Pegging the Interest Rate on Bank Reserves: A Resolution of New Keynesian Puzzles and Paradoxes

Behzad Diba
Georgetown University

Olivier Loisel
CREST

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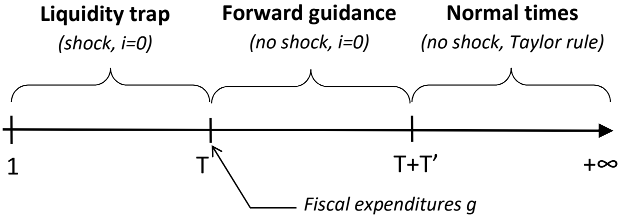
Motivation and overview

- The Great Recession has led central banks to **temporarily peg** their policy rates near zero.
- The New Keynesian (NK) literature has **puzzling and paradoxical implications** under a temporary interest-rate peg:
  - forward-guidance puzzle,
  - fiscal-multiplier puzzle,
  - paradox of flexibility,
  - paradox of toil.
- This paper offers a **resolution** of these puzzles and paradoxes based on a **simple and possibly minimal departure** from the basic NK model.
- This departure involves the central bank pegging the interest rate on bank reserves (IOR rate) – as central banks did in reality.

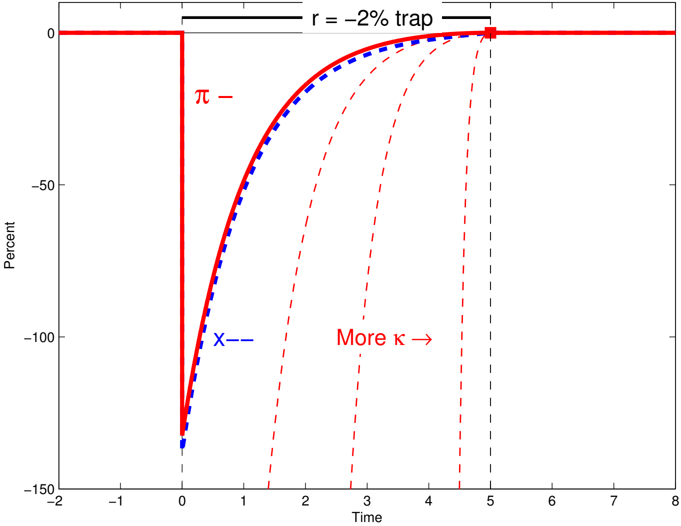
Three limit puzzles and paradox



Puzzle or paradox	Experiment	Outcome
Forward-guidance puzzle I	$T \longrightarrow +\infty$	$y_1, \pi_1 \longrightarrow -\infty$
Forward-guidance puzzle II	$T' \longrightarrow +\infty$	$y_1, \pi_1 \longrightarrow +\infty$
Fiscal-multiplier puzzle	$T \longrightarrow +\infty$	$\partial y_1/\partial g, \partial \pi_1/\partial g \longrightarrow +\infty$
Paradox of flexibility	$\theta \longrightarrow 0$	$y_1, \pi_1, \partial y_1/\partial g, \partial \pi_1/\partial g \longrightarrow \pm\infty$

- **Stark discontinuity** in the permanent-peg or flexible-price limit.

Forward-guidance puzzle and paradox of flexibility



Source: Cochrane (2017a).

Resolution of the puzzles and paradoxes

- The source of these limit puzzles and paradox lies in the basic NK model's property of exhibiting **indeterminacy under a permanent interest-rate peg**.
- Indeterminacy arises because the central bank pegs the interest rate on a bond serving **only as a store of value** (Canzoneri and Diba, 2005).
- In our model, the central bank pegs the interest rate on bank reserves, which serve to **reduce the costs of banking**.
- Our model delivers **determinacy under a permanent IOR-rate peg**, even under perfectly flexible prices, and therefore solves the limit puzzles and paradox.
- For a related reason, our model can also **solve the paradox of toil** (which says that positive supply shocks are contractionary under a temporary interest-rate peg).

Literature on the NK puzzles and paradoxes

- **Identification**: Carlstrom, Fuerst, and Paustian (2015); Cochrane (2017a); Del Negro, Giannoni, and Patterson (2015); Eggertsson (2010, 2011, 2012); Eggertsson, Ferrero, and Raffo (2014); Eggertsson and Krugman (2012); Farhi and Werning (2016); Kiley (2016); Werning (2012); Wieland (2016).

- **Empirical evidence**: Cohen-Setton, Hausman, and Wieland (2017); Garín, Lester, and Sims (2017); Wieland (2016).

- **Attenuations**: Angeletos and Lian (2016); Del Negro, Giannoni, and Patterson (2015); Farhi and Werning (2017); McKay, Nakamura, and Steinsson (2016); Wiederholt (2015).

- **Resolutions**: Angeletos and Lian (2016); Cochrane (2017a, 2017b); Gabaix (2016); García-Schmidt and Woodford (2015).

Original contribution

1. We **solve all** three limit puzzles and paradox with a **simple** departure from the basic NK model.

2. We solve them even for an **arbitrarily small** departure (i.e. arbitrarily small banking costs).

3. We still solve or attenuate them for a **vanishingly small** departure, and also solve the paradox of toil in that case, thus
   - providing an equilibrium-selection device in the basic NK model,
   - closing the gap between the basic sticky-price and sticky-information models.

4. Our resolution of the puzzles and paradoxes **preserves** two standard implications of the basic NK model in normal times:
   - the Fisher effect,
   - the Taylor principle (under a corridor system).

Outline of the presentation

Households

- The representative household (RH) consists of **workers** and **bankers**.

- RH's intertemporal **utility function** is

$$U_t = \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[u\left(c_{t+k}\right) - v\left(h_{t+k}\right) - v^b \left(h_{t+k}^b\right) \right] \right\}.$$

- Bankers use their own labor h_t^b and reserves m_t to produce loans:

$$\ell_t = f^b \left(h_t^b, m_t\right),$$

where f^b can be in particular **any CES function**.

- We can invert f^b and rewrite bankers' labor disutility as $v^b(h_t^b) = \Gamma(\ell_t, m_t)$.

- The FOCs imply $I_t^\ell > I_t^b$ (because $\Gamma_\ell > 0$) and $I_t^b > I_t^m$ (because $\Gamma_m < 0$).

Firms

- Firms are monopolistically competitive and owned by households.
- They use workers' labor to produce output:

$$y_t = f(h_t).$$

- They have to **borrow their nominal wage bill** $P_t \ell_t = W_t h_t$ in advance from banks, at the gross nominal interest rate I_t^ℓ.
- Prices are **sticky** à la Calvo (1983), with a degree of price stickiness $\theta \in [0, 1)$.

Government and steady state

- The **monetary authority** has two independent instruments:
  - the (gross) nominal interest rate on reserves $I_t^m \geq 1$,
  - the quantity of nominal reserves $M_t > 0$.
- The **fiscal authority** sets exogenously real fiscal expenditures $g_t \geq 0$, and fiscal policy is **Ricardian**.
- We assume that
  - I_t^m is set **exogenously** around some steady-state value $I^m \in [1, \beta^{-1})$,
  - $\mu_t \equiv M_t / M_{t-1}$ is set **exogenously** around the steady-state value $\mu = 1$.
- **Proposition**: *There is a unique steady state, and this steady state has zero inflation.*
- We log-linearize the model around this steady state.

Four equilibrium conditions I

1. Profit maximization by firms leads to the **Phillips curve**

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \kappa_y \widehat{y}_t - \kappa_m \widehat{m}_t - \kappa_g \widetilde{g}_t,$$

where

- $\kappa_y > 0$ and $\kappa_g > 0$ depend (positively) on $\Gamma_{\ell\ell}$,
- $\kappa_m > 0$ depends (positively) on $|\Gamma_{\ell m}|$.

2. RH's indifference between b_t and m_t leads to the **interest-rate-spread equation**

$$\underbrace{i_t^b - i_t^m}_{\text{marginal opportunity cost of holding reserves}} = \underbrace{\delta_y \widehat{y}_t - \delta_m \widehat{m}_t - \delta_g \widetilde{g}_t}_{\text{marginal benefit of holding reserves}},$$

where

- $\delta_y > 0$ and $\delta_g > 0$ depend (positively) on $|\Gamma_{\ell m}|$,
- $\delta_m > 0$ depends (positively) on Γ_{mm}.

Four equilibrium conditions II

3. The Euler equation gives the standard **IS equation**

$$\widehat{y}_t = \mathbb{E}_t \left\{ \widehat{y}_{t+1} \right\} - \frac{1}{\sigma} \mathbb{E}_t \left\{ i_t^b - \pi_{t+1} \right\} + \widetilde{g}_t - \mathbb{E}_t \left\{ \widetilde{g}_{t+1} \right\}.$$

4. The (first difference of the) **reserve-market-clearing condition** is

$$\pi_t = -\Delta \widehat{m}_t + \widehat{\mu}_t.$$

- Using these four equilibrium conditions, we get the **dynamic equation** in reserves:

$$\mathbb{E}_t\{L\mathcal{P}(L^{-1})\widehat{m}_t\} = i_t^m + \mathbb{E}_t\{\mathcal{Q}_\mu(L^{-1})\widehat{\mu}_t\} + \mathbb{E}_t\{\mathcal{Q}_g(L^{-1})\widetilde{g}_t\}.$$

- **Lemma**: *The roots of $\mathcal{P}(X)$ are three real numbers ρ, ω_1, and ω_2 such that $0 < \rho < 1 < \omega_1 < \omega_2$.*

Local-equilibrium determinacy

- **Proposition**: *Setting exogenously I_t^m and μ_t ensures local-equilibrium determinacy.*
- The spread equation can be viewed as a **shadow Wicksellian rule** for i_t^b.
- This rule ensures determinacy **only** because our assumptions on f^b imply that

$$\delta_m \kappa_y - \delta_y \kappa_m > 0.$$

- This inequality corresponds to the **Taylor principle** (the nominal interest rate should react by more than one-to-one to the inflation rate in the long run):

$$\Delta i^b \underset{\uparrow}{=} \delta_y \Delta \widehat{y} - \delta_m \Delta \widehat{m} \underset{\uparrow}{=} \left(\delta_y \frac{\kappa_m}{\kappa_y} - \delta_m \right) \Delta \widehat{m} \underset{\uparrow}{=} \frac{\delta_m \kappa_y - \delta_y \kappa_m}{\kappa_y} \pi.$$

spread equation — Phillips curve — reserve-market-clearing condition

Forward-guidance puzzle

- Consider the **basic NK model**, and assume that the central bank
  - pegs $i_t^b = \underline{i}^b < 0$ between 1 and T,
  - follows the rule $i_t^b = \phi \pi_t$ from $T+1$ onwards, where $\phi > 1$.

- Since a permanent i_t^b peg generates **indeterminacy**, the dynamic system between 1 and T has an "**excess stable eigenvalue**," so that the economy explodes backward in time starting from the terminal conditions $\widehat{y}_{T+1} = \pi_{T+1} = 0$:

$$\lim_{T \to +\infty} (\widehat{y}_1, \pi_1) = (+\infty, +\infty) .$$

- Now consider **our model**, and assume that the central bank pegs
  - $i_t^m = \underline{i}^m < 0$ between 1 and T,
  - $i_t^m = 0$ from $T+1$ onwards.

- Since a permanent i_t^m peg delivers **determinacy**, the dynamic system between 1 and T has **no excess stable eigenvalue**, so that

$$\lim_{T \to +\infty} (\widehat{y}_1, \pi_1) = (\widehat{y}_1^p, \pi_1^p) .$$

Fiscal-multiplier puzzle

- Consider the same experiment as before, and assume in addition that the government announces at date 1 that $\widetilde{g}_T = \widetilde{g}^* > 0$ and $\widetilde{g}_t = 0$ for $t \neq T$.

- In the **basic NK model**, for the same reason as before, we get

$$\lim_{T \to +\infty} \left(\frac{\partial \widehat{y}_1}{\partial \widetilde{g}^*}, \frac{\partial \pi_1}{\partial \widetilde{g}^*} \right) = (+\infty, +\infty) .$$

- This result still obtains when $\widetilde{g}_T = \xi^T \widetilde{g}^*$ with $0 \ll \xi < 1$: news about **vanishingly distant** and **vanishingly small** fiscal expenditures can have **exploding** effects today.

- In **our model**, for the same reason as before, we get

$$\lim_{T \to +\infty} \left(\frac{\partial \widehat{y}_1}{\partial \widetilde{g}^*}, \frac{\partial \pi_1}{\partial \widetilde{g}^*} \right) = (0, 0) .$$

Paradox of flexibility

- Consider the same experiments as before, but now make $\theta \to 0$ holding T constant.

- In the **basic NK model**, we get

$$\lim_{\theta\to 0}\left(\widehat{y}_1, \pi_1, \frac{\partial \widehat{y}_1}{\partial \widetilde{g}^*}, \frac{\partial \pi_1}{\partial \widetilde{g}^*}\right) = (+\infty, +\infty, +\infty, +\infty)\,.$$

- The reason is that the system's **excess stable eigenvalue goes to zero** as $\theta \to 0$: under an i_t^b peg, as prices become more and more flexible,
  - the effects of shocks die out more and more quickly forward in time,
  - the economy explodes more and more quickly backward in time.

- In **our model**, for the same reason as before, we get

$$\lim_{\theta\to 0}\left(\widehat{y}_1, \pi_1, \frac{\partial \widehat{y}_1}{\partial \widetilde{g}^*}, \frac{\partial \pi_1}{\partial \widetilde{g}^*}\right) = \left(\widehat{y}_1^f, \pi_1^f, \frac{\partial \widehat{y}_1^f}{\partial \widetilde{g}^*}, \frac{\partial \pi_1^f}{\partial \widetilde{g}^*}\right).$$

Arbitrarily small departure

- Our model solves the limit puzzles and paradox **for any given**
  - (dis)utility and production functions u, v, v^b, f, and f^b,
  - structural-parameter values $\beta \in (0,1)$, $\varepsilon > 0$, and $\theta \in [0,1)$,
  - policy-parameter values $I^m \in [1, \beta^{-1})$ and $g \geq 0$.

- Now replace v^b by γv^b (and hence Γ by $\gamma\Gamma$), where $\gamma > 0$ is a scale parameter.

- **Proposition**: *As $(I^m, \gamma) \to (\beta^{-1}, 0)$ with $(\beta^{-1} - I^m)/\gamma$ bounded away from zero and infinity, the steady state and reduced form of our model converge towards the steady state and reduced form of the corresponding basic NK model.*

- Thus, even an **arbitrarily small** departure from the basic NK model is enough to solve the limit puzzles and paradox.

Vanishingly small departure

- Consider a **sequence of models** converging towards the basic NK model, each of them solving the limit puzzles and paradox.

- Consider the **limit of equilibrium outcomes** along this sequence, for any given
  - duration of the IOR-rate peg T,
  - degree of price stickiness θ.

- This limit coincides with a **particular equilibrium** (out of an infinity of equilibria) of the basic NK model under a temporary, followed by a permanent, interest-rate peg.

- Thus, we provide an **equilibrium-selection device** in the basic NK model, which is reminiscent of the one developed in the global-games literature (Carlsson and van Damme, 1993; Morris and Shin, 1998, 2000).

Our selected equilibrium

- We show that our **selected equilibrium**
  - exhibits neither the fiscal-multiplier puzzle nor the paradox of flexibility,
  - exhibits an **attenuated** form of the forward-guidance puzzle: inflation and output grow *linearly* with the duration of the peg, not *exponentially*.

- We relate this attenuation of the forward-guidance puzzle to **price-level stationarity** ($p_\infty = p_0$) under a temporary IOR-rate peg ($i_t^m = i_t^b = i^*$ for $1 \leq t \leq T$):

$$\widehat{y}_1 = \widehat{y}_\infty - \frac{Ti^*}{\sigma} + \frac{p_\infty - p_1}{\sigma} = -\frac{Ti^*}{\sigma} - \frac{\pi_1}{\sigma}.$$

- We also show that our selected equilibrium does not exhibit the **paradox of toil**, and relate this feature to **inflation inertia** in our model.

Paradox of toil

- Consider the **basic NK model**, and assume that the central bank
  - pegs $i_t^b = 0$ between 1 and T,
  - follows the rule $i_t^b = \phi \pi_t$ from $T+1$ onwards, where $\phi > 1$.
- Consider a cost-push shock $\widehat{\varphi}^* > 0$ between 1 and T:

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} + \sigma^{-1}\mathbb{E}_t\{\pi_{t+1}\}, \quad \text{(IS)}$$

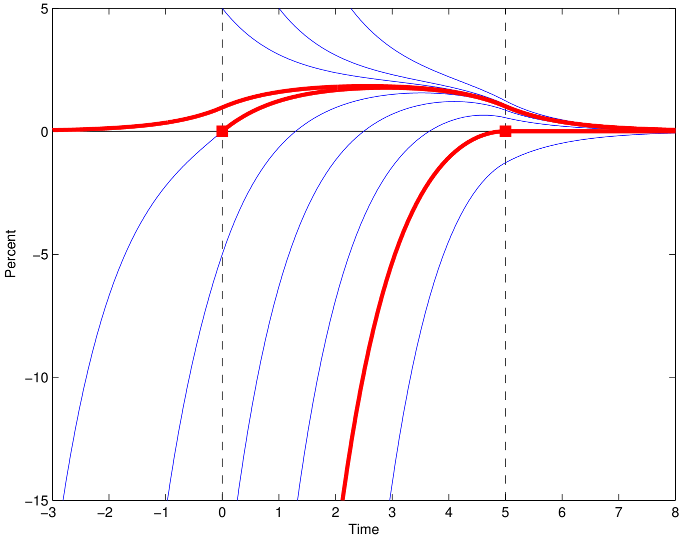
$$\pi_t = \beta\mathbb{E}_t\{\pi_{t+1}\} + \kappa\widehat{y}_t + \kappa_\varphi\widehat{\varphi}^*. \quad \text{(PC)}$$

- We get sequentially:
  - $\pi_{T+1} = \widehat{y}_{T+1} = 0$ (from the rule at dates $t \geq T+1$),
  - $\pi_T > 0$ and $\widehat{y}_T = 0$ (from IS and PC at date T),
  - $\pi_{T-1} > 0$ and $\widehat{y}_{T-1} > 0$ (from IS and PC at date $T-1$)...
- In our selected equilibrium, we have $\pi_{T+1} < 0$ because of the **inertia introduced by the state variable** (the stock of reserves), and hence $\widehat{y}_{T+1} < 0$, and hence $\widehat{y}_T < 0$ (from IS at date T); and we show that $\widehat{y}_t < 0$ for $1 \leq t \leq T$.

Comparison with other equilibria

- Our selected equilibrium differs from the **standard equilibrium**, which exhibits all four puzzles and paradoxes.
- It also differs from Cochrane's (2017a) **backward-stable and no-inflation-jump equilibria**:
  - our equilibrium exhibits (a weak form of) the forward-guidance puzzle,
  - our equilibrium makes inflation negative at the start of a liquidity trap.
- It belongs to the set of **local-to-frictionless equilibria** (Cochrane, 2017a), as it does not exhibit the paradox of flexibility.
- It behaves like the equilibrium of Mankiw and Reis's (2002) **sticky-information model** in terms of exhibiting or not the puzzles and paradoxes (Carlstrom, Fuerst, and Paustian, 2015; Kiley, 2016).
- So it brings the canonical sticky-price model **at par with** its sticky-information cousin in terms of their ability to solve or attenuate the four puzzles and paradoxes.

Standard, backward-stable, and no-inflation-jump equilibria



Source: Cochrane (2017a).

Discounting models

- We consider the class of "**discounting models**" with a reduced form of type

$$\widehat{y}_t = \xi_1 \mathbb{E}_t \{\widehat{y}_{t+1}\} - \frac{1}{\sigma} \mathbb{E}_t \left\{ i_t^b - \xi_2 \pi_{t+1} \right\},$$
$$\pi_t = \beta \xi_3 (\theta) \mathbb{E}_t \{\pi_{t+1}\} + \kappa (\theta) [\widehat{y}_t - \xi_4 (\theta) \mathbb{E}_t \{\widehat{y}_{t+1}\}],$$

where $\beta \in (0,1)$, $\sigma > 0$, $(\xi_1, \xi_2) \in (0,1]^2$, and, for all $\theta \in (0,1)$, $\xi_3(\theta) \in (0,1]$, $\xi_4(\theta) \in [0,1)$, and $\kappa(\theta) > 0$, with $\lim_{\theta \to 0} \kappa(\theta) = +\infty$.

- This class of models nests, as **special cases**,
  - the basic NK model, with $\xi_1 = \xi_2 = \xi_3(\theta) = 1$ and $\xi_4(\theta) = 0$,
  - Gabaix's (2016) benchmark model, with $\xi_2 = 1$ and $\xi_4(\theta) = 0$,
  - Angeletos and Lian's (2016) model.

- The last two models have been proposed to **solve or attenuate the forward-guidance puzzle**.

Comparison with discounting models I

- We highlight four properties of discounting models (thus generalizing Cochrane, 2016), and we show how our model is different:

  1. these models do not solve the **paradox of flexibility**;
  2. they require a **sufficiently large departure** from the basic NK model to solve the forward-guidance puzzle;
  3. they cannot solve the forward-guidance puzzle without generating a **negative** long-term relationship between π_t and i_t^b;

     by contrast, our model generates the **Fisher effect**, i.e. a **one-to-one** long-term relationship between π_t and i_t^b;

Comparison with discounting models II

4. these models cannot solve the forward-guidance puzzle without having non-standard and far-reaching implications for equilibrium determinacy in "**normal times**;"

   by contrast, in our model, under a **corridor system**, the spread equation becomes $\widehat{m}_t = (\delta_y/\delta_m)\widehat{y}_t - (\delta_g/\delta_m)\widetilde{g}_t$, and the Phillips curve can be rewritten as

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \underbrace{\left(\frac{\delta_m \kappa_y - \delta_y \kappa_m}{\delta_m}\right)}_{>0} \widehat{y}_t - \underbrace{\left(\frac{\delta_m \kappa_g - \delta_g \kappa_m}{\delta_m}\right)}_{>0} \widetilde{g}_t;$$

   therefore, the reduced form of our model is then **isomorphic** to the basic NK model's reduced form for any given rule for i_t^b;

   as a consequence, our model then inherits all the **standard implications** of the basic NK model for equilibrium determinacy in normal times.

Robustness check #1: Endogenous nominal reserves

- In our benchmark model, the stock of nominal reserves is **exogenous**.
- We endogenize it by considering the rule $M_t = P_t \mathcal{R}(P_t, y_t)$, with $\mathcal{R}_P < 0$ and $\mathcal{R}_y \leq 0$.
- The steady state is still unique, and we derive a simple sufficient **condition for determinacy** under a permanent IOR-rate peg.
- **This condition is met**
  - arguably for all relevant calibrations and all values of θ (paradox of flexibility),
  - necessarily for (I^m, γ) sufficiently close to $(\beta^{-1}, 0)$ (basic-NK-model limit).
- The shadow rule for i_t^b is still **Wicksellian**:

$$i_t^b \underset{\substack{\uparrow \\ \text{spread equation}}}{=} i_t^m + \delta_y \widehat{y}_t - \delta_m \widehat{m}_t - \delta_g \widetilde{g}_t \underset{\substack{\uparrow \\ \text{nominal-reserves rule}}}{=} i_t^m + \delta_y \widehat{y}_t - \delta_m \left(-\nu_P \widehat{P}_t - \nu_y \widehat{y}_t \right) - \delta_g \widetilde{g}_t .$$

Robustness check #2: Household cash

- In our benchmark model, the central bank controls **bank reserves**; but in reality, it controls the **monetary base** (bank reserves and cash).
- We introduce **household cash**, through a cash-in-advance constraint, into our benchmark model.
- Again, the steady state is still unique, and we derive a simple sufficient **condition for determinacy** under a permanent IOR-rate peg.
- Again, **this condition is met**
  - arguably for all relevant calibrations and all values of θ (paradox of flexibility),
  - necessarily for (I^m, γ) sufficiently close to $(\beta^{-1}, 0)$ (basic-NK-model limit).
- Again, the shadow rule for i_t^b is still **Wicksellian**:

$$i_t^b \underset{\uparrow}{=} i_t^m + \delta_y \widehat{y}_t - \delta_m \widehat{m}_t - \delta_g \widetilde{g}_t \underset{\uparrow}{=} i_t^m + \delta_y \widehat{y}_t - \delta_m \left[\frac{1}{1-\alpha_c} \widehat{\left(\frac{M_t}{P_t} \right)} - \frac{\alpha_c}{1-\alpha_c} \widehat{c}_t \right] - \delta_g \widetilde{g}_t .$$

spread equation — reserve-market-clearing condition

Summary

- Our model solves, even for an **arbitrarily small** departure from the basic NK model,
  - the forward-guidance puzzle,
  - the fiscal-multiplier puzzle,
  - the paradox of flexibility.
- It still solves or attenuates them for a **vanishingly small** departure, and also solves the paradox of toil in that case, thus
  - providing an **equilibrium-selection device** in the basic NK model,
  - **closing the gap** between the basic sticky-price and sticky-information models.
- It **preserves** two standard implications of the basic NK model in normal times:
  - the Fisher effect,
  - the Taylor principle (under a corridor system).
- Our resolution is essentially **robust** to
  - the endogenization of nominal reserves,
  - the introduction of household cash.