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Abstract:

In an overlapping generations maximization framework with consumers, whose information on uncertain future income realizations is front-loaded, a closed form aggregate consumption function with CRRA preferences is derived. To have a closed form solution we assume that consumers solve their intertemporal optimization problem sequentially. First they assess risk-adjusted life-time wealth and then the optimal consumption path. The derived model captures precautionary saving, which is dependent on the human to non-human wealth ratio. On aggregate level, after accounting for habit formation, the model is able to explain both the short-run (e.g. the excess sensitivity and the excess smoothness puzzle) and long-run stylized facts of the U.S. consumption data.

Keywords: Consumption, Information, Habit Persistence, Precautionary Saving

JEL classification: D11, D12, D82, E21

Non-technical summary

In this paper we derive in an overlapping generation framework a closed form consumption function with CRRA preferences and income uncertainty and estimate it in the aggregate U.S. data. To have the closed form solution we assume that the consumer solves her intertemporal utility maximization problem sequentially so that she first evaluates her non-human and human wealth accounting for income uncertainty. Thereafter, in the second stage, conditional on the risk adjusted life-time resource constraint, the consumer determines the optimal planned path of consumption.

An additional benefit of the sequential approach is that it allows us to decompose imperfect information on a future income stream into a component related to the stochastic properties of available income data as well as one measuring the amount of period-specific information that consumers have on future income innovations. Accordingly, we deviate from normal convention, which assumes either that (i) the consumer has no period-specific information on future income realizations (e.g. the income generating process is a random walk), or that (ii) the consumer can perfectly anticipate, at least, the trend development of her future income stream. Our approach is more general and contains the above-mentioned two cases as limiting polar cases. More explicitly, we assume that the amount of period-specific information can be front-loaded so that, with the lengthening of the projection horizon, expected income changes converge to those implied by some stochastic time-series process, which for simplicity is assumed to be a random walk. Furthermore, we connect this framework to Blanchard's (1985) overlapping generation model with the positive probability of death and complement it to include also habit formation. After aggregating, we end up with a dynamic consumption function where current consumption depends on one period lead and lag of consumption as well as on fundamental variables, i.e. on real non-human wealth, current-period real labor income and the real interest rate as well as on the determinants of time-varying precautionary saving.

In general, the data compatibility of our estimations is good. The model captures precautionary saving and, hence, is in better accordance with micro-data evidence and the results of the numerical analysis of a fully rational optimizing consumer than, for instance, the hybrid of the LC/PIH and Keynesian model. It also explains both excess smoothness and sensitivity puzzles.

According to our estimation results consumers adjust their expected human wealth around 35-40 % downwards for precautionary reasons in evaluating their life-time budget constraint. According to our results consumers have also a lot of period-specific information on future income realizations although this is strongly front-loaded concentrating on the first 1.5 years and their planning horizon (life-expectancy) is somewhat above 40 years. Also non-time separability in the utility function with habit formation parameter around 0.9 is strongly supported by the data.

1. Introduction

Following the seminal paper by Hall (1978), the bulk of empirical research on aggregate consumption has concentrated on estimating the Euler equation. Hall's argument that consumption follows a random walk and, hence, that the future development of consumption cannot be forecast largely undermined the interest in "structural" consumption function research. However, empirical evidence since Flavin(1981), Campbell (1987), Deaton (1987) and Campbell and Deaton (1989) have called Hall's result into serious question and this evidence can be summed up by two puzzles, i.e. the excess sensitivity and the excess smoothness puzzle. [1, 2] Accordingly, for both policy and forecast purposes aggregate consumption function research is still well motivated and needed. There is, however, a difficult problem coupled with the "structural" consumption function approach, i.e. no general closed-form solution to the stochastic dynamic programming problem of the utility maximizing consumer can be presented, when labor income is uncertain. Since Zeldes (1989), efforts to circumvent this difficulty have created a vast literature on numerical micro-level analysis of utility maximizing consumers. This research has markedly increased the understanding on a consumer's behaviour under uncertainty, but, by the same token, it has widened the gap between the micro- and macro-level consumption analysis.

As is well known, dynamic programming gives a closed form solution to the consumer's maximization problem only under assumptions that the utility function is quadratic, resulting in certainty equivalence, or that the utility function exhibits constant absolute risk aversion (CARA). From the empirical point of view, both of these functions are generally regarded unrealistic. Therefore, when coupled with empirically more realistic constant relative risk-averse (CRRA) preferences, in macro-data studies income uncertainty has been either excluded or, equally unrealistically, bypassed by assuming that labor income risk is perfectly correlated with the capital-income risk (Campbell and Mankiw, 1989; Fuhrer, 2000). Both of these assumptions imply that the consumption function does not contain precautionary saving. This contradicts both micro-data evidence and the results of the numerical analysis of a fully rational optimizing consumer, which suggest that income uncertainty results in precautionary saving. Nor does currently the most popular approach to modelling aggregate consumption, the hybrid of the Life Cycle/ Permanent Income Hypothesis (LC/PIH) and the Keynesian model as proposed by Campbell and Mankiw (1989), capture precautionary saving. In addition, for solving the excess sensitivity puzzle this model makes three extreme

[1] The excess sensitivity puzzle refers to the excess sensitivity of consumption to income predicted on the basis of lagged information (Flavin 1981). The excess smoothness puzzle refers to the lower volatility of consumption than that of labor income (Deaton 1987; Campbell and Deaton, 1989).

[2] Some writers as e.g. Carroll (2001) argue that Euler equation estimation methods are incapable of producing econometrically consistent estimates of utility function parameters and conclude that empirical estimation of consumption Euler equations should be abandoned.

assumptions: (i) part of the consumers are fully rational LC/PIH consumers while (ii) the rest of consumers are income constrained with consumption equalling income and (iii) the income share of each group is constant. However, disregarding the unrealism of these assumptions, the merit of this approach is that, especially, after supplemented by habit formation as in Fuhrer (2000), the estimated consumption function is able to capture satisfactorily many stylized features of the aggregate data; the excess sensitivity and smoothness of consumption and a gradual and hump-shaped response of consumption to an income shock.

In this paper we derive a solved-out aggregate consumption function with CRRA preferences that is also able to capture the aforementioned stylized features of the macro-data as well as precautionary saving as a response to income uncertainty. This consumption function is derived in a single optimisation framework without dividing consumers into optimizing and non-optimizing sub-groups. The key assumption is that in the mental frames of consumers the optimization problem is solved sequentially rather than simultaneously, which is an implicit assumption in the dynamic programming approach.[3] A familiar example of sequential approach elsewhere in economics is the analysis of the portfolio selection problem, where the choice of optimal portfolio is separated from the rest of the household's optimization problem (Markowitz, 1952). We assume that in the first stage, the consumer evaluates her risk-adjusted non-human and human wealth conditional on uncertain lifespan and labor income. Thereafter, in the second stage, conditional on the risk adjusted life-time resource constraint, the consumer determines her optimal planned path of consumption.

With uncertain labor income, the transformation of the optimization problem from simultaneous to sequential simplifies the solution of the problem essentially, because the tools of deterministic optimization can be applied and a closed form solution can be found also with the CRRA utility function. An additional benefit of the sequential approach is that it allows us to decompose imperfect information on a future income stream into the component related to the stochastic properties of the past income stream as well as the component measuring the amount of period-specific information that consumers have on future income innovations. As regards the latter component, we deviate from normal convention, which assumes either that (i) the consumer has no period-specific information on future income realizations (e.g. the income generating process is a random walk), or that (ii) the consumer can perfectly anticipate, at least, the trend development of her future income stream. Our approach is more general and contains the above-mentioned two cases as limiting polar cases. More explicitly, we assume that the amount of period-specific information can be front-loaded so that, with the lengthening of the projection horizon, expected income changes

[3] Hence, sequential approach to the consumer's optimisation problem can also be thought as a simplifying rule in line with bounded rationality. For instance, Akerlof and Yellen (1985), Akerlof (2002), Thaler (1994, 2000) and Gabaix and Laibson (2000) have argued that, rather than being fully rational, agents may use nearly rational decision strategies summarising information and making choices based on simplified mental frames.

converge to those implied by some stochastic time-series process, which for simplicity is assumed to be a random walk.

Furthermore, we connect this framework to Blanchard's (1985) overlapping generation model with the positive probability of death and complement it to include also habit formation. After aggregating, we end up with a dynamic consumption function where current consumption depends on one period lead and lag of consumption as well as on fundamental variables, i.e. on real non-human wealth, current-period real labor income and the real interest rate as well as on the determinants of time-varying precautionary saving. As the estimated specification can be presented in terms of the parameters of the underlying utility function, death probability and the determinants of precautionary saving, the empirical relevance of many underlying hypothesis can be tested and the consumption function containing quite general dynamics can be estimated. We apply the derived consumption function to aggregate U.S. consumption data. We find that the specified consumption function fits the data well and the estimates of the underlying deep parameters are reasonable.

The structure of this paper is as follows. Section 2 discusses the effects of income uncertainty, front-loaded information and habit persistence on the life-time budget constraint. In Section 3 we derive the solved-out aggregate consumption function. The U.S. aggregate consumption data is presented in Section 4, our empirical results are presented in Section 5 and Section 6 concludes.

2. The life-time budget constraint under uncertain life-time, uncertain labor income and front loaded information

All consumers face uncertainty regarding their future labour income as well as the length of their life-time. We assume that the consumers solve their intertemporal utility maximization problem sequentially so that in the first stage they evaluate their risk adjusted expected life-time wealth. Thereafter, by using the risk-adjusted wealth as a resource constraint they solve their optimal planned consumption path.

2.1. Risk adjusted life-time wealth

As in Blanchard (1985), each agent faces a constant probability of death π.[4] Agents are selfish in a sense that they have no bequest motive. To remove involuntary bequests resulting from uncertainty about death, access to fair annuity markets are assumed, i.e. agents can

[4] In the limiting case of $\pi = 0$, everything that follows collapses to the conventional infinite horizon framework of a single representative agent.

contract with life insurance companies to receive a payment contingent on their death.[5] Denoting by $V_{k,t}$ the end of period t real non-human wealth of an agent born in the beginning of period k, agents in the k^{th} cohort may contract to receive a payment $(\pi/(1-\pi))V_{k,t-1}$, if they do not die and pay $V_{k,t-1}$, if they do die in the beginning of period t. Hence, no bequests to younger generations are left. Now the dynamic budget constraint of identical individuals in the k^{th} cohort is:

$$V_{k,t} = (1+r_t)\left(\frac{V_{k,t-1}}{1-\pi} + y_{k,t} - c_{k,t}\right) \tag{1}$$

In (1) $y_{k,t}$ is labour income (net of taxes minus transfers), $c_{k,t}$ is consumption and r_t is the real interest rate in period t. For each consumer the dynamic budget constraint (1) implies the following (ex-post) intertemporal budget constraint:

$$\sum_{i=0}^{\infty} R_{t,t+i}(1-\pi)^i c_{k,t+i} = \frac{V_{k,t-1}}{1-\pi} + \sum_{i=0}^{\infty} R_{t,t+i}(1-\pi)^i y_{k,t+i} \tag{2}$$

where $R_{t,t+i} = (1+r_t)\Big/\prod_{j=0}^{i}(1+r_{t+j})$.

We treat the real interest rate deterministic and, for notational reasons, constant in our theoretic analysis. Accordingly $R_{t,t+i} = R^i$ when $i \geq 1$. However, to allow the separation of the equilibrium long-term interest rate effect from that of the short-term rate in our empirical application, we denote $(1-\overline{R})^{-1} = \sum_0^\infty R^i \left(= \sum_0^\infty R_{t,t+i}\right)$.

Now, with an uncertain income stream (2) can be written in the form:

$$\sum_{i=0}^{\infty} R^i (1-\pi)^i c_{k,t+i} = \frac{V_{k,t-1}}{1-\pi} + \underbrace{\sum_{i=0}^{\infty} R^i (1-\pi)^i E_t y_{k,t+i}}_{H_{k,t}^E} + \sum_{i=1}^{\infty} R^i (1-\pi)^i v_{k,t+i} = W_{k,t} \tag{3}$$

where $v_{k,t+i} = y_{k,t+i} - E_t y_{k,t+i}$, E_t is the expectation operator, $H_{k,t}^E$ expected human wealth and $W_{k,t}$ total (ex post) wealth . We see that (3) contains also the present value of stochastic income innovations $v_{k,t+i}$, the size of which is not known ex ante. However, in the following we show that a deterministic risk-adjusted life time-wealth equivalent to the ex-ante uncertain wealth can be derived, if the stochastic properties of the income generation process are

[5] An alternative and also actuarially fair way to distribute the wealth of deceased agents would be to assume 100 per cent inheritance tax, the revenue of which is allocated among those living as lump-sum transfers or via lotteries. One could envisage that this allocation mechanism mimics intergenerational bequests under the assumption that all agents have equal probability to inherit. Nevertheless, we follow Blanchard (1985) and assume perfect annuity markets, because, as documented e.g. by Warshawsky (1988), U.S. households have had access to private annuity markets, at least, since early 1900s. However, apparently due to the adverse selection problem, the workings of these markets have been imperfect and the size small as discussed by Friedman and Warshawsky (1988, 1990). We return to the empirical implications of this issue in Section 4.

known. We also show that this risk-adjusted wealth is separable to expected wealth component ($V_{k,t-1}/(1-\pi)+H_{k,t}^{E}$) and the risk-adjustment component, i.e. the present value of the planned precautionary saving, if the functional form of the utility function is CRRA (or CARA). However, before that we discuss the information set available to the consumer concerning her future income realisations. In that regard we deviate from the convention that the consumer has either perfect foresight or that, besides the knowledge of the stochastic properties of the past income stream, the consumer has no additional information on her future income realisations. Following Willman (2003) we assume that the consumer may have a lot of period-specific information concerning future income changes although this information may be strongly front-loaded.

2.2. Front loaded information and the expected human wealth

Denote by $x_{k,t+i}$ the set of the ex-ante possible outcomes of the future realizations of income changes $\Delta y_{k,t+i}$. Assume that there are two kinds of information concerning future income realizations, i.e. information on the stochastic properties of the past income stream, which for simplicity is assumed to follow a random walk, and period-specific information on future income realizations. The amount of period-specific information concerning the period *t+i* is measured by the parameter γ_{t+i}, which can range from zero to one. The closer to zero (one) γ_{t+i}, the less (more) the amount of period-specific information is available on period *t* concerning the income realization on period *t+i*. Denote by $f(x_{k,t+i}\,|\,\gamma_{t+i})$ the conditional probability density function containing both time series and period specific information available to the consumer on period *t* about future realization $\Delta y_{k,t+i}$. It is apparent that the distribution function $f(\cdot|\gamma_{t+i})$ is asymmetric being the more skewed towards the actual realisation, the closer to unity γ_{t+i} is As suggested by Marron and Wand (1992) the asymmetric density function $f(\cdot|\gamma_{t+i})$ can quite flexibly be presented as a mixture of two normally distributed density function as follows:

$$f(x_{k,t+i}\,|\,\gamma_{t+i}) = \gamma_{t+i}g(x_{k,t+i}) + (1-\gamma_{t+i})h(x_{k,t+i}) \qquad (4)$$

where the density function $g(x_{k,t+i})$, related to the period specific information, is normally distributed with mean $\Delta y_{k,t+i}$ and variance $(1-\gamma_{t+i})\sigma_g^2$ and the density function $h(x_{k,t+i})$, related to the random walk realizations, is normally distributed with mean zero and variance σ_h^2. The weights γ_{t+i} and $1-\gamma_{t+i}$ show the shares of probability mass related to period-specific information and to the random-walk process, respectively. Under these assumptions it is straightforward to show that the mean and variance of the function $f(\cdot|\gamma_{t+i})$ are:

$$\int_{-\infty}^{\infty} x_{k,t+i} f(x_{k,t+i} \,|\gamma_{t+i}) dx_{k,t+i} \quad = \gamma_{t+i}\Delta y_{k,t+i} \tag{5a}$$

$$\int_{-\infty}^{\infty} (x_{k,t+i} - \gamma_{t+i})^2 f(x_{k,t+i} \,|\gamma_{t+i}) dx_{k,t+i} = (1-\gamma_{t+i})[\sigma_h^2 + \gamma_{t+i}(\sigma_g^2 + \Delta y_{k,t+i})] \tag{5b}$$

We see that, when $\gamma_{t+i} \to 1$, then the mean of $f(\cdot|\gamma_{t+i})$, i.e. the expected income realisation, approaches to $\Delta y_{k,t+i}$ and the variance to zero, i.e. to the limiting case of perfect foresight. Correspondingly, when $\gamma_{t+i} \to 0$, then $f(\cdot|\gamma_{t+i})$ coincides with $h(\cdot)$ with zero mean and variance σ_h^2.

To be empirically applicable, one additional assumption concerning the distribution of the period-specific information over the planning horizon is required. For that purpose, it is natural to assume that the information content is wider concerning income changes in the near future than regarding longer planning horizons. To be more explicit we assume that the information parameter γ_{t+i} is determined by the following simple process $\gamma_{t+i} = \gamma^i$. Now on the basis of (5a) $E_t y_{k,t+i} = y_{k,t} + \sum_{j=1}^{i} \gamma^j \Delta y_{k,t+j}$ and, hence, in (3) expected human wealth can be defined in terms of future realisations as follows:

$$H_{k,t}^E = \sum_{i=0}^{\infty} R^i (1-\pi)^i \left(y_{k,t} + \sum_{j=1}^{i} \gamma^j \Delta y_{k,t+j} \right) = \frac{1-(1-\pi)\overline{R}\gamma}{1-(1-\pi)\overline{R}} \sum_{i=0}^{\infty} [R(1-\pi)\gamma]^i y_{k,t+i} \tag{6}$$

The important property of relation (6) is that the average size of expected human wealth $H_{k,t}^E$ is practically unaffected by the size of the information parameter γ. This results from the fact that with $\overline{R} = R$ the term $\frac{1-(1-\pi)\overline{R}\gamma}{1-(1-\pi)\overline{R}} \sum_{i=0}^{\infty} [R(1-\pi)\gamma]^i = 1/[1-(1-\pi)R]$, which is independent from γ. Therefore, while a decrease in γ reduces the size of the discounted income term $\sum_{j=1}^{i} [R(1-\pi)\gamma]^j y_{k,t+j}$ and makes the dependency of $H_{k,t}^E$ from future income stream more front loaded, it also increases the size of the scaling factor $\frac{1-(1-\pi)\overline{R}\gamma}{1-(1-\pi)\overline{R}}$, which compensates the effect of γ on the discounted income term.

2.3. Expected risk-adjusted wealth

We assume that, in making risk adjustment to her uncertain expected life-time wealth the consumer applies the conventional Arrow-Pratt approach to risk, i.e. the consumer defines the deterministic equivalent for which she would be willing to exchange her expected, but risky, life-time wealth. However, in deriving the relevant risk adjustment to the wealth it matters how often the consumer repeats her maximization process. If the consumer maximized her intertemporal utility only once, for instance, in the beginning of period *t* basing on the information available at that point of time, then the properties of the discounted term $\sum_{i=1}^{\infty}[R(1-\pi)]^{i} v_{k,t+i}$ would summarize the uncertainty related to wealth. However, it is clear that consumers repeat the optimization task quite frequently and for expositional convenience we assume that the consumer repeats her utility maximization process in the beginning of each period. Then the statistical properties of the expected wealth based on differences in information content between two successive periods determine the relevant uncertainty measure of wealth.

For that purpose, we define the period *t* expected human wealth conditional for both the information available in the beginning of period *t* and in the beginning of period *t+1*:[6]

$$H_{k,t}^{E} = y_t + R(1-\pi)E_t H_{k,t+1} \tag{7}$$

$$E_{t+1}H_{k,t} = y_t + R(1-\pi)E_{t+1}H_{k,t+1} \tag{8}$$

The difference of (8) and (7) determines the impact of the increment of period *t+1* information on period t expected human wealth, if it were available already in the beginning of period *t* , i.e.[7]

$$E_{t+1}H_{k,t} - H_{k,t}^{E} = R(1-\pi)\left[E_{t+1}H_{k,t+1} - E_t H_{k,t+1}\right] \tag{9}$$

It is straightforward to see that, if no period specific information is available $(\gamma = 0)$ and income in log terms follows random walk (or almost identically $\Delta y_{k,t+1} / y_{k,t} = \varepsilon_{k,t+1}$ with

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[6] It is quite straightforward to extend the analysis to account for the possibility that the time frequency in data is higher (e.g. one quarter) than the length of period (n quarters) that is relevant for determining the size of risk adjustment to human wealth. In this case

$$\begin{aligned} E_{t+n}H_{k,t} &= H_{k,t}^{E} + \sum_{i=0}^{n-1}[R(1-\pi)]^{i}\left(1-\gamma^{i}\right)\Delta y_{t+i} + R^{n}(1-\pi)^{n}\left[E_{t+n}H_{k,t+n} - E_t H_{k,t+n}\right] \\ &= H_{k,t}^{E}\left\{\begin{aligned} &1 + \frac{1-R(1-\pi)}{1-R(1-\pi)\gamma}\sum_{0}^{n-1}[R(1-\pi)]^{I}\left(1-\gamma^{i}\right)\alpha_{i-1}\varepsilon_{t+i} + \\ &\frac{R^{n}(1-\pi)^{n}}{1-R(1-\pi)\gamma}\left[\sum_{0}^{n-1}\frac{\left(1-\gamma^{1+i}\right)}{(R(1-\pi)\gamma)^{i}}\alpha_i\varepsilon_{t+1+i} + \frac{1-\gamma^{n}}{(R(1-\pi)\gamma)^{n-1}}\sum_{0}^{\infty}\alpha_{n+i}\varepsilon_{t+n+1+i}\right]\end{aligned}\right\} \end{aligned} \tag{12a}$$

If, n=1, (12a) reduces to equation (12).

[7] We could also say that whilst H_t^E is period t expectation, $E_{t+i}H_t$ is its period *t+i* realization and H_t is its realization when $i \to \infty$.

$\varepsilon_{k,t} \sim N(0, \sigma_\varepsilon^2)$), then the right-hand side of (9) equals $\frac{R(1-\pi)y_{k,t}}{1-R(1-\pi)}\varepsilon_{k,t+1} =$ $R(1-\pi)H_{k,t}^{E}\varepsilon_{k,t+1}$ and, hence, (9) reduces to a random walk as follows:

$$E_{t+1}H_{k,t} = H_{k,t}^{E}\left(1 + R(1-\pi)\varepsilon_{k,t+1}\right) \tag{10}$$

However, if the consumer has also period specific information, as we discussed in previous section, then equation (6) implies that the square bracket term on the right hand side of (9) is equal to:

$$\begin{aligned} E_{t+1}H_{k,t+1} - E_t H_{k,t+1} &= \sum_{i=0}^{\infty}\left[R(1+\pi)\right]^i y_{k,t+1} + \sum_{j=1}^{i}\gamma^j y_{k,t+j}\varepsilon_{k,t+1+j} \\ &\quad - \sum_{i=0}^{\infty}\left[R(1+\pi)\right]^i y_{k,t}\left(1+\gamma\varepsilon_{k,t+1}\right) + \sum_{j=1}^{i}\gamma^j y_{k,t+j}\varepsilon_{k,t+1+j} \\ &= \frac{1-\gamma}{1-\overline{R}(1-\pi)}\sum_{i=0}^{\infty}\left[R(1+\pi)\right]^i y_{k,t+i}\varepsilon_{k,t+1+i} \end{aligned} \tag{11}$$

Now, with the expected human wealth $H_{k,t}^{E}$ being determined by (6), equation (9) can be written in the form:

$$E_{t+1}H_{k,t} = H_{k,t}^{E}\left(1 + \frac{\overline{R}(1-\pi)(1-\gamma)}{1-\overline{R}(1-\pi)\gamma}\sum_{i=0}^{\infty}\alpha_{k,i}\varepsilon_{k,t+1+i}\right) \tag{12}$$

where $\alpha_{k,i} = \frac{\left[R(1+\pi)\gamma\right]^i y_{k,t+i}}{\sum_{i=0}^{\infty}\left[R(1+\pi)\gamma\right]^i y_{k,t+i}}$ and, hence, $\sum_{i=0}^{\infty}\alpha_{k,i} = 1$. It is straightforward to see that (12) reduces to (10), when no period specific information is available $(\gamma = 0)$. Likewise, we see that the more period specific information is available (the closer to unity γ), the smaller the addition of information and its impact on the expected human wealth. With $\gamma=1$, (12) coincides with perfect foresight.

Equations (12) and (10) define the expected human wealth including the related uncertainty in terms of one period addition in information content. The next step is to derive the deterministic wealth equivalent, $\widetilde{W}_{k,t} = V_{k,t-1}/(1-\pi) + \widetilde{H}_{k,t}$, for which the consumer would be willing to exchange her wealth containing uncertain human wealth as defined by (12), i.e.

$$E_{t+1}W_{k,t} = \underbrace{V_{k,t-1}/(1-\pi) + H_{k,t}^{E}}_{W_{k,t}^{E}}\left(1 + \sum_{i=0}^{\infty}\phi_{k,i}\varepsilon_{k,t+1+i}\right).$$

where $\phi_{k,i} = \frac{\overline{R}(1-\pi)(1-\gamma)}{1-\overline{R}(1-\pi)\gamma}\alpha_{k,i}$ with $0 \le \sum_{i=0}^{\infty}\phi_{k,i} \le R(1-\pi)$.

Utility equivalence, using the second order Taylor expansion, requires:

$$u\left(\widetilde{W}_{k,t}\right)= E_t\left\{u\left(W_{k,t}^E + H_{k,t}^E \sum_{i}^{\infty} \phi_{k,i}\varepsilon_{k,t+1+i}\right)\right\}$$

$$\approx u\left\{W_{k,t}^E\right\}+\frac{1}{2}u''\left\{W_{k,t}^E\right\}E_t\left\{\left(H_{k,t}^E \sum_{i=0}^{\infty} \phi_i\varepsilon_{k,t+1+i}\right)^2\right\} \tag{13}$$

where $u(\cdot)$ denotes the utility function and $u''(\cdot)$ its second derivative. Assume the utility function to be that of the CRRA:

$$u\left(\widetilde{W}_{k,t}\right)=\begin{cases}\dfrac{1}{1-\theta}\widetilde{W}_{k,t}^{1-\theta} & 0<\theta<1 \quad 0 \text{ or } \theta>1\\ \log\left(\widetilde{W}_{k,t}\right) & \theta=1\end{cases} \tag{14}$$

where θ is the coefficient of constant relative risk aversion. Now equation (10) implies the following solution for the risk-adjusted human wealth:[8]

$$\widetilde{H}_{k,t}=\begin{cases} H_{k,t}^E\left\{1-\left(\dfrac{H_{k,t}^E}{W_{k,t}^E}\right)^{-1}\left[1-\left(1-\dfrac{(1-\theta)\theta}{2}\left(\dfrac{H_{k,t}^E}{W_{k,t}^E}\right)^2\sigma_\varepsilon^2\sum_{i=0}^{\infty}\phi_i^2\right)^{\frac{1}{1-\theta}}\right]\right\} & ;0<\theta<1 \quad \text{or} \quad \theta>1\\ H_{k,t}^E\left[1-\left(\dfrac{H_{k,t}^E}{W_{k,t}^E}\right)\dfrac{\sigma_\varepsilon^2}{2}\sum_{i=0}^{\infty}\phi_i^2\right] & ;\theta=1\end{cases}$$

$$=H_{k,t}^E\Lambda_{k,t}\left(\theta,\frac{H_{k,t}^E}{W_{k,t}^E},\sigma_\varepsilon^2\sum_{i=0}^{\infty}\phi_i^2\right) \tag{15}$$

Relation (15) shows that the risk adjusted human wealth can be presented as the multiplicand $\Lambda_{k,t}$ of the expected human wealth , which, except the special case of the logarithmic utility function, is nonlinear function of risk aversion, the variance of stochastic labour income and the human wealth to total wealth ratio. These results are in line with the analytical results by Skinner (1988) and the numerical simulation results by Zeldes (1989) for a consumer with a

[8] In ending up the solution of $\widetilde{H}$ corresponding the logarithmic utility function we applied the approximation $\log\left(\widetilde{W}/W^E\right)\approx\dfrac{\widetilde{H}-H^E}{W^E}$.

finite life-time.[9] Table 1 shows some suggestive numbers for Λ_k, when no period specific information exists and $R = 0.96$.

Table 1. Some examples of Λ_k

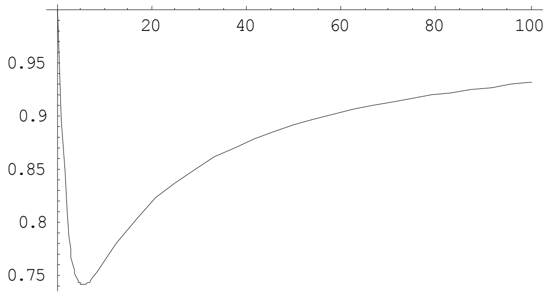
	$\theta = 1$	$\theta = 3$	$\theta = 5$	$\theta = 10$
$\sigma_\varepsilon = 0.15$ $H_k^E / W_k^E = 1$	0.990	0.970	0.954	0.929
$\sigma_\varepsilon = 0.3$ $H_k^E / W_k^E = 1$	0.959	0.895	0.860	0.841
$\sigma_\varepsilon = 0.5$ $H_k^E / W_k^E = 1$	0.891	0.769	0.741	0.763
$\sigma_\varepsilon = 0.5$ $H_k^E / W_k^E = 0.75$	0.916	0.798	0.750	0.744
$\sigma_\varepsilon = 0.5$ $H_k^E / W_k^E = 0.5$	0.943	0.847	0.785	0.735

Table 1 shows that the size of risk adjustment to human wealth depends positively on the size of risk aversion θ and income uncertainty σ_ε. This is also the case with respect to the human wealth to total wealth ratio when the size of relative risk aversion is in the range of 1-5, which is conventionally thought to represent the realistic range of risk aversion. Somewhere above this range a further increase in the ratio H_k^E / W_k^E starts decreasing the risk adjustment. For instance, in Table 1 in the range of $0 < \theta \le 5$ the increase of the ratio H_k^E / W_k^E from 0.5 (the bottom row) to unity (the third row), the size of risk adjustment increases, while with $\theta = 10$ it decreases. This reflects the fact that risk adjustment depends highly nonlinearly on

[9] We can also apply the sequential approach to the consumer's utility maximization problem under CARA preferences and compare the implied risk adjustment to that given by the stochastic optimization approach. These approaches results in almost identical risk adjustments. For instance, if income generating process follows a random walk for levels, $y_{k,t+1} = y_{k,t} + \varepsilon_{k,t+1}$, the constant absolute risk aversion is θ_A and the subjective discount rate equals the risk-free real interest rate, then the size of risk adjustment to wealth equals $\left(\frac{R}{1-R}\right)\frac{\theta_A}{2}\sigma_\varepsilon^2$, when the stochastic optimization technique is used. (see Caballero, 1990). In applying our sequential approach this adjustment is otherwise the same but multiplied by R (marginally below unity).

the size risk aversion and it has a maximum with a finite value of risk aversion parameter θ (typically in the neighbourhood of 10) as is shown by Graph 1, which corresponds to the third row of Table 1.

Figure 1. The dependency of Λ_k (vertical axis) on θ (horizontal axis).



Now we can present the risk-adjusted life-time budget constraint, which is relevant for the determination of the consumer's optimal (planned) consumption path, as follows:

$$\sum_{i=0}^{\infty} R^i (1-\pi)^i c_{k,t+i} = \frac{V_{k,t-1}}{1-\pi} + \Lambda_{k,t} H_{k,t}^E = \widetilde{W}_{k,t} \qquad (16)$$

2.4. The life-time budget constraint with habit formation

We next modify the life-time budget constraint to account for habit formation. The habit formation implies non-separability in utility over time. In internal-habit models, habit depends on a household's own past consumption and the household takes account of this when choosing how much to consume as e.g. in Muellbauer (1988), Muellbauer and Lattimore (1995), Sundaresan (1989) and Constantinides (1990).[10] Hence, habit-formation introduces "reference dependence" into a conventional expected utility analysis, i.e. in common with prospect theory of Kahneman and Tversky (1979) and Tversky and Kahneman (1991), the carrier of utility is, besides the level of consumption, a consumption gain or loss compared to some reference level of consumption. The simplest treatment of habit is to replace the $c_{k,t+i}$ argument in the utility function by $c^*_{k,t+i} = c_{k,t+i} - ac_{k,t+i-1}$, where the parameter a measures habit persistence with $a>0$ and the term $ac_{k,t+i-1}$ is the time-varying habit level of

[10] In external-habit models such as those in Abel (1990, 1999) and Campbell and Cochrane (1995), habit depends on aggregate consumption being independent from an agent's own decisions.

consumption.[11] Following Muellbauer (1988) (see also Appendix 1), after substituting $c^*_{k,t+i} + ac_{k,t+i-1}$ for $c_{k,t+i}$, the life-time resource constraint (16) can be modified to the form:

$$\sum_{i=0}^{\infty}\left[R(1-\pi)\right]^i c^*_{k,t+i} = \left(1-(1-\pi)a\overline{R}\right)\widetilde{W}_{k,t} - a\cdot c_{k,t-1} \tag{17}$$

3. Derivation of the aggregate consumption function

In this section we derive the optimal consumption rule for consumers in cohort *k*. Then we aggregate across cohorts and show that aggregate consumption can be expressed in terms of aggregate non-human and risk-adjusted human wealth.

3.1. The optimal consumption rule and the aggregate consumption function

Corresponding our earlier risk-analysis we assume that the consumer's preferences can be described by the CRRA utility function. Hence, subject to the budget constraint (17), each consumer belonging to the cohort *k* maximizes her expected intertemporal utility:

$$\max E_t\left[U_{k,t}\right] = E_t\left(\sum_{i=0}^{\infty}\left(\frac{1}{1+\rho}\right)^i u\left(c^*_{k,t+i}\right)\right) = \sum_{i=0}^{\infty}\left(\frac{1-\pi}{1+\rho}\right)^i \frac{\left(c^*_{k,t+i}\right)^{1-\theta}}{1-\theta} \tag{18}$$

where parameter ρ is the rate of subjective time preference. The resulting first-order condition of maximization is:

$$c^*_{k,t+i} = c^*_{k,t}\left((1+\rho)^i R^i\right)^{-\frac{1}{\theta}} \tag{19}$$

After substituting this condition to the risk-adjusted life-time budget constraint (17) we obtain:

$$c_{k,t} = \kappa\left[1-(1-\pi)a\overline{R}\right]\left(\frac{V_{k,t-1}}{1-\pi} + \Lambda_{k,t}H^E_{k,t}\right) + a(1-\kappa)c_{k,t-1} \tag{20}$$

[11] The requirement $c^*_{k,t} > 0$ sets an upper bound to the habit parameter, i.e. $a < c_{k,t}/c_{k,t-1}$, which in micro-data can be well below unity. However, because our main focus is on macro-level consumption and our aggregation is based on equally distributed labor income, we think that the appropriate reference for the feasible upper bound of a is the minimum plausible relative change of aggregate consumption, which in a quarterly data is quite close to unity. A corresponding multiplicative power function formulation used e.g. by Fuhrer (2000) avoids the upper bound problem and it would be an appropriate choice, if the log approximation of the life-time budget constraint were used as in Campbell and Mankiw (1989). However, the log approximation, besides being problematic in aggregation, requires that the consumption-wealth ratio is constant (stationary) over the life-cycle, which condition violates both micro-data evidence and theoretic implications of overlapping generation models.

$$\text{where } \kappa = \begin{Bmatrix} 1 - \dfrac{(1-\pi)(1+\bar{r})^{\frac{1}{\theta}-1}}{(1+\rho)^{\frac{1}{\theta}}} & , \rho \neq \bar{r} \text{ and } \theta \neq 1 \\ \dfrac{\rho+\pi}{1+\rho} \quad , & , \rho = \bar{r} \text{ or } \theta = 1 \end{Bmatrix} \approx \frac{1}{\theta}\rho + \left(1 - \frac{1}{\theta}\right)\bar{r} + \pi$$

We see that via $\Lambda_{k,t}$ the consumption function (20) implies precautionary saving out of expected human wealth $H^E_{k,t}$.

Assume next that the size of each cohort when born is π. Accordingly, in period t the size of cohort born in period k is $\pi(1-\pi)^{t-k}$ and the size of population is $\sum_{k=-\infty}^{t}\pi(1-\pi)^{t-k} = 1$. This results in the following aggregation rule: $z_t = \sum_{k=-\infty}^{t}\pi(1-\pi)^{t-k} z_{k,t}$ and $z_{t-1} = \sum_{k=-\infty}^{t-1}\pi(1-\pi)^{t-1-k} z_{k,t-1}$ with $z = \{c, H^E, \Lambda, V\}$. Following Blanchard (1985) we also assume that, except for across generation differences in stochastic income innovations, labor income is equally distributed across population. Now the aggregation of (20) gives:[12]

$$c_t = \kappa\left[1-(1-\pi)a\bar{R}\right]\left(\frac{V_{t-1}}{1-\pi} + \Lambda_t H^E_t\right) + a(1-\kappa)c_{t-1} \tag{21}$$

An interesting implication of aggregation is that, although over the life-cycle of an individual consumer $\Lambda_{k,t}$ is time varying and strongly related to development of the expected human to total wealth ratio $\left(H^E_{k,t}/W^E_{k,t}\right)$, this does not necessarily imply that on the aggregate level Λ_t should be non-stationary. In fact, under conventional assumptions of overlapping generation modelling, saving and wealth accumulation across cohorts follow the same life-time profile (except for discrepancies resulting from stochastic income innovations) and, hence, the aggregate $\left(H^E_t/W^E_t\right)$ remains practically constant. Therefore, in the following our maintained hypothesis is that Λ_t is stationary.[13]

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[12] To end up with (21) the Taylor approximation:

$$\sum_{k=-\infty}^{t}\pi(1-\pi)^{t-k}\Lambda_{k,t}H^E_{k,t} \approx \Lambda_t H^E_t + \sum_{k=-\infty}^{t}\pi(1-\pi)^{t-k}\Lambda_t\left(H^E_{k,t} - H^E_t\right) + \sum_{k=-\infty}^{t}\pi(1-\pi)^{t-k}H^E_t\left(\Lambda_{k,t} - \Lambda_t\right) = \Lambda_t H^E_t$$

is used.

[13] In general, however,, it is possible that variation in the aggregate non-human to human wealth ratio affects the marginal propensity to consume out of expected human wealth. If true, this would introduce an additional nonlinearity into the wealth channel. However, besides being difficult to identify, the impact of this nonlinearity on aggregate consumption can be thought to be of second-order magnitude. This is very different from the role of the non-human to human wealth ratio in explaining cross-sectional differences in consumption, where this ratio can be thought to play the role of first order importance.

To express (21) in terms of observable variables, we forward (21) by one period and take expectations. Thereafter we multiply it by $R_t(1-\pi)\gamma$, subtract it from (21) and utilise the aggregate-level dynamic budget constraint $V_t = R_t^{-1}\left[V_{t-1} + y_t - c_t\right]$ implied by (1) (compare to Blanchard, 1985; Gali, 1990). We end up with equation:

$$\left\{1+\gamma\left[(1-\pi)(1-\kappa)aR_t - \kappa\left(1-(1-\pi)a\overline{R}\right)\right]\right\}c_t = (1-\pi)\gamma\, R_t E_t c_{t+1} + a(1-\kappa)c_{t-1}$$

$$+\left(1-(1-\pi)\overline{R}a\right)\kappa\left\{\left(\frac{1}{1-\pi}-\gamma\right)\left(V_{t-1}+y_t\right)+\left(\frac{\Lambda_t\left(1-(1-\pi)\overline{R}\gamma\right)}{1-(1-\pi)\overline{R}}-\frac{1}{1-\pi}\right)y_t\right\} \qquad (22)$$

Equations (22) covers a wide range of alternative cases. If $\pi = 0$, the overlapping generations framework reduces to that of infinitely living representative agent framework. If no habit formation exists $(a = 0)$, then consumption is determined by the beginning of period financial wealth and the future income stream. Consumption is the more forward-looking, the closer to unity the information parameter γ. In the opposite polar case with $\gamma = 0$ consumers have no period-specific information and consumption is determined by the beginning of period financial wealth, current labor income and possibly the lagged consumption.

In the case of no habit persistence and perfect information, i.e. $a = 0$ and $\theta = \gamma = 1$, implying that also $\Lambda = 1$, equations (21) reduce to the aggregate discrete-time version of Blanchard's (1985) overlapping generation consumption function including besides the next period consumption also the beginning of period non-human wealth. The introduction of wealth effect reflects the fact that due to the positive probability of death the expected future income is discounted by higher than the market rate of interest and accounts for feed-back effects from precautionary savings. The precautionary savings effect is strengthened the more the risk adjustment parameter Λ is below unity in (22). The downward deviation of γ from unity, i.e. the period-specific information is incomplete and front-loaded, introduces current period labor income into the relation and the model is able to display "excess sensitivity" of consumption to income (as well as its mirror image of excess smoothness). The habit persistence, i.e. $a > 0$, introduces lagged consumption into the specification, which may markedly increase the ability of the model to display "excess smoothness" and, in general, the ability to track the short-run dynamics of the aggregate data.

4. Description of the data

In our empirical estimation we use the quarterly data of the U.S. economy covering the period from 1952q1 to 2004q4. Appendix 2 contains a detailed description of the data and their sources.

A well known complication in applying the Permanent Income/Life-Cycle theory to actual data is that these theories do not apply to purchases of durable goods. Therefore, following general practice durables expenditures are excluded in our definition of private consumption, which consists of consumption expenditures on nondurable goods and services. However, as durables are included by households' budget constraint, the consistent treatment of durables requires that they are accounted for by nonhuman real wealth. Hence, our nonhuman wealth equals the end of period household net worth, which includes all financial wealth, housing wealth and consumer durables. Our labor income is measured net of taxes and contains also the labor-income component of the proprietors' (self-employed) income.[14] As e.g. in Blanchard (1997) and Gollin (2002) this is done by using compensation per employee as a shadow price of labor of self-employed workers. Corresponding the consumption concept nominal labor income and non-human wealth are deflated by the deflator of the consumption of nondurable goods and services. Our interest rate variable is the 3-month U.S. Treasury bill rate.

5. Estimation results

Before estimation our dynamic specifications (22) it is useful to examine, whether our data fulfills the long-run co-integration properties implied by the underlying theory. Therefore, we first estimate the long-run equilibrium relation between consumption, labor income and non-human wealth.

5.1. Steady-state consumption equation

The steady state relation implied by specification (21), with stationary Λ_t, can be written in the form:[15]

$$c_t = \frac{\kappa\Lambda}{\bar{r}+\pi} y_t + \kappa V_{t-1} \qquad (23)$$

Hence, corresponding to (23) we estimate the following long-run relation:

$$\frac{c_t}{y_t} = \alpha + \kappa \frac{V_{t-1}}{y_t} + \nu_t . \qquad (24)$$

[14] Our definition of labor income deviates form that used e. g. by Lettau and Ludvigson (2004) and Palumbo et al. (2006) and Rudd and Whelan (2006), who did not include the labor income of self-employed persons. However, as consumption covers the consumption of self-employed persons, also labor income should account for the labor income component of self-employed persons. This is also in accordance with general practice to calculate the labor income share, eg. in the context of estimating the Cobb-Douglas production function, where the part of proprietors' (self-employed) income is classified as labor income. See e.g. a discussion by Krueger (1999) and Gollin (2002).

[15] In ending up with (23) we used the approximation $1-R(1-\pi)\approx r+\pi$

Co-integration requires that the residual v_t is stationary. If this requirement is fulfilled, then parameter κ gives the estimate for the long-run MPC out of non-human wealth. Further, under a generally used assumption of $\rho = \bar{r}$ the death probability (or the expected remaining life-time) can be solved as a difference κ and $\bar{r}$.[16] In addition, α reduces unambiguously to the estimate of risk adjustment parameter Λ.

Estimation results of (24) are presented in the first column of Table 2.[17] As the Dickey-Fuller t ratio test statistic (DF-ttest) in Column 1 indicates, the data does not fulfil the co-integration requirements. However, a closer examination of the residual of Column 1 equation shows very interesting time-profile (see the upper panel of Figure 2). There is quite abrupt upward shift in the level of residual in 1974 that the wealth-income ratio is not able to explain. [18]

In order to be able to explain the observed level shift in the data, the specifications (24) should contain a corresponding downward shift in the risk adjustment parameter Λ, which was assumed to be constant in estimating the equation of Column (1). This raises the question, is there anything that would explain a permanent decrease in average income risk since the mid-70s. In fact, the answer to this question is positive. As discussed by Feldstein (1974) and Feldstein and Seligman (1981), there was a substantial increase in private pension coverage and the funding requirements in 1974, when Employee Retirement Income Security Act (ERISA) was imposed. Several researchers have documented the importance of uncertain lifespan on life-cycle consumption, either in theoretical models (Yaari, 1965; Davies, 1981; Abel, 1985; Hubbard, 1984; and Kotlikoff et al., 1986) or empirical studies (Hubbard and Judd, 1987; and Hurd 1989), when the working of annuity insurance market is imperfect. Hence, in the absence of a strong bequest motive, perfecting insurance arrangements, as apparently the increased pension coverage of ERISA did, can markedly lower precautionary saving. Regarding the working of private annuity markets in the United States, as shown by Friedman and Warshawsky (1988, 1990), they are small and imperfect, i.e. premiums above actuarially fair prices are high. Hence, under these conditions, the improved coverage of the private pension system in 1974 should have decreased precautionary saving. To account for

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[16] With $\rho = \bar{r}$, $\kappa \approx \frac{1}{\theta}\rho + \left(1 - \frac{1}{\theta}\right)\bar{r} + \pi = \bar{r} + \pi$.

[17] Table 2 presents only DF-ttest statistics, because the application of the augmented DF-test (allowing for several lags) did not improve the statistical significance of the test statistic.

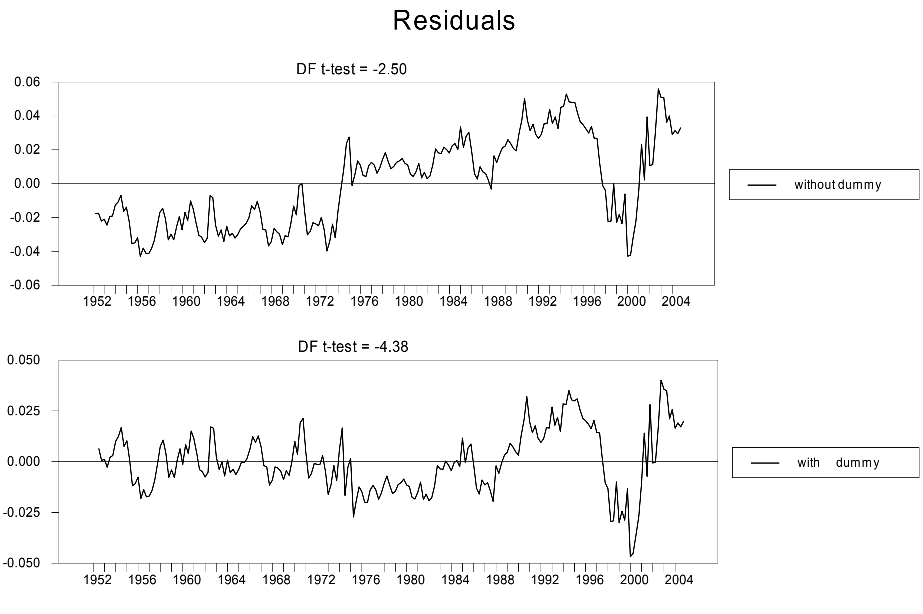
[18] Also Rudd and Whelan (2006) identify the lack of cointegration between log consumption, log wealth and log labor income. However, in their data the residual does not show as clear-cut regime shift as in our data. As discussed in the footnote 18, the main difference between their data and ours is that their labor income did not contain the labor income of self-employed persons. This mainly explains the more abrupt level shift of residual in our data than in theirs. The fact that (23) is specified for consumption-income and wealth-income ratios (instead for their logarithms) plays, at most, a minor role. Anyway, in estimating (23) with the regime-shift dummy included, proved to be statistically significant and improved essentially the residual properties, even if self-employed labor income was excluded from total labor income.

Table 2. **Empirical estimates of long-run equation**

	1952:1-2004:4		1974:3-2004:4
	(1)	(2)	(3)
α	0.5739 (0.0174)	0.6370 (0.0107)	0.6336 (0.0133)
α_{DUM}	-	-0.0441 (0.0022)	-
κ	0.0164 (0.0008)	0.0144 (0.005)	0.0146 (0.0006)
Annualised mpc_V	0.066	0.058	0.058
$\overline{R}^2$	0.689	0.893	0.844
DF-ttest	-2.50	-4.38***	-2.95

*** denotes that the DF t ratio test is significant at a significance level of 1% .

Figure 2. The residuals of columns (1) and (2) equations of Table 2.



the shift in the average propensity to consume we constructed a level-shift dummy (1 until 1974Q2 and 0 thereafter) and added it to the estimated long-run equation.

These results are presented in Column (2) of Table 2 and the corresponding residual in the lower panel of Figure 1. Now the unit root of residuals is rejected at a significance level of 1 % (see Phillips and Ouliaris, 1990). In addition, as column (3) shows, estimated parameters remain practically the same, when equation is estimated in the post 1974Q2 period.[19] Also the implied long-run marginal propensity to consume out of non-human wealth (0.058) is well in the range reported in many earlier studies [see e.g Palumbo et al. (2006)]. This exceeds markedly the generally used estimate of the equilibrium real interest rate of 4%, and is in line with the view that consumers' planning horizon is finite. Under the additional assumption that the subjective discount rate and the real interest rate are equal, the annualised π would be 1.8% implying the expected remaining life-time (or the planning horizon) of 56 years. We see also that the estimate of α is well below unity, which under the assumption of $\rho = r$ implies the risk adjustment of 36 % to the expected human wealth.

Hence, we can conclude that, supplemented with the level-shift dummy in 1974, which may reflect the impact of the improved coverage of the private pension system on precautionary saving, the US data fulfills co-integration requirements satisfactorily. In addition, the parameter estimates of the steady state equation look quite reasonable being in line with our theoretic framework.

5.2. Dynamic consumption function

Before estimating equations (22) we have to expres composite parameters κ in terms of its determinants i.e. $\kappa = 1 - \frac{(1-\pi)(1+\bar{r})^{\frac{1}{\theta}-1}}{(1+\rho)^{\frac{1}{\theta}}}$. However, the formula determining κ contains four parameters of which two, i.e. the subjective discount rate ρ and the relative risk aversion parameter θ, do not appear elsewhere in the equation and they cannot be identified in estimating (22). Therefore, we follow a general practice and assume that the subjective discount rate equals the equilibrium real interest rate. Under this assumption risk aversion parameter θ disappears from estimated specification and $\kappa = \frac{\bar{r}+\pi}{1+\bar{r}}$. We assume that the subjective discount rate $\rho = \bar{r} = 0.01$ corresponding the annual rate of 4 percent. To eliminate

[19] These estimation results support, at least indirectly, the view that the major part of households'savings is related to precautionary rather than to bequest motive. Also micro-data evidence by Auerbach and Kotlikoff (1987) and Hurd (1989) does not support a bequest motive. Bernheim (1991), in turn, reports that households increase their life insurance purchase as a means of providing bequests for children in response to the government's provision of social security annuities, but also according to his results the life insurance offset appears to be quite modest.

heteroscedasticity we divide both sides of equations by current period labor income. Hence, in estimating the equation with the method of instrumental variables it can be written as follows:

$$E_t\left\{\left\{1+\frac{\gamma}{1.01}\left[(1-\pi)^2 aR_t-(0.01+\pi)\left(1-\frac{(1-\pi)a}{1.01}\right)\right]\right\}\frac{c_t}{y_t}-(1-\pi)\gamma\frac{R_t c_{t+1}}{y_t}-a(1-\pi)\frac{c_{t-1}}{y_t}\right.$$

$$\left.-\left(\frac{1.01-(1-\pi)a}{1.01}\right)\left(\frac{0.01+\pi}{1.01}\right)\left\{\left(\frac{1}{1-\pi}-\gamma\right)\left(\frac{V_{t-1}}{y_t}+1\right)+\left(\frac{\Lambda_t(1.01-(1-\pi)\gamma)}{0.01+\pi}-\frac{1}{1-\pi}\right)\right\}z_t\right\}=0 \tag{25}$$

where z_t is a vector of instruments.

In estimating (25) we allow income uncertainty and, hence, Λ_t to be time varying. As discussed by Carroll (1992), income uncertainty may be closely linked to unemployment risk, which we measure by the change of the unemployment rate U_t.[20] Also, as was discussed in previous Section, the enlargement of the coverage of the private pension system in 1974 may have decreased the uncertainty motivating precautionary saving which could explain the observed level-shift in the saving ratio. We take this into account by a level shift-dummy *D74q2* which equals one until 1974q2 and zero thereafter. Hence, we use the following parameterisation for defining the time varying risk adjustment term:

$$\Lambda_t=\Lambda_0+\Lambda_U\left(U_t-U_{t-i}\right)+\Lambda_{1974}D74q2 \quad ; \qquad 0\le\Lambda_0\le 1 \text{ and } \Lambda_U,\Lambda_{1974}\le 0 \tag{26}$$

In estimating we use the generalised method of moments (GMM), as described in Hansen (1982) and Hansen and Singleton (1982). Following the general practice in the GMM estimations, Hansen's J statistic of over-identifying restrictions together with associated p-values is used as a main statistical criterion in evaluating the data compatibility and the goodness of fit of estimated equations. To take into account serial correlation of residuals, the modified Bartlett weights proposed by Newey and West (1987) are used in calculating the weighting matrix of the minimized objective function.[21] Also standard errors, reported in parenthesis, are Newey-West corrected.

Our estimation period covers the interval 1954Q2 – 2004Q4 and the estimation results are presented in Table 3. We estimate both the finite and infinite horizon $(\pi=0)$ version of equation (25). In the latter alternative the long-run *mpc* out of non-human wealth is constrained to equal the predetermined real interest rate of 4% (annually) as

[20] It is possible that in addition or alternatively to the change of unemployment also the level of unemployment affects (with appositive sign) on income uncertainty. We also examined this alternative, but no statistically significant impact was found (results not reported here).

[21] To account for the possibility that also the measurement error ε_t is serially correlated, the lag length was allowed to be determined by data and the lag length of 5 quarters were ended up.

Table 3. Estimation results of the U.S. consumption function

	1954:2-2004:4				1974:3-2004:4			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
γ	0.8308 (0.0863)	0.7621 (0.0776)	0.8214 (0.0845)	0.7331 (0.0871)	0.7892 (0.0868)	0.7376 (0.0815)	0.5986 (0.1193)	0.6631 (0.1027)
π	0	0.0058 (0.0013)	0	0.0058 (0.0013)	0	0.0051 (0.0012)	0	0.0058 (0.0015)
a	0.8974 (0.0513)	0.8493 (0.0513)	0.9116 (0.0472)	0.8871 (0.0462)	0.9163 (0.0403)	0.8949 (0.0397)	0.9829 (0.0205)	0.9354 (0.0310)
Λ_0	0.7508 (0.0390)	0.6158 (0.0334)	0.7556 (0.0392)	0.6252 (0.0338)	0.7633 (0.0354)	0.6454 (0.0350)	0.9014 (0.1317)	0.6505 (0.0368)
Λ_{1974}	-0.0626 (0.0186)	-0.0483 (0.0090)	-0.0558 (0.0185)	-0.0443 (0.0097)	-	-	-	-
Λ_U	-	-	-0.0224 (0.0113)	-0.0123 (0.0055)	-	-	-0.0453 (0.0380)	-0.0165 (0.0080)
κ mpc_V	0.0099 0.0396	0.0156 0.0629	0.0099 0.0396	0.0156 0.0629	0.0099 0.0396	0.0149 0.0600	0.0099 0.0396	0.0157 0.0631
Lifetime (in years)	∞	42.89	∞	42.74	∞	48.95	∞	42.40
J-test p-value	10.40 [0.794]	10.9140 [0.6928]	8.96 [0.776]	8.6712 [0.7307]	-	9.6741 [0.7856]	10.06 [7580]	9.8206 [0.7085]

Note: Standard errors are presented in parenthesis and the p-values of the Hansen's J-test in square brackets. Instruments are: Constant, 3 and 4 periods lags of the consumption to income ratio, from 2 to 4 periods lags of the non-human wealth to income ratio and the real long term interest rate,- from 4 to 8 periods lags of real short-term interest rate, 1, 2 and 4 period lags of the 4-period change of the unemployment rate, 3 period lag of the unemployment rate and Dummy D74Q2 in the full sample estimations. Lifetime (in years) = $1/\left((1+\pi)^4-1\right)$; $\kappa=(\bar{r}+\pi)/(1+\bar{r})$ and $mpc_V=4*\kappa(1-a(1-\pi)/(1+\bar{r}))/((1-a(1-\kappa))(1-\pi))$

Table 3 shows. All parameter estimates are correctly signed and statistically significant (except unemployment uncertainty parameter Λ_U in the infinite horizon equation of column 7). On a statistical basis, the data compatibility of estimated equations is good and, especially in specifications allowing positive death probability, parameter estimates show stability when estimated both over the full-sample and the sub-sample. These equations imply that the long-run marginal propensity to consume *(mpc)* out of non-human wealth is in the range of 0.059-0.063 which is very well in line with our preliminary long-run estimates exceeding by around 50% the corresponding predetermined *mpc'*s of estimated infinite horizon equations. The death probability estimates vary in the range of 0.005-0.0058, which implies the life expectancy of 42.5-50 years. These estimates are well in line with our priors and quite close to that proposed by the estimated long-run equation.[22]

In infinite life-time specifications the lower *mpc* out of non-human wealth is compensated by higher estimates of Λ_0 than those of the finite horizon equations. Estimates of Λ_0 in Table 3 indicate that the risk adjustment to the certainty equivalent human wealth is around 25% in equations estimated for infinitely living consumers (except in column 7) and 35-40 % in equations estimated for finitely living consumers. In addition, as Λ_{1974} estimates show, until the 1974 this adjustment was about 4-5 percentage points larger.

If we take the annualised standard error of income change as a relevant reference for income uncertainty, then risk adjustments, especially, in equations allowing positive death probability may look quite high. For instance, according to Abowd and Card (1989) the cross-sectional standard deviation of the annual change of the logarithmic earnings varied between 0.36-0.47 in the PSID panel data, which on the bases of Table 1 might suggest 15-25 % risk-adjustment, if relative risk aversion ranges between 3-5 and a smaller adjustment outside this range.

There are, at least, the following two explanations for the relatively large deviations of the estimates of Λ_0 (and the sum $\Lambda_0 + \Lambda_{1974}$) from unity. Firstly, income uncertainty over longer than one year (or one period in our theoretic setting) may be relevant for determining the size of risk adjustment to human wealth, which would accordingly increase the size of the relevant standard error of income growth.[23] Secondly, our specification does not take into account the fact that people will retire, if alive, in around their sixties or a bit later, after which their income level markedly decreases. It is quite likely that part of the deviation of Λ_0 from unity is explained by this fact.[24]

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[22] In applying overlapping-generation approach with the constant probability of death, a conventional practice is to assume as if consumers were born at age 20 with life expectancy of around 40-50 years. See e.g. the discussion in Laxton et al. (1998).

[23] This is the direct implication of equation (12a) presented in footnote 6.

[24] For instance, Gali (1990) to capture this effect assumed that labor supply (and labor income) declines geometrically over the lifetime of the consumer. He estimated the rate of decline to range between 0.3 and 0.8 per

Our estimation results also support the hypothesis that unemployment risk, measured by the change of the unemployment rate, affects the size of the risk adjustment in human wealth. Further, our results imply that households possess a lot of period-specific information on future income changes. The point estimates of γ are well below unity (around 0.66-0.76 in finite horizon equations) deviating significantly both from zero and unity. This implies that period-specific information is heavily front-loaded with the major part of information concentrating on the nearest 1.5 years. Likewise, the very marked deviation of γ from zero explains for its part the "excess smoothness" of consumption, allowing permanent income (human wealth) and consumption to be smoother than measured income (Campbell and Deaton, 1989). However, a more important explanation comes from habit persistence, which hypothesis is strongly supported by the data. Estimates for the habit persistence parameter a are high, i.e. in the neighbourhood of 0.9. This is well in line with the estimates of 0.8-0.9 presented e.g. by Fuhrer (2000) for the US economy.

6. Conclusions

In this paper we first derived a closed form consumption function with CRRA preferences, habit persistance and income uncertainty in an overlapping generation framework and then estimated in the aggregate U.S. data.

To have the closed form solution we assumed that the consumer solves her intertemporal utility maximization problem sequentially so that she first evaluates non-human and human wealth accounting for income uncertainty. We extended conventional analysis by allowing the consumer to have a lot of period specific information on future income realizations in addition to information regarding the stochastic properties of the past income stream. We assumed that based on all this information the risk-averse consumer makes a risk- adjustment to expected life-time wealth. We showed that the risk adjustment is a highly nonlinear function of the relative risk aversion, income uncertainty and the ratio of expected human to non-human wealth.

In the second stage, conditional on the risk adjusted expected wealth, the consumer was assumed to determine her planned optimal consumption path. After allowing habit persistence we aggregated the consumption rule across generations and applied the derived aggregate consumption function to the U.S. data.

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cent annually. This kind of life-time income profiles, coupled with e.g. the 4% real interest rate and 2% death probability, would imply the downward revision of human wealth by 5-11 percent compared to the case of constant labor income.

In general, our estimated equations fit the data well and they are able to solve both excess smoothness and sensitivity puzzles. They also capture precautionary saving and, hence, are in better accordance with micro-data evidence and the results of the numerical analysis of a fully rational optimizing consumer than, for instance, the hybrid of the LC/PIH and Keynesian model.

According to our estimation results consumers adjust their expected human wealth around 35-40 % downwards for precautionary reasons in evaluating their life-time budget constraint. In addition, our estimation results support the hypothesis that precautionary savings depend positively on unemployment risk (measured by the change of the observed unemployment rate). Our results also imply that consumers have a lot of period-specific information on future income realizations although this is strongly front-loaded concentrating on the first 1.5 years and their life-expectancy is somewhat above 40 years. Also non-time separability in the utility function with habit formation parameter around 0.9 is strongly supported by the data.

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APPENDIX 1. Life-time resource constraint under habit persistence

Define period t consumption as

$$c_t = c_t^* + a c_{t-1} \quad \text{(A.1.1)}$$

and life-time budget constraint

$$\sum_{i=0}^{\infty} R_{t,t+i} (1-\pi)^i c_{t+i} = W_t \quad \text{(A.1.2)}$$

(A.1.1) implies for the left-hand side of (A.1.2):

$$\begin{aligned}
\sum_{i=0}^{\infty} R_{t,t+i} (1-\pi)^i c_{t+i} = {} & c_t^* + a c_{t-1} \\
& + \left(c_{t+1}^* + a c_t^* + a^2 c_{t-1}\right) R_{t,t+1} (1-\pi) \\
& + \left(c_{t+2}^* + a c_{t+1}^* + a^2 c_t^* + a^3 c_{t-1}\right) R_{t,t+2} (1-\pi)^2 + \ldots\ldots \\
& + \left(c_{t+k}^* + a c_{t+k-1}^* + \ldots + a^k c_t^* + a^{k+1} c_{t-1}\right) R_{t,t+k} (1-\pi)^k + \ldots\ldots \\
& = \left(a c_{t-1} + \sum_{i=0}^{\infty} R_{t,t+i} (1-\pi)^i c_{t+i}^* \right) \sum_{i=0}^{\infty} a^i R_{t,t+i} (1-\pi)^i \\
& \approx \left(a c_{t-1} + \sum_{i=0}^{\infty} R_{t,t+i} (1-\pi)^i c_{t+i}^* \right) \frac{1}{1-(1-\pi) a \overline{R}} \quad \text{(A.1.3)}
\end{aligned}$$

Hence (A.1.2) can be written in the form,

$$\sum_{i=0}^{\infty} R_{t,t+i} (1-\pi)^i c_{t+i}^* = \left(1 - a(1-\pi)\overline{R}\right) W_t - a c_{t-1} \quad \text{(A.1.4)}$$

APPENDIX 2: Data description

Consumption and consumption prices: Private Consumption at current price is defined as sum of seasonally adjusted NIPA personal per capita consumption expenditures on nondurables and services (Table 7.1. Selected Per Capita Product and Income Series in Current and Chained Dollars). The corresponding constant price measure is obtained using a Fisher chain-aggregation formula that replicated the procedure used by the Bureau of Economic Analysis in producing chained dollar volume estimates. These series are divided by four to turn the NIPA annualised levels to correspond quarterly levels. Deflator is obtained as the ratio of current and constant price series.

Wealth: Nonhuman wealth is defined as household net worth and is taken from the Flow of Funds Accounts of the Board of Governors of the Federal Reserve System, Table B.100.

After-tax labor income: Labor income is defined as wages and salaries (inc. supplements) + imputed labor income to self-employed persons + transfer payments – labor taxes. This data is directly from NIPA Table 2.1 except imputed labor income to self-employed. This component is calculated by assuming that the average wage rate can be used as a shadow price for compensation per self-employed person. This implies that the imputed labor income of self-employed workers equals wage and salary income multiplied by the ratio of the number of self-employed workers to the number of full-time equivalent employees (NIPA Tables 6.7B-D: Self-employed persons, and 6.5B-D: Full-time equivalent employees by industry). As these figures are available only annually, quarterly values are assumed to equal annual averages.[25] Labor taxes are defined as [wages and salaries (inc. supplements) + imputed labor income to self-employed persons / wages and salaries (inc. supplements) + proprietors' income + rental income + interest and dividend income] times personal current taxes. The constructed labor income series is divided by four to turn the NIPA annualised levels to correspond quarterly levels.

Interest rate: The nominal interest rate variable is the 3-month Treasury bill secondary market rate. Our source is the Federal Reserve Board, Statistics: Releases and historical data. The real interest rate is obtained subtracting from the nominal rate (scaled to the quarterly rate) one period lead of inflation defined in terms of the consumption deflator of nondurable goods and services.

Population: Population is from the same NIPA table as per capita consumption (Table 7.1: Selected Per Capita Product and Income Series in Current and Chained Dollars).

Unemployment rate: The unemployment rate is the seasonally adjusted series published by the U.S. Department of Labor, Bureau of Labor Statistics

[25] Also a Hodrick-Prescott filtered variant was tried, but results remained practically unchanged.

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