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In 2010 all ECB publications feature a motif taken from the €500 banknote.

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2 Bank of England, Threadneedle Street, London EC2R 8AH, United Kingdom; e-mail: rupert.devincent-humphreys@bankofengland.co.uk

3 European Central Bank, Kaiserstrasse 29, D-60311 Frankfurt am Main, Germany; email: josep_maria.puigvert@ecb.europa.eu

Address
Kaiserstrasse 29
60311 Frankfurt am Main, Germany

Postal address
Postfach 16 03 19
60066 Frankfurt am Main, Germany

Telephone
+49 69 1344 0

Internet
http://www.ecb.europa.eu

Fax
+49 69 1344 6000

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CONTENTS

Abstract

This paper presents a set of probability density functions for Euribor outturns in three months' time, estimated from the prices of options on Euribor futures. It is the first official and freely available dataset to span the complete history of Euribor futures options, thus comprising over ten years of daily data, from 13 January 1999 onwards. Time series of the statistical moments of these option-implied probability density functions are documented until April 2010. Particular attention is given to how these probability density functions, and their associated summary statistics, reacted to the unfolding financial crisis between 2007 and 2009. In doing so, it shows how option-implied probability density functions could be used to contribute to monetary policy and financial stability analysis.

JEL classification: C13; C14; G12; G13;

Keywords: financial market; probability density functions; options; financial crisis

1 Introduction

Forward interest rates reflect the market's aggregate risk-neutral expectation of spot interest rates in the future.[4] From the prices of options on interest rate futures, it is possible to construct the entire probability distribution for the interest rate in the future. And because that probability distribution describes the (risk-neutral) likelihood the market ascribes to all possible outcomes, it provides a quantitative measure of the market's assessment of the risks around the forward rate, in terms of both magnitude and directional bias. Therefore, such option-implied probability density functions (PDFs) constitute a natural complement to the many other financial market indicators already considered by central banks and monetary policy practitioners. For instance, the yield curve - an important component of monetary policy transmission - is influenced by how short-term rates are expected to evolve over time. Furthermore, option-implied PDFs can provide an easily-accessible tool for visualizing how the market reacts to specific events, and may thus contribute to both monetary policy and financial stability analysis.

A number of methods of constructing these option-implied PDFs have already been developed. They have been classified and compared by Bliss and Panigirtzoglou (2000) into five groups: stochastic process methods, implied binomial trees, option-implied PDF approximating function methods, finite-difference methods, and implied volatility smoothing methods. To date there has been a large discussion on the different possible methods and the differences among them. As for instance, Campa, Chang and Reider (1997) compared implied binomial trees, smoothed implied volatility smile and a mixture of lognormal methods. Coutant, Jondeau and Rockinger (1999) compared single lognormal, mixtures of lognormals, Hermite polynomials and maximum entropy methods. In general, although these methods might differ in the very tails of the distribution, there is generally no major difference in the central section of the estimated PDFs. And arguably it is the central section of the PDFs which is more likely to be useful for monetary policy purposes, in contrast to financial stability analysis, where there may be greater focus on the tails of the distribution.

This paper uses a non-parametric technique, based on the Bliss and Panigirtzoglou (2000) and the Cooper (2000) results, to estimate the option-implied PDFs. This method was preferred because, according to the previous authors it is much more stable than other techniques and avoids the possible existence of ”spikes” in the distribution. In fact, Cooper states, that by using the non-parametric technique, the small errors in the prices cause only small local errors

[4] At short horizons, where term premia are likely to be negligible, forward rates could be therefore represent a good approximation of the market's actual expectation of interest rates in the future.

in the estimated probability density function while for other techniques, e.g. the mixture of lognormals, the errors can be sufficient for the minimisation to reach very different parameter estimates with large changes in the shape of the estimated probability density function. The results produced by the non-parametric technique are not materially different to those of other existing techniques. The dataset which this methodology produces may be considered the first official Euribor dataset large enough for practitioners and researchers to extract useful information in support of their macroeconomic analysis.[5] In particular, such option-implied PDFs have not been studied in detail during periods of financial crisis where arguably they may be the most useful.

This paper presents an analysis of probability density functions for Euribor outturns in three months' time, from 13 January 1999, when options on Euribor futures first started trading, until April 2010. With more than ten years of daily data, this provides a comprehensive picture of the market's *quantitative* assessment of the risks around interbank rates in the future. Importantly, this dataset includes periods of prolonged stability as well as periods of turbulence. The evolution of market interest rates is a key component of the transmission of monetary policy, and so such a comprehensive, quantitative assessment may be a natural complement to the wide range of financial market indicators already considered by monetary policy makers.

In addition, we study in detail in Section 2 the evolution of the options on 3-month Euribor futures trading volume. The trading volume for this ten year data period is analysed by option type, maturity date, year and moneyness type. The study of the trading volume based on these types of categories allows us to select the most liquid strikes and to avoid the possible bias that could be introduced to the option-implied PDF by selecting the whole set of option strikes.

The remainder of the paper is organised as follows: Section 2 describes the type of data that is used and the way that the data is filtered. Section 3 sets out the estimation technique that is used to compute the implied probability density functions, in addition tries to describe how the results that we obtained can be replicated by using built-in MATLAB functions. Section 4 shows how information from option-implied probability density functions can be used to inform and add value to economic analysis. Section 5 describes in detail how the Euribor implied PDFs reacted to the unfolding financial crisis between 2007 and 2009. In doing so, it demonstrates how implied PDFs can provide timely and quantitative indicators of not only the amount of uncertainty around forward Euribor, but the directional bias within that. Section six concludes.

[5] Option-implied PDFs for GBP Libor, since 1988 are published by the Bank of England here: http://www.bankofengland.co.uk/statistics/impliedpdfs/index.htm

2 The Data

The data used in this paper refer to the daily settlement prices on futures on the 3-month Euribor and on options on the 3-month Euribor futures. These daily settlement prices are published by Euronext.liffe, formed in January 2002 from the takeover of the London International Financial Futures and Option Exchange (LIFFE). According to LIFFE, these contracts were developed in response to the economic and monetary union within Europe, and the emergence of Euribor as the key cash market benchmark within Europe's money markets. Since its launch, LIFFE's Euribor contracts have come to dominate the euro denominated short-term interest rate (STIR) derivatives market, capturing over 99% of the market share; they are now the most liquid and heavily traded euro-denominated STIR contracts in the world. Delivery months for the 3-month Euribor futures contracts are March, June, September and December; the last trading day is two business days prior to the third Wednesday of the delivery month, and the delivery day is the first business day after the last trading day. The Exchange Delivery Settlement Price (EDSP) is based on the European Bankers Federations' Euribor Offered Rate (EBF Euribor) for 3-month Euro deposits at 11.00 Brussels time on the last trading day. The settlement price will be 100.000 minus the EBF Euribor Offered Rate rounded to three decimal places. The minimum size price movement is 0.05, which equates to EUR 12.50. The data can be downloaded directly from the internet website via the following link http://www.liffe.com/reports/eod?item=Histories.

Bliss and Panigirtzoglou (2000) state that out-of-the-money calls (puts) tend to be more liquid than puts (calls) of the same strike. We began by analysing the trading volume for all Euribor options since the first day of trading, 13 January 1999. In absolute terms, 81% of the options are traded out-of-the-money whereas only 18% are traded in-the-money. Furthermore, some of the in-the-money options are traded not independently, but as part of a bundled trading strategy, e.g. straddles or strangles, which combine options out-of-the-money with options in-the-money. With this confirmation, we also applied our methodology to those option prices which were either at- or out-of-the money, but not in-the-money.

Trading was much more concentrated in the options contracts maturing in nine months or less. These accounted for more than the 85% of the total trading. For those option contracts for longer maturities, in which trading was very seldom, perhaps even going untraded some days, the settlement prices were directly assigned by LIFFE. Finally, and as may be expected, the number of traded contracts has increased steadily since this instrument was first introduced, with the most trading in the most recent years.

Table 1: Trading volume descriptive statistics (in number of transactions)

	Contract expiring in 3 months or less					
	CALL			**PUT**		
	At the money	In the money	Out of the money	At the money	In the money	Out of the money
Volume Traded	1,165,039	25,522,170	81,332,548	529,526	8,302,374	34,738,846
Percentage Traded	1.1%	22.2%	76.7%	1.2%	19.1%	79.7%
Maximum	104,500	561,194	1,675,662	41,540	339,499	437,522
Maximum Date	11Apr2008	13Oct2008	09Jan2010	10Apr2008	25Sep2008	14Oct2008
Mean	412	8,318	28,760	187	2,936	12,284
Std. Dev.	3,367	23,258	65,531	1,745	9,704	25,452
Volume per option type		106,019,757			43,570,746	
Total Volume	**149,590,503 (26.7%)**					
	Contract expiring between 3 and 6 months					
	CALL			**PUT**		
	At the money	In the money	Out of the money	At the money	In the money	Out of the money
Volume Traded	720,182	24,323,437	108,736,808	508,490	12,966,668	57,225,845
Percentage Traded	0.5%	18.2%	81.3%	0.7%	18.3%	80.9%
Maximum	116,436	226,775	592,700	58,900	421,595	595,077
Maximum Date	10Aug2007	07Feb2008	02Nov2009	22Apr2008	15Feb2007	23Jun2009
Mean	255	8,601	38,450	180	4,585	20,235
Std. Dev.	3,092	17,925	54,940	2,043	14,088	36,799
Volume per option type		133,780,427			70,701,003	
Total Volume	**204,481,430 (36.5%)**					
	Contract expiring between 6 and 9 months					
	CALL			**PUT**		
	At the money	In the money	Out of the money	At the money	In the money	Out of the money
Volume Traded	322,074	13,060,952	66,188,484	330,656	8,646,703	40,888,158
Percentage Traded	0.4%	16.4%	83.2%	0.7%	17.3%	82.0%
Maximum	38,901	286,500	449,700	51,396	222,122	880,283
Maximum Date	02May2007	04Mar2010	11Jan2010	02May2007	19Jul2007	11Dec2009
Mean	114	4,618	23,405	117	3,058	14,458
Std. Dev.	1,205	13,912	39,838	1,427	9,918	35,774
Volume per option type		79,571,510			49,865,517	
Total Volume	**129,437,027 (23.1%)**					

	Contract expiring between 9 months and 1 year					
	CALL			**PUT**		
	At the money	In the money	Out of the money	At the money	In the money	Out of the money
Volume Traded	100,805	3,992,276	30,327,167	140,485	3,041,959	14,115,998
Percentage Traded	0.3%	11.6%	88.1%	0.8%	17.6%	81.6%
Maximum	19,520	240,900	765,140	36,200	183,000	326,210
Maximum Date	17Mar2003	03Mar2010	10Dec2003	14Sep2006	12Sep2006	27Feb2007
Mean	36	1,412	10,724	50	1,076	4,992
Std. Dev.	482	6,846	30,284	825	5,553	14,310
Volume per option type		34,420,248			17,298,442	
Total Volume			**51,718,690 (9.2%)**			

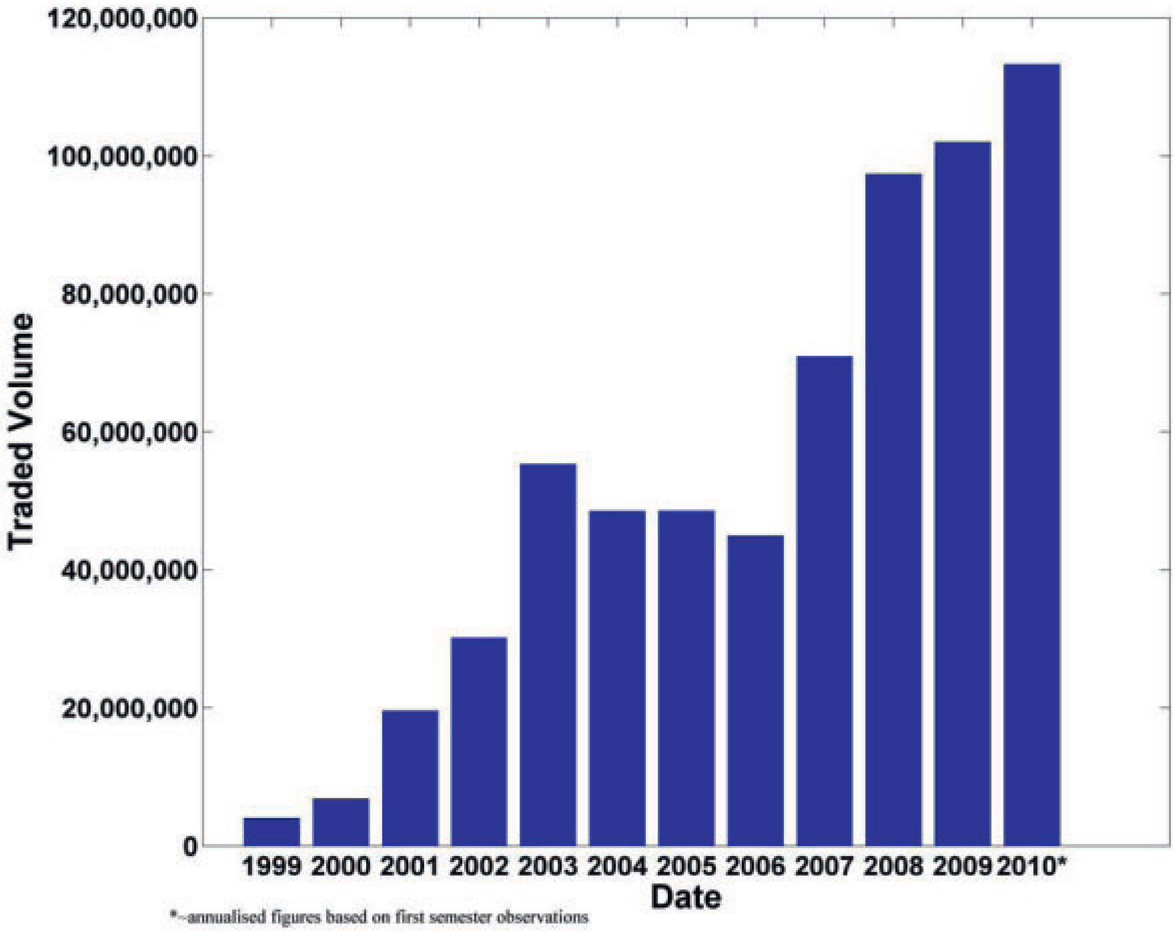
	Contract expiring between 1 year and 1 year and three months					
	CALL			**PUT**		
	At the money	In the money	Out of the money	At the money	In the money	Out of the money
Volume Traded	40,370	976,863	7,681,420	38,400	663,486	3,813,320
Percentage Traded	0.5%	11.2%	88.3%	0.9%	14.7%	84.5%
Maximum	5,500	24,800	120,900	10,000	64,000	237,900
Maximum Date	21Nov2005	03Dec2002	12Sep2003	07Sep2005	22Dec2009	10Dec2009
Mean	14	345	2,716	14	235	1,348
Std. Dev.	210	1,348	8,127	244	1,469	6,375
Volume per option type		8,698,653			4,515,206	
Total Volume			**13,213,859 (2.4%)**			

	Contract expiring between 1 year and three months and 1 year and six months					
	CALL			**PUT**		
	At the money	In the money	Out of the money	At the money	In the money	Out of the money
Volume Traded	15,750	441,905	3,472,557	18,550	409,348	1,589,324
Percentage Traded	0.4%	11.2%	88.4%	0.9%	20.3%	78.8%
Maximum	3,250	50,010	170,500	3,250	24,010	80,000
Maximum Date	31Aug2007	27May2003	05Aug2004	31Aug2007	22Dec2009	14Dec2009
Mean	6	156	1,228	7	145	562
Std. Dev.	93	1,160	6,274	107	833	2,746
Volume per option type		3,930,212			2,017,222	
Total Volume			**5,947,434 (1.1%)**			

	Contract expiring between 1 year and six months and 1 year and nine months					
	CALL			**PUT**		
	At the money	In the money	Out of the money	At the money	In the money	Out of the money
Volume Traded	10,895	271,009	1,707,171	13,475	336,323	1,044,415
Percentage Traded	0.5%	13.6%	85.8%	1.0%	24.1%	74.9%
Maximum	4,500	7,500	1,62,750	4,500	12,000	87,350
Maximum Date	17Mar2010	08Jan2008	09Jul2009	17Mar2010	07Jan2010	09Dec2009
Mean	4	96	604	5	119	369
Std. Dev.	106	423	2,662	121	568	2,370
Volume per option type		1,989,075			1,394,213	
Total Volume			**3,383,288 (0.6%)**			

	Contract expiring between 1 year and nine months and 2 years					
	CALL			PUT		
	At the money	In the money	Out of the money	At the money	In the money	Out of the money
Volume Traded	15,129	293,580	1,599,628	13,029	415,902	871,069
Percentage Traded	0.8%	15.4%	83.8%	1.0%	32.0%	67.0%
Maximum	2,500	23,500	124,000	2,500	27,000	34,000
Maximum Date	29May2007	29Jan2010	05Mar2009	29May2007	10Feb2006	20Aug2009
Mean	5	104	566	5	147	308
Std. Dev.	95	605	3,535	88	778	1,470
Volume per option type		1,908,337			1,300,000	
Total Volume			**3,208,337 (0.6%)**			

Figure 1: Total traded volume for all contracts per year



In addition, three other types of quality assurance check are made on the price date. First, a basic plausibility check: any option prices that are either zero or negative are immediately rejected. The second check is founded in option-pricing theory. In order to yield non-negative probability estimates, a call price function should be both monotonic and convex. In practice, this may not be the case if the difference between the 'true' price of options with adjacent strikes is less than the minimum tick size, or if there are sufficiently large variations in the bid-ask spread. So any option prices that do not meet these monotonicity and convexity requirements are also excluded. Finally, if after the application of the preceeding two filters, there are less than three out-of-the-money option prices for a particular expiry date, then no PDF will be estimated for that expiry date.

3 Methodology

3.1 Fixed-expiry probability density functions

The non-parametric technique used in this paper to derive the PDF is based on both Bliss and Panigirtzoglou (2000) and Cooper (2000). These two articles make use of the Breeden and Litzenberger (1978) result which states that the implicit interest rate probabilities can be inferred from the second partial derivative of the call price function with respect to the strike price.

The Breeden and Litzenberger result follows from the Cox and Ross (1976) pricing model, and is set out below:

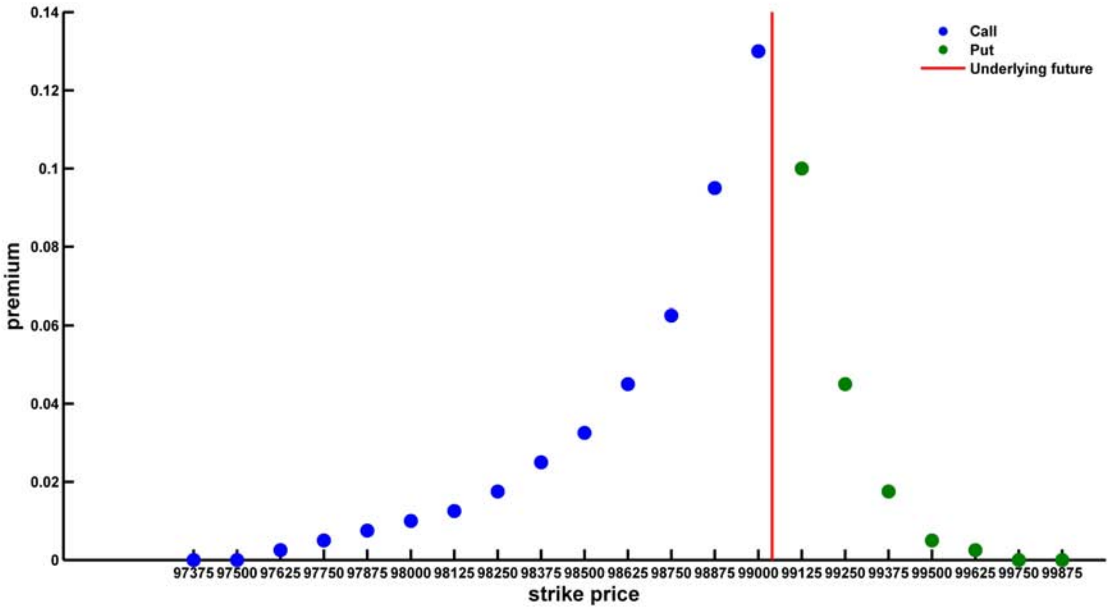
$$C(F_t, K, \tau) = e^{-r\tau} \int_K^\infty f(F_t)(F_t - K)dF_T \tag{1}$$

$$\frac{\partial C(F_t, K, \tau)}{\partial K} = -e^{-r\tau} \int_K^\infty f(F_t)dF_T \tag{2}$$

$$\frac{\partial^2 C(F_t, K, \tau)}{\partial^2 K} = e^{-r\tau} f(F_t) \tag{3}$$

where C is the call function, K is the option's strike price, r is the risk-free rate, F_t is the value of the underlying future at time t and $f(F_T)$ is the probability density function which describes the possible outturns for the underlying futures at time T. The option's time to maturity, t is equal to $T - t$. Of course, equation (3) cannot be applied directly to obtain $f(F_T)$, because we only observe option prices for a discrete set of strike prices, rather than a twice-differentiable continuum. So in practice, the task of estimating a PDF using the Breeden and Litzenberger result amounts to estimating a twice-differential call price function (Chart 2).

Figure 2: Out-of-the-money calls and puts (March 2010 contract); 27 October 2009



However, taking the second derivative of a call price function estimated directly, by interpolating through the discrete set of option *premium* vs. *strike price* data, can sometimes lead to unstable or inaccurate estates of the PDF. Instead, Bliss and Panigirtzoglou (2000), following the results derived from Malz (1997) and Shimko (1993), suggested that better results can be obtained if the option *premium vs. strike price* data are first transformed into implied *volatility vs. delta* values before interpolating. This procedure is described in more detail below.

The first step is to transform the option prices into implied volatilities. Implied volatilities are computed by numerically solving for the value of σ which solves the Black (1976) futures options pricing model, for each option contract:[6]

$$C(F_t, K, \tau) = e^{-r\tau}(F_0\Theta(\frac{ln(\frac{F_0}{K}) + \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}) + K\Theta(\frac{ln(\frac{F_0}{K}) - \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}})) \tag{4}$$

In the second step, the *implied volatilities* are used to calculate the *delta* values.

$$\delta(F_t, K, \tau) = \frac{\partial C(F_t, K, \tau)}{\partial K} = e^{-r\tau}F_0\Theta(\frac{ln(\frac{F_0}{K}) + \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}})) \tag{5}$$

where Θ is the standard normal cumulative distribution function. In our implementation, we use the two built-in MATLAB functions, *blsimpv* and *blsdelta*, to calculate the *implied volatilities*

[6]See, for instance, Hull (2000) for an overview of option pricing, and related quantities.

and deltas, respectively.

Next, the *premium* vs. *strike price* data are interpolated using a cubic smoothing spline, which minimizes:

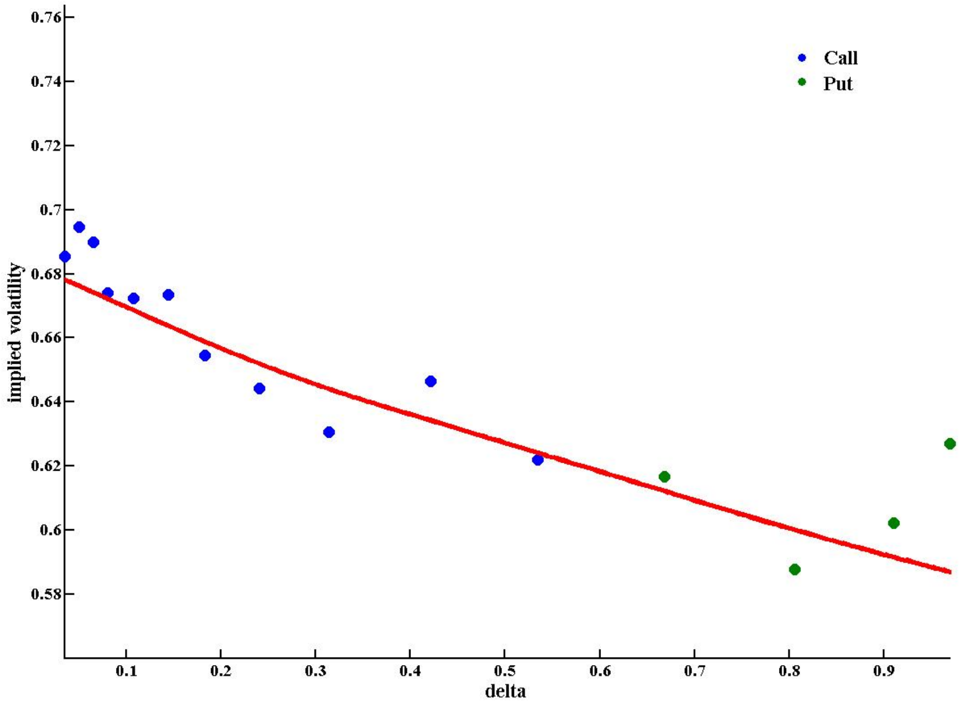
$$\lambda \sum_{i=1}^{n} \omega_i(\sigma_i - g(\delta_i))^2 + (1-\lambda)\int g^2(t)^2 dt \tag{6}$$

where λ is the smoothing roughness parameter, equal to 0.99[7] , *delta* is the Black-Scholes *delta* and represents the x-axis of the spline, σ is the Black-Scholes *sigma* and represents the y-axis of the spline and the weights ω_i are calculated using $\omega_i = \frac{\nu_i^2}{mean(\nu_i^2)}$ where ν_i is Black-Scholes *vega*. The value of *vega* is almost negligible for options which are deep out-of-the-money and deep in-the-money and sequentially increases as we get near-the-money. In particular, it reaches a maximum for at-the-money options. Hence, the ω_i used in (6) place most weight on near-the-money options, and therefore lesser weight on away-from-the-money options. This is consistent with using these PDFs to support monetary policy analysis, where interest is likely to lie in the centre of the distribution, i.e. close to the underlying interest rate, rather than the distribution's tails. Chart 3 shows the interpolated volatility smile, as a function of *delta*. Although *delta* can take values between 0 and $exp(r\tau)$, the traded contracts may not span that complete range. Therefore, the smoothing spline is extrapolated outside the range of traded price points with a second order polynomial, i.e. a quadratic equation, using the MATLAB built-in *fnxtr()* function. As a result of the extrapolation, the piecewise cubic curve obtained using interpolation is extended with a quadratic curve at each endpoint so that the full *delta* range is covered.

Note that although we are using the Black-Scholes formulae, we do not assume that the assumptions of the Black-Scholes option pricing paradigm - in particular the implicit underlying asset price dynamics - hold true. They merely provide convenient transformation which allows the option data to be interpolated in a way that produces more stable results. That transformation is then later undone.

[7]The optimal smoothing roughness parameter is the one that minimizes the observed deltas with the fitted deltas by the smoothing spline.

Figure 3: Delta-implied volatility smile for the out-of-the-money calls and puts (March 2010 contract); 27 October 2009



In the next step, the interpolated volatility smile is transformed back from *volatility* vs. *delta* values to *premium* vs. *strike price* values. This is done by evaluating the interpolated volatility smile at 1000 equally-spaced *delta* values between zero and one using the MATLAB function *fnval()*. The 1000 *delta* values are then transformed back into strike prices using the inverse of equation (5):

$$exp((\frac{\sigma_{ATM}^2}{2}\tau) + log(F_0) - \sigma\sqrt{\tau}\Theta^{-1}(\delta exp(r\tau))) \tag{7}$$

(7) where Θ^{-1} is the inverse of the cumulative density function of a standardised *Normal* distribution. The implied volatility values of the spline are translated back into call prices using the Black-Scholes option pricing equation (4). Chart 4 shows a fitted call price function, and a fitted put price function. In order to calculate the second derivative of the call function, we fit cubic polynomials through triplets of consecutive *(strike price, call price)* pairs; from the coefficients of the fitted polynomials we evaluate the second derivative, which gives us the PDF (chart 5).

Figure 4: Fitted call and put option function for the out-of-the-money calls and puts (March 2010 contract); 27 October 2009

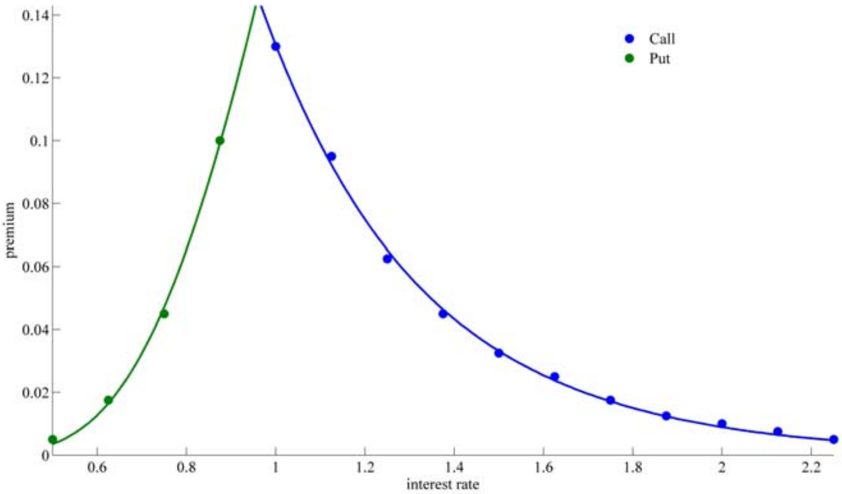
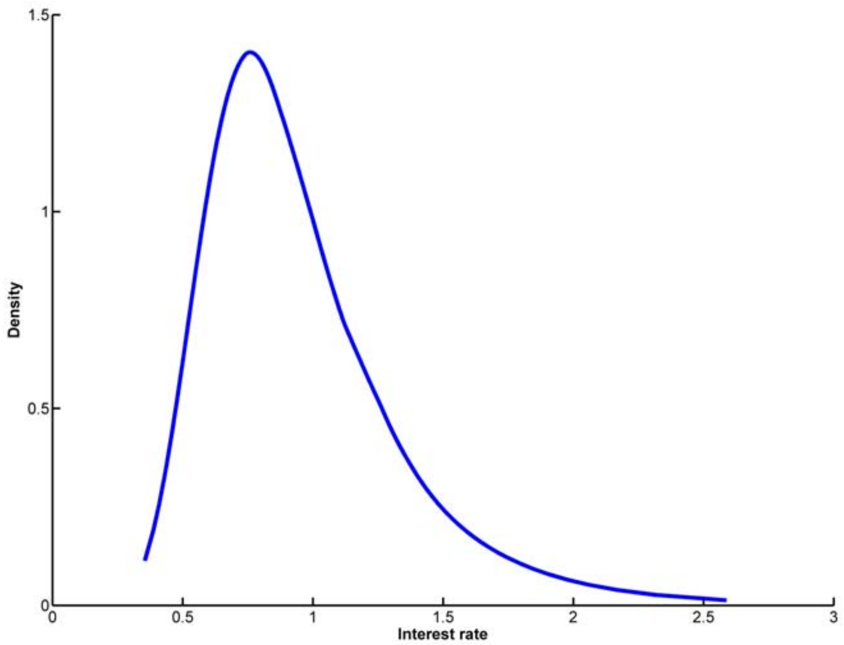


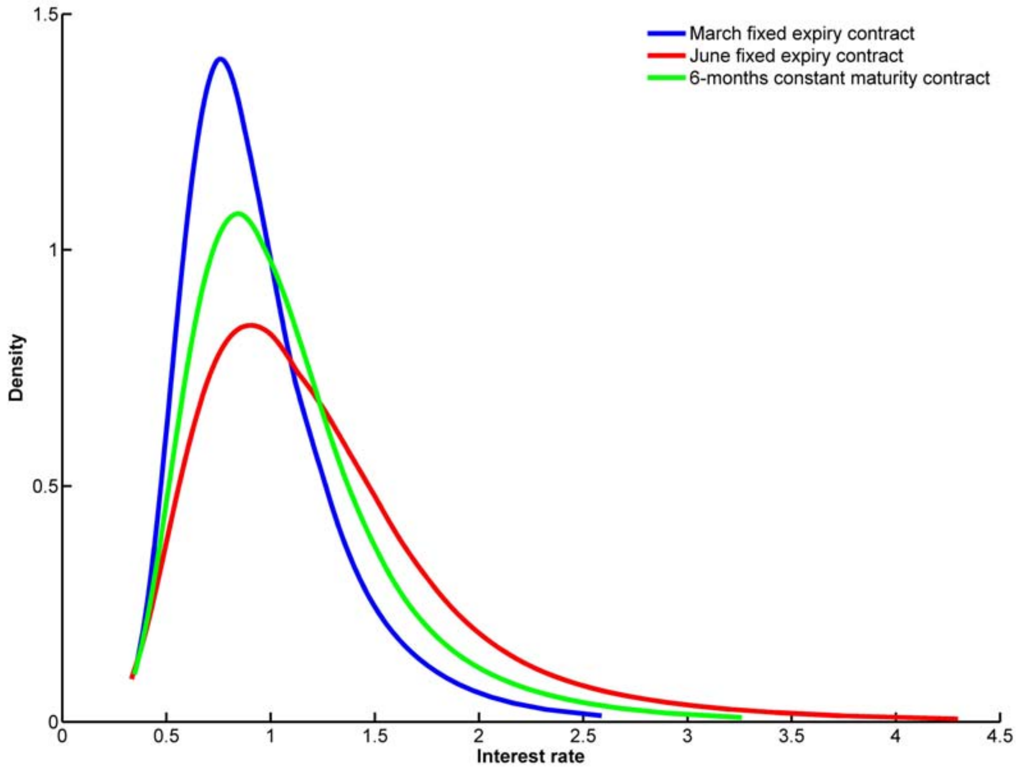
Figure 5: Fixed expiry PDF for the March 2010 contract on 27 October 2009



3.2 Constant maturity probability density functions

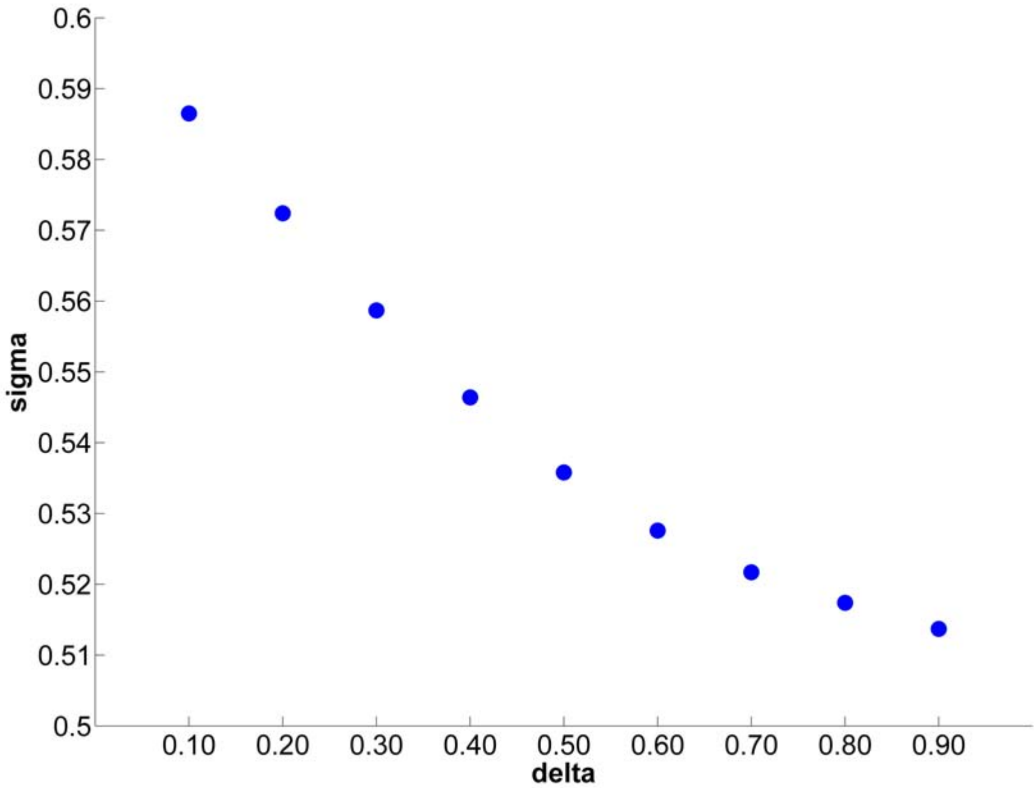
A total of eight option contracts on the three-month Euribor futures are traded daily on LIFFE. Each of these eight contracts expires on the same day as the underlying future contract cycle of March, June, September or December. As each option contract gets closer to the expiry date, the uncertainty about possible future Euribor outcomes declines. Therefore, the amount of uncertainty embodied by the PDF also tends to decline as we approach the expiry date. In particular, very little trading, if any, typically takes place on the days immediately prior to the expiry date. This regular time-to-maturity feature makes it very difficult to compare PDF statistics on the same fixed expiry contract over time. A possible solution to this time pattern is to estimate constant maturity PDFs interpolating over the eight fixed expiry PDFs. Based on this interpolation we calculate three-month, six-month, nine-month, one-year and one-year and six-months constant maturity contracts. For any given day, each of these PDFs always represents the same constant period ahead.

Figure 6: Interpolation of the 6-month constant maturity PDF on 27 October 2009



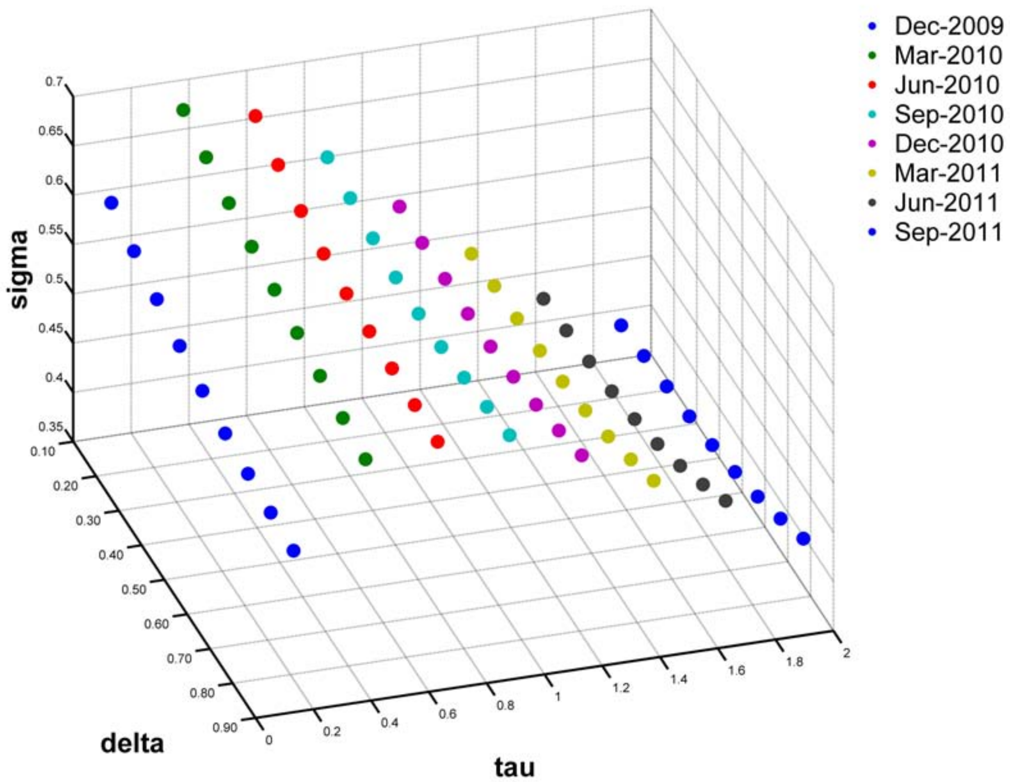
The method does not interpolate directly over the PDFs but over the implied volatility curves with the same *delta* but with different maturities. The advantage of doing it this way is that the same *delta* but for contracts expiring in different dates is always defined by the non-parametric technique. In addition, the delta always ranges between 0 and 1. In detail, to construct the constant maturity PDF a vector containing the nine *delta values* from 0.1 to 0.9, with a step-width of 0.1, is first created for every fixed expiry contract. For each *delta* in this vector, the value of the corresponding *sigma* is then calculated by evaluating the previously-estimated volatility smiles. This is done by using the grid of 1000 two-component points defined in the previous section, where the first coordinate is the *delta* and the second is the *sigma*. From this grid, the nine *sigmas* are calculated using linear interpolation.

Figure 7: Nine points delta-sigma space for the December 2009 contract; 27 October 2009



For each of the nine *deltas*, the value of the *sigmas* for different times to maturity (the fixed expiry ones) is calculated.

Figure 8: Nine delta-sigma space for all the fixed expiry contracts; 27 October 2009



For each of the nine *deltas*, a smoothing spline is constructed by interpolating the sigmas of all the fixed expiry contracts. For each of the nine splines, we evaluate them in the constant maturity values in order to get the corresponding *sigmas* at these points.

After that, for each constant horizon we already have all the required data: nine *deltas*, nine *sigmas*, *tau*, risk-free interest rate, and the underlying value, which is obtained by interpolating the two closest underlying contracts with a smoothing spline. Later on, the *deltas* are converted into strikes, and the premium of every *artificially-created* option is calculated using the Black-Scholes model. Then, the non-parametric model is used again in order to calculate the PDF, as defined in the previous section. Summarizing, the exactly ATM implied volatility is calculated, a 1000-point *delta* grid is generated, a 1000-point *sigma* grid is calculated using splines, then the *deltas* are transformed back into the strike space to calculate the premium, and finally the constant maturity PDF is calculated.

Figure 9: Nine delta-sigma space for all the fixed expiry and constant maturity contracts; 27 October 2009

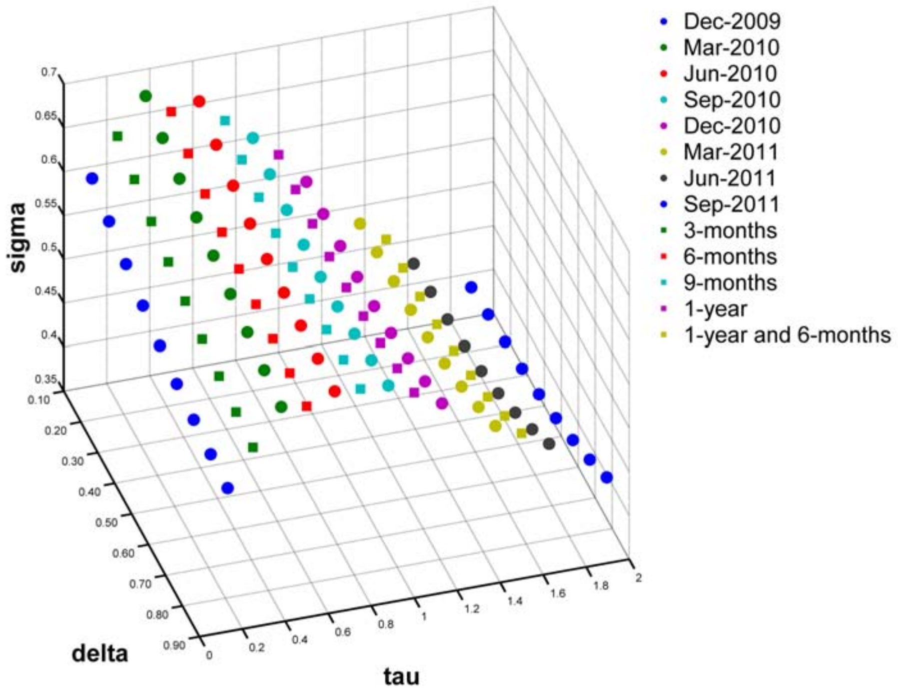


Figure 10: Three-Dimension Probability density function of the Three-month constant maturity PDFs

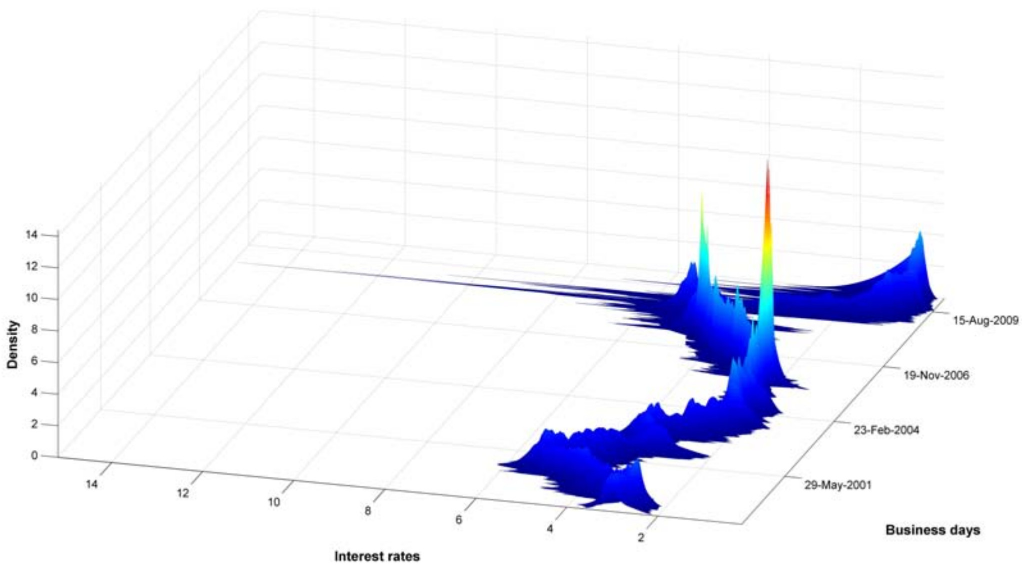
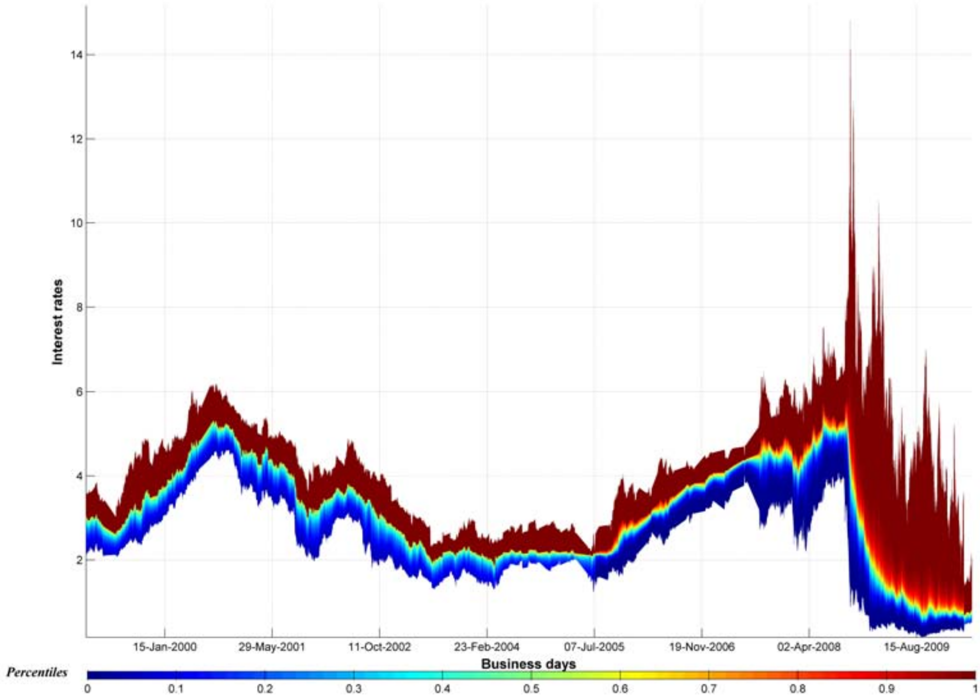


Figure 11: Projection in the density plane of the 3D Probability density function of the Three-month constant maturity PDFs



4 How can we use option-implied PDF derived indicators?

This chapter provides a number of examples to demonstrate how option-implied PDFs may be able to enhance our analysis. Indicators derived from implied PDFs may be better quality than those derived from (single) option prices. Furthermore, option-implied PDFs may offer the possibility of new indicators, e.g. the most likely outturn implied by option prices (i.e. the mode of the implied-distribution). Finally, PDFs are a powerful communication tool: they provide a concise, visual summary of risk and uncertainty - both magnitude and directional bias - embodied in option prices. Being able to visualize the distribution can be particularly useful when the associated risk parameters are changing rapidly. However, it must be remembered that option-implied PDFs are risk neutral. That caveat, and the fact that Euribor PDFs pertain to the inter-bank rate, not the policy rate are first discussed below.

4.1 Two important caveats to interpreting Euribor PDFs

It is imperative to be clear from the outset what Euribor option-implied PDFs do, or more importantly, do not, tell us.

4.1.1 Option-implied PDFs are risk neutral

Option-implied PDFs estimated under a Black-Scholes option-pricing derivation (such as this one) are by construction risk-neutral. The option-implied PDF represents the set of probabilities under which the expectation of the terminal asset price must be discounted by the risk-free rate, in order to equate with the market price. Such PDFs correspond to the probabilities that an investor would have if he were risk-neutral, but the agents that price the options might in fact be risk averse. If that were the case, risk premia would lead to differences in both the location and shape of the risk-neutral and actual distributions. The extent of such differences is likely to vary with both asset class and maturity.

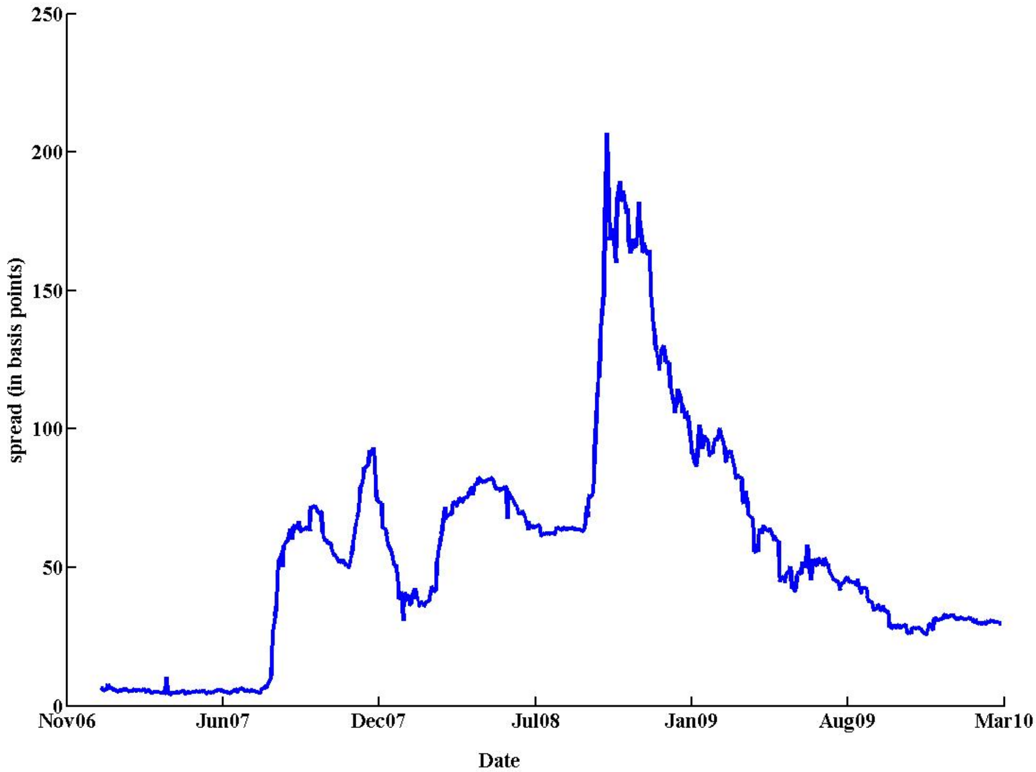
Different techniques can be used to transform the risk-neutral PDFs implied by options into estimates of the actual distribution. Bliss and Panigirtzoglou (2004) and Alonso et al (2006) exploit the fact that the risk-neutral and actual distributions are related to each via the marginal rate of substitution of the representative investor to define the functional form of the transformation. They then estimate the parameters of that transformation function for different assumed forms of the utility function by maximising the forecasting ability of the transformed PDFs. In contrast, Liu et al (2004) following Fackler and King (1990), define their transformation in terms of the beta function. The additional flexibility of the beta function might better align the transformed PDFs with the pattern of past outturns, but perhaps at the cost of economic insight. Estimating the actual PDFs from the risk-neutral PDFs is outside the scope of this paper; instead it focuses directly on the option-implied PDFs themselves. The issue of how best to extract the actual distribution of possible asset price outcomes in the future is one that would merit further research.

4.1.2 Euribor PDFs pertain to the inter-bank rate, not the policy rate

Rates on overnight index swaps (OIS) are considered to provide the best market-based indication of market participants' expected path of average official policy rates. This is because although OIS may still include a premium to compensate for term risk and liquidity risk, the element that compensates for credit risk is minimal: it pertains to only overnight, rather than three-month, credit risk.

Before the financial turbulence, the spread between Euribor and Eonia had been small and stable (Chart 12): over H1 2007 it averaged 5.3 basis points, with a standard deviation of 0.7 basis points. At that time, therefore, the path of forward Euribor could also be considered a reasonable (if slightly upward biased) proxy of the market's expectations of average future policy rates. More importantly, the stability of the Euribor-Eonia spread meant that the risks around future Euribor outturns, as captured by Euribor PDFs, were driven by the perceived risks around the outlook for expected policy rates, rather than the outlook for that spread. But that spread became large and volatile with the onset of the financial turbulence, reaching over 200 basis points at its peak. This means that Euribor PDFs no longer characterise the risks purely around expected policy rates. Instead, they can be thought of as conflating the risks around central expectation for both the official policy rate and the inter-bank credit spread.[8] This does not diminish the value of Euribor PDFs because Euribor is still a fundamental element of the transmission mechanism.

Figure 12: 3-month Euribor and 3-month forward Eonia sport rate spread



[8]Bank of England (2009) provides an indicative illustration as to how the PDF for the expected average policy rate and the PDF for the spread could be separated, but only if a simplifying assumption is made about the functional forms of both distributions.

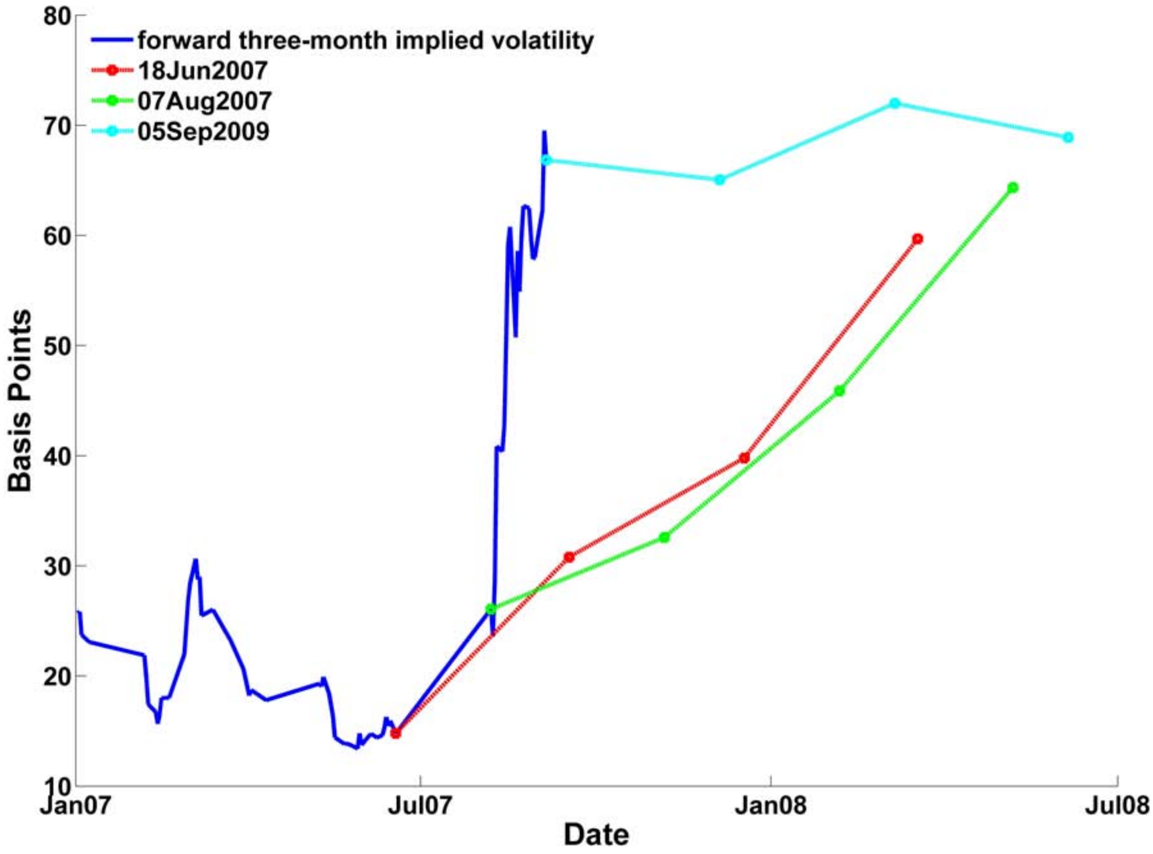
4.2 PDFs may provide better quality indicators

Indicators of risk and uncertainty derived from the implied-PDF may be better quality than the more-common equivalent indicators: at-the-money implied volatility and the so-called risk-reversal. This is because the implied-PDF incorporates information from all available options, whereas implied volatility is based on only one strike, and the risk reversal only two. However, because of the nature of the risk reversal, the quality improvement may be greater for this measure.

The at-the-money implied volatility is simply the value of σ in equation 4 required to equate the Black-Scholes price with the observed market price for the option whose strike price is closest to the prevailing underlying futures price. It represents the standard deviation of the underlying asset's returns distribution, and therefore measures the amount of uncertainty embedded in the option price. In the Black-Scholes option pricing paradigm, because this relates to the underlying asset rather than the option, it does not vary with strike price. In practice this is not the case: implied volatility typically varies with strike price, giving rise to volatility smiles or smirks (Chart 3). Because at-the-money implied volatility simply measures the implied volatility of a single contract, it does not take into account the influence of the other data on the overall shape of the estimated PDF.

It is worth noting that an analogy can be made between PDF-implied uncertainty, which represents the annualised volatility over the remaining life of the option, and a spot interest rate. Extending that analogy, forward implied-uncertainty can be constructed from a term structure of spot implied-uncertainty in the same way as forward interest rates are computed. For instance, the 1-year spot implied-uncertainty represents the average of the three-month implied-uncertainty for the four periods beginning immediately and in three, six and nine months' time. Decomposing the one-year spot implied-uncertainty into the set of forward values (Chart 13) makes it clear that the pickup seen in the one-year implied volatility seen at the outbreak of the crisis can be largely be attributed to near-term uncertainty: the change in the nine-month-forward implied volatility was significantly less than the change in spot implied volatility.

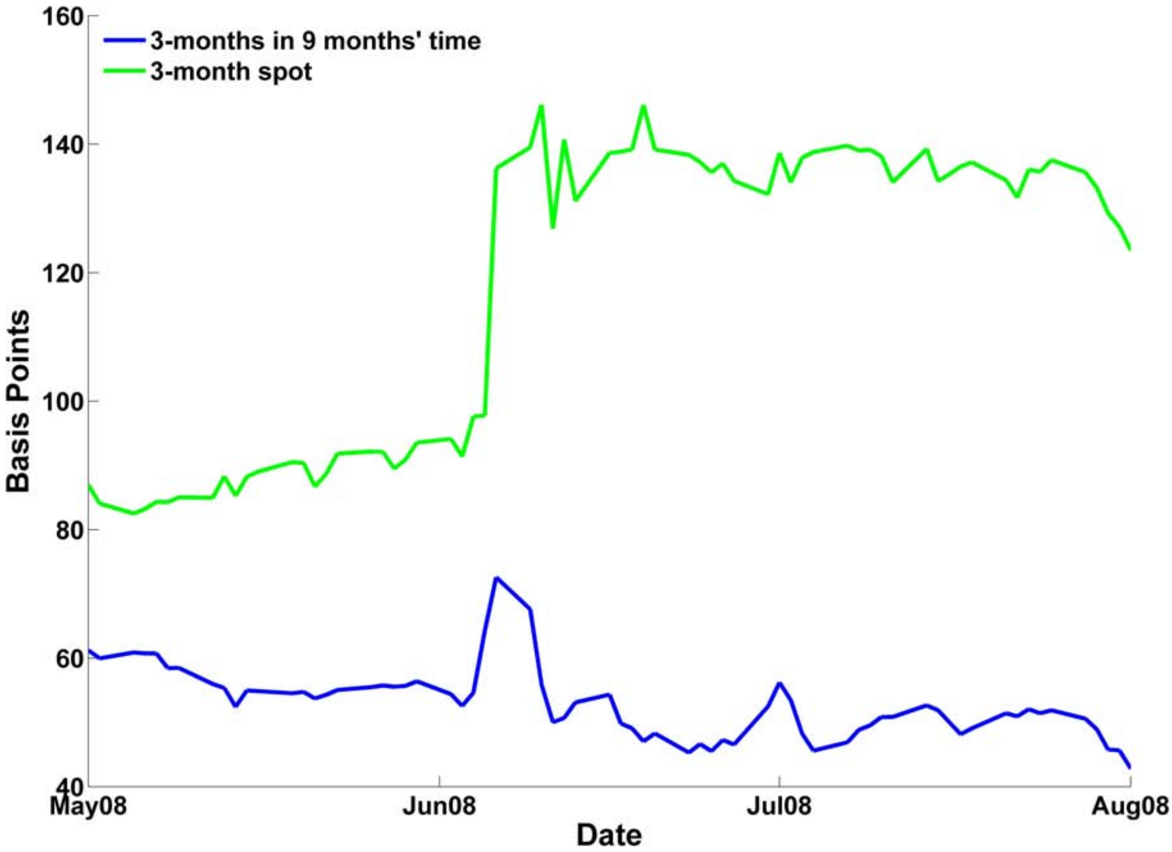
Figure 13: Three-month implied volatilities for the spot, three, six and nine month forward contracts



Developments in forward implied-uncertainty between June and August 2008 are also interesting. Implied uncertainty increased on 6 June, shortly after the Governing Council press conference that was interpreted by the market as hawkish. However it is interesting to observe in Chart 14 that although short-term uncertainty swiftly reverted to around its previous levels, longer-term uncertainty remained elevated, even after the increase in the key interest rates announced at the July Governing Council Press conference.

One measure of balance of risks often cited by market participants is the so-called 'risk-reversal'. This measures the difference in implied volatility between a call and a put option that are equally out of the money. In other words, it measures the slope of the implied-volatility smile, which in turn is related to the skewness of the distribution. However, the skewness of the option-implied PDF may be considered a better measure since it incorporates information from all the strikes for which options trade, not just two.

Figure 14: Forward implied uncertainty in the selected period



Moreover, the fact that the risk reversal is the absolute difference between two implied volatilities means that changes in the overall level of implied volatility can cause changes in the risk reversal to misstate changes in the balance of risks, in the economic sense outlined in Lynch et al (2004). The onset of the financial market turbulence in 2007 is a case in point. Chart 15 shows that the outbreak of financial turbulence was accompanied by a sharp downward movement in the 3-month risk reversal. [9] If this were the only indicator considered, then one might think that there had been a similarly large movement in the market's perception of the balance of risks. But Chart 15 also shows that at the same time the general level of implied volatility also increased substantially. Chart 16 shows the PDFs for the two volatility smiles. Although the standard deviation increased, there was little change in the statistical skewness, and therefore the economic balance of risks. This highlights the value added by estimating

[9]Note that in the market, Euribor options, and therefore risk reversals, only exist for the fixed schedule of quarterly expiry dates. The 'three-month risk reversals' presented here are derived from the interpolated volatility smiles, as per Chart 24.

PDFs from the full range of available options, rather than simple summary indicators based only on a limited number of prices.

Figure 15: Implied volatility smile for the three-month constant maturity Euribor PDFs on selected dates

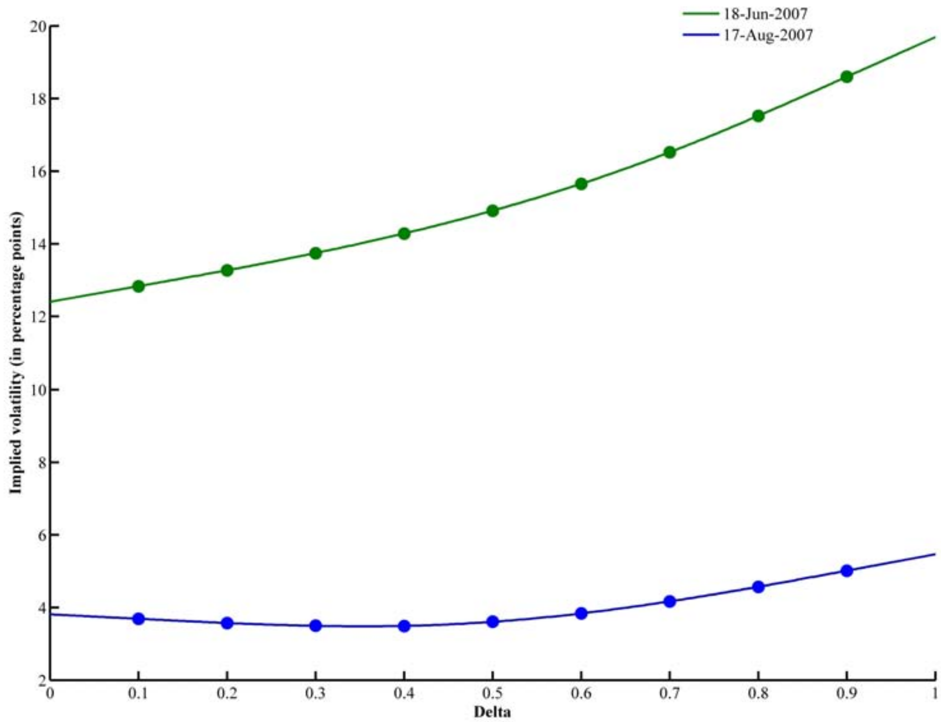
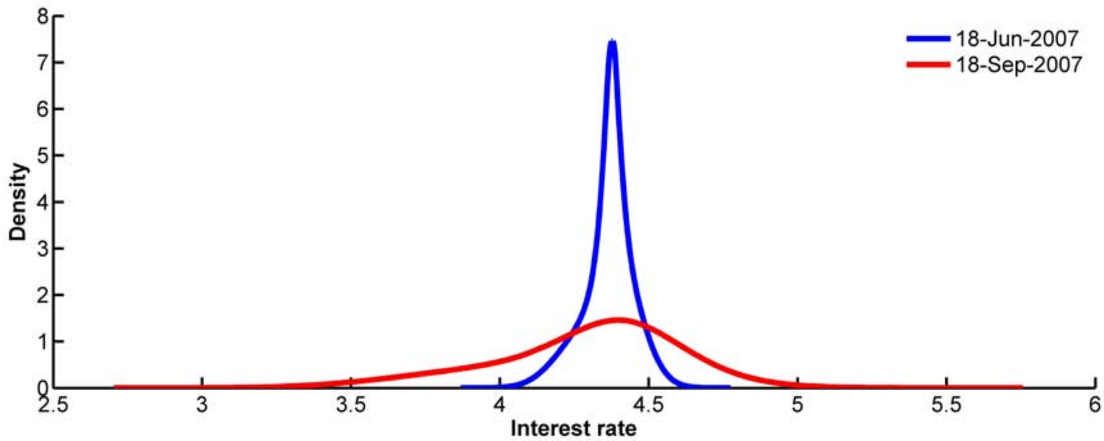


Figure 16: Three-month constant maturity PDFs on selected dates



4.3 PDFs may offer new information

Theoretically, the mean of the PDF - the risk-neutral expectation of the outturn - is equal to the futures rate, by definition.[10],[11] So differences in the mean of the PDF can simply be observed from movements in the (interpolated) futures rates. However, without information about the skewness of the distribution, one cannot determine whether differences in the futures rate are simply because the whole distribution has undergone a shape-preserving translation, or whether the weight on one of the tails has increased.

It may therefore be useful also to consider differences in the mode of the distribution before interpreting differences in the mean (the futures rate). Note that this applies to interpreting differences across maturity as well as changes in one (constant) maturity over time. The PDFs estimated for 30 October 2009 in Chart 17 are a good example. Because of the strong positive skewness, the mean of the one-year PDF is notably higher than that of the three-month PDF. However, the modes are not so different. Chart 18 compares the mean path for Euribor, i.e. the futures curve, with the modal path implied by options. This shows that the most likely outcomes implied by options prices were for much weaker rises in Euribor over the coming year than suggested by the futures curve. That may be of interest to policy makers, although the caveat about risk-neutrality should be borne in mind.

[10] Although this methodology does not impose that condition.

[11] Ignoring the small difference between a forward rate and a futures rate that arises because of the margin requirement for exchange-traded futures.

Figure 17: Three-month and one-year constant maturity PDFs for 30 Oct. 2009

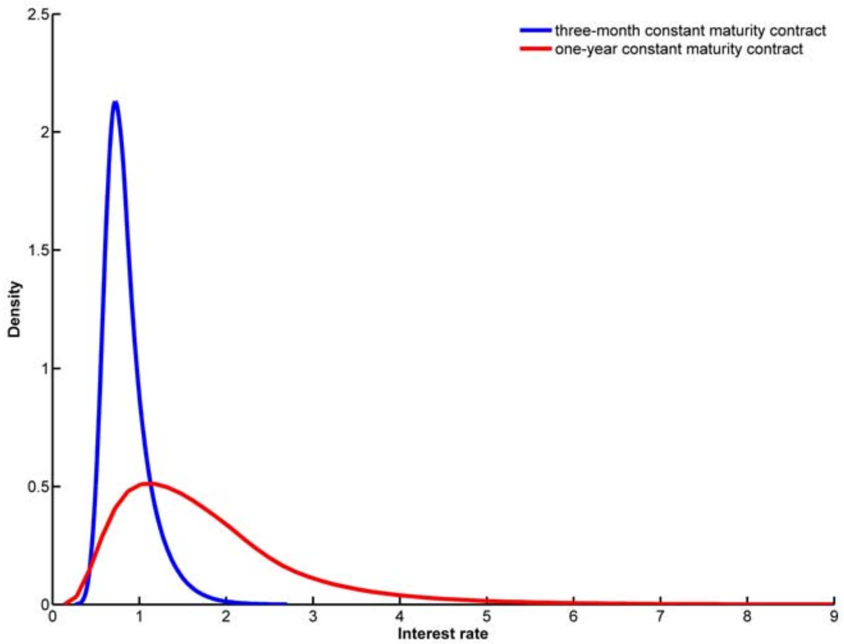
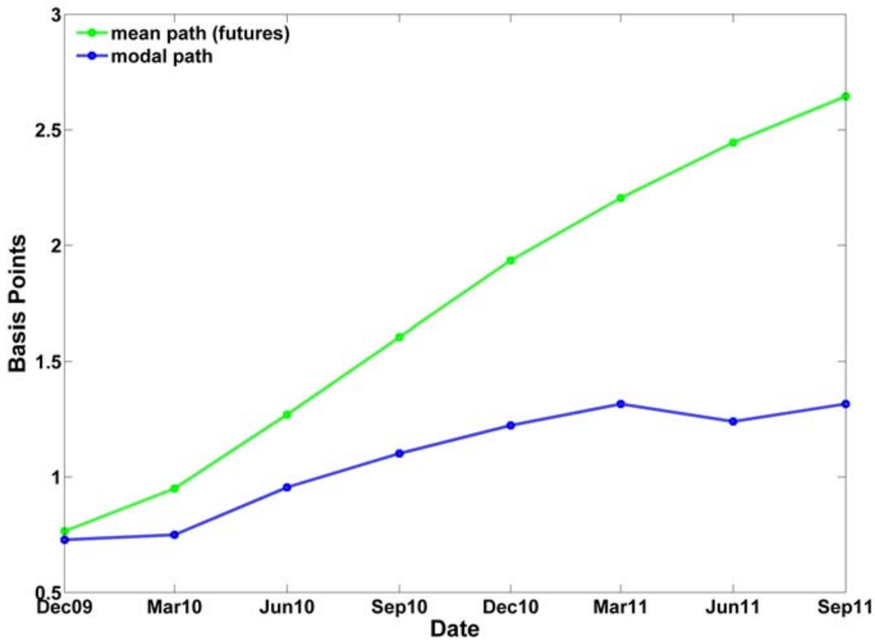


Figure 18: Mean and mode interest rate paths



4.4 PDFs are a powerful tool for conveying information on risk and uncertainty

The autumn of 2008 was especially tumultuous, but two events stand out: the failure of Lehman Brothers on 15 September and the internationally-coordinated monetary policy actions on 8 October. Option-implied PDFs are a powerful tool for succinctly capturing how such events affect market participants' views on the likely evolution of Euribor. They may also be used to assess the extent to which option prices anticipated such events.

The failure of Lehman Brothers led to material changes in the three-month-ahead Euribor distribution (Chart 19). While there had been little movement in the PDF in the preceding week, Euribor option prices assigned a significantly greater weight to interest rate outturns much less than the prevailing forward rate. And that left-tail continued to grow. Stress in the cash markets increased markedly too and the spread between forward Euribor and Eonia increased. But while it could also be argued, that the large negative skew reflects in part the view that the Euribor-Eonia spread could be much narrower than the forward spread, the sheer magnitude of the left tail suggests that it is also likely to reflect beliefs about future policy rates. However, it is difficult to be sure that such developments were not influenced by changes in risk aversion. For instance, if investors' intrinsic assessment of the actual probabilities of such outturns had not changed, but rather they decided that they would now require protection (in the form of options) against outturns at that particular probability, then that would also increase the estimated risk-neutral probabilities.

On 8 October, as part of internationally-coordinated monetary policy action, the ECB announced that, from the operation settled on 15 October, the weekly main refinancing operations will be carried out through a fixed rate tender procedure with full allotment at the interest rate on the main refinancing operation, i.e. 3.75%.[12] That rate was 50 basis points below the minimum bid rate affirmed at the previous Governing Council meeting on 2 October. An examination of the Euribor PDFs in the days leading up to that announcement and shortly afterwards reveals two interesting observations (Chart 20). First, it appears as if the impact of both the 2 October Press Conference or the 8 October announcement on the option-implied Euribor distribution was small compared to that of the accumulation of news during the intervening days (in particular, over the weekend). The fact that even by 7 October, the PDF had shifted so much to the left, and become more negatively skewed, suggests that market participants were already placing more weight on Euribor outturns in three months time being

[12] It was also announced on 8 October that, as of 9 October, the ECB will reduce the corridor of standing facilities from 200 basis points to 100 basis points around the interest rate on the main refinancing operation. Further details on both announcements can be found at: http://www.ecb.europa.eu/press/pr/date/2008/html/pr081008_2.en.html

much less than the current forward rate, even though the precise timing and details of the 8 October announcement took the market by surprise. The second interesting observation is that although the bulk of the implied three-month Euribor distribution continued to move towards lower interest rates, there was no movement in the tail of the distribution. One possible explanation is that, despite the unprecedented events of the preceding month, market participants still did not attach any weight to the possibility that Euribor would be 2% or less in three months' time.

Figure 19: Three-month constant maturity Euribor PDFs before and after the failure of Lehman Brothers on 15 September 2008

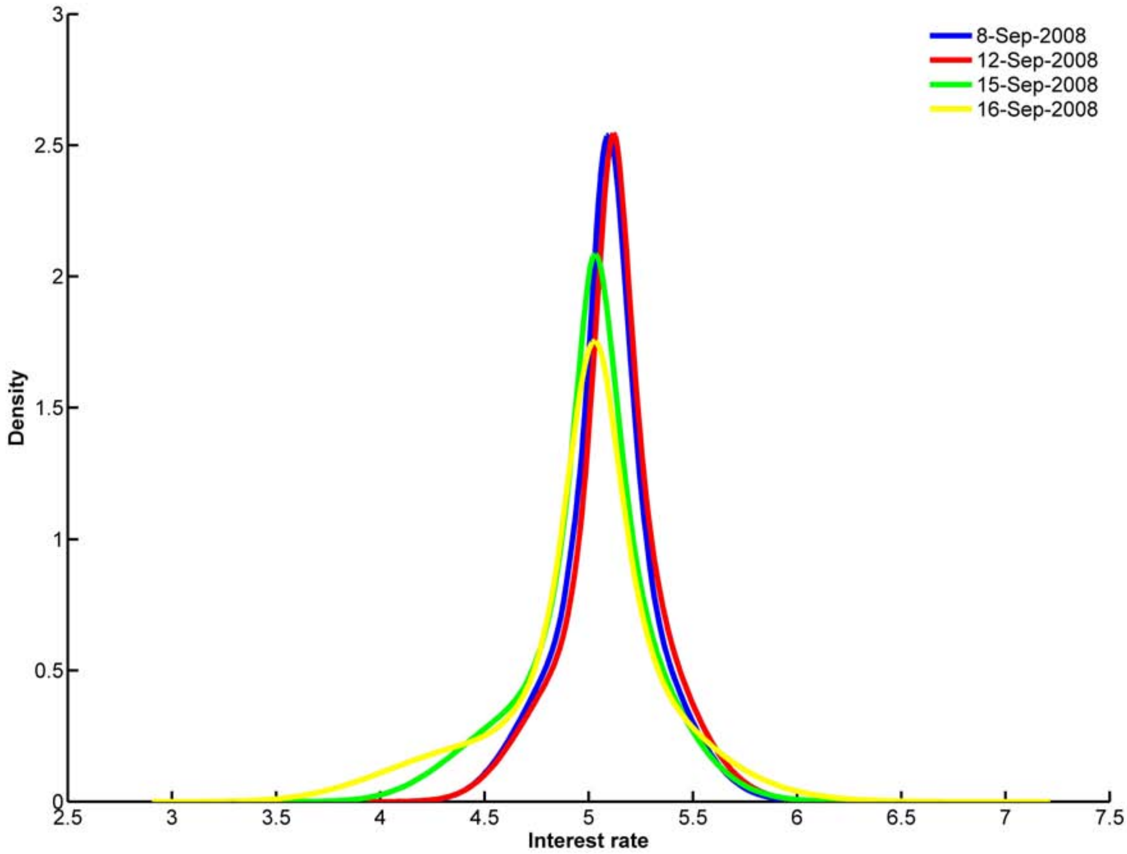
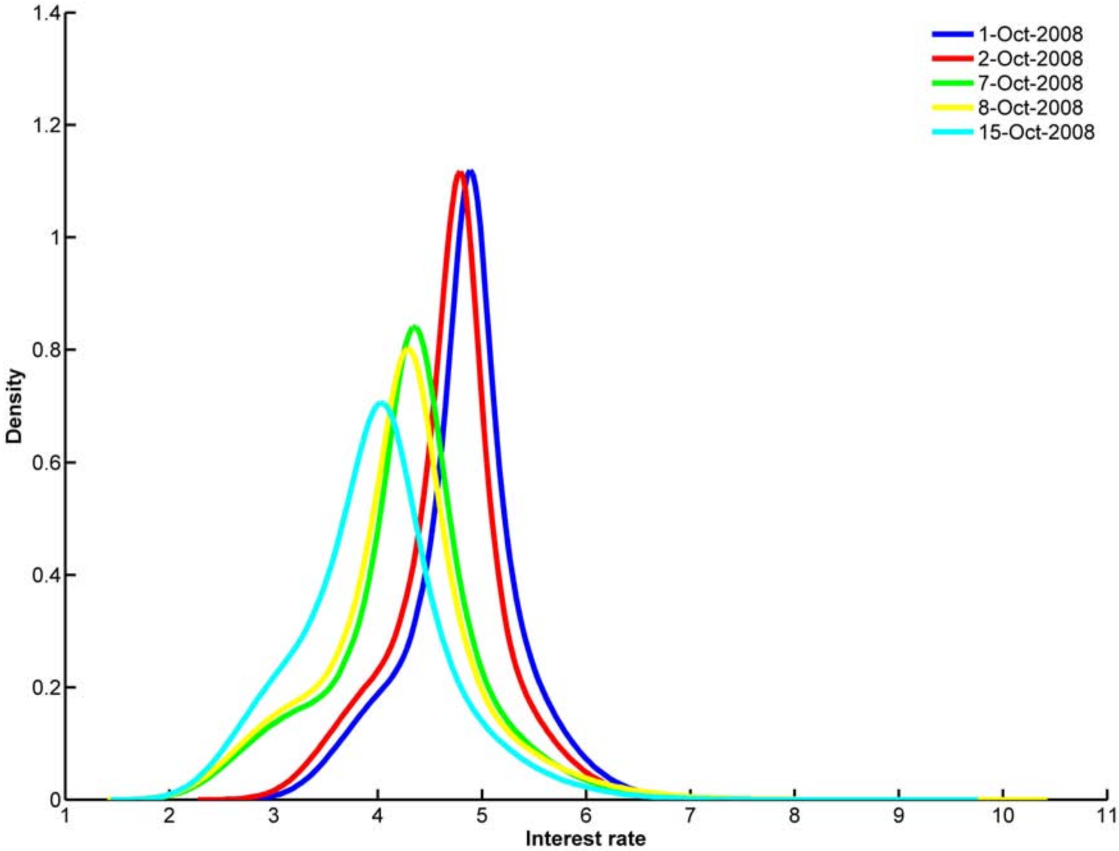


Figure 20: Three-month constant maturity Euribor PDFs before and after the change in monetary policy on 8 October 2008



5 The evolution of option-implied PDF statistics during the financial crisis

This final section documents in detail how Euribor PDFs reacted to the unfolding financial crisis between 2007 and 2009. In doing so, it demonstrates how the higher moments of the option-implied PDFs can provide timely and quantitative indicators of not only the amount of uncertainty around forward Euribor, the mean of the PDF, but the directional bias within that.

The data are introduced in Charts 21 to 25, to provide a general overview, and then two specific episodes are discussed in more detail. Chart 21 first shows the mean of option-implied distributions, which is simply equal to the forward rate, around which the risks are measured.

Charts 22-25 then present two measures of the amount of uncertainty and two measures of its directional bias. For both uncertainty and skewness, two types of measures are shown: one constructed directly from option prices and another one based on higher moments of the PDF. Note that the options price data in early 2007 did not always meet the quality criteria outlined in section 2 to estimate PDFs. The following two episodes are then examined more closely:

1. The onset of financial market turbulence
2. February to August 2008 : the tension between declining demand and rising prices

Figure 21: Mean of the Three-month and One-year Euribor constant maturity PDFs from 2007

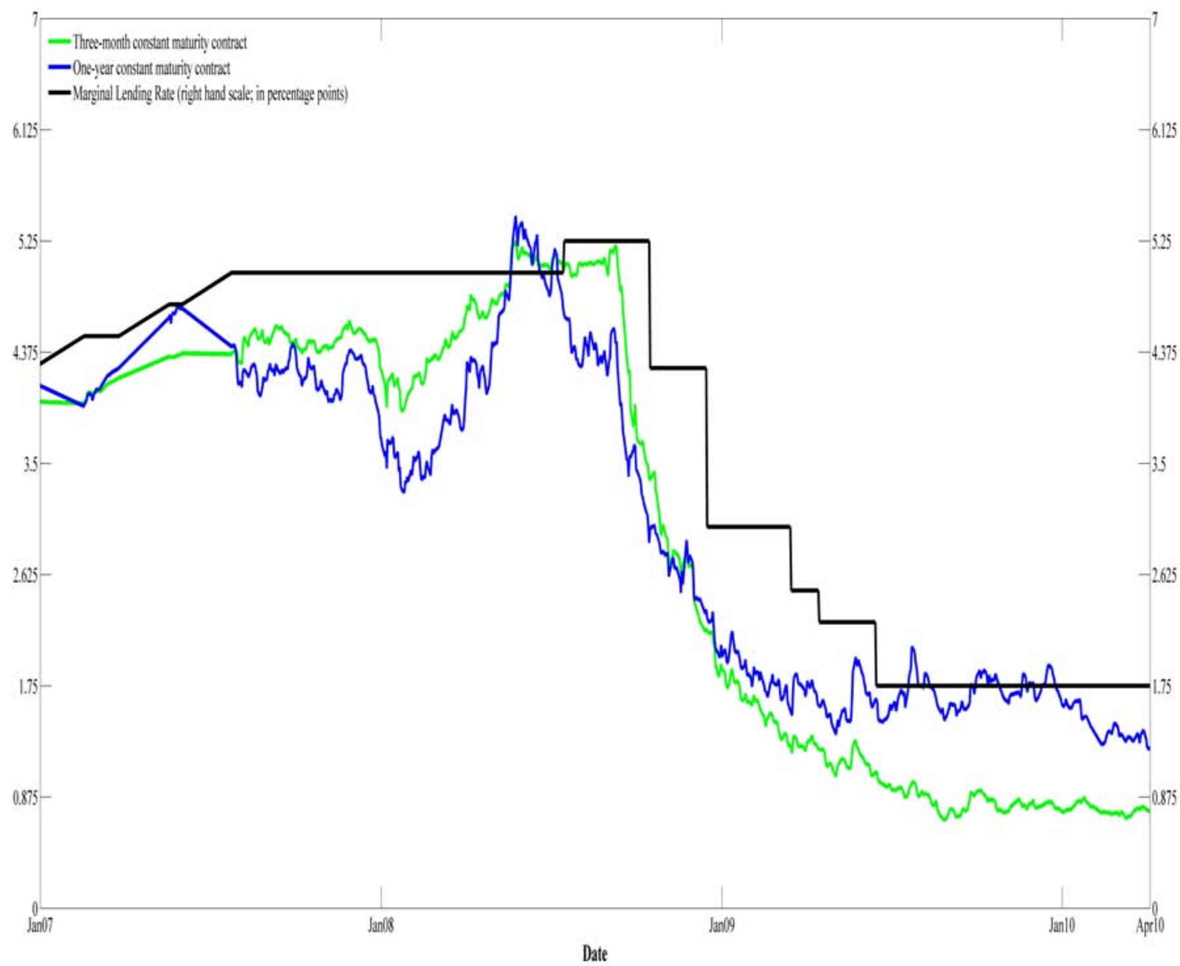


Figure 22: Standard Deviation of the Three-month and One-year Euribor constant maturity PDFs

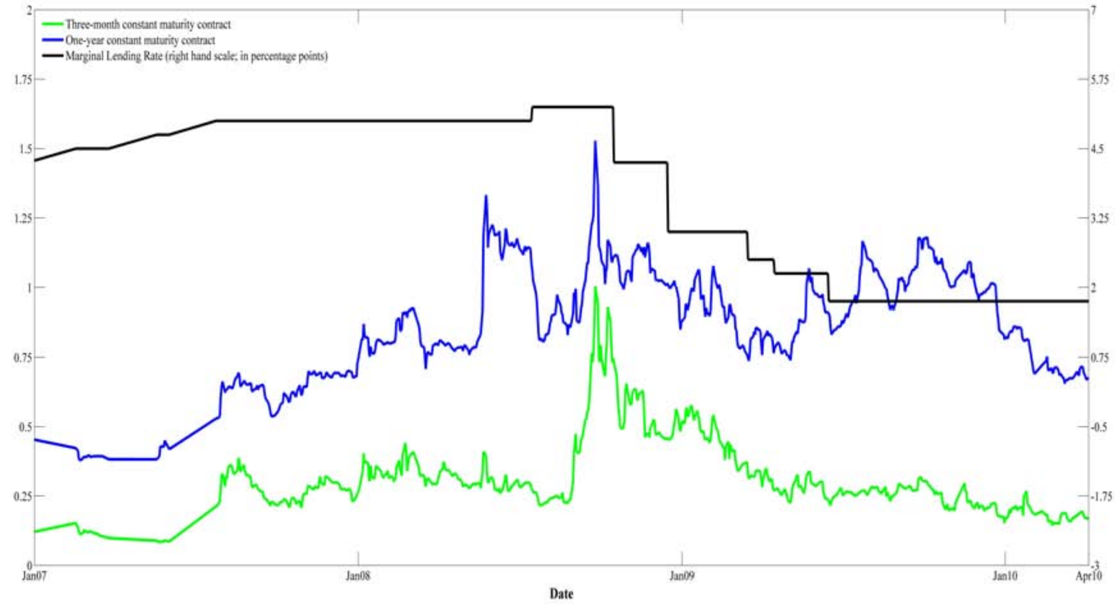


Figure 23: Implied Volatility of the Three-month and One-year Euribor constant maturity PDFs

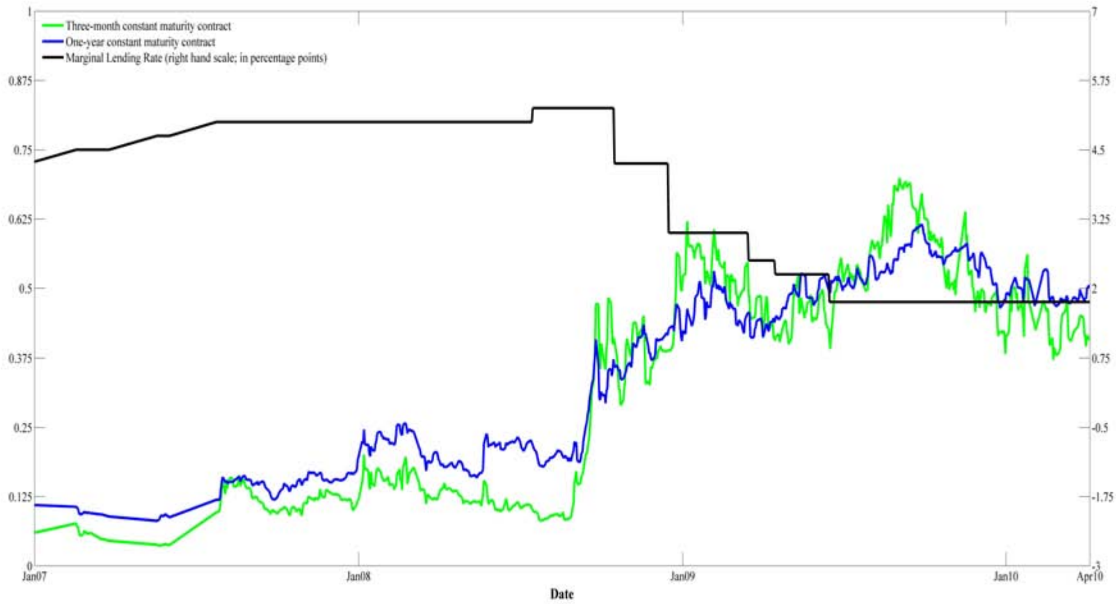


Figure 24: Skewness of the Three-month and One-year Euribor constant maturity PDFs

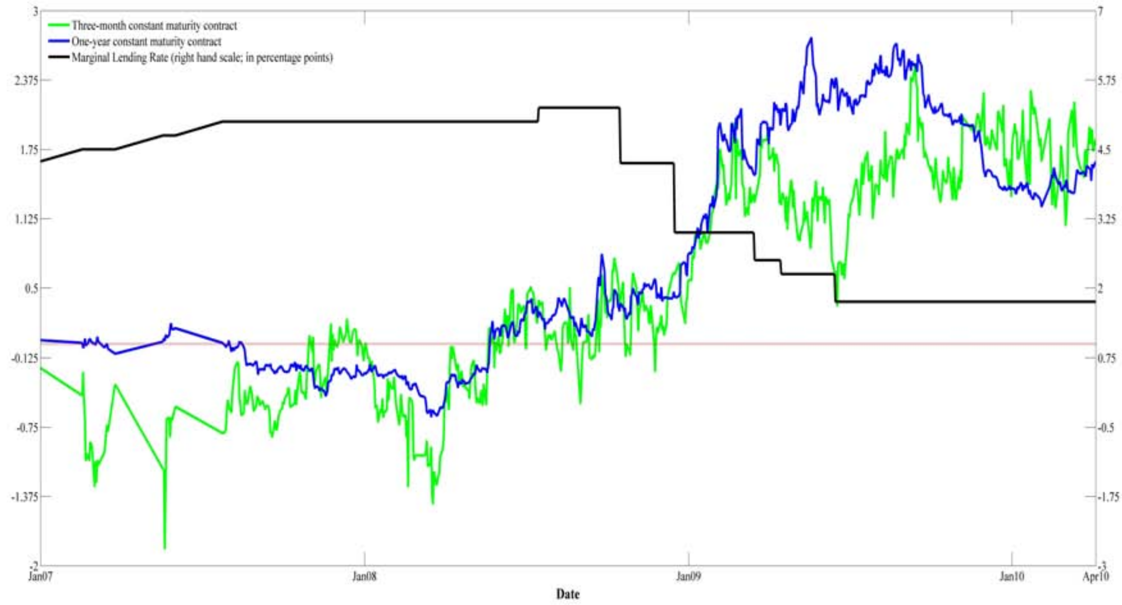
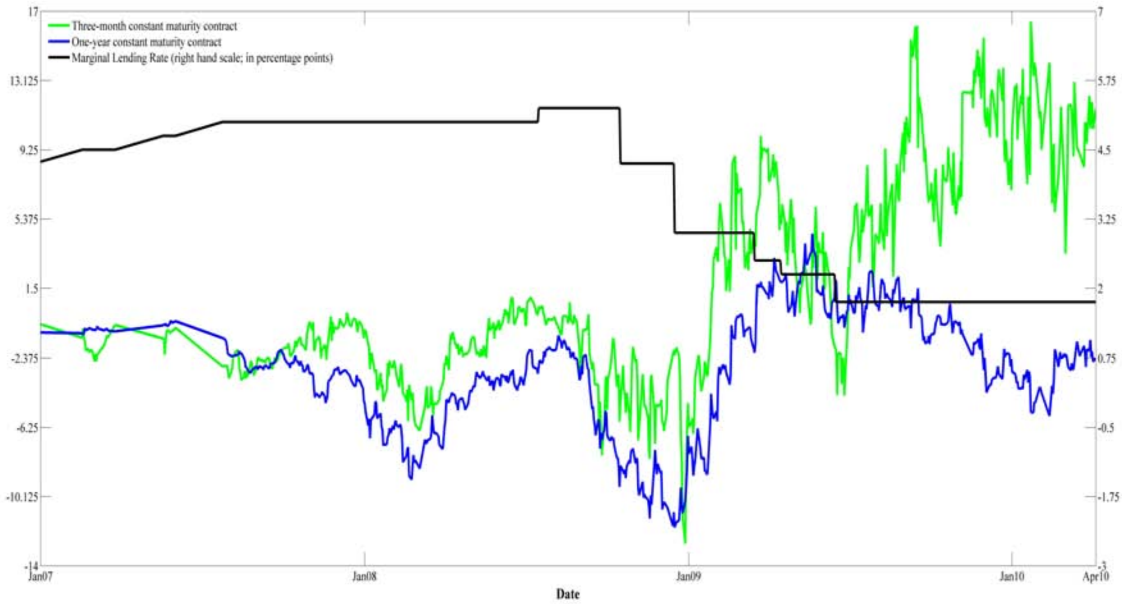


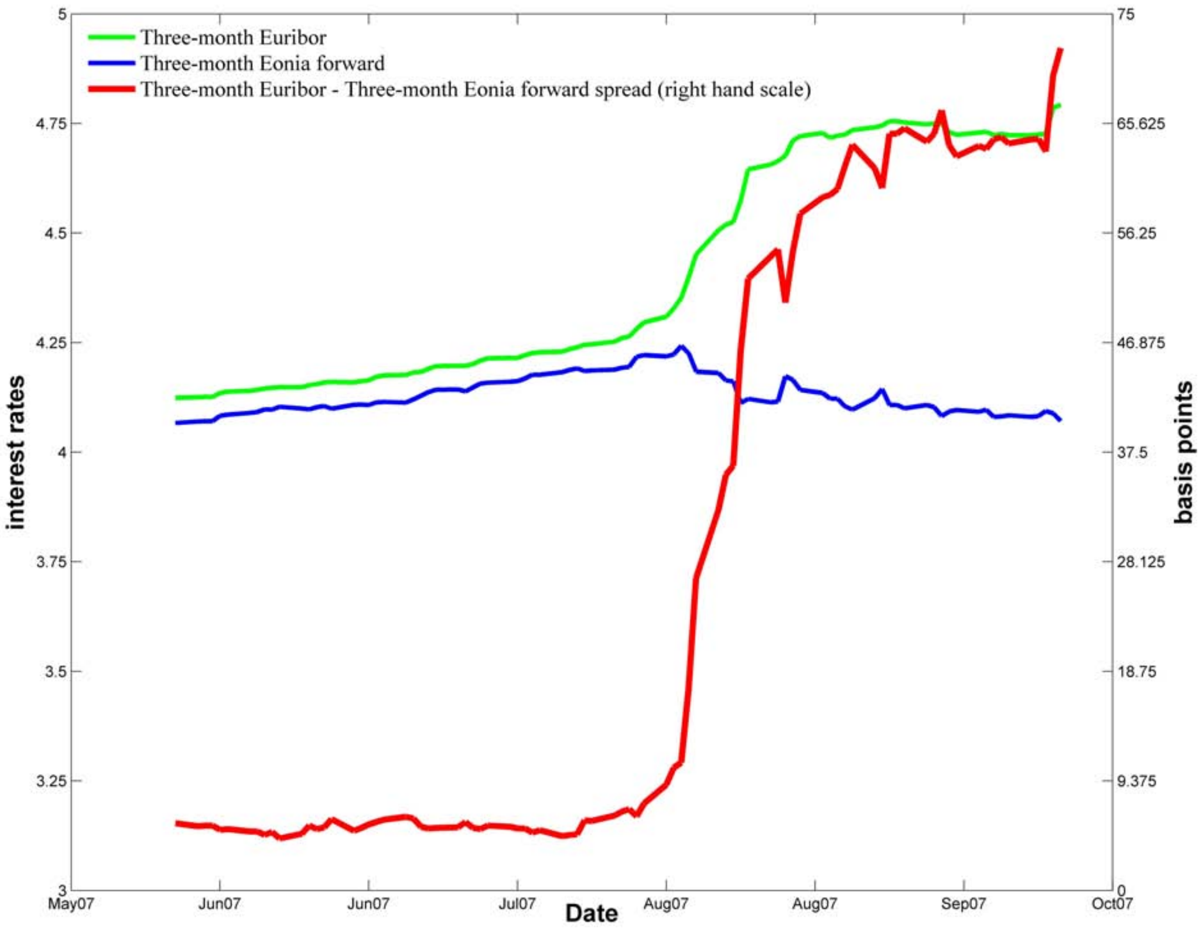
Figure 25: Risk Reversal of the Three-month and One-year Euribor constant maturity PDFs



5.1 The onset of financial market turbulence

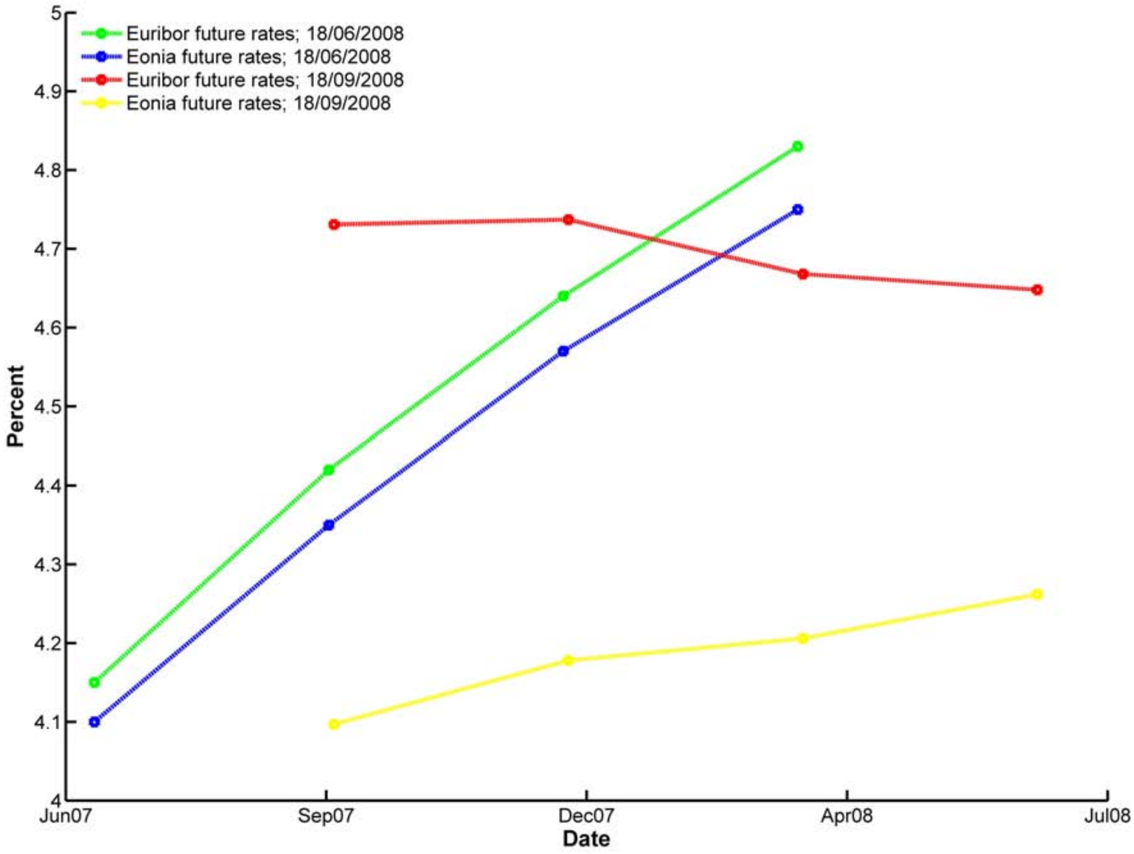
One striking feature of the onset of financial market turbulence was the dislocation that occurred in short-term money markets on 9 August, when Euribor and the Eonia rate diverged (Chart 26). Both the Euribor and Eonia curves flattened, but the Euribor curve lying unusually far above the Eonia curve (Chart 27). The fact that the spread between the red and yellow lines narrows, suggests that the market expected the situation of abnormally high three-month Euribor-Eonia spreads to ease only slowly, over the coming year.

Figure 26: Three-month interest rates, three-month Eonia forward, and the spread between them



Option-implied Euribor PDFs offer insight on the market's assessment of the risks around the Euribor curve. Chart 27 shows the estimated three-month PDFs before and after the onset

Figure 27: Euribor and Eonia Three-month forward curves, before and after the money market dislocation



of market turbulence. The moments of these PDFs, and additional information, are presented in Table 2. These data show how the width of both distributions increased considerably, reflecting in part the abrupt and unprecedented divergence from Eonia swap rates and ensuing uncertainty about the speed and magnitude of any subsequent convergence. Chart 13 already demonstrated that this increase in width was predominantly a near-term phenomenon. However, as already noted in section 4.1.1, these PDFs are risk-neutral so an increase in width could be because of an increase in risk aversion as well as an increase in the actual amount of risk. In this context both factors may well have played a role. Movements in the skewness of the distribution indicate how market participants perceive the balance of risks to be changing.[13] At short horizons there

[13]Here, 'balance of risks' has the precise economic meaning as set out in Lynch (2004). In summary it is the difference between expected conditional losses, depending on whether the outturn is greater or less than the central estimate, for an agent with rates forecast error with a quadratic loss function.

was little change in the balance of risks. However, at longer horizons the balance of risks moved to the downside. This suggests that market participants placed more weight on outturns much less than the prevailing forward rate at that time. And that could be consistent with an even more rapid return to more normal spread levels than the interest rate curves alone suggested.

Figure 28: Three-month constant maturity Euribor PDFs, before and after the onset of market turbulence

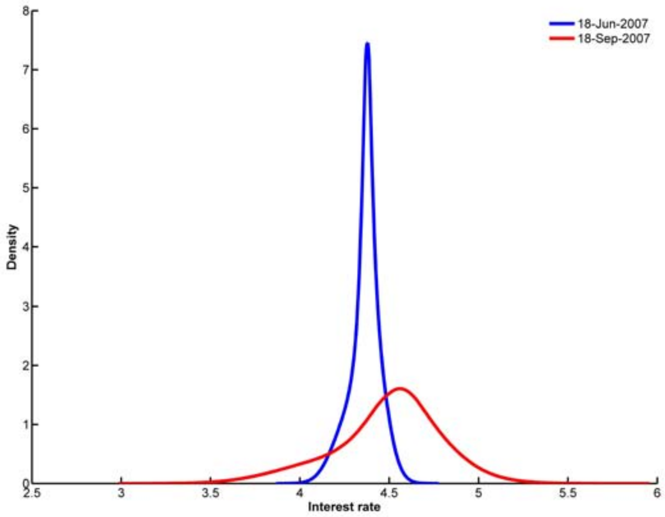


Table 2: Moments of the Three-month constant maturity in Chart 28, and related information

Moments	**3-months**	**1-year**	**change**
Mean	4.36	4.5	0.14
Standard deviation (percentage points)	0.08	0.3	0.22
Skewness	-0.55	-0.47	-0.08
Memo			
Implied Volatility	0.03	0.13	0.1
Implied Volatility (basis points)	0.15	0.58	0.43
Forward rates from the Euribor spot rate	4.42	4.74	0.32
Forward rates from the EONIA spot rate	4.35	4.18	-0.17
Euribor - Eonia (basis points)	7	56	49

So the onset of financial market turbulence led market participants to reappraise their view on longer-term rates, and their assessment of the uncertainty around shorter-term rates. These developments were captured by movements in option-implied Euribor distributions. In particular, the standard deviation and skewness of these distributions inform us about the quantity and balance of risk, subject to the risk-neutral caveat. One advantage of these indicators is that they are a quantitative measure, and can therefore be used to put the latest developments into an historical context. The movements in these option-implied indicators directly following the outbreak of the financial market turbulence did not appear to be exceptionally notable compared to their own history. That, and the fact that implied uncertainty did not change much at longer horizons, may suggest that the market did not, at first, believe that the overall impact of the turbulence would be severe.

5.2 February to August 2008: a tension between declining demand and rising prices

By February 2008 it was becoming clear that despite the possible demand implications of the financial crisis, risks to inflation over the medium term were still to the upside, and forward rates began to increase once more. The rise in 3-month forward Eonia over February to May broadly unwound the policy cuts that had been implicitly priced in during January, whereas, as presented in Chart 29, Euribor rose significantly above its year-end level, thus widening the gap between these two interest rates. However, as we show in Chart 30, the balance of risks around forward Euribor moved significantly to the downside between January and April. So although market participants were revising up their central expectation for Euribor outturns, they were initially still attaching increasing weight to Euribor outcomes below the forward rate. The latter can be observed in Chart 31. This may reflect views on either the ECB policy rate in the future, or the evolution of the Euribor-EONIA spread.

Figure 29: Three-month Euribor and three-month forward EONIA

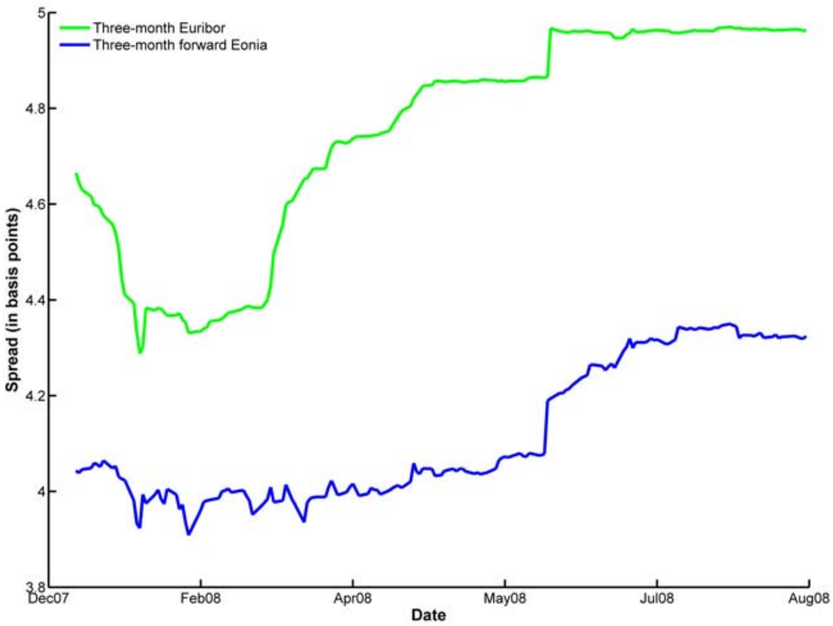


Figure 30: Skewness of the Three-month Euribor constant maturity PDF

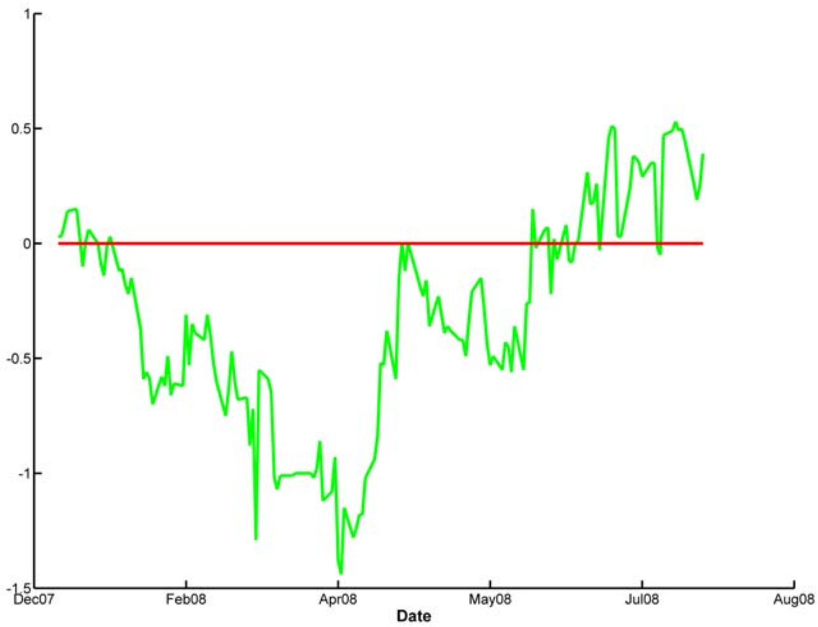
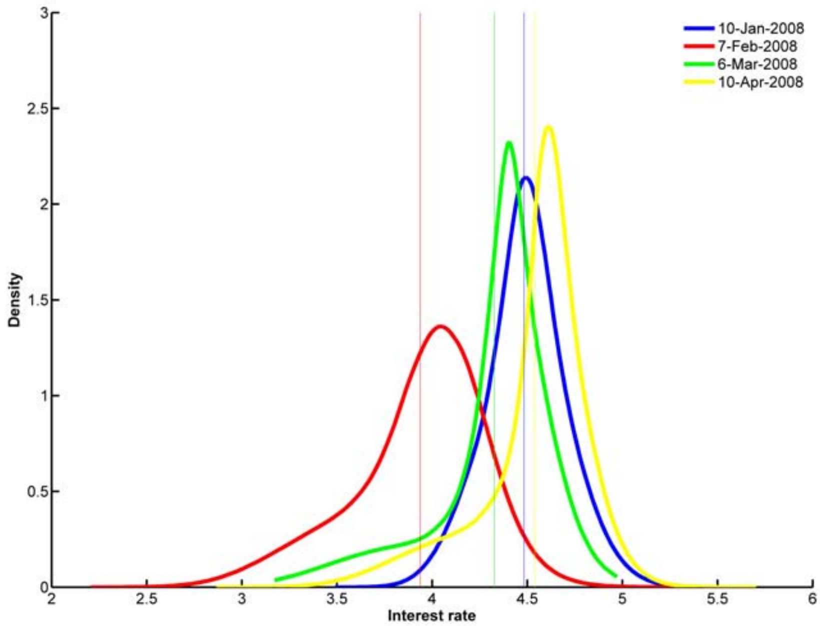


Figure 31: Three-month constant maturity Euribor PDFs on selected dates



6 Concluding remarks

This paper has shown how the methodology for extracting probability distributions from the prices of financial options, as first developed by Bliss and Panigirtzoglou (2000) and Cooper (2000), can be applied to Euribor. Using this methodology, we have estimated probability distributions for Euribor outturns three months in the future; the resulting dataset which is to be made publicly available via the ECB's Statistical Data Warehouse comprises over ten years of daily data. These PDFs provide a timely and quantitative indication of the market's assessment of the risks around forward Euribor: not just how much uncertainty there is, but precisely how that is distributed over different possible outturns. These can be used to analyse trends such as the extent to which the balance of risks is skewed to the upside or the downside, or to analyse how specific events affected the entire spectrum of views. Therefore, this indicator may appeal to those interested in monetary policy or financial stability. Moreover, such a comprehensive dataset, spanning the complete history of the euro particularly valuable because that gives the context which the current situation, or recent developments may be compared against, and provides a benchmark to help judge whether the current situation is 'normal' or 'extreme'. For most of the euro's history, the balance of risks around Euribor was driven primarily by the perceived balance of risks around the key policy rate. However, following the financial market turbulence of August 2007 that originated in the US sub-prime mortgage market, and the exceptional consequences for banking systems worldwide, expectations of interbank rates in the future diverged from expectations of the policy rate. Therefore, Euribor PDFs must be interpreted more carefully during this period because they combine a view on possible future values of the policy rate with possible future values of the Euribor-Eonia spread. Nevertheless, they still provide a good quantitative indication of the balance of risks around this key part of the monetary policy transmission mechanism.

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