m_{s-1}\frac{1+R^M_{s-1}}{1+\pi_s}+b_{s-1}\frac{1+R_{s-1}}{1+\pi_s}+w_s h_s+d_s=c_s+b_s+m_s \qquad (3)$$

for all s, where m is real balances, b is real holdings of the benchmark asset, d is real income from other sources, w is the real wage, and $\pi_s=(P_s-P_{s-1})/P_{s-1}$ is inflation. In time period t, the household knows w_t, R^M_t, R_t, and π_t with perfect certainty, but not the future values of these variables. Under these assumptions, the optimality conditions for the period t variables are determined from the following:

$$\eta=u_c(c_t,m_t)w_t \qquad (4)$$

$$\frac{u_m(c_t,m_t)}{u_c(c_t,m_t)}=\frac{R_t-R^M_t}{1+R_t} \qquad (5)$$

$$u_c(c_t,m_t)=\beta(1+R_t)E_t\left[\frac{u_c(c_{t+1},m_{t+1})}{1+\pi_{t+1}}\right] \qquad (6)$$

where u_m and u_c denote partial derivatives.

Equation (5) could be interpreted as either a money demand curve in implicit form or as an LM relation, where the right hand side is the opportunity cost of holding money relative to consumption in the current period.[9] Equation (6) is the standard Euler equation for consumption, which could be interpreted as a non-linear IS relation that contains current and future real balances. For related analysis and further discussion, see McCallum and Nelson (1999), McCallum (2001), and Ireland (2004).

If the instantaneous utility function, u, is the sum of two utility functions, one containing only money and the other containing only

[9] Dutkowsky and Atesoglu (2001) consider dynamic microfoundations for the conventional static double-log money demand function.

consumption, *i.e.* $u(c, m) = U(c) + V(m)$, then the IS relation excludes current and future real balances and takes the form: $U'(c_t) = \beta E\left[(1 + r_t)U'(c_{t+1})\right]$, where $(1 + r_t) \equiv (1 + R_t)/(1 + \pi_{t+1})$. Preference shocks could also be incorporated by modifying instantaneous utility in (1) to be $a_t u(c_t, m_t/e_t)$, where a_t and e_t are stochastic shocks. An a_t shock is usually interpreted as an IS curve shock, since it would alter (6) but not (5), and an e_t shock is usually interpreted as an LM curve shock.

2.2 Multiple Monetary Assets

Next, we modify the instantaneous utility function in (1) to include multiple monetary assets. Barnett (1978, 1980, 1982) used a similar model to derive user cost prices of interest-bearing monetary assets and provided the relevant economic theory underlying monetary aggregation. In particular, Barnett (1980, 1982) showed that aggregation of a set of monetary assets requires that the assets be weakly separable from all other variables in the representative household's utility function meaning that these assets are separated from all other decision variables by a sub-utility function. Further, the representative household will only view the corresponding price and quantity aggregates as being the price and quantity of a single elementary good if the sub-utility function is linearly homogeneous. Thus, the use of monetary aggregates in an economic model is equivalent to assuming homothetic weak separability. Since our primary goal in this paper is to test for additive separability between money and consumption, it is preferable to work with a set of monetary assets rather than employ a monetary aggregate, which itself requires separability assumptions.

Let $u(c, \mathbf{m})$ be a function of consumption and a vector of ℓ monetary assets $\mathbf{m} = (m_1, ..., m_\ell)$. Let R_{it} be the nominal interest rate on the i^{th} monetary asset. The modified optimality conditions for the period t variables are obtained by replacing m_t with $\mathbf{m}_t$ in (4) and (6) and replacing (5) with a set of corresponding conditions for each asset:

$$\eta = u_c(c_t, \mathbf{m}_t)w_t \tag{7}$$

$$\frac{u_{m_i}(c_t, \mathbf{m}_t)}{u_c(c_t, \mathbf{m}_t)} = \frac{R_t - R_{it}}{1 + R_t} \text{ for all } i = 1, ..., \ell \tag{8}$$

$$u_c(c_t, \mathbf{m}_t) = \beta(1 + R_t)E_t\left[\frac{u_c(c_{t+1}, \mathbf{m}_{t+1})}{1 + \pi_{t+1}}\right] \tag{9}$$

where u_{m_i} denotes the partial derivative of u with respect to the i^{th} monetary asset. In this case, if u is the sum of two utility functions, one containing only monetary assets and the other containing only consumption, *i.e.* $u(c, \mathbf{m}) = U(c) + V(\mathbf{m})$, then the IS relation will exclude current and future monetary assets. In general, there is not a single LM relation.

Although (8) are optimality conditions from a forward-looking household's expected lifetime utility maximization problem, they are also first-order necessary conditions from a static utility maximization problem of the following form:

$$\underset{c,\mathbf{m}}{MAX} \left\{ u(c, \mathbf{m}) : p_t c + \boldsymbol{\gamma}_t \mathbf{m} = Y_t \right\} \tag{10}$$

where $\boldsymbol{\gamma}_t = (\gamma_{1t}, ..., \gamma_{\ell t})$ is a vector of nominal *user costs* for the monetary assets with $\gamma_{it} = p_t(R_t - R_{it})/(1 + R_t)$ and Y_t is the optimal expenditure on the current period variables as determined from the household's lifetime utility maximization problem; see Barnett (1978, 1980). Moreover, when the household reoptimizes in subsequent time periods (in period $t + 1$ for example), the optimal solution will continue to obey updated versions of the optimality conditions including (8). Thus, in any future period, the optimal solutions for consumption and real balances in that same period will also be consistent with the solution to a static optimization of the form described by (10) with correspondingly updated prices and user costs, but with u time invariant.[10]

2.3 Testing the Utility Function for Separability

We now describe a non-parametric revealed preference method for testing a dataset for consistency with additive separability. Revealed preference methods can be used to determine if a dataset consisting of observations on prices and quantities of a set of goods is consistent with utility maximization as well as various separability assumptions. In general, such methods can be applied to time series data under the assumption of strong time separability of the lifetime utility function (as assumed here and in Ireland, 2004).

Our approach will be based on first determining, using tests of the Generalized Axiom of Revealed Preference (GARP), whether or not time series data on monetary assets and consumption are consistent with a utility maximization problem in the form of (10). If GARP is satisfied, we then apply a numerical test to determine if the data can be rationalized by an instantaneous utility function that is *additively separable*, as defined by Varian (1983), between real money balances and

[10] This result is often emphasized in the literature on monetary aggregation theory; See, for example, Barnett (1980). An analogous result also holds in the simpler model of the previous section.

consumption. Specifically, we test the assumption that some monotonic transformation, f, of the instantaneous utility function, u, is the sum of two different utility functions, U and V, which contain only consumption and real monetary assets respectively: *i.e.* $f(u(c, \mathbf{m})) = U(c) + V(\mathbf{m})$.

If additive separability is rejected, then it is clear that real money balances enter the IS curve, since no monotonic transformation of the instantaneous utility function can render it into the sum of two utility functions separating consumption and real balances. As described above, however, the ability to exclude real money balances from the IS curve requires the assumption that the instantaneous utility function is actually additive, meaning that $u(c, \mathbf{m}) = U(c) + V(\mathbf{m})$, not just additively separable, implying that $f(u(c, \mathbf{m})) = U(c) + V(\mathbf{m})$ for some monotonic function f. This distinction would make no difference with respect to the intra-temporal optimality conditions, however. In essence, therefore, we are testing a *necessary* (but, not sufficient) condition for the exclusion of real balances from the IS curve.

The addition of an IS curve shock, a_t (described above), would not affect the validity of (10), on which our empirical analysis will be based, although it does affect the Euler equation for consumption. Thus, our test is not affected by such shocks. Our test cannot be extended, however, to allow for shocks to the LM relation(s) described by (5) and (8), since these would affect the validity of (10).

3 Non-Parametric Test of Additive Separability

Let $(\mathbf{p}^i, \mathbf{c}^i)$ and $(\boldsymbol{\gamma}^i, \mathbf{m}^i)$ represent data on prices and quantities for two sets of goods, where $i = 1, ..., n$ denotes a particular observation. Following our empirical application, we will refer to these two sets as consumption goods and monetary assets respectively. The notation, $\mathbf{c}^i$, denotes an observed quantity vector for the consumption goods and $\mathbf{p}^i$ denotes the corresponding prices. Similarly, the notation, $\mathbf{m}^i$, denotes an observed quantity vector for the monetary assets and $\boldsymbol{\gamma}^i$ denotes the corresponding vector of user costs. In this section, we focus on results for a vector of consumption goods to be consistent with standard notation in the revealed preference literature (see, for example, Varian, 1983). Nevertheless, the results presented here remain valid if there is only a single consumption good, with the price and quantity data for that good (p^i and c^i respectively) replacing the vectors $\mathbf{p}^i$ and $\mathbf{c}^i$ in the main theorems. Similarly, the results would also remain valid if there is only a single monetary asset.

Varian (1982) proves that a data set is consistent with maximization of a utility function, $u(\mathbf{c}, \mathbf{m})$, given observed prices and user costs if and only if the data satisfy GARP. Thus, if the combined data $(\mathbf{p}^i, \boldsymbol{\gamma}^i; \mathbf{c}^i, \mathbf{m}^i)$

$i = 1, ..., n$ satisfies GARP, then the observed quantities and prices are consistent with a utility maximization problem in the form of (10).

3.1 Background

Let $\mathbf{c}$ and $\mathbf{m}$ denote arbitrary bundles of consumption goods and monetary assets. Following Varian (1983), we say that a utility function, $u(\mathbf{c}, \mathbf{m})$ is *additively separable* if there is some monotonic transformation f such that

$$f\left(u(\mathbf{c}, \mathbf{m})\right) = U(\mathbf{c}) + V(\mathbf{m}) \tag{11}$$

Varian (1982, 1983) developed non-parametric conditions for a given dataset to be consistent with utility maximization, weak separability, and additive separability. In regards to additive separability, Varian (1983) proved the following theorem:

Varian's Theorem for Additive Separability.[11]

The following two conditions are equivalent:

(i) *There exist two concave, monotonic, continuous utility functions, U and V, whose sum rationalizes the data $(\mathbf{p}^i, \mathbf{c}^i)$ and $(\boldsymbol{\gamma}^i, \mathbf{m}^i)$ $i = 1, ..., n$*

(ii) *There exist numbers U^i, V^i, $\lambda^i > 0$ such that;*

$$U^i \leq U^j + \lambda^j \mathbf{p}^j (\mathbf{c}^i - \mathbf{c}^j) \tag{12}$$

$$V^i \leq V^j + \lambda^j \boldsymbol{\gamma}^j (\mathbf{m}^i - \mathbf{m}^j) \tag{13}$$

for all $i, j = 1, ..., n$.

Condition (i) concerns the ability to rationalize the data $(\mathbf{p}^i, \mathbf{c}^i)$ and $(\boldsymbol{\gamma}^i, \mathbf{m}^i)$ $i = 1, ..., n$ with an additively separable utility function. The sum of U and V is said to *rationalize* the data if $U(\mathbf{c}^i) + V(\mathbf{m}^i) \geq U(\mathbf{c}) + V(\mathbf{m})$ for all $\mathbf{c}$ and $\mathbf{m}$ such that $\mathbf{p}^i\mathbf{c} + \boldsymbol{\gamma}^i\mathbf{m} \leq \mathbf{p}^i\mathbf{c}^i + \boldsymbol{\gamma}^i\mathbf{m}^i$ for all $i = 1, ..., n$: *i.e.* if the observed bundles provided at least as much utility as all other bundles that would have required the same or less expenditure to purchase.

Condition (ii) will be used to formulate our test for additive separability. The numbers U^i and V^i can be interpreted as measuring the utility derived from consumption goods and monetary assets respectively at each data point: *i.e.* U^i measures $U(\mathbf{c}^i)$ and V^i measures $V(\mathbf{m}^i)$ for all i. The numbers λ^i can be shown to measure the marginal utility of expenditure at each data point, or, equivalently, the Lagrange multiplier associated with the budget constraint at each data point. In Appendix 1, we provide a heuristic argument to motivate condition (ii) and the

[11] See Varian (1983), Theorem 6.

interpretation of the numbers described by it. As a convenience to the reader, in Appendix 2 we provide a brief discussion of weak separability and weak separability testing.

3.2 Test Procedure

Varian's theorems for weak separability and additive separability both involve sets of inequality constraints, but the weak separability conditions can also be stated in terms of GARP. Varian programmed a weak separability test based on the corresponding conditions in the widely used computer program NONPAR.[12] Varian (1983, p. 108) remarked, however, that he was "...unable to find a convenient combinatorial condition that is necessary and sufficient for additive separability."

In this paper, we formulate a new numerical test of Varian's conditions for additive separability, which is based on the solution to a linearly constrained optimization problem. The problem is to minimize

$$F = \sum_{i=1}^{n} \left(\frac{\lambda^i - \mu^i}{\mu^i} \right)^2$$

in the numbers U^i, V^i, λ^i, μ^i subject to the following two sets of linear inequality constraints:

$$U^i \leq U^j + \lambda^j \mathbf{p}^j (\mathbf{c}^i - \mathbf{c}^j) \tag{14}$$

$$V^i \leq V^j + \mu^j \boldsymbol{\gamma}^j (\mathbf{m}^i - \mathbf{m}^j) \tag{15}$$

for all $i, j = 1, ..., n$. The numbers U^i, V^i, λ^i, μ^i are all constrained to be strictly positive for all $i = 1, ..., n$. If F can be minimized to exactly zero, then $\lambda^i \equiv \mu^i$ for all i and (14) and (15) are equivalent to the necessary and sufficient non-parametric conditions for additive separability laid out in (ii) of Varian's theorem for additive separability.

The existence of a solution to (14) is equivalent to the condition that the data $(\mathbf{p}^i, \mathbf{c}^i)$ $i = 1, ..., n$ satisfy GARP.[13] Similarly, the existence of a solution to (15) is equivalent to the condition that the data $(\boldsymbol{\gamma}^i, \mathbf{m}^i)$ $i = 1, ..., n$ satisfy GARP. Thus, two necessary conditions for additive separability are that the $(\mathbf{p}^i, \mathbf{c}^i)$ and $(\boldsymbol{\gamma}^i, \mathbf{m}^i)$ data *separately* satisfy GARP. In addition, as discussed above, the combined data $(\mathbf{p}^i, \boldsymbol{\gamma}^i; \mathbf{c}^i, \mathbf{m}^i)$ $i = 1, ..., n$ for both sets of quantities and prices must also satisfy GARP. These results suggest the following three step test procedure:

[12] See Swofford and Whitney (1994), Fleissig and Whitney (2003), Jones, Dutkowsky, and Elger (2005), and Jones, Elger, Edgerton, and Dutkowsky (2005) for recent advances in weak separability testing.

[13] See Afriat's theorem in Varian (1983, p. 100).

Step 1 - Test the combined data $(\mathbf{p}^i, \boldsymbol{\gamma}^i; \mathbf{c}^i, \mathbf{m}^i)$ $i = 1, ..., n$ for GARP. If this condition is violated, then the data are inconsistent with maximization of a utility function (separable or otherwise).

Step 2 - Test the data $(\mathbf{p}^i, \mathbf{c}^i)$ and $(\boldsymbol{\gamma}^i, \mathbf{m}^i)$ $i = 1, ..., n$ separately for GARP. If either of these conditions are violated, then the data are inconsistent with maximization of an additively separable utility function.

Step 3 - Solve the linear inequality constrained minimization problem and determine if F can be minimized to zero. If it cannot be minimized to zero, then the data are inconsistent with maximization of an additively separable utility function. If the conditions in Steps 1 and 2 are satisfied and F can be minimized to zero, then the data are consistent with maximization of an additively separable utility function.

We can interpret the minimized value of the objective function, $\hat{F}$, if the necessary GARP conditions in Step 2 are satisfied. If the $(\mathbf{p}^i, \mathbf{c}^i)$ data satisfy GARP, then U^i and λ^i can be interpreted as measuring the utility provided by the consumption goods at each observation and the marginal utility of expenditure on them, respectively (see Varian, 1983, p. 101). Similarly, if the $(\boldsymbol{\gamma}^i, \mathbf{m}^i)$ data satisfy GARP, then V^i and μ^i can be interpreted as measuring the utility provided by the monetary assets at each observation and the marginal utility of expenditure on them. If the utility function is additively separable, then these measures of marginal utility of expenditure can be made the same for all data points. The minimized objective function, $\hat{F}$, measures the sum of squared proportional deviations between the two measures of marginal utility. Additive separability is rejected unless $\hat{F}$ is zero. We also show in our empirical analysis that, when additive separability is rejected, influential observations can be identified from non-zero values of the individual deviations, $(\lambda^i - \mu^i)/\mu^i$ for $i = 1, ..., n$, obtained from the minimization.[14]

This test procedure is similar to a weak separability test proposed by Swofford and Whitney (1994). In their test, an objective function is minimized subject to a set of linear and non-linear inequality constraints. A group of goods is weakly separable from all other goods if and only if their objective function can be minimized to zero. If a feasible solution to the constraints exists, but their objective function cannot be minimized to zero, then the test is interpreted as finding incomplete adjustment of expenditure on the weakly separable goods. Swofford and Whitney (1994) use the results from the minimization to measure the average

[14] This is similar to the idea of looking at bilateral GARP violations. Unlike GARP, however, the test does not directly identify pairs of observations that are causing violations of additive separability.

amount of incomplete adjustment, see also Jones, Elger, Edgerton, and Dutkowsky (2005) for further discussion.

3.3 Possible Extensions

A well-known limitation of the non-parametric approach to demand analysis is that it suffers from "inadequate statistical procedures and goodness-of-fit metrics" (Gross, 1995, p. 701). For example, Varian (1982) proved that a dataset can be rationalized by a well-behaved utility function if and only if it satisfies GARP. If a dataset violates GARP, it would be useful to have a metric to determine how severe the violations are and/or if those violations are statistically significant. The standard GARP test is, however, not a hypothesis test in the usual statistical econometric sense. Rather, GARP is better characterized as checking whether or not the observed data is consistent with utility maximization without resorting to any stochastic specification. We can reinterpret the GARP test in statistical terms by assuming that the observed quantity data are equal to true quantity data plus random errors (*e.g.* measurement errors). The corresponding null hypothesis would be that the true data satisfies GARP; see Varian (1985) for further discussion. There are several approaches to extending conventional non-parametric methods in this direction.

Varian (1985) proposes a statistical test based on computing minimal adjustments to the observed data required to eliminate violations of a given revealed preference axiom. Varian's approach has several limitations. First, in terms of testing GARP, the adjustments are based on solving a mathematical programming problem with a large number of non-linear inequality constraints (the number of these constraints is a function of the square of the sample size). Consequently, the method is impractical for large datasets and can be time consuming to use (see Jones and de Peretti, 2005 for further discussion). Second, the method also requires the tester to specify the variance of the random errors. de Peretti (2005) proposes an alternative method, which is also based on adjusting the observed data; see Jones and de Peretti (2005) for a comparison of these two approaches.

Gross (1995) proposes an alternative approach based on partitioning the dataset into two subsets referred to as CS and VS. The violator subset (VS) contains the observations that are causing most of the violations, while the consistent subset (CS) is its complement. A test statistic can be formulated by estimating the fraction of expenditure that is wasted by the VS observations in maximizing utility consistent with CS. Gross recommends using bootstrap methods to estimate the distribution of the test statistic.

Fleissig and Whitney (2005) propose two methods for assessing the statistical significance of GARP violations. Their methods are based on adding measurement errors to the observed data to create a large number of perturbed datasets. Suppose that a dataset violates GARP in an empirical application. In their *least lower bound test*, measurement errors are added to the quantity data to produce a perturbed dataset, which is then tested for GARP. This step is repeated a large number of times to produce a test statistic. The test statistic is the fraction of perturbed datasets that do not violate GARP. If more than $\alpha\%$ of the perturbed datasets have no GARP violations, then the null hypothesis of utility maximization cannot be rejected (see Fleissig and Whitney 2005: 359).

These methods are all directly applicable to evaluating the significance of GARP violations.[15] An important contribution to this literature would be to extend these methods to allow us to evaluate the significance of violations obtained from non-parametric separability tests (either from standard weak separability tests or from our new additive separability test). de Peretti (2006), for example, extends his earlier work to consider the significance of violations of weak separability.[16] In addition, Jones, Dutkowsky, and Elger (2005) and Jones, Elger, Edgerton, and Dutkowsky (2005) suggest and implement a unified approach to testing weak separability, which synthesizes a joint test of the necessary and sufficient conditions based on Swofford and Whitney (1994) with the measurement error approach proposed by Varian (1985). Future research along these lines is clearly warranted, but we do not pursue the issue further in this paper.

4 Comparisons with Alternative Approaches

In this section, we compare and contrast our non-parametric test with an alternative parametric approach used by Ireland (2004). In Ireland's model, households are assumed to maximize a strongly time separable lifetime utility function:

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(a_t u(c_t,\frac{m_t}{e_t})-\eta h_t\right)\right] \tag{16}$$

where a_t and e_t are preference shocks (representing shocks to the IS curve and LM curve respectively). The rest of the model is a relatively standard New Keynesian general equilibrium model. The first-order ap-

[15] Varian (1985) also described how to evaluate the significance of violations of the weak axiom of cost minimization (WACM).

[16] See also Swofford and Whitney (1994, pp. 246-248).

proximation around the steady state equilibrium leads to the following linear IS equation:

$$\widehat{y}_t - E_t\widehat{y}_{t+1} + \omega_1(\widehat{r}_t - E_t\widehat{\pi}_{t+1}) - \omega_1(\widehat{a}_t - E_t\widehat{a}_{t+1}) = \omega_2\left((\widehat{m}_t - \widehat{e}_t) - (E_t\widehat{m}_{t+1} - E_t\widehat{e}_{t+1})\right) \tag{17}$$

where y is output, r is the gross nominal interest rate, and π is inflation. The superscript "ˆ" indicates log deviations from the steady state.

In this linear approximation, $\omega_2 \neq 0$ and real balances enter the IS curve if and only if the cross partial derivative of the instantaneous utility function, u_{12}, is non-zero, meaning that the utility function is non-separable between consumption and real balances. If this coefficient is different from zero, real balances affect the marginal rate of substitution between current and future consumption. The magnitude of the coefficient suggests the empirical importance of such non-separabilities near the steady state.

Ireland (2004) estimates (17) by maximum likelihood using quarterly per-capita US data between 1980 and 2001, proxying real balances with the ratio of $M2$ to the GDP deflator. On the basis of this estimation, Ireland reports that the ω_2 coefficient is not significantly different from zero. Andres, Lopez-Salido and Valles (2001) apply essentially the same approach to quarterly euro area data between 1980 and 1999. They also find ω_2 to be statistically insignificant, however, they obtain an implausibly low income elasticity of money demand. They proxy real balances using $M3$. By contrast, Kremer, Lombardo, and Werner (2003) conduct a similar analysis for Germany using data between 1970 and 1998, and find evidence of an active role of real balances (again proxied with $M3$).

Kremer, Lombardo, and Werner (2003) also describe problems which arise in the estimation of general equilibrium, rational expectations models. They note that most of the research in this area imposes the assumption of saddle path stability on the model to rule out indeterminate solutions. The authors, therefore, consider the minimum state variable (MSV) solution technique to allow for only one particular type of solution under multiple stable paths. In contrast, ensuring saddle-path stability is not an issue in our approach, because our test is based on a static optimization problem derived from the household's intra-temporal optimality conditions.

The non-parametric approach followed in this paper offers several advantages. First, the test does not make any assumptions about the functional form of the utility function and does not involve the use of linear approximations around steady state. Second, as noted by Swofford and Whitney (1994), statistical tests require more observations than the number of estimated parameters. In contrast, non-parametric revealed preference tests can be run on any number of observations. The ability to test over relatively short time periods is an advantage when it is not possible to construct economic data in a consistent way over longer time periods, which is the case for euro area data. Third, we work directly with consumption data rather than a broader output measure. Fourth, we work with disaggregate data for sets of monetary assets as well as for monetary aggregates and our analysis explicitly considers the own rates of return on interest bearing assets. In contrast, studies in this literature typically measure real balances using relatively broad monetary aggregates, *e.g.* $M2$ or $M3$, but do not consider own rates of remuneration on the monetary assets included in these aggregates, some of which are highly correlated with market interest rates.[17] Finally, as noted above, we do not need to impose equilibrium determinacy since our test is based on a static optimization problem.

At the same time, there are also some limitations in our approach. First, as discussed previously, we cannot incorporate preference shocks to the LM relationship(s) in the analysis, which is a limitation of our approach relative to Ireland's. In other respects, however, the non-parametric test approach is fully consistent with forward-looking agents maximizing expected lifetime utility subject to the usual inter-temporal budget constraints (including IS curve shocks).

A second shortcoming, also discussed previously, relates to the fact that our approach is based on the household's intra-temporal optimality conditions rather than the inter-temporal conditions as in Ireland (2004). Any monotonic transformation of the instantaneous utility function generates the same intra-temporal optimality conditions. Consequently, we can only test whether a monotonic transformation of the instantaneous utility function is equal to the sum of two utility functions: one containing only real balances and the other containing only consumption. If this condition is rejected, then real balances will certainly enter into the IS curve, since no monotonic transformation of the instantaneous utility function can render it into the sum of two utility functions separating consumption and real balances. But, if the condition is not rejected, we cannot necessarily conclude that real balances are excluded from the IS

[17]The role of the own rate of money is often considered in empirical work on the demand for broad aggregates; see, for examples, Carlson, Hoffman, Keen, and Rasche (2000) and Calza, Gerdesmeier, and Levy (2001). Belongia and Ireland (2006) explicitly consider the role of the own rate of return on money in monetary policy transmission. Also, Divisia monetary aggregates have been widely advocated, because they account for interest earned on monetary assets, see Stracca (2004) and Hancock (2005).

curve (in other words, the condition we test is necessary, but not sufficient, for excluding real balances).[18] Thus, a rejection of the condition would be a stronger result than that obtained using Ireland's approach, whereas a failure to reject would be a weaker result.

5 Empirical Results

5.1 Data Description

In this section, we test the assumption of additive separability between consumption and the monetary assets that make up the $M3$ monetary aggregate using euro area data compiled by the ECB. We work with data on total private consumption (from Eurostat) and four monetary assets using quarterly data for the period 1991Q1 to 2005Q1.[19] The monetary asset quantity and interest rate data are updated versions of those described in Stracca (2004). The assets are described in Table 1.

Table 1: Components of M3

CC	Currency
OD	Overnight Deposits
SD	Other Short Term Deposits*
MS	Marketable Instruments**

* Mainly time and savings deposits
** Repurchase agreements, money market fund shares, and money market paper and debt securities issued with maturities of up to 2 years.

We choose the $M3$ monetary aggregate, because it plays a prominent role in the ECB's monetary policy strategy. Before 1999, the monetary data are aggregated across euro area countries using the irrevocable parities of the legacy currencies with the euro of December 31, 1998. The interest rate on CC is zero. The interest rates on overnight deposits and other short-term deposits are denoted as ROD and RSD respectively. The own rate on marketable instruments is proxied by RST, which is a representative short-term rate (the 3 month inter-bank lending rate). The monetary asset stocks are converted to real terms using the private

[18] For example, the two-good Cobb-Douglas utility function is additively separable, since the logarithm of it is a linear function of the logs of the quantities of the two goods. Barnett and Choi (1989) provide a three good parametric utility function, which can be made either weakly or strongly (in our terminology additively) separable depending on whether or not certain interaction parameters are non-zero.

[19] Harmonised Eurostat data for private consumption are available only from 1991. Moreover, high quality data for dissaggregated consumption is unavailable at the quarterly frequency.

consumption expenditures deflator ($PCON$) from Eurostat. The real monetary asset stocks and real private consumption (CON) are both converted to per-capita terms using an estimate of the euro area population (POP) obtained from the European Commission.[20]

The opportunity cost of real per-capita consumption is $PCON$. In order to define opportunity cost variables for the monetary assets, we define a benchmark rate of return, $BENCH$, as RST plus a liquidity premium of 0.8% per annum, following the method employed by Stracca (2004). The nominal user cost of currency is $BENCH/(1 + BENCH)$ multiplied by $PCON$. The nominal user cost of an interest bearing monetary asset is defined as $PCON(BENCH - OWN)/(1+BENCH)$, where OWN is the own rate on the particular asset (ROD, RSD, or RST).

The real per-capita monetary asset stocks (defined on a quarterly basis) are displayed in Figure 1. The corresponding expenditure shares for each of the four monetary assets relative to total expenditure on monetary assets are shown in Figure 2.

[insert Figures 1 and 2 here]

The combined expenditure share for all of the components of $M3$ relative to total expenditure on consumption and monetary assets is shown in Figure 3. It is notable that the expenditure share for the monetary components is not negligible, but it is relatively small, indicating that spending on monetary services is not a very sizeable part of overall expenditure. In addition, the expenditure share is trending downward in the first half of the sample period.

[insert Figure 3 here]

5.2 Test Results

We begin our analysis by running the non-parametric test for additive separability over the full sample 1991Q1 to 2005Q1. Step 1 of the test is to check the data for consistency with utility maximization, which is accepted (the combined data for both consumption and monetary assets satisfies GARP). Step 2 checks the necessary conditions for additive separability. The monetary asset and user cost data also satisfies GARP.[21]

[20] Quarterly population figures are interpolated from annual figures.

[21] Incidentally, this finding casts some doubt on the need to incorporate money demand shocks into econometric models that are used to explain the euro area data. Our results indicate that apparently unexplained fluctuations in money growth that have been noted in the euro area over recent years are *not* necessarily due to an underlying shock in preferences.

We work with data on total private consumption rather than a vector of consumption goods, so the other GARP condition in step 2 is trivially satisfied. The minimization problem in Step 3 is solved using the double precision IMSL library routine DLCONG in FORTRAN. The results from the minimization problem are displayed in Table 2. In the table, we provide the minimized value of the objective function, $\hat{F}$, multiplied by 10000. The data are consistent with additive separability if the minimized objective function is zero. The convergence properties of numerical optimization procedures imply, however, that this condition can only be approximately satisfied in empirical applications. In practice, therefore, we check whether or not $\hat{F}$ is less than a very small number (*e.g.* 1.0E-018) , which is approximately zero.

Table 2: Results for M3

Sample Period	$\hat{F}$ mult. by 10000
1991Q1 to 2005Q1	245.2014
1993Q1 to 2005Q1	9.3632
1993Q1 to 2005Q1*	0.0000**

* 3 observations are deleted

** $\hat{F} \leq$ 1.0E-018

The results indicate that additive separability is rejected for the full sample with $\hat{F} = 0.02452$.[22] We then analyzed the values of the percentage deviations between the two measures of marginal utility of expenditure, $100(\lambda^i - \mu^i)/\mu^i$, to determine which observations are causing us to reject. These deviations are depicted for all observations in Figure 4. The figure shows that our rejection of additive separability seems to be largely attributable to the first eight quarters of the sample, where there are non-trivial deviations (both positive and negative). The figure also shows non-trivial deviations for three pairs of adjacent quarters: 1997Q2 and 1997Q3; 2001Q4 and 2002Q1; and 2004Q1 and 2004Q2.

[insert Figure 4 here]

To determine if the rejection can be entirely attributed to observations in 1991 and 1992, we deleted the first eight quarters and ran the test again over the period 1993Q1 to 2005Q1. The results indicated that additive separability is still rejected, although the objective function was reduced ($\hat{F} = 0.000936$). We then deleted three additional quarters as well (one from each of the above pairs) and reran the test procedure

[22] We also ran the weak separability test from Fleissig and Whitney (2003), which indicated that the monetary assets in $M3$ are weakly separable from consumption for the full sample.

again. In this case, the solver found a feasible solution that minimized the objective function to approximately zero (*i.e.* less than or equal to 1.0E-018) indicating that additive separability is accepted.[23]

Thus, strictly speaking, our deterministic test rejects additive separability for the euro area indicating that real balances enter into the IS curve. If we consider the results from the 1993Q1 to 2005Q1 sub-sample in terms of economic significance, however, they indicate that the data are largely consistent with additive separability between the monetary assets that make up the $M3$ monetary aggregate and total private consumption. In particular, it is notable that the assumption is accepted if we remove just three quarters of data. Moreover, one of the adjacent pairs that is causing us to reject (2001Q4 and 2002Q1) corresponds to the currency changeover when the national currencies were replaced by the euro currency. The apparent significance of the currency changeover for our empirical results will be reinforced by additional findings in the next section and so merits some discussion. One possible interpretation is that incompatibility between the data prior to and immediately following the currency changeover could be playing a role in this case. The effect of the currency changeover on the CC data is apparent in both Figures 1 and 2, which depict the quantity and expenditure share for currency in the upper left panel of the two figures respectively.[24] The fact that the violations are limited to two quarters, while the euro cash changeover was a relevant phenomenon for several quarters (and was of course largely anticipated), appears to favour an interpretation based on data problems over an alternative one based on the behavioral response of economic agents. Nevertheless, such an interpretation is not critical for our main point.[25]

5.3 Robustness Analysis

We ran a number of additional tests to check our results for robustness. First, we ran the test for the full sample using a single monetary ag-

[23]The two observations in each of these three pairs appear to be interacting with each other to cause violations, but they do not appear to be interacting across pairs. We determined this by eliminating the observations in a stepwise fashion.

[24]Note that by data problems we do not refer to the data on *total* currency in circulation, which are of high quality, but rather to possible changes in the *geographic distribution* of currency, for which no reliable data are available (for example, cash held in Eastern Europe).

[25]In addition, fully harmonised monetary data are available only from 1997Q3 onwards. For prior periods, monetary data have been reconstructed by euro area national central banks. The data before and after 1997Q3 are comparable, but are not of the same quality. Thus, data issues may also play a role for 1997Q2 and 1997Q3, although the effect on the data is not as immediately evident in Figures 1 and 2.

gregate in place of the set of four monetary assets to be comparable to parametric test results based on aggregate data. We used both the standard simple sum $M3$ monetary aggregate and the corresponding Divisia monetary aggregate both in real per-capita terms (see Stracca, 2004 for further empirical analysis of these aggregates). The opportunity cost variables for the monetary aggregates are constructed by dividing total per capita expenditure on the monetary assets by the corresponding quantity aggregate.

The data for consumption and the monetary aggregate satisfy GARP in Step 1 for both the Divisia and simple sum aggregates. Step 2 is trivially satisfied, since there is only a single monetary good and a single consumption good. The results from Step 3 indicate that additive separability is rejected in both cases for the full sample period. We graph the percentage deviations in Figure 5 for the tests based on Divisia and simple sum $M3$, which can be easily compared with Figure 4. In the figure, the solid line is based on Divisia and the dashed line is based on simple sum. The results for the Divisia aggregate are very similar to the results based on the disaggregated data, except that there are no deviations that correspond exactly to the currency changeover. This finding is consistent with the fact that the currency changeover has a noticeable effect on the data for CC, but it has much less effect on the monetary aggregates. In contrast, the results for the simple sum aggregate are less similar to the results for the disaggregated data.

[insert Figure 5 here]

Second, we ran our test using the narrower collection of monetary assets in the $M1$ monetary aggregate, which consists of CC and OD. The GARP conditions in Steps 1 and 2 are satisfied for this alternative dataset. Pretesting (using Fleissig and Whitney's weak separability test) indicated that weak separability of the $M1$ assets is rejected for the full sample, but not for the sub-sample from 1993Q1 to 2005Q1. Since weak separability is a necessary condition for additive separability, we chose to limit our analysis to the shorter sub-sample. The results from Step 3 of the test are reported in Table 3. The results indicated that additive separability is rejected and the objective function is higher than for the corresponding results in Table 2 for the 1993Q1 to 2005Q1 sub-sample. The percentage deviations, $100(\lambda^i - \mu^i)/\mu^i$, for this sub-sample for tests based on both $M1$ and $M3$ are graphed in Figure 6 for comparison. In the figure, the solid line is based on $M3$, the dashed line is based on $M1$, and the shaded region highlights the currency changeover (2001Q4 and 2002Q1).

As shown in the figure, the percentage deviations have the same pattern for tests based on $M1$ and $M3$, but the deviations for the currency changeover (2001Q4 and 2002Q1) are larger for the test results based on $M1$ than for $M3$. This finding is consistent with the fact that currency has a higher expenditure share relative to the complete set of monetary assets for $M1$ than for $M3$. Another minor difference is that the test results for $M1$ point to an additional pair of observations (2003Q3 and 2003Q4) with small positive and negative deviations of approximately equal absolute magnitude.

Table 3: Results for M1

Sample Period	$\hat{F}$ mult. by 10000
1993Q1 to 2005Q1	69.2862
1993Q1 to 2005Q1*	0.0000**

* 4 observations are deleted

** $\hat{F} \leq$ 1.0E-018

Similar to our analysis based on $M3$, we deleted four additional quarters (one from each of the four pairs) and reran the test procedure again. In this case, the solver found a feasible solution that minimized the objective function to approximately zero (*i.e.* less than or equal to 1.0E-018) indicating that additive separability is accepted. Taken together, the results based on the components of $M1$ are very similar to the results based on the components of $M3$, but provide slightly weaker support for additive separability.

[insert Figure 6 here]

5.4 Test Results for an Extended Sample Period

In previous sections, we have reported evidence for the sample period from 1991 to 2005. Our decision to focus on this sample period was motivated by the availability of high quality and harmonised national accounts data for the euro area from that year onwards. The issue of data quality is of paramount importance, since we test for exact optimization of an additively separable utility function. The non-parametric separability test we use does not account for stochastic features of the data, such as random measurement errors. Nevertheless, it would be useful to be able to directly compare the results from our non-parametric tests with studies focussing on euro area money demand, most of which start from the early 1980's; See, for examples, Calza, Gerdesmeier and Levy (2001) and Brand and Cassola (2004). In addition, Andres, Lopez-Salido and Valles (2001) use data for the 1980's and 1990's.

To facilitate such comparisons, we use extended data for private consumption and the consumption deflator from the database for the ECB's Area Wide Model (see Fagan, Henry and Mestre 2001). The monetary data used in previous sections can be extended back to 1980Q2 and the Area-Wide Model database contains data up until 2003Q4. Thus, we are able to test for additive separability using data from 1980Q2 until 2003Q4, although several caveats are in order. First, it should be noted that the national accounts data for the 1980's have been reconstructed in the best possible way, but still imply the aggregation of largely heterogeneous data for the individual countries, not necessarily consistent with the ESA95 standard, and the need of interpolating some missing data for some (smaller) countries at the beginning of the sample period. Second, the extended consumption data used in these tests do not coincide exactly with the consumption data used in previous sections even when the sample periods overlap (*i.e.* from 1991 through 2003). Thus, we should exercise caution in when interpreting the results of our analysis of the extended sample period.

The extended sample period has 95 quarterly observations. As above, Step 1 of the additive separability test is to check the data for consistency with utility maximization, which is accepted (the combined data for both consumption and monetary assets satisfies GARP). Step 2 checks the necessary conditions for additive separability. The monetary asset and user cost data for the components of the $M3$ monetary aggregate also satisfies GARP over the extended sample period. Again, the other GARP condition in step 2 is trivially satisfied, since we work with total private consumption. Thus, the necessary conditions for additive separability between the $M3$ assets and consumption are satisfied. Pretesting (again using Fleissig and Whitney's weak separability test) indicated that weak separability of the $M3$ assets is rejected for the full sample, but not for the slightly shortened sample period 1981Q2 to 2003Q4. Since weak separability is a necessary condition for additive separability, we chose to limit our analysis to 1981Q2 to 2003Q4. Step 3 of the test is to solve the minimization problem and check whether or not the minimized value of the objective function, $\hat{F}$, is approximately zero.

Not surprisingly, given our previous results, additive separability is rejected for the extended sample. We found a cluster of very large percentage deviations, $100(\lambda^i - \mu^i)/\mu^i$, in the period from 1991Q1 through 1993Q1 with the largest ones being for 1991Q4 and 1992Q1. These results suggest testing for additive separability over two shorter sample periods: 1981Q2 to 1990Q4 and 1993Q1 to 2003Q4.

The results for 1993Q1 to 2003Q4 are, again not surprisingly, very similar to the corresponding results in Table 2 for the slightly longer period 1993Q1 to 2005Q1.[26] In particular, the rejection of additive separability is largely attributable to non-trivial deviations for two quarters, 2001Q4 and 2002Q2, surrounding the euro currency launch. The results are less supportive of additive separability if 1991 and 1992 are included and there are large deviations in those two years. Additive separability is also rejected by the test for the earlier period from 1981Q2 through 1990Q4. Interestingly, the results indicate that there are large percentage deviations throughout the early 1980's, but not after 1985Q1. The percentage deviations for the 1980's are depicted in Figure 7 along with the percentage deviations for the 1990's for comparison. Figure 8 shows the same percentage deviations on a different scale and omits the observations prior to 1985Q2.

[insert Figures 7 and 8 here]

Several salient conclusions arise from our analysis of the extended dataset, which complement our previous interpretations. First, the data are not consistent with additive separability over the full sample period from 1980 to 2003 and observations in 1991 and 1992 stand out in the analysis. Second, the data are much more consistent with additive separability if only the period from 1993 onwards is considered. Specifically, although strictly speaking the test rejects additive separability for this shorter sample period, the rejection seems to be mainly attributable to a couple of data points surrounding the launch of the euro currency. Third, additive separability appears to be a much less reasonable assumption for the earlier sample period from 1980 through 1990.

6 Conclusions

In this paper we have developed a non-parametric test for additive separability between consumption and real balances, which we applied to the euro area using data from 1991 to 2005. The main result from our test is that additive separability is rejected for the euro area, but the rejection is largely attributable to observations in 1991 and 1992. After 1992, the rejections are caused by only a few pairs of observations including observations surrounding the launch of the euro currency. The revealed preference violations of additive separability do not appear to mirror a truly "behavioral" non-separability between consumption and real balances involving most observations in the sample period. Rather, the conditions for additive separability appear to be respected most of the time in the sense that (excluding 1991 and 1992) they are not rejected if

[26] The monetary asset and user cost data are exactly the same as in previous analysis. The consumption data differ somewhat, however.

just three quarterly observations are eliminated from the dataset. Nevertheless, the fact that the failure of separability after 1992 is caused by only a few observations may not necessarily be seen as reassuring, especially since we have not yet developed a criterion to assess the importance/significance of violations of additive separability detected by our new non-parametric test. Clearly, the relationship between these specific observations and the violations of additive separability is subject to interpretation. One possible interpretation is that measurement problems or inconsistencies in the data are playing a role in at least some cases, as suggested by the significance of the currency changeover in our empirical results, but other more structural interpretations could also be advanced. Nevertheless, such interpretations are not critical to our main point.

In the context of the literature on non-parametric separability tests, it is clear that an important direction for further research is to develop metrics for evaluating the severity and/or significance of violations of the hypothesis of exact optimization of a (weakly or additively) separable utility function. A body of work has explored the related problem of assessing the statistical significance of violations of GARP (*i.e.* the hypothesis of utility maximization); see, for recent examples, de Peretti (2005), Jones and de Peretti (2005), and Fleissig and Whitney (2005). de Peretti (2006) extends his earlier work to evaluate the statistical significance of violations of weak separability. It is possible that one of these methods could also be extended to evaluate the statistical significance of violations of additive separability found by our new test. Ideally, however, we would not restrict ourselves to just the question of statistical significance. Instead, to a central bank conducting monetary policy, the more important question is whether or not violations of additive separability are significant from an economic perspective (see McCallum, 2001 for some analysis along these lines).

All in all, the findings of this study largely reinforce earlier claims in the literature that perfect non-separability between consumption and real balances is implausible, but that non-separabilities may not be very important empirically. In other words, the evidence against the hypothesis that real balances and consumption are additively separable is far from compelling. Therefore, it appears that, all in all, little may lost if the euro area (theoretical) IS curve, and hence the Phillips curve as well, are studied abstracting from money, at least from the early 1990s onwards.

For robustness, we also ran our test using data for the 1980's. Unlike the period after 1992, we find systematic violations of the hypothesis of additive separability especially in the first part of the 1980's, which corresponds to a period of relatively high inflation. Although highly speculative, one might conjecture that additive separability could be regime-dependent; *i.e.* it may prevail in a credible low-inflation regime, but not in a regime of high and volatile inflation. If true, this would reinforce the intuition that monetary variables contain more information and are, consequently, more interesting when there are more pronounced changes in inflation and money growth than in periods when such changes are subdued (see, for example, arguments along these lines in Estrella and Mishkin, 1997, pp. 300-301). This is an intriguing conjecture, which could be taken up in future research by analyzing data from different countries and sample periods.

Appendix 1 - Condition for Additive Separability

Condition (ii) from Varian's theorem for additive separability can be motivated using a heuristic argument. See Varian (1983, p. 101) and Swofford and Whitney (1994, p. 237-9) for very similar arguments in the context of utility maximization and weak separability. Suppose that the data maximize the sum of two concave, monotonic, and *differentiable* utility functions, U and V; *i.e.* $\mathbf{c}^i$ and $\mathbf{m}^i$ maximize $U(\mathbf{c})+V(\mathbf{m})$ subject to $\mathbf{p}^i\mathbf{c}+\boldsymbol{\gamma}^i\mathbf{m} = \mathbf{p}^i\mathbf{c}^i+\boldsymbol{\gamma}^i\mathbf{m}^i = Y^i$ for all $i = 1, ..., n$. The Lagrangian (for the i^{th} observation) would be $\mathcal{L}^i = U(\mathbf{c}) + V(\mathbf{m}) + \lambda^i(Y^i - \mathbf{p}^i\mathbf{c} + \boldsymbol{\gamma}^i\mathbf{m})$. The corresponding first-order necessary conditions would be as follows:

$$\triangledown U(\mathbf{c}^i) = \lambda^i \mathbf{p}^i \tag{18}$$

$$\triangledown V(\mathbf{m}^i) = \lambda^i \boldsymbol{\gamma}^i \tag{19}$$

for all $i = 1, ..., n$. The standard properties of differentiable concave functions imply

$$U(\mathbf{c}^i) \leq U(\mathbf{c}^j) + \triangledown U(\mathbf{c}^j)(\mathbf{c}^i - \mathbf{c}^j) \tag{20}$$

$$V(\mathbf{m}^i) \leq V(\mathbf{m}^j) + \triangledown V(\mathbf{m}^j)(\mathbf{m}^i - \mathbf{m}^j) \tag{21}$$

for all $i, j = 1, ..., n$. By combining (18) and (19) with (20) and (21) respectively, we obtain

$$U(\mathbf{c}^i) \leq U(\mathbf{c}^j) + \lambda^j \mathbf{p}^j(\mathbf{c}^i - \mathbf{c}^j) \tag{22}$$

$$V(\mathbf{m}^i) \leq V(\mathbf{m}^j) + \lambda^j \boldsymbol{\gamma}^j(\mathbf{m}^i - \mathbf{m}^j) \tag{23}$$

for all $i, j = 1, ..., n$. The conditions in (ii) are obtained by setting $U^i \equiv U(\mathbf{c}^i)$ and $V^i \equiv V(\mathbf{m}^i)$ for all i. Thus, the numbers U^i and V^i represent the utility levels produced by consumption goods and the monetary assets respectively at the i^{th} observation. The numbers λ^i represent the marginal utility of total expenditure at the i^{th} observation, since it is equivalent to the Lagrange multiplier for the budget constraint. The heuristic derivation of these conditions hinged on differentiability, but Varian's proof of the theorem does not require that assumption.

Appendix 2 - Weak versus Additive Separability

In this paper, we focus on testing the utility function, $u(\mathbf{c}, \mathbf{m})$, for additive separability. The utility function is *additively separable* if there exists a monotonic transform, f, such that $f(u(\mathbf{c}, \mathbf{m})) = U(\mathbf{c}) + V(\mathbf{m})$. A widely tested condition is that the utility function is weakly separable. Specifically, the utility function is *weakly separable* in $\mathbf{m}$ if there exists a macro function, $\overline{u}$, and a sub-utility function, V, such that $u(\mathbf{c}, \mathbf{m}) = \overline{u}(\mathbf{c}, V(\mathbf{m}))$.

Varian (1983) proves that a dataset can be rationalized by a weakly separable utility function if and only if there exist numbers U^i, V^i, λ^i, $\mu^i > 0$ such that;

$$U^i \leq U^j + \lambda^j \mathbf{p}^j(\mathbf{c}^i - \mathbf{c}^j) + \lambda^j (V^i - V^j)/\mu^j \tag{24}$$

$$V^i \leq V^j + \mu^j \boldsymbol{\gamma}^j (\mathbf{m}^i - \mathbf{m}^j) \tag{25}$$

for all $i, j = 1, ..., n$.[27] The latter set of inequalities is often called the *Afriat inequalities*. These conditions are equivalent to the conditions that the data $(\boldsymbol{\gamma}^i, \mathbf{m}^i)$ $i = 1, ..., n$ satisfy GARP and that the data $(\mathbf{p}^i, 1/\mu^i; \mathbf{c}^i, V^i)$ $i = 1, ..., n$ satisfy GARP for some choice of indexes (V^i, μ^i) that satisfy the Afriat inequalities. Thus, weak separability tests can be formulated by first using a numerical algorithm to construct indexes satisfying the Afriat inequalities and then testing the data $(\mathbf{p}^i, 1/\mu^i; \mathbf{c}^i, V^i)$ $i = 1, ..., n$ for GARP. See Fleissig and Whitney (2003) for a recent weak separability test based on a linear programming algorithm.

Additive separability implies that the utility function is *blockwise weakly separable*: *i.e.* it is weakly separable (simultaneously) in both $\mathbf{c}$ and $\mathbf{m}$, but the converse is not true. It is, therefore, a considerably more restrictive condition than weak separability.

[27] See Varian (1983), Theorem 3.

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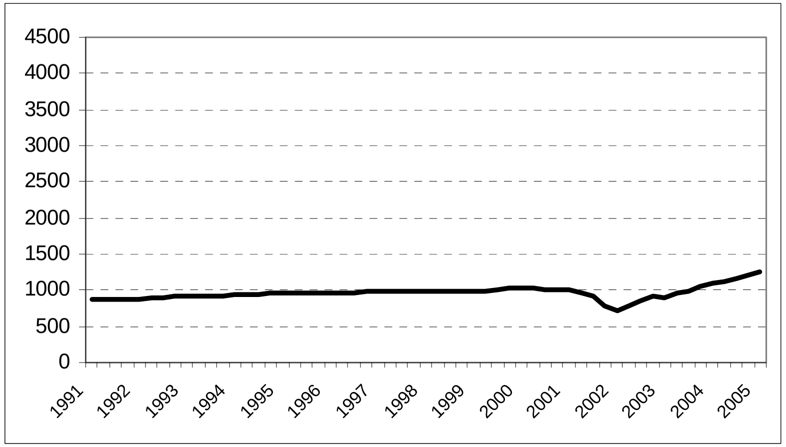
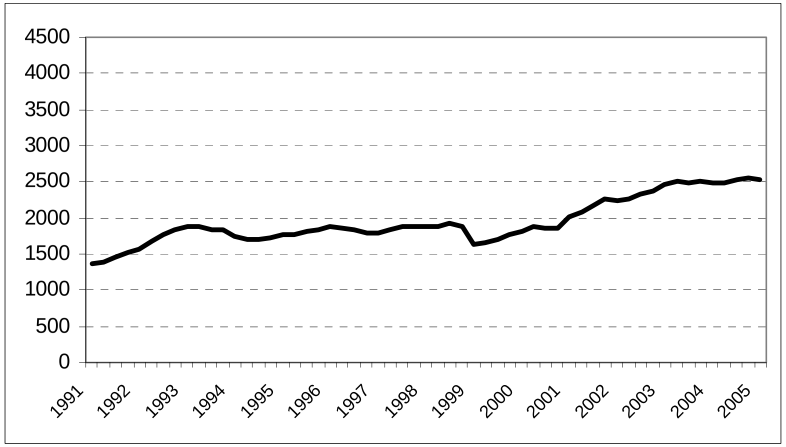
Figure 1: Monetary Asset Stocks

Real Per-Capita Terms

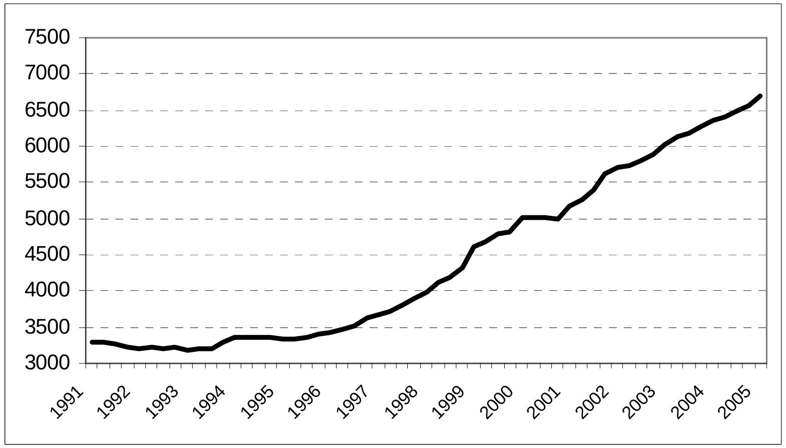
CC: Currency



MS: Marketable Instruments



OD: Overnight Deposits



SD: Short-Term Deposits

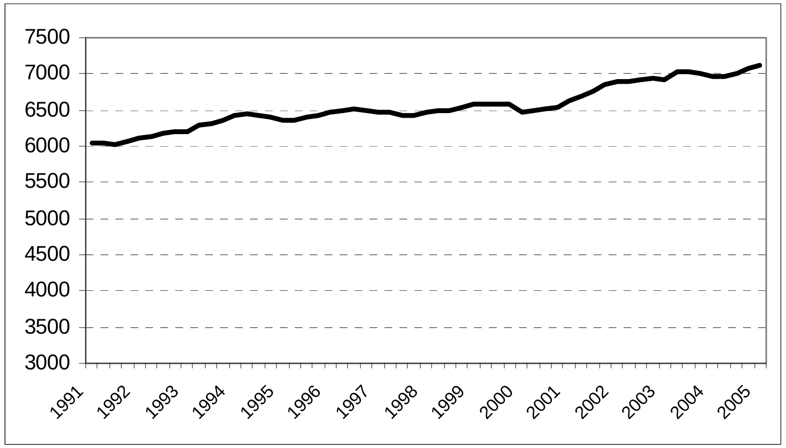
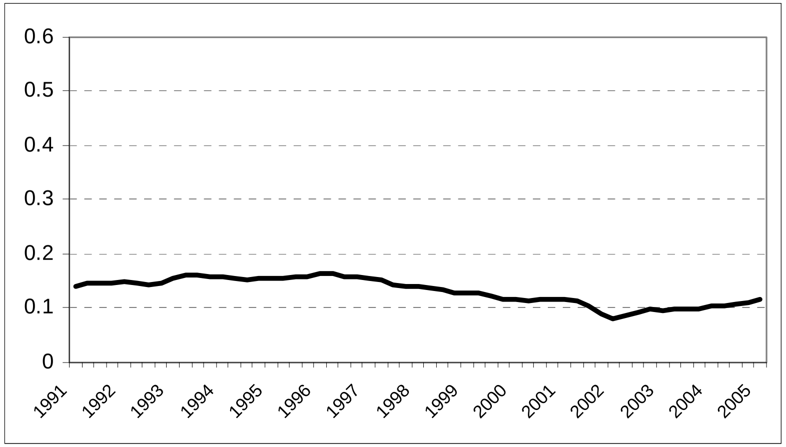


Figure 2: Expenditure Shares of the Four Monetary Assets

Relative to Total Expenditure on M3

CC: Currency



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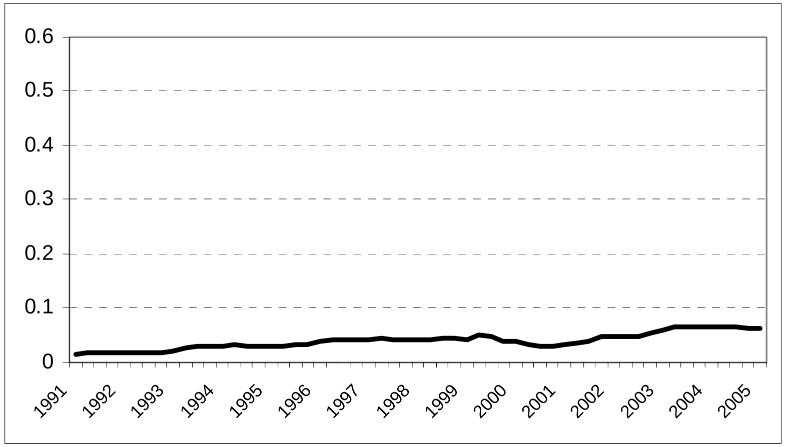

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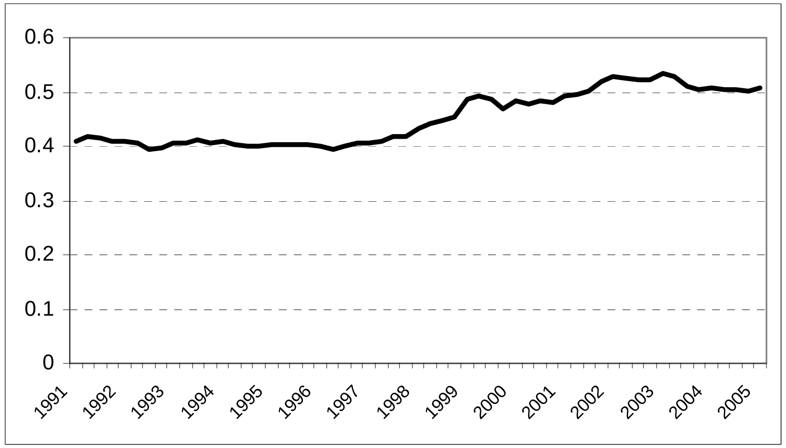

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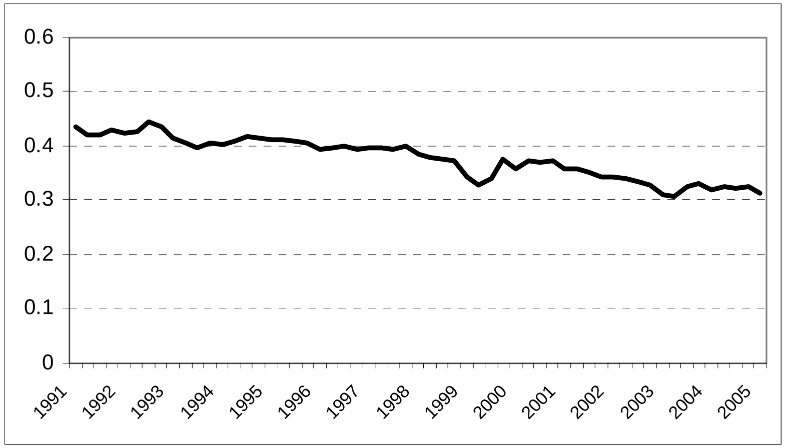


Figure 3: Expenditure Share of the Components of M3

Relative to Expenditure on Consumption and Money

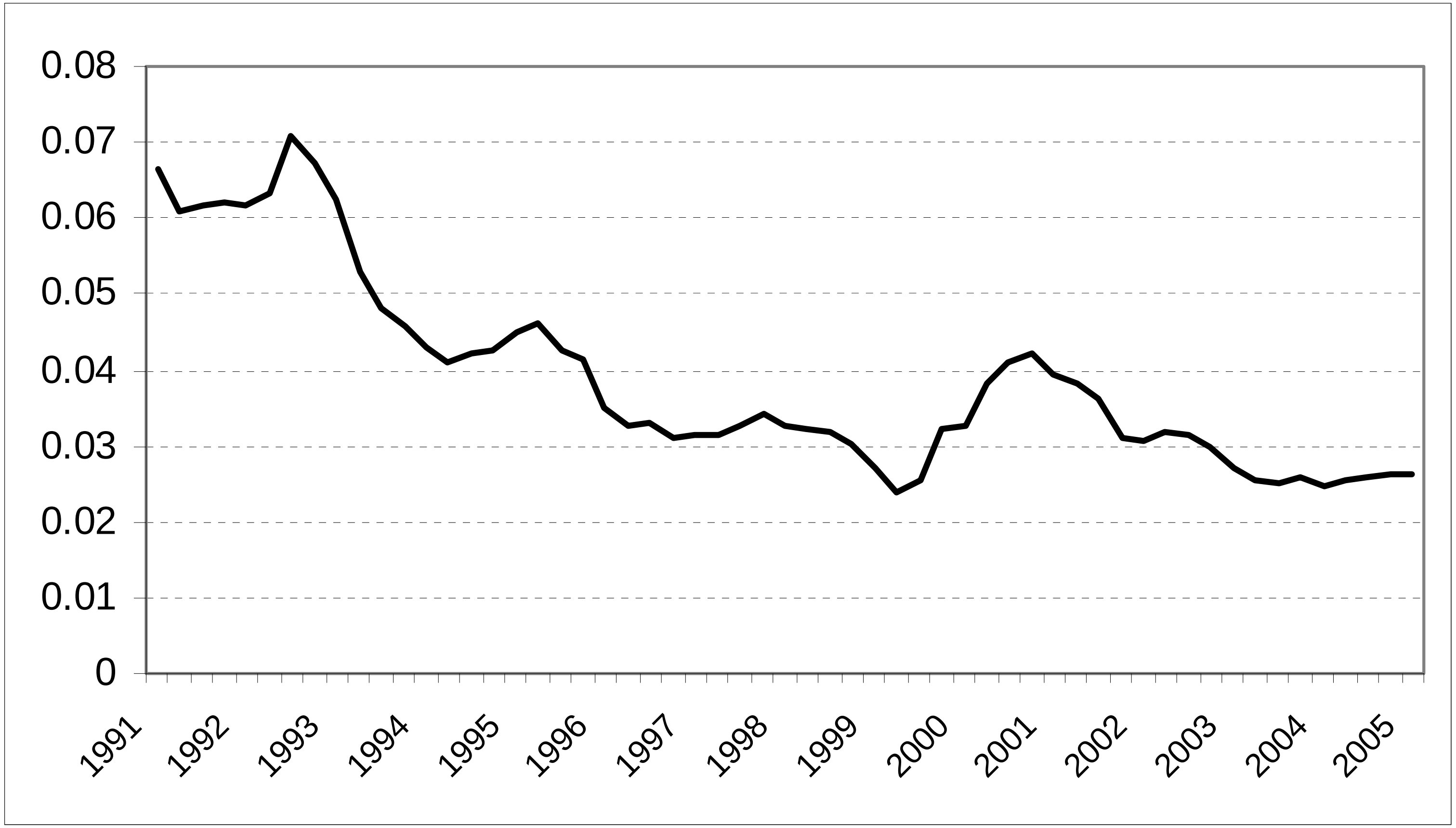
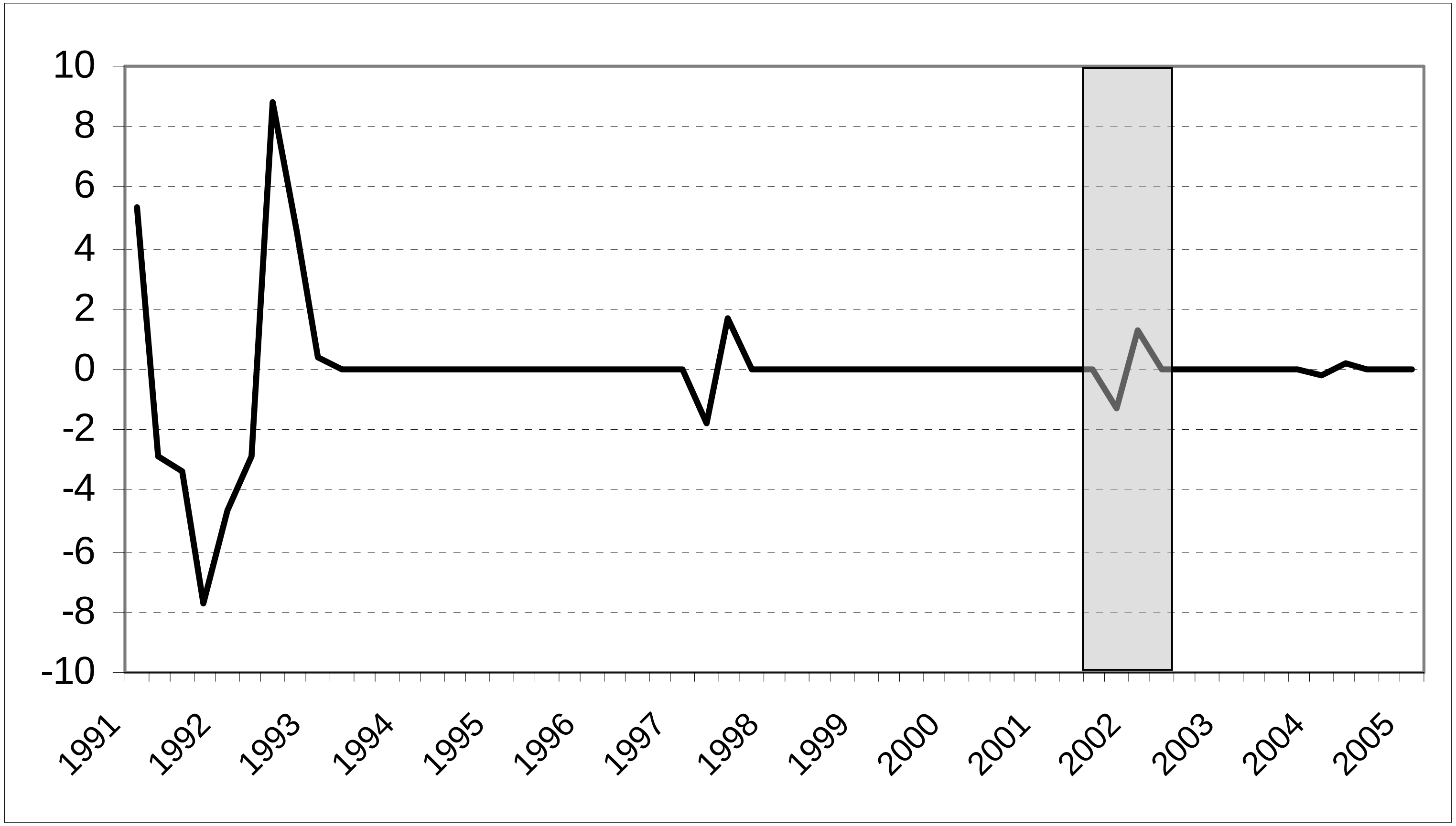


Figure 4: Percentage Deviations of Lambda from Mu

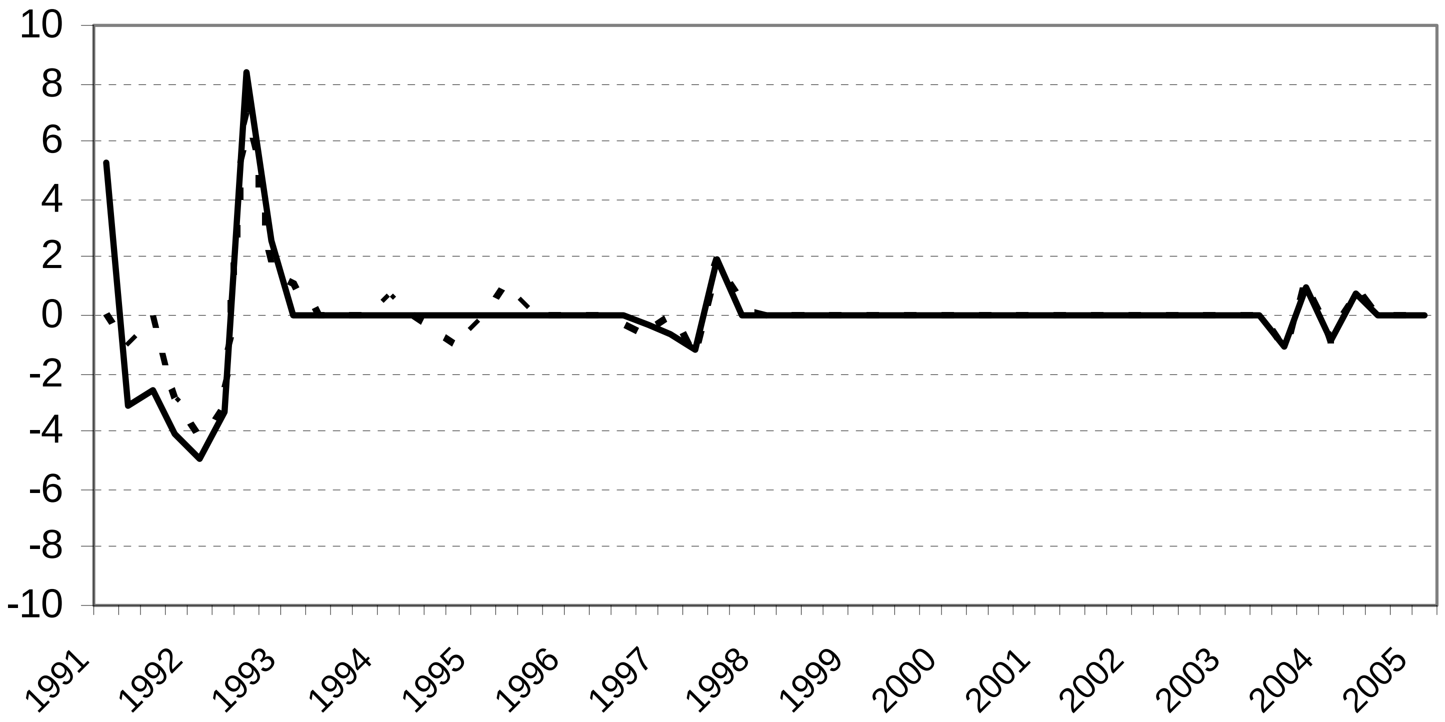
Components of M3 - 1991Q1 to 2005Q1



Note: The shaded region denotes the currency changeover.

Figure 5: Percentage Deviations of Lambda from Mu

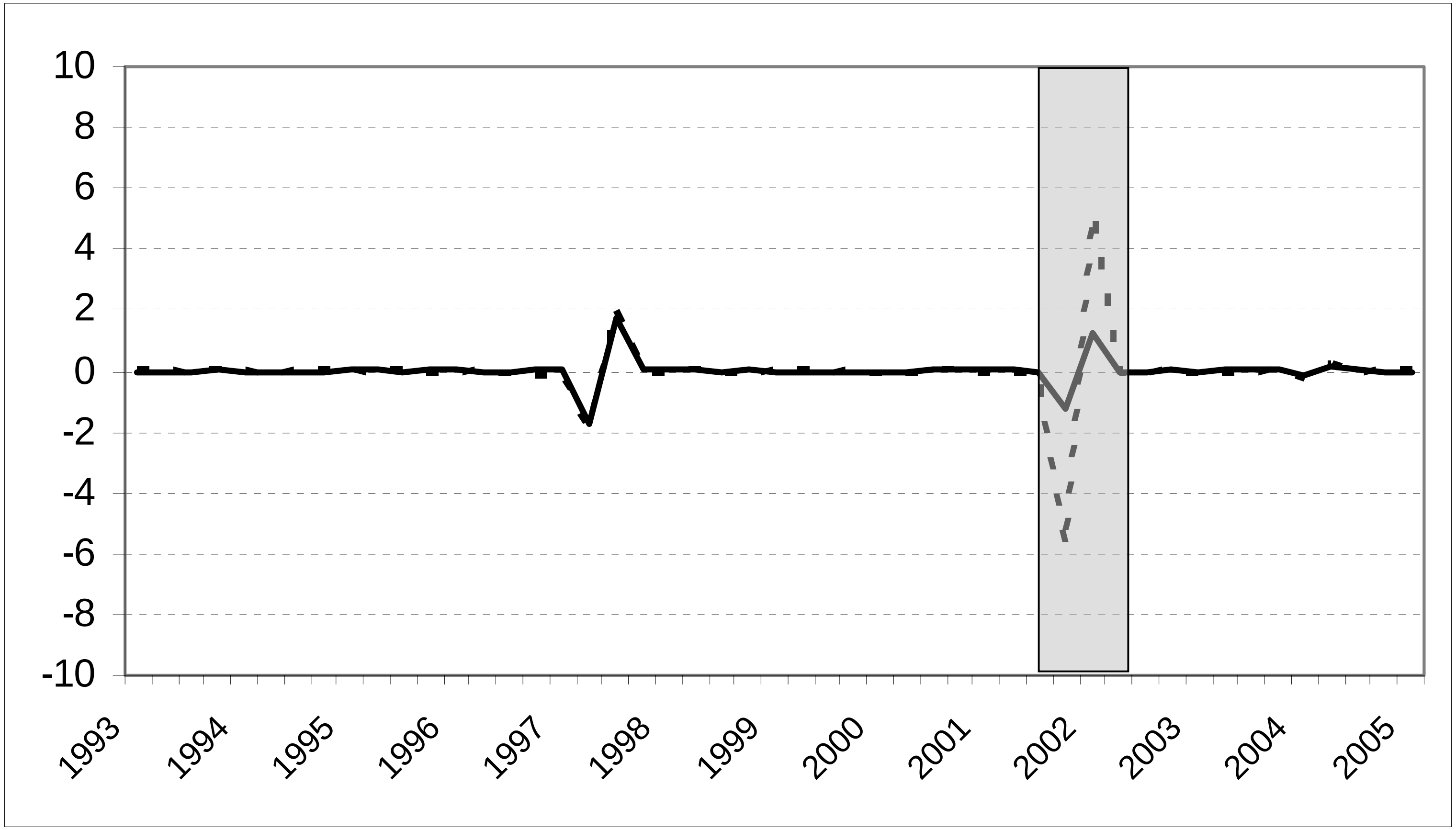
Divisia and Simple Sum M3 - 1991Q1 to 2005Q1



Note: Percentage deviations are solid for Divisia *M3* and dashed for simple sum *M3*.

Figure 6: Percentage Deviations of Lambda from Mu

Components of M1 and M3 - 1993Q1 to 2005Q1



Notes: Percentage deviations are solid for *M3* and dashed for *M1*. The shaded region denotes the currency changeover.

Figure 7: Percentage Deviations of Lambda from Mu

Components of M3

1981Q2 to 1990Q4 versus 1993Q1 to 2003Q4

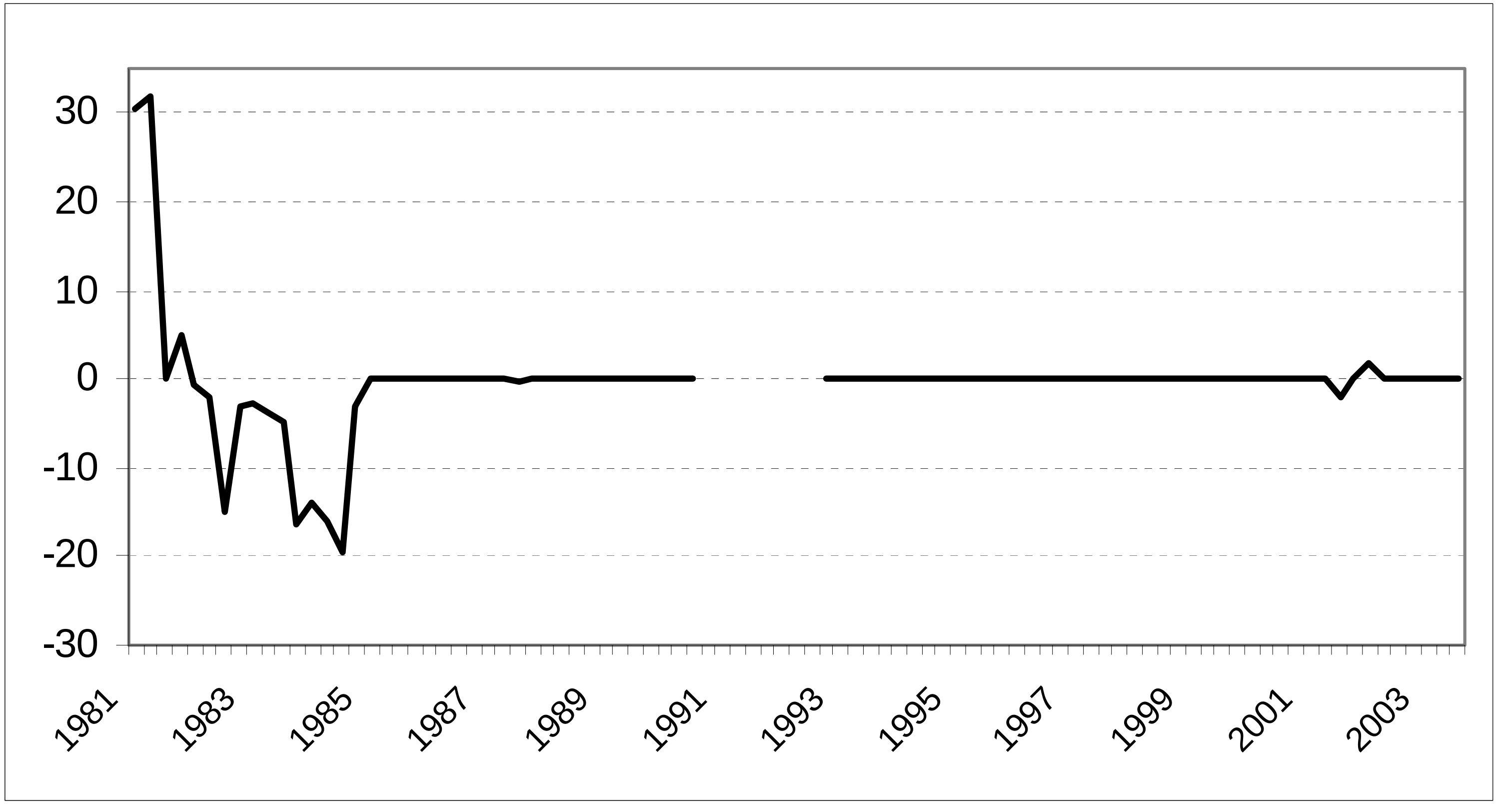
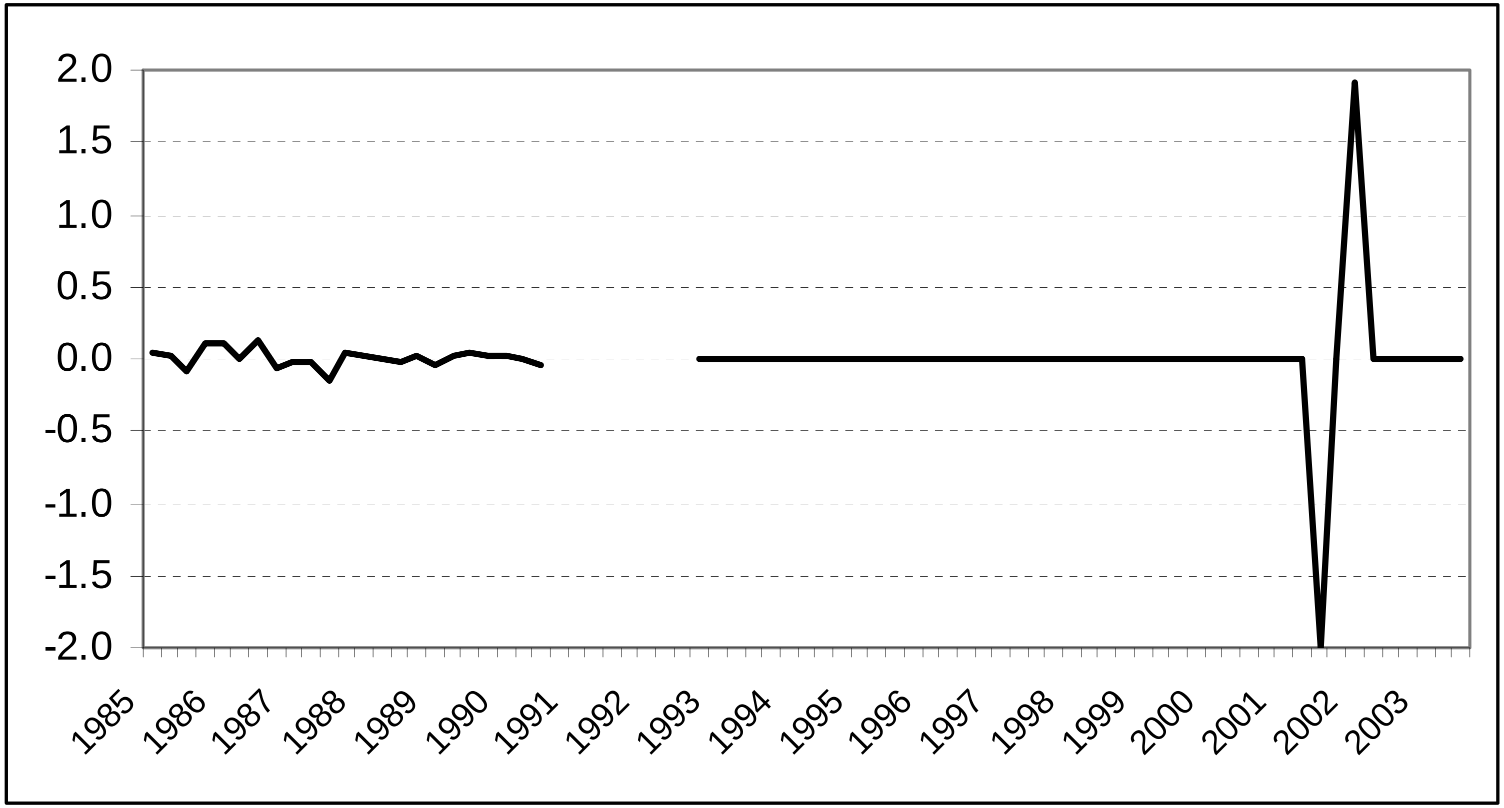


Figure 8: Percentage Deviations of Lambda from Mu

Components of M3

1981Q2 to 1990Q4 versus 1993Q1 to 2003Q4

(*Alternative scale, observations before 1985 omitted from figure*)



Note: Percentage deviations up to and including 1985Q1 are omitted to permit comparison using the alternative scale (±2%).

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