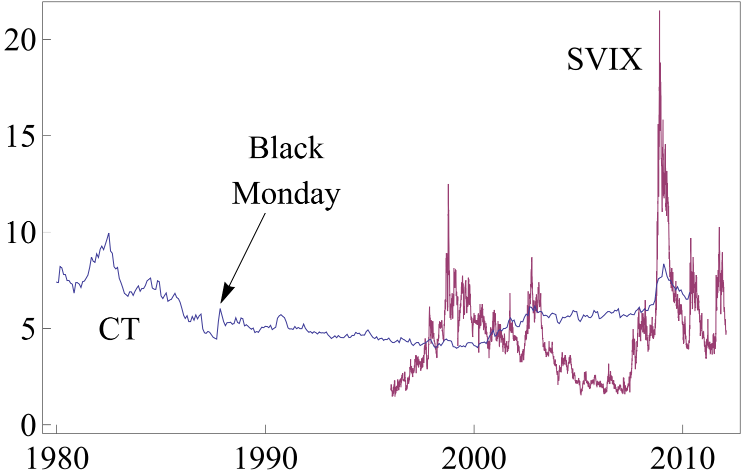
Volatility, Valuation Ratios, and Bubbles: An Empirical Measure of Market Sentiment

Can Gao   Ian Martin

October, 2019

Two views of the equity premium

Based on valuation ratios (Campbell and Thompson, *RFS*, 2008) and on index option prices (Martin, *QJE*, 2017)



Outline

- Very roughly, think of D/P as revealing $\mathbb{E}\,R - \mathbb{E}\,G$, and interest rates and option prices as revealing $\mathbb{E}\,R$; then the gap between the two reveals $\mathbb{E}\,G$

Specifically, today:

1. Relate dividend yields to expected returns and dividend growth using a twist on the Campbell–Shiller methodology
2. Introduce a bound on expected returns based on interest rates and option prices
3. Derive a bound on expected dividend growth by playing off (1) against (2)

Campbell–Shiller decomposition (1)

Notation: log dividend yield $dp_t = \log(D_t/P_t)$; log return r_{t+1}; log dividend growth g_{t+1}

- Campbell and Shiller (1988) famously showed that, up to a linearization,

$$dp_t = \frac{k}{1-\rho} + \sum_{i=0}^{\infty} \rho^i \, \mathbb{E}_t \left[r_{t+1+i} - g_{t+1+i} \right] \quad \text{where} \quad \rho \approx 0.97$$

- These are expected log returns, not expected returns
- Low expected log returns may be consistent with *high* expected returns if returns are volatile, right-skewed, or fat-tailed
- All three plausibly true in late 1990s, so the distinction between log returns and simple returns matters

Campbell–Shiller decomposition (2)

$$dp_t = \frac{k}{1-\rho} + \sum_{i=0}^{\infty} \rho^i \left(r_{t+1+i} - g_{t+1+i}\right) \underbrace{- \frac{\rho(1-\rho)}{2} \sum_{i=0}^{\infty} \rho^i \left(dp_{t+1+i} - \overline{dp}\right)^2}_{\text{second order term} \approx -0.145 \text{ in late '90s}}$$

- In the late '90s dp_t was 2.2 sd below its mean (using CRSP data 1947–2017)
- Ignoring the second order term is equivalent to understating $\mathbb{E}_t\, r_{t+1+i} - g_{t+1+i}$ by 14.5 pp for one year, 3.1 pp for five years, or 1.0 pp for 20 years
  - In long sample, 1871–2015, numbers are even bigger: 25.3 pp for one year, 5.5 pp for five years, 1.8 pp for 20 years, or 1.0 pp for ever
- Thus the CS decomposition may "cry bubble" too soon

An alternative approach (1)

- Campbell and Shiller loglinearize

$$r_{t+1} - g_{t+1} = dp_t + \log\left(1 + e^{-dp_{t+1}}\right)$$

- We start, instead, from

$$r_{t+1} - g_{t+1} = y_t + \log\left(1 - e^{-y_t}\right) - \log\left(1 - e^{-y_{t+1}}\right)$$

where

$$y_t = \log\left(1 + \frac{D_t}{P_t}\right)$$

- y_t, unlike dp_t, is in natural units: if $D_t/P_t = 2\%$ then $y_t = 1.98\%$ whereas $dp_t = -3.91$

An alternative approach (2)

Result

We have the loglinearization

$$y_t = (1-\rho)\sum_{i=0}^{\infty}\rho^i\left(r_{t+1+i} - g_{t+1+i}\right)$$

where $\rho = e^{-\bar{y}} \approx 0.97$.

On average, this relationship holds ***exactly****—no linearization needed:*

$$\bar{y} = \bar{r} - \bar{g}$$

An alternative approach (3)

- We have already seen that the Campbell–Shiller approximation may lead one to conclude too quickly that the market is bubbly, as

$$dp_t < -\frac{k}{1-\rho} + \sum_{i=0}^{\infty} \rho^i \, \mathbb{E}_t \left(r_{t+1+i} - g_{t+1+i} \right)$$

- Our variant is a **conservative** diagnostic for bubbles. If y_t is *far from its mean* then

$$y_t \geq (1-\rho) \sum_{i=0}^{\infty} \rho^i \, \mathbb{E}_t \left(r_{t+1+i} - g_{t+1+i} \right)$$

  - *Far from its mean*: $\mathbb{E}_t \left[\left(y_{t+i} - \bar{y} \right)^2 \right] \leq \left(y_t - \bar{y} \right)^2$ for all $i \geq 0$
  - In AR(1) case, "*far*" means "one standard deviation"

Information in valuation ratios (1)

- If y_t follows an AR(1) with autocorrelation ϕ_y,

$$\mathbb{E}_t\left(r_{t+1} - g_{t+1}\right) = \text{constant} + \frac{1 - \rho\phi_y}{1 - \rho} y_t$$

- In the unit root case $\phi_y = 1$, we have $y_t = \mathbb{E}_t\left(r_{t+1} - g_{t+1}\right)$
- So we use y_t to forecast $r_{t+1} - g_{t+1}$
- We estimate the regression freely, but results are almost identical if we estimate ρ and ϕ_y from time series, then use the formula above
- AR(1) is not critical: key is that we have a forecast of $\mathbb{E}_t\, y_{t+1}$. Will show AR(k) later

Information in valuation ratios (2)

RHS_t	LHS_{t+1}	$\widehat{a_0}$	$s.e.$	$\widehat{a_1}$	$s.e.$	R^2
y_t	$r_{t+1} - g_{t+1}$	-0.067	[0.049]	3.415	[1.317]	7.73%
	r_{t+1}	-0.018	[0.050]	3.713	[1.215]	10.51%
	$-g_{t+1}$	-0.049	[0.028]	-0.298	[0.812]	0.32%
dp_t	$r_{t+1} - g_{t+1}$	0.417	[0.146]	0.107	[0.042]	7.58%
	r_{t+1}	0.500	[0.138]	0.114	[0.041]	9.92%
	$-g_{t+1}$	-0.083	[0.085]	-0.007	[0.024]	0.19%

Table: S&P 500, annual data, 1947–2017, dividends reinvested monthly at CRSP 30-day T-bill rate. Hansen–Hodrick standard errors.

- Relative importance of r and g is sample specific: g more important in long sample. But coefficient estimates for $r - g$ are stable

Information in options (1)

- We start from an identity

$$\mathbb{E}_t\, r_{t+1} = \frac{1}{R_{f,t+1}}\, \mathbb{E}_t^* \left(R_{t+1} r_{t+1}\right) - \operatorname{cov}_t \left(M_{t+1} R_{t+1}, r_{t+1}\right)$$

- M_{t+1} is an SDF. Risk-neutral $\mathbb{E}_t^*$ satisfies $\frac{1}{R_{f,t+1}}\, \mathbb{E}_t^* \left(X_{t+1}\right) = \mathbb{E}_t \left(M_{t+1} X_{t+1}\right)$
- We assume that $\operatorname{cov}_t \left(M_{t+1} R_{t+1}, r_{t+1}\right) \leq 0$
  - Similar to the negative correlation condition of Martin (2017)
  - Loosely, requires that investors are sufficiently risk-averse wrt R_{t+1}
  - Holds in Campbell–Cochrane (1999), Bansal–Yaron (2004), Barro (2006), Wachter (2013), Bansal et al. (2014), Campbell et al. (2016), ...
- We then have

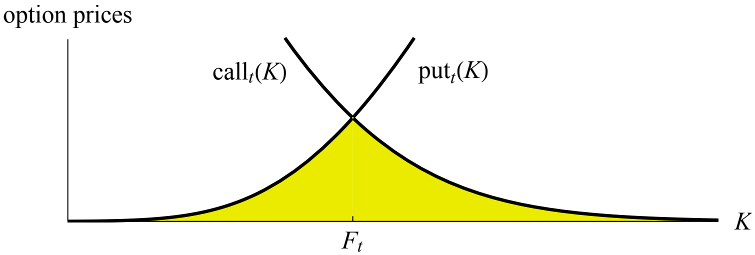
$$\mathbb{E}_t\, r_{t+1} \geq \frac{1}{R_{f,t+1}}\, \mathbb{E}_t^* \left(R_{t+1} r_{t+1}\right)$$

Information in options (2)

$$\mathbb{E}_t\, r_{t+1} \geq \frac{1}{R_{f,t+1}}\, \mathbb{E}^*_t\left(R_{t+1} r_{t+1}\right)$$

- Doesn't require that the market is complete
- Doesn't require any distributional assumptions (eg lognormality)
- Allows for the presence of constrained and/or irrational investors
- Holds with equality for a log investor who chooses to hold the market
- This investor's perspective works well empirically for forecasting
  - the market as a whole (Martin, *QJE*, 2017)
  - individual stocks (Martin and Wagner, *JF*, 2019)
  - currencies (Kremens and Martin, *AER*, 2019)

Information in options (3)



- Using the result of Breeden and Litzenberger (1978), we show

$$\frac{1}{R_{f,t+1}} \mathbb{E}_t^* \left(R_{t+1} r_{t+1} \right) = r_{f,t+1} + \underbrace{\frac{1}{P_t} \left\{ \int_0^{F_t} \frac{\text{put}_t(K)}{K} \, dK + \int_{F_t}^{\infty} \frac{\text{call}_t(K)}{K} \, dK \right\}}_{\text{LVIX}_t}$$

- This gives the lower bound $\mathbb{E}_t \, r_{t+1} - r_{f,t+1} \geq \text{LVIX}_t$
- Bootstrapped p-value for the mean of $r_{t+1} - r_{f,t+1} - \text{LVIX}_t$ being *negative* is 0.097

A sentiment index

- Putting the pieces together,

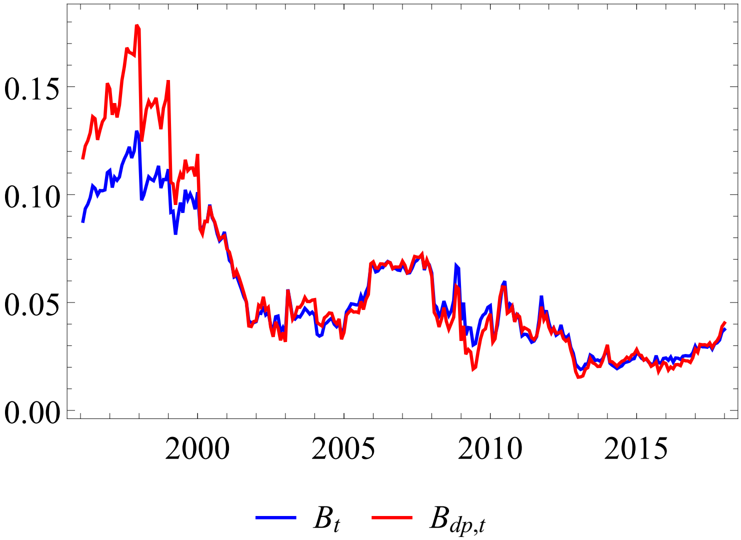
$$\begin{aligned}\mathbb{E}_t\, g_{t+1} &= \mathbb{E}_t\left(r_{t+1} - r_{f,t+1}\right) + r_{f,t+1} - \mathbb{E}_t\left(r_{t+1} - g_{t+1}\right) \\ &\geq \underbrace{\mathrm{LVIX}_t + r_{f,t+1} - \mathbb{E}_t\left(r_{t+1} - g_{t+1}\right)}_{B_t}\end{aligned}$$

- We replace $\mathbb{E}_t\left(r_{t+1} - g_{t+1}\right)$ by the forecast based on y_t:

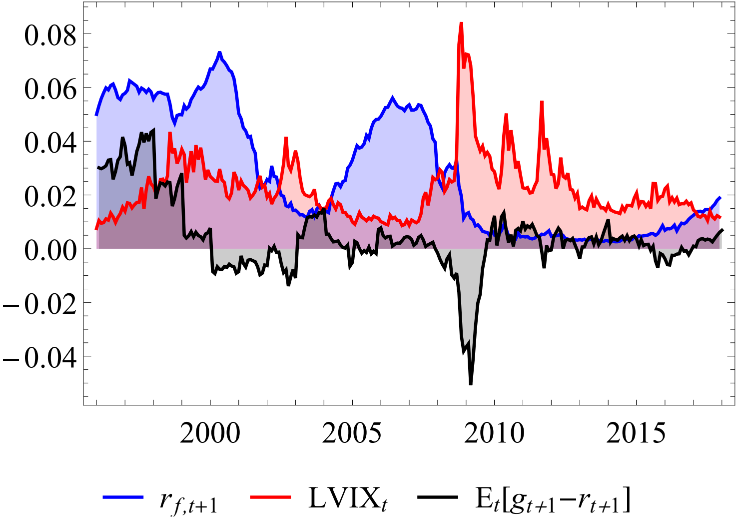
$$B_t = \mathrm{LVIX}_t + r_{f,t+1} - \left(\widehat{a}_0 + \widehat{a}_1 y_t\right)$$

  with $\widehat{a}_0$ and $\widehat{a}_1$ calculated on a rolling basis so B_t is observed at t
- The bound $\mathbb{E}_t\, g_{t+1} \geq B_t$ relies on two key assumptions:
  - the modified NCC
  - a stable statistical relationship between valuation ratios and $r - g$

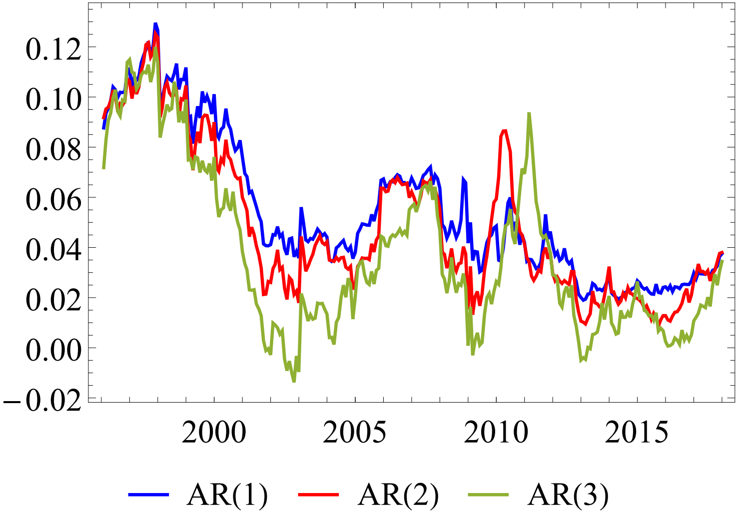
The sentiment index



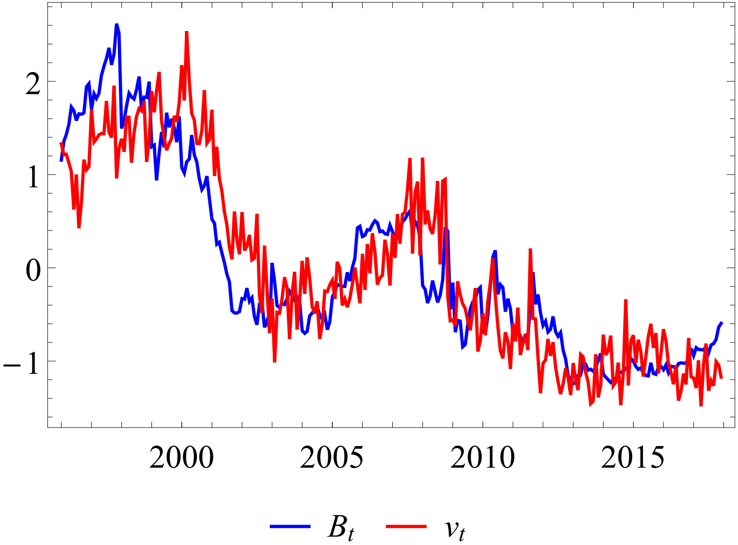
The three components of the sentiment index, B_t



Allowing y_t to follow an AR(k)



Sentiment index vs. detrended volume (1)



Sentiment index vs. detrended volume (2)

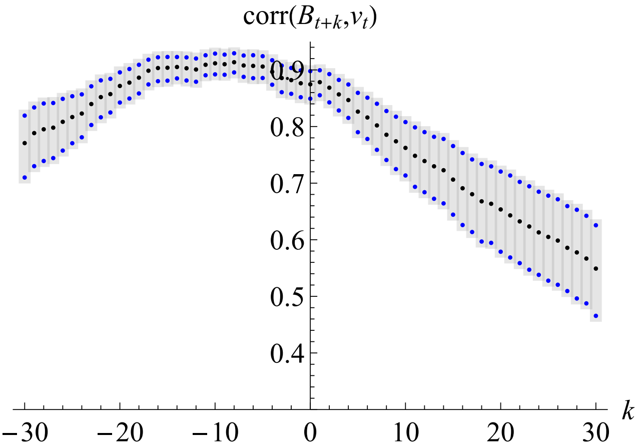


Figure: Correlation between B_{t+k} and detrended volume at time t.

Sentiment index vs. crash probability index (1)

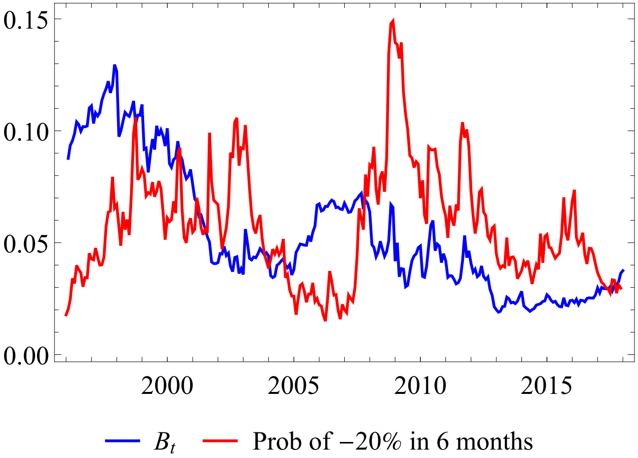


Figure: B_t and crash probability (Martin, 2017)

Sentiment index vs. crash probability index (2)

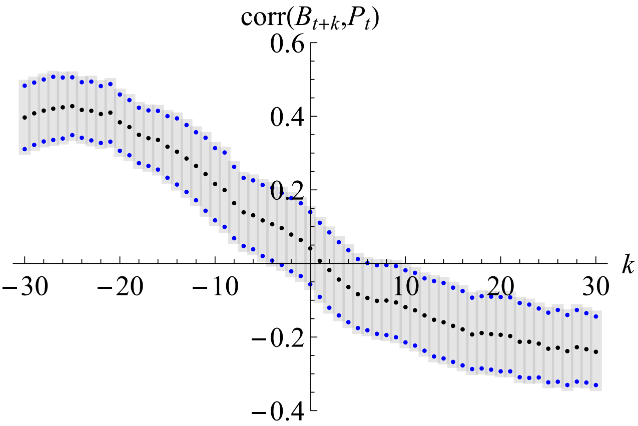


Figure: Correlation between B_{t+k} and crash probability at time t.

Conclusion

- Volatility and valuation ratios have long been linked to bubbles
- We use some theory to make the link quantitative
- We have tried to make choices in a conservative way to avoid "crying bubble" prematurely, and/or overfitting
- Signature of a bubble: valuation ratios, volatility, and interest rates are simultaneously high